

SIMPLIFIED DESIGN OF  
SLENDER REINFORCED CONCRETE  
COLUMNS

by  
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This report is intended to be used only by professional personnel competent to evaluate the significance and limitations of the report and who accept responsibility for the application of the report.

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## CHAPTER 1

### INTRODUCTION

#### 1.1 General

The task of accounting for slenderness effects in reinforced concrete columns can be a tedious experience. The designer may have difficulty in making initial assumptions, which may lead to inefficient iteration. Therefore, a design procedure providing good initial assumptions or leading to a direct solution is desirable. The purpose of this report is to develop design aids and methods of solution for slender reinforced concrete columns in accordance with American Concrete Institute Standard 318-77, Building Code Requirements for Reinforced Concrete (ACI 318-77) (1).

#### 1.2 Code Equations

The following is a summary of the design equations from ACI 318-77, Section 10.11 - "Approximate evaluation of slenderness effects."

$$M_c = \delta M_2 \quad \text{ACI Eq. (10-6)}$$

in which  $M_c$  is the "factored moment to be used for design of compression member";  $M_2$  is the "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive"; and  $\delta$  is the "moment magnification factor."

$$\delta = \frac{C_m}{1 - (P_u / \phi P_c)} \quad \text{ACI Eq. (10-7)}$$

in which  $C_m$  is "a factor relating actual moment diagram to an equivalent

uniform moment diagram";  $P_c$  is the "critical load";  $P_u$  is the "factored axial load at given eccentricity  $\leq \phi P_n$ ";  $P_n$  is the "nominal axial load strength at given eccentricity"; and  $\phi$  is the "strength reduction factor."

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \quad \text{ACI Eq. (10-8)}$$

in which  $EI$  is the "flexural stiffness of compression member";  $k$  is the "effective length factor for compression members"; and  $\ell_u$  is the "unsupported length of compression member."

$$EI = \frac{(E_c I_g / 5) + E_s I_{se}}{1 + \beta_d} \quad \text{ACI Eq. (10-9)}$$

or conservatively

$$EI = \frac{E_c I_g / 2.5}{1 + \beta_d} \quad \text{ACI Eq. (10-10)}$$

in which  $E_c$  is the "modulus of elasticity of concrete";  $E_s$  is the "modulus of elasticity of reinforcement";  $I_g$  is the "moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement";  $I_{se}$  is the "moment of inertia of reinforcement about centroidal axis of member cross section"; and  $\beta_d$  is the "ratio of maximum factored dead load moment to maximum factored total load moment, always positive."

Section 10.11.5.3 states, "In Eq. (10-7), for members braced against sidesway and without transverse loads between supports  $C_m$  may be taken as

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \quad \text{ACI Eq. (10-11)}$$

but not less than 0.4. For all other cases,  $C_m$  shall be taken as 1.0."  $M_1$  is the "value of smaller factored end moment on compression member calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature"; and  $M_2$  is the "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive."

For unbraced frames, Section 10.11.6.2 states, "In Eq. (10-7),  $P_u$  and  $P_c$  shall be replaced by the summations  $\Sigma P_u$  and  $\Sigma P_c$  for all columns in a story."

### 1.3 Object and Scope

The object of this report is to develop design methods and design aids for slender reinforced concrete columns of known geometry, without requiring an initial estimate of the reinforcing ratio. For single columns (including columns in braced frames), the design method yields direct solutions for the reinforcing ratio. For unbraced frames, the design method provides an approximate solution for the reinforcing ratio by treating columns within a story as a single equivalent column.

The design aids are in accordance with ACI 318-77 and include five column configurations: 1) rectangular tied columns with equal reinforcement in all faces, 2) rectangular tied columns with equal reinforcement in the end faces (parallel to the neutral axis), 3) rectangular tied columns with reinforcement in the lateral faces (perpendicular to the neutral axis), 4) rectangular spiral columns, and 5) circular spiral columns.

Only uniaxial bending is considered.

## CHAPTER 2

### FORMULATION OF THE TECHNIQUE

#### 2.1 General

The development of slender reinforced concrete column design aids involves the incorporation of slenderness effects into the column interaction diagrams. The most useful design aids allow the interaction diagrams to be expressed in terms of the axial load and the (unmagnified) moment obtained from an elastic frame analysis. Design aids of this type can be constructed in a straight forward manner, as will be demonstrated in the following sections, because slenderness effects are manifested in the form of a magnified moment, and the degree of magnification is a function of axial load and geometry, which are known for each point on an interaction diagram.

Conventional interaction diagrams (2) generalize the axial and bending capacities in terms of stress to negate the requirement for individual interaction diagrams for specific cross sections. The generalized capacities, the axial load index and the moment index, are  $\frac{\phi P_n}{A_g}$  and  $\frac{\phi M_n}{A_g h}$ , respectively. This generalized approach is, also, desirable for slender column interaction diagrams. To do this, the slenderness effects must be expressed in terms consistent with the generalized capacities. Since conventional interaction diagrams are plotted for specific reinforcing ratios for each column configuration, in this development, expressions for the moment magnifier are, also, written as functions of the reinforcing ratio and the column configuration, as well as the column slenderness. By dividing the bending capacity of a section by the corresponding moment magnifier, axial load index-moment index relations are used to produce interaction diagrams

expressed in terms of the axial load index,  $\frac{\phi P_n}{A_g}$ , and an index reflecting the unmagnified moment,  $\frac{M_2 C_m}{A_g h}$ .

## 2.2 Single Columns

Expressions for the critical load and the corresponding moment magnifier, written consistently with the axial load and moment indices, are developed in this section.

The expression for the critical load is

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \quad \text{ACI Eq. (10-8)}$$

EI may be taken as either

$$EI = \frac{(E_c I_g / 5) + E_s I_{se}}{1 + \beta_d} \quad \text{ACI Eq. (10-9)}$$

or

$$EI = \frac{E_c I_g / 2.5}{1 + \beta_d} \quad \text{ACI Eq. (10-10)}$$

Selecting the larger value for EI from ACI Eqs. (10-9) and (10-10) results in a larger critical load, giving a more economical design. ACI Eq. (10-9) is dependent on the column reinforcement. When this equation is used for design, the conventional approach is to (1) assume a reinforcing ratio, (2) calculate the critical load, (3) calculate the moment magnifier, (4) magnify the moments, (5) select the reinforcing ratio from the conventional interaction diagrams, (6) compare the reinforcing ratio with the assumed value, and (7) revise, if necessary, by repeating steps



(1) through (6). To eliminate the need for assuming an initial reinforcing ratio and the subsequent revisions to the solution, the moment magnifier,  $\delta$  (which depends upon  $P_c$ , and thus the reinforcing ratio), is incorporated into the interaction diagrams.

To express the critical load in terms consistent with the axial index, the flexural stiffness,  $EI$ , must be "generalized." The contribution of the reinforcing steel to the moment of inertia,  $I_{se}$ , may be written as

$$I_{se} = C_{se} \rho A_g \gamma^2 h^2 \quad (1)$$

in which  $C_{se}$  is a characteristic coefficient for each reinforcing configuration;  $\rho$  is the total reinforcing ratio  $= \frac{A_{st}}{A_g}$ ;  $\gamma$  is the ratio of the distance between the centroids of the reinforcement in opposite faces parallel to the neutral axis of bending to the column depth,  $h$ , as shown in Fig. B.3, and is called the geometric index;  $A_g$  is the gross area of the cross section; and  $h$  is the depth of the cross section (i.e., the dimension perpendicular to the neutral axis of bending). For different reinforcing configurations,  $C_{se}$  may be shown (2) to be:

1.  $\frac{1}{6}$  for rectangular columns with equal reinforcement distributed in each face as a thin plate,
2.  $\frac{1}{4}$  for rectangular columns with equal reinforcement distributed in the end faces,
3.  $\frac{1}{12}$  for rectangular columns with equal reinforcement distributed in each lateral face as a thin plate,
4.  $\frac{1}{8}$  for rectangular or circular columns with the reinforcement distributed as a uniform cylinder.

The moment of inertia of the gross cross section,  $I_g$ , may be expressed

as

$$I_g = C_g A_g h^2 \quad (2)$$

in which  $C_g$  is a characteristic coefficient based on the shape of the gross cross section. For rectangular and circular gross cross sections,  $C_g$  is  $\frac{1}{12}$  and  $\frac{1}{16}$ , respectively.

Using ACI Eq. (10-9), EI may be written

$$EI = \frac{\alpha A_g h^2}{1 + \beta_d} \quad (3)$$

in which  $\alpha = \frac{E_c C_g}{5} + E_s C_{se} \rho \gamma^2$ .

The critical load is generalized as the critical stress,  $\frac{P_c}{A_g}$ , which can be written as

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha h^2}{(1 + \beta_d)(k \ell_u)^2} \quad (4)$$

To further generalize the expression, the slenderness ratio,  $\frac{k \ell_u}{r}$ , is introduced.  $r$ , the radius of gyration of the gross cross section may be expressed as  $C_r h$ , in which  $C_r$  is a characteristic coefficient based on the shape of the gross cross section. For rectangular and circular cross sections,  $C_r$  is  $\sqrt{\frac{1}{12}} \approx .2887$  and  $\frac{1}{4}$ , respectively. ACI 318-77 allows the former coefficient to be approximated as 0.30. However, to be conservative, the theoretical value is used in this development.

The critical stress may now be written as

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha}{(1 + \beta_d) C_r^2 \left( \frac{k \ell_u}{r} \right)^2} \quad (5a)$$

or

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha}{C_r^2 \lambda^2} \quad (5b)$$

in which  $\lambda = \sqrt{1 + \beta_d} \frac{k\ell_u}{r}$  and is called the modified slenderness ratio. For a given cross sectional shape, reinforcing ratio,  $\rho$ , and modified slenderness ratio,  $\lambda$ , the critical stress,  $\frac{P_c}{A_g}$ , is constant.

The moment magnifier,  $\delta = \frac{C_m}{1 - (P_u / \phi P_c)} \geq 1.0$ , may be expressed in terms of the "factored stress,"  $\frac{P_u}{A_g}$ , and the critical stress.

$$\delta = \frac{C_m}{1 - \left( \frac{P_u / \phi P_c}{A_g / A_g} \right)} \geq 1.0 \quad (6a)$$

Since the factored axial load,  $P_u$ , must be less than or equal to the design axial strength,  $\phi P_n$ , equating the factored load to the design strength ( $P_u = \phi P_n$ ) will provide a conservative approximation for the moment magnifier,  $\delta$ .

$$\delta = \frac{C_m}{1 - \left( \frac{\phi P_n / \phi P_c}{A_g / A_g} \right)} \geq 1.0 \quad (6b)$$

This form of the expression for  $\delta$  is consistent with the generalized column capacities.

The moment index of conventional interaction diagrams is written in terms of the design moment, ( $\phi P_n e = \phi M_n$ ), divided by the product  $A_g h$ . Rewriting ACI Eq. (10-6) in similar terms gives

$$\frac{M_c}{A_g h} = \delta \frac{M_2}{A_g h} \quad (7)$$

in which  $M_c$  is the magnified factored column moment used for design ( $M_c = M_u = \phi M_n$ ), and  $M_2$  is the larger of the factored end moments obtained from the elastic frame analysis. Dividing both sides of Eq. (7) by the moment magnifier and substituting  $\phi M_n$  for  $M_c$  gives

$$\frac{M_2}{A_g h} = \frac{1}{\delta} \frac{\phi M_n}{A_g h} \quad (8)$$

By using the expression for the moment magnifier, Eq. (6b), Eq. (8) may be rewritten.

$$\frac{M_2}{A_g h} = \frac{1 - \left( \frac{\phi P_n}{A_g} / \frac{\phi P_c}{A_g} \right)}{C_m} \frac{\phi M_n}{A_g h} \quad (9)$$

Since  $C_m$  varies with the particular column constraints and loading, it is more useful to incorporate  $C_m$  on the left side of the equation in a "slender column moment index," which is defined as

$$\frac{M_2 C_m}{A_g h} = \left[ 1 - \left( \frac{\phi P_n}{A_g} / \frac{\phi P_c}{A_g} \right) \right] \frac{\phi M_n}{A_g h} \quad (10)$$

To develop a slender column interaction diagram, the axial load and moment indices,  $\frac{\phi P_n}{A_g}$  and  $\frac{\phi M_n}{A_g h}$ , are determined for various locations of the neutral axis for a particular column geometry and reinforcing ratio, in the normal manner. The value of the moment index is then modified using Eq. (10) to obtain the slender column moment index. The locus of points

represented by the axial load index and the slender column moment index represent the interaction diagram for the column. Since  $\frac{P_c}{A_g}$  is a function of the modified slenderness ratio,  $\lambda$ , as well as geometry and material properties, separate interaction diagrams must be constructed for each combination of  $\lambda$ ,  $f_y$ ,  $f'_c$ , column shape and steel arrangement. The details of the interaction diagrams are discussed in Chapter 3.

When  $C_m$  is less than 1, the moment magnifier can be less than 1. Therefore, the moment magnifier should be checked to prevent the design of an inadequately reinforced column. Rewriting Eq. (8), the moment magnifier is

$$\delta = \frac{\phi M_n / A_g h}{M_2 / A_g h} \quad (11a)$$

or in terms of the slender column moment index,  $\frac{M_2 C_m}{A_g h}$ ,

$$\delta = C_m \frac{\phi M_n / A_g h}{M_2 C_m / A_g h} \quad (11b)$$

Once the reinforcing ratio,  $\rho$ , has been determined from the appropriate slender column interaction diagram, the corresponding magnified moment index,  $\frac{\phi M_n}{A_g h}$ , may be back calculated by entering the interaction diagram for a short column (modified slenderness ratio,  $\lambda = 0$ ) with the given axial index,  $\frac{\phi P_n}{A_g}$ , and reinforcing ratio. The moment magnifier is then checked using Eq. (11b).

### 2.3 Multiple Columns in Unbraced Frames

Since the lateral movement of unbraced frames is dependent upon the total lateral stiffness of the structure, ACI 318-77 requires the moment

magnifier,  $\delta$ , to be based on all of the columns in a given story. For slender columns in unbraced frames, the approach is to approximate the unbraced story with a single equivalent column subject to a single equivalent load. The moment magnifier,  $\delta$ , for the equivalent column is then determined as outlined in Section 2.2 for single columns. The equivalent column moment magnifier approximates the value of  $\delta$  for the story and is applied to the column moments,  $M_2$ , to obtain approximate column reinforcing ratios using short column interaction diagrams ( $\lambda = 0$ ). A more accurate moment magnifier is then calculated for the columns in the story in the normal manner, using the approximate reinforcing ratios obtained during the first trial. If necessary, the design is revised.

The development of the equivalent column approach follows:

For  $n$  columns in a single story, the moment magnifier is

$$\delta = \frac{1}{1 - \frac{\sum_{i=1}^n P_{ui}}{\phi \sum_{i=1}^n P_{ci}}} \quad (12)$$

The moment magnifier is maximized when the summation of the factored axial loads is maximized. If all columns are simultaneously loaded to their design strengths,  $\sum_{i=1}^n (\phi P_n)_i$  may be substituted for  $\sum_{i=1}^n P_{ui}$  in Eq. (12) giving

$$\delta = \frac{1}{1 - \frac{\sum_{i=1}^n (\phi P_n)_i}{\phi \sum_{i=1}^n P_{ci}}} \quad (13)$$

or in terms of stresses,

$$\delta = \frac{1}{1 - \frac{\sum_{i=1}^n (\phi P_n)_i / \sum_{i=1}^n A_{gi}}{\phi \sum_{i=1}^n P_{ci} / \sum_{i=1}^n A_{gi}}} \quad (14)$$

The concept of the "equivalent column" is now introduced to obtain an equivalent column modified slenderness ratio,  $\lambda_m$ , axial load index,  $\frac{(\phi P_n)_m}{A_{gm}}$ , and moment index  $\frac{(M_2/h)_m}{A_{gm}}$ . Each of these is derived below.

From Eq. (14), the equivalent column axial load index is defined as

$$\frac{(\phi P_n)_m}{A_{gm}} = \frac{\sum_{i=1}^n (\phi P_n)_i}{\sum_{i=1}^n A_{gi}} \quad (15)$$

The interaction diagrams establish a relationship between the axial load index and the moment index for values of the reinforcing ratio. This relationship may be defined as

$$\frac{\phi P_n}{A_g} = f\left(\frac{M_2}{A_g h}, \rho\right) \quad (16)$$

$C_m$  is not shown because, for an unbraced frame,  $C_m$  is unity. Using the design assumptions of ACI 318-77, Section 10.2 (1) (linear strain distribution and equivalent rectangular concrete stress block) for a specific strain distribution, the axial load index and moment index ( $\lambda = 0$ ) vary

linearly with the reinforcing ratio,  $\rho$ . This may be observed by studying values of the axial load index and moment index in which the reinforcement furthest from the extreme compression fiber is at zero strain (or stress), at half of the yield strain (or stress), and at the yield strain (or balanced condition) (2). By inspection of the slender column interaction diagrams, the curves appear to be flat between the axial load index axis and the balanced condition, particularly for high modified slenderness ratios,  $\lambda$ , for which the moment magnification is the greatest. The curves are approximately uniformly spaced, where ACI Eq. (10-9) (as opposed to ACI Eq. (10-10)) is used to calculate the flexural stiffness,  $EI$ .

Approximating the interaction diagrams as a family of straight lines, Eq. (16) is rewritten

$$\frac{\phi P_n}{A_g} = C_2 \frac{M_2}{A_g h} + C(\rho) \quad (17)$$

in which  $C_2$  is the slope of the lines and  $C(\rho)$  is the intercept of the axial load index axis. The axial load index axis intercept,  $C(\rho)$ , may be written as a linear equation,

$$C(\rho) = C_1 \rho + C_0 \quad (18)$$

in which  $C_1$  is the change in the intercept divided by the change in the reinforcing ratio and  $C_0$  is the intercept constant. Combining Eqs. (17) and (18) gives

$$\frac{\phi P_n}{A_g} = C_2 \frac{M_2}{A_g h} + C_1 \rho + C_0 \quad (19)$$



Eq. (19) multiplied by  $A_g$  is

$$\phi P_n = C_2 M_2/h + C_1 \rho A_g + C_0 A_g \quad (20)$$

For a column,  $i$ , the axial load capacity, moment capacity, reinforcing ratio, and gross cross section area are denoted by  $(\phi P_n)_i$ ,  $(M_2/h)_i$ ,  $\rho_i$  and  $A_{gi}$ , respectively. Taking the summation of  $n$  columns, Eq. (20) is

$$\sum_{i=1}^n (\phi P_n)_i = C_2 \sum_{i=1}^n (M_2/h)_i + C_1 \sum_{i=1}^n \rho_i A_{gi} + C_0 \sum_{i=1}^n A_{gi} \quad (21)$$

The second term on the right side of Eq. (21) must be modified to derive an expression in terms of an equivalent column:

$$\sum_{i=1}^n (\phi P_n)_i \geq C_2 \sum_{i=1}^n (M_2/h)_i + \frac{C_1}{n} \sum_{i=1}^n \rho_i \sum_{i=1}^n A_{gi} + C_0 \sum_{i=1}^n A_{gi} \quad (22)$$

The effect of the modification can be determined using Chebyshev's Inequality (3):

$$n \sum_{j=1}^n a_j b_j \geq \left( \sum_{j=1}^n a_j \right) \left( \sum_{j=1}^n b_j \right) \quad (23)$$

Applying Chebyshev's Inequality to the second summation on the right side of Eq. (21) shows that the modification underestimates the summation of axial load capacity. While this particular simplification is somewhat unconservative, additional modifications will help insure that the overall procedure is satisfactory.

Defining the mean reinforcing ratio as

$$\rho_m = \frac{1}{n} \sum_{i=1}^n \rho_i \quad (24)$$

substituting Eq. (24) into Eq. (22), and dividing by the summation of gross cross sectional areas gives the following inequality:

$$\frac{\sum_{i=1}^n (\phi P_n)_i}{\sum_{i=1}^n A_{gi}} \geq \frac{C_2 \sum_{i=1}^n (M_2/h)_i}{\sum_{i=1}^n A_{gi}} + \frac{C_1 \rho_m \sum_{i=1}^n A_{gi}}{\sum_{i=1}^n A_{gi}} + \frac{C_0 \sum_{i=1}^n A_{gi}}{\sum_{i=1}^n A_{gi}} \quad (25a)$$

or

$$\frac{\sum_{i=1}^n (\phi P_n)_i}{\sum_{i=1}^n A_{gi}} \geq C_2 \frac{\sum_{i=1}^n (M_2/h)_i}{\sum_{i=1}^n A_{gi}} + C_1 \rho_m + C_0 \quad (25b)$$

As defined by Eq. (15), the equivalent column axial load index is

$$\frac{(\phi P_n)_m}{A_{gm}} = \frac{\sum_{i=1}^n (\phi P_n)_i}{\sum_{i=1}^n A_{gi}} \quad (15)$$

Similarly, the equivalent column moment index is defined from Eq. (25b) as

$$\frac{(M_2/h)_m}{A_{gm}} = \frac{\sum_{i=1}^n (M_2/h)_i}{\sum_{i=1}^n A_{gi}} \quad (26)$$

The equivalent column modified slenderness ratio is consistently derived with the previously defined equivalent column axial and moment indices. For  $n$  columns, the summation of critical

loads is

$$\sum_{i=1}^n P_{ci} = \sum_{i=1}^n \frac{\pi^2 (EI)_i}{(k\ell_u)_i^2} \quad (27)$$

Based on ACI Eq. (10-9), EI varies linearly with reinforcing ratio and may be written similarly to Eq. (3) for the  $i$ th column as

$$(EI)_i = \frac{(a_2 + b_2 \rho_i) A_{gi} h_i^2}{1 + \beta_{di}} \quad (28)$$

in which  $a_2 = \frac{E_c C_g}{5}$  and  $b_2 = E_s C_{se} \gamma^2$ . For simplicity, the geometric index,  $\gamma$ , is treated as constant for all columns in a story. Combining Eq. (28) and Eq. (27) gives

$$\sum_{i=1}^n P_{ci} = \pi^2 \sum_{i=1}^n \frac{(a_2 + b_2 \rho_i) A_{gi} h_i^2}{(1 + \beta_{di}) (k\ell_u)_i^2} \quad (29a)$$

or

$$\sum_{i=1}^n P_{ci} = \frac{\pi^2}{C_r^2} \sum_{i=1}^n \frac{(a_2 + b_2 \rho_i) A_{gi}}{\lambda_i^2} \quad (29b)$$

or

$$\sum_{i=1}^n P_{ci} = \frac{\pi^2}{C_r^2} \left[ a_2 \sum_{i=1}^n \frac{A_{gi}}{\lambda_i^2} + b_2 \sum_{i=1}^n \rho_i \frac{A_{gi}}{\lambda_i^2} \right] \quad (29c)$$

Eq. (29c) can be conservatively modified using Chebyshev's Inequality (3) to give the following inequality:

$$\sum_{i=1}^n P_{Ci} \geq \frac{\pi^2}{C_r^2} \left[ a_2 + \frac{b_2}{n} \sum_{i=1}^n \rho_i \right] \sum_{i=1}^n \frac{A_{gi}}{\lambda_i^2} \quad (30a)$$

This modification helps to counteract the lack of conservatism in Eq. (22). The summation of the reinforcing ratio in Eq. (30a) is consistent with the mean reinforcing ratio, Eq. (24), used in describing the equivalent column axial load index and moment index interaction. Eq. (30a) is re-written

$$\sum_{i=1}^n P_{Ci} \geq \frac{\pi^2}{C_r^2} [a_2 + b_2 \rho_m] \sum_{i=1}^n \frac{A_{gi}}{\lambda_i^2} \quad (30b)$$

or

$$\sum_{i=1}^n P_{Ci} \geq \frac{\pi^2}{C_r^2} \alpha_m \sum_{i=1}^n \frac{A_{gi}}{\lambda_i^2} \quad (30c)$$

in which  $\alpha_m = a_2 + b_2 \rho_m = E_c C_g / 5 + E_s C_s \rho_m \gamma^2$  (see Eq. (3)). Eq. (30c) written in terms of the equivalent column critical stress is

$$\frac{\sum_{i=1}^n P_{Ci}}{\sum_{i=1}^n A_{gi}} \geq \frac{\pi^2}{C_r^2} \alpha_m \frac{\sum_{i=1}^n \frac{A_{gi}}{\lambda_i^2}}{\sum_{i=1}^n A_{gi}} \quad (31)$$

Eq. (5) written for the equivalent column critical stress and denoted with subscript, m, is

$$\frac{P_{cm}}{A_{gm}} = \frac{\pi^2 \alpha_m}{C_r^2 \lambda_m^2} \quad (32)$$

Equating the right sides of Eqs. (31) and (32) and solving for the equivalent column modified slenderness ratio,  $\lambda_m$ , is

$$\lambda_m = \sqrt{\frac{\sum_{i=1}^n A_{gi}}{\sum_{i=1}^n A_{gi}/\lambda_i^2}} \quad (33)$$

The geometric index,  $\gamma$ , is treated as a constant in the preceding derivation. For many applications, the geometric index is a constant or very close to a constant. The moment of inertia is influenced by the square of the geometric index. However, the bending capacity for a particular axial load is influenced by the change in moment arm of the reinforcement which for end face reinforcement is linear with the geometric index. Since the moment of inertia and the bending capacity have different order relationships with the geometric index, the mean column geometric index is arbitrarily taken as

$$\gamma_m = \frac{\sum_{i=1}^n A_{gi} \gamma_i}{\sum_{i=1}^n A_{gi}} \quad (34)$$

The equivalent column parameters, axial load index, moment index, modified slenderness ratio, and geometric index, are defined in Eqs. (15), (26), (33), and (34), respectively. The equivalent column is treated as a single unbraced column. The approximate moment magnifier is determined from the procedure leading to Eq. (11b). The approximate moment magnifier is applied to individual column moments and the columns are designed using

the short column interaction diagrams ( $\lambda = 0$ ). The actual moment magnifier for the story is calculated using the columns just designed, and, if necessary, the designs are revised.

## CHAPTER 3

## RESULTS

## 3.1 Slender Column Interaction Diagrams

Slender column interaction diagrams are plotted for column types as follows:

1. rectangular tied columns with equal reinforcement in all faces
2. rectangular tied columns with equal reinforcement in the end faces (parallel to the neutral axis)
3. rectangular tied columns with reinforcement in the lateral faces (perpendicular to the neutral axis)
4. rectangular spiral columns
5. circular spiral columns

The diagrams are limited to the following range of parameters:

1. material strengths,  $f'_c = 4, 5, 6$  ksi and  $f_y = 60$  ksi
2. geometric index,  $\gamma = 0.6, 0.75, 0.9$
3. modified slenderness ratio,  $\lambda = 0, 30, 45, 55, 65, 70, 75, 80, 85, 90, 95, 100$ .

Designation of the interaction diagrams is similar to the ACI Design Handbook (3) designation. The first character represents the column type, as shown in Fig. B.3, followed by the concrete compressive strength,  $f'_c$ , the yield strength of the reinforcement,  $f_y$ , the geometric index,  $\gamma$ , and the modified slenderness ratio,  $\lambda$  (i.e., Type  $f'_c - f_y \gamma - \lambda$ ).

The values of the modified slenderness ratios,  $\lambda$ , are selected such that the change in  $\lambda^2$  for consecutive values is approximately equal (the critical stress is inversely proportional to  $\lambda^2$ ). Linear interpolation

between values of the slenderness index is conservative, as shown in Fig. 3.1. ACI 318-77 limits the slenderness ratio to a maximum of 100 using the provisions of Section 10.11. As  $\beta_d$  approaches 1.0 the maximum allowable modified slenderness ratio approaches  $100 \sqrt{2}$ . However, the diagrams plotted are limited to a maximum value of  $\lambda$  of 100.

The column strengths are based on linear strain distributions and equivalent rectangular concrete stress distributions as described in Section 10.2 of ACI 318-77. The reinforcement is characterized using a uniform thin wall distribution. Interaction diagrams are plotted in 1 percent increments of the reinforcing ratio,  $\rho$ , from 1 to 8 percent. Strength reduction factors,  $\phi$ , of 0.7 for tied columns and 0.75 for spiral columns in compression, and 0.9 for members in pure bending and tension are incorporated in the interaction diagrams. The tension portion of the column capacity interaction has been included for the short column interaction diagrams (slenderness index,  $\lambda = 0$ ). For column splice considerations, points are plotted on the interaction curves at which the reinforcement furthest from the extreme compression fiber is at zero strain (or stress), at half the yield strain (or stress), and at the yield strain (or balanced condition).

Discontinuities in the shape of the interaction curves may be observed in the region of small eccentricities. In portions of this region, the equivalent rectangular concrete stress distribution extends over the entire cross section. The change in the axial load and moment indices is a result of the changing stress distribution in the reinforcement only. For higher eccentricities, the equivalent rectangular concrete stress distribution extends over a portion of the cross section and influences the change in the axial load and moment indices. A transition between these



two portions is expected. Discontinuities in the shape develop because too few points are plotted in this region. Columns, however, are designed with minimum eccentricities greater than the eccentricities corresponding to the discontinuities, and therefore, the discontinuities are of no consequence to design.

## CHAPTER 4

## EXAMPLES

## 4.1 Braced Column

The slender column interaction diagram approach is compared to the procedure illustrated in the ACI Design Handbook "Columns Example 2 - Selection of reinforcement for a rectangular tied column with bars on four faces (slenderness ratio found to be above critical value)." "Columns Example 2" is stated as follows:

For a 22 x 20 in. rectangular tied column with bars equally distributed along four faces, select the reinforcement

Given: Loading

Factored axial load  $P_u = 560$  kips

Factored end moment at top of column  $M_u = M_2 = +3920$  in.-kips

Dead load moment unfactored at top of column  $M_d = 1120$  in.-kips

Factored end moment at bottom of column  $M_1 = +2940$  in.-kips

No transverse loading on member

Materials

Compressive strength of concrete  $f'_c = 4$  ksi

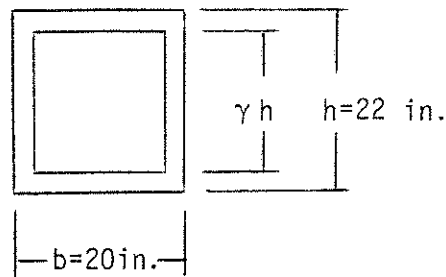
Yield strength of reinforcement  $f_y = 60$  ksi

Nominal maximum size of aggregate = 1 in.

Design conditions

Unsupported length of column  $\ell_u = 27.5$  ft.

Column braced against sidesway



#### 4.1.1 Slender Column Interaction Diagram Method:

Step 1. Determine column section size (same as Columns Example 2)

Given:  $h = 22\text{ in.}$

$b = 20\text{ in.}$

Step 2. Check whether slenderness effect must be considered (same as Columns Example 2 or alternately)

a. Calculate the slenderness ratio

For column braced against sidesway,  $k = 1.0$

$$r = \sqrt{\frac{I}{A}} h = \sqrt{\frac{I}{A}} 22 = 6.35\text{ in.}$$

$$\frac{k\ell_u}{r} = \frac{1.(27.5 \times 12)}{6.35} = 52.0$$

b. Compare  $\frac{k\ell_u}{r}$  with  $34-12 M_1/M_2$

$$34-12 M_1/M_2 = 34-12 (2940/3920) = 25$$

$$\frac{k\ell_u}{r} > 34-12 M_1/M_2$$

Slenderness effects must be considered.

Step 3. Determine reinforcing ratio and steel required.

a. Determine the axial load index and moment index

$$\frac{P_u}{bh} = \frac{560}{20(22)} = 1.27\text{ ksi}$$

Since there is no transverse load on the column,

$$C_m = 0.6 + 0.4(M_1/M_2) = 0.6 + 0.4 \left( \frac{2940}{3920} \right) = 0.9$$

$$\frac{M_2 C_m}{bh^2} = \frac{3920(.9)}{20(22)^2} = 0.364 \text{ ksi}$$

- b. Determine appropriate interaction diagram(s).

$$\text{Estimate } \gamma \approx \frac{h-5}{h} = \frac{22-5}{22} = 0.77$$

$$\text{Use } \gamma = 0.75$$

Calculate the modified slenderness ratio

$$\beta_d = (1.4M_d)/M_u = \frac{1.4(1120)}{3920} = 0.4$$

$$\lambda = \sqrt{1+\beta_d} \frac{k\ell_u}{r} = \sqrt{1+0.4} 52.0 = 61.5$$

Interpolate between modified slenderness ratios of 55 and 65 for  $f'_c = 4$  ksi,  $f_y = 60$  ksi,  $\gamma = 0.75$ , and equal reinforcing in all faces.

- c. From interaction diagram for  $\lambda = 65$ ,  $\rho = 0.034$ .

From interaction diagram for  $\lambda = 55$ ,  $\rho = 0.027$

Interpolating for  $\lambda = 61.5$

$$\rho = 0.027 + (0.034 - 0.027) \frac{61.5 - 55}{65 - 55} = 0.032$$

- d. Since  $C_m$  is less than 1.0, check moment magnifier.

From interaction diagram for  $\lambda = 0$  with  $\frac{\phi P_n}{bh} = \frac{P_u}{bh}$

$$\text{at } \rho = 0.032, \quad \frac{\phi M_n}{bh^2} = 0.59$$

$$\delta = C_m \frac{\phi M_n}{bh^2} / \frac{M_2 C_m}{bh^2} = 0.9(0.59/0.364) = 1.46 > 1.0.$$

- e. Compute steel required

$$A_{st} = \rho bh = 0.032(20)22 = 14.1 \text{ in}^2$$

Step 4. Select reinforcement (same as Columns Example 2)

## 4.1.2 ACI Design Handbook Approach:

Step 1. Determine column section size

Given:  $h = 22$  in.

$b = 20$  in.

Step 2. Check whether slenderness ratio is less than critical value

a. Compute  $M_1/M_2$  and read critical value of  $k\ell_u/h$  from

Columns 1

$$M_1/M_2 = \frac{2940}{3920} = 0.75$$

Critical  $k\ell_u/h = 7.5$

b. Determine  $k$

For columns braced against sidesway,  $k = 1.0$

c. Compute  $k\ell_u/h$  and compare with critical value

$$k\ell_u/h = 1.0(27.5)12/22 = 15 > 7.5$$

Slenderness effects must be considered.

d. Determine moment magnification factor

Determine appropriate moment magnifier graph.

For rectangular tied column with  $f'_c = 4$  ksi, use

Columns 5.2

$$\text{Compute } \beta_d = (1.4M_d)/M_u = 1.4(1120)/3920 = 0.4$$

$$\text{Compute } P_u(1+\beta_d)/A_g = 560(1.4)/(22 \times 20) = 1.78 \text{ ksi}$$

Since there is no transverse load, compute  $C_m$  from

$$C_m = 0.6 + 0.4(M_1/M_2) = 0.6 + 0.4(2940/3920) = 0.9$$

$$\text{Estimate } \gamma \approx \frac{h-5}{h} = \frac{22-5}{22} = 0.77 \approx 0.75$$

Determine appropriate interaction diagram.

For rectangular tied column with steel along four faces,

$f'_c = 4$  ksi,  $f_y = 60$  ksi, and  $\gamma \approx 0.75$ , use R4-60.75.

Assume trial values of $\rho$	0.02	0.03	0.04
Read $h_e/h$	1.00	1.10	1.20
Compute $k\ell_u/h_e = k\ell_u/h \div h_e/h$	15.00	13.60	12.50
Read $\delta/C_m$ from graph	1.91	1.64	1.49
Compute $\delta = C_m(\delta/C_m)$	1.72	1.48	1.34
e. Compute $\delta M_u$ , in-kips	6740	5800	5250

Step 3. Determine reinforcing ratio and steel required.

a. Compute  $P_u/A_g = 560/440 = 1.27$  ksi

Assumed $\rho$	0.02	0.03	0.04
----------------	------	------	------

b. Compute  $\delta M_u/A_g h$

	0.696	0.599	0.542
--	-------	-------	-------

c. Read  $\rho$  for  $P_u/A_g$  and

$\delta M_u/A_g h$	0.042	0.032	0.027
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Compare with assumed

$\rho$	$\neq 0.02$	$\approx 0.03$	$\neq 0.04$
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Repeat from Step 2d, assuming  $\rho_g = 0.033$

Read  $h_e/h = 1.13$

Compute  $k\ell_u/h_e = 13.3$

Read  $\delta/C_m = 1.60$

Compute  $\delta = 1.44$

Compute  $\delta M_u = 5640$  in-kips

Compute  $\delta M_u/A_g h = 0.583$

Read  $\rho = 0.031 \approx 0.033$

Use  $\rho = 0.031$

d. Compute required  $A_{st} = \rho_g A_g = 0.031 \times 440 = 13.6 \text{ in}^2$

Step 4. Select reinforcement

### 4.1.3 Discussion

The proposed method shows considerable computational savings over the method outlined in the ACI Design Handbook. The Handbook procedure is an iterative process requiring initial assumptions for the reinforcing ratio, followed by successful trials. The slender column interaction method is a direct method, requiring no initial assumptions regarding the reinforcing ratio. The example calculations show that the proposed method provides results in a single solution which can be achieved only after several iterations using the method illustrated in the ACI Design Handbook.

### 4.2 Unbraced Frame

To illustrate the slender column interaction diagram approach, the example below has been condensed as follows:

An unbraced structure consisting of 5 columns requires consideration of slenderness effects. The columns have been preliminarily sized. Combining the maximum live load with dead load and lateral load will control the design of all columns. A summary of all columns is as follows:

Column	1	2	3	4	5
b, in.	20	20	20	20	20
h, in.	20	24	28	26	22
$\gamma$ assumed	.75	.75	.75	.75	.75
$\lambda = \sqrt{1+\beta_d} k \ell_u / r$	80	75	65	70	85
$P_u$ , kips	240	576	1008	1040	704
$M_2$ , in.-kips	2375	4026	5529	5189	4543

$f'_c = 4$  ksi,  $f_y = 60$  ksi

Determine the reinforcing ratio of each column.

Step 1. Determine the values of the equivalent column parameters for slenderness, axial load, and moment

a. equivalent column modified slenderness ratio

$$\lambda_m = \sqrt{\frac{\sum_{i=1}^5 A_{gi}}{\sum_{i=1}^5 A_{gi}/\lambda_i^2}}$$

$$\sum_{i=1}^5 A_{gi} = 20(20) + 20(24) + 20(28) + 20(26) + 20(22) = 2400 \text{ in}^2$$

$$\sum_{i=1}^5 A_{gi}/\lambda_i^2 = 20(20)/80^2 + 20(24)/75^2 + 20(28)/65^2 + 20(26)/70^2 + 20(22)/85^2 = 0.4474$$

$$\lambda_m = \sqrt{2400/0.4474} = 73.24$$

b. Equivalent column axial load index in which  $\frac{P_u}{A_g} = \frac{\phi P_n}{A_g}$

$$\sum_{i=1}^5 \phi P_{ni} = 240 + 576 + 1008 + 1040 + 704 = 3568 \text{ kips}$$

$$\frac{(\phi P_n)_m}{A_{gm}} = \frac{\sum_{i=1}^5 (\phi P_n)_i}{\sum_{i=1}^5 A_{gi}} = \frac{3568}{2400} = 1.49 \text{ ksi}$$

c. Equivalent column moment index

$$\sum_{i=1}^5 M_{2i}/h_i = 2375/20 + 4026/24 + 5529/28 + 5189/26 + 4543/22 = 890 \text{ kips}$$

$$\frac{(M_2/h)_m}{A_{gm}} = \frac{\sum_{i=1}^5 M_{2i}/h_i}{\sum_{i=1}^5 A_{gi}} = \frac{890}{2400} = 0.371 \text{ ksi}$$



Step 2. Determine the equivalent column reinforcing ratio by interpolation between appropriate slender column interaction diagrams using the axial load index and moment index calculated in Step 1.

For  $f'_c = 4$  ksi,  $f_y = 60$  ksi and modified slenderness ratio of

$$\lambda = 70 \quad \rho_m = 0.047$$

$$\lambda = 75 \quad \rho_m = 0.052$$

By interpolation

$$\lambda = 73.24 \quad \rho_m = 0.050$$

Step 3. Determine the moment index for a modified slenderness ratio of zero given the equivalent column reinforcing ratio, and calculate the equivalent column moment magnifier.

a. For  $f'_c = 4$  ksi,  $f_y = 60$  ksi and  $\lambda = 0$ .

$$\frac{M_{cm}}{A_{gm}} = .74$$

$$b. \delta_m = \frac{M_{cm}/A_{gm}}{M_{gm}/A_{gm}} = 0.74/0.371 = 1.99$$

Step 4. Determine preliminary reinforcing ratios and calculate corresponding moment magnifier

Column	1	2	3	4	5
a. Calculate $P_u/bh$	0.60	1.20	1.80	2.00	1.60
Calculate $M_{2C_m}/bh^2$	.297	.349	.353	.384	.469
Calculate $M_c/bh^2$	.591	.695	.702	.764	.933
Determine $\rho$ from	0.031	0.041	0.051	0.061	0.071
$f'_c = 4$ ksi, $f_y = 60$ ksi,					
$\gamma = 0.75$ , $\lambda = 0$					

## b. Calculate critical stresses

From Critical Stress Aid (Fig. B.3)

For  $f'_c = 4$  ksi,  $K_c = 7116$ For rectangular columns with reinforcement in all faces and  $f_y = 60$  ksi,  $K_s/\rho\gamma^2 = 572400$ 

$$P_c/A_g = 2 K_c/\lambda^2 \quad \text{for } K_c > K_s$$

$$P_c/A_g = (K_c + K_s)/\lambda^2 \quad \text{for } K_c < K_s$$

$$\text{in which } K_c = \frac{\pi^2 E_c C_g}{5 C_r^2} \quad \text{and} \quad K_s = \frac{\pi^2 E_s C_{se} \rho \gamma^2}{C_r^2}$$

Column	1	2	3	4	5
$K_c =$	7116	7116	7116	7116	7116
Calculate $K_s$	9981	13201	16421	19640	22860
	$>K_c$	$>K_c$	$>K_c$	$>K_c$	$>K_c$
$P_c/bh$	2.671	3.612	5.571	5.460	4.149
$P_c$	1069	1734	3120	2839	1826
$\sum_{i=1}^5 P_c$	$= 1069 + 1734 + 3120 + 2839 + 1826 = 10588$ kips				

$$\delta = \frac{1}{1 - \frac{\sum_{i=1}^5 P_u}{\phi \sum_{i=1}^5 P_c}} = \frac{1}{1 - \frac{3568}{.7(10588)}} = 1.93 < 1.99$$

For design, the procedure can be terminated. However, to evaluate the equivalent column approximation more closely, the moment magnifier is further refined.

Step 5. Revise magnified moments, revise column reinforcing ratios and recalculate moment magnifier

- a. Use the average of the equivalent column moment magnifier and the moment magnifier calculated from the preliminary reinforcing ratios.

$$\text{Assume } \delta = \frac{1.99+1.93}{2} = 1.96$$

b. Column	1	2	3	4	5
Calculate $M_c/bh^2$	.582	.684	.692	.753	.919
$P_u/bh$	0.60	1.20	1.80	2.00	1.60
Determine $\rho$	0.030	0.040	0.050	0.060	0.070
c. Calculate $K_s$	9659	12879	16099	19319	22538
	$>K_c$	$>K_c$	$>K_c$	$>K_c$	$>K_c$
$P_c/bh$	2.621	3.555	5.495	5.395	4.104
$P_c$	1048	1706	3077	2805	1806

$$\sum_{i=1}^5 P_c = 1048 + 1706 + 3077 + 2805 + 1806 = 10442 \text{ kips}$$

$$\delta = \frac{1}{1 - \frac{\sum_{i=1}^5 P_u}{\phi \sum_{i=1}^5 P_c}} = \frac{1}{1 - \frac{3568}{.7(10442)}} = 1.954 \approx 1.96$$

The equivalent column moment magnifier overestimates the actual moment magnifier by less than 2 percent. Although assuming a constant geometric index,  $\gamma$ , the example is exaggerated by using diverse loadings on 5 different column cross sections. Since the results are favorable for this example, the method should provide good estimates of the reinforcing ratio,  $\rho$ , for more uniform geometry and loading.

## CHAPTER 5

### SUMMARY AND CONCLUSIONS

#### 5.1 Summary

For single slender reinforced concrete columns (braced or unbraced), a technique to obtain the reinforcing ratio without requiring an initial estimate of the reinforcing ratio is presented. By reducing unbraced frames to equivalent single columns, slenderness effects in unbraced frames may be estimated. The provisions of ACI 318-77, Section 10.11, "Approximate evaluation of slenderness effects," are utilized.

Conventional interaction diagrams are modified by a non-dimensional parameter characterizing column slenderness. The abscissa is modified to an index reflecting the (unmagnified) moment obtained from an elastic frame analysis,  $M_2$ . The slender column interaction diagrams provide direct solutions for the reinforcing ratio for single columns (braced or unbraced).

#### 5.2 Conclusions

Slender interaction diagrams, applied to the design of single slender columns (braced or unbraced), lead directly to the solutions of the reinforcing ratio. The designer does not have to contend with making initial reinforcing ratio assumptions or with making iterative calculations. For braced frames with slender columns, the equivalent column technique provides useful approximate reinforcing ratios. The slender column interaction diagram approach will be useful to designers of reinforced concrete structures.

## REFERENCES

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3. Handbook of Mathematical Functions, National Bureau of Standards, Appl. Math. Ser. No. 55, 1964.

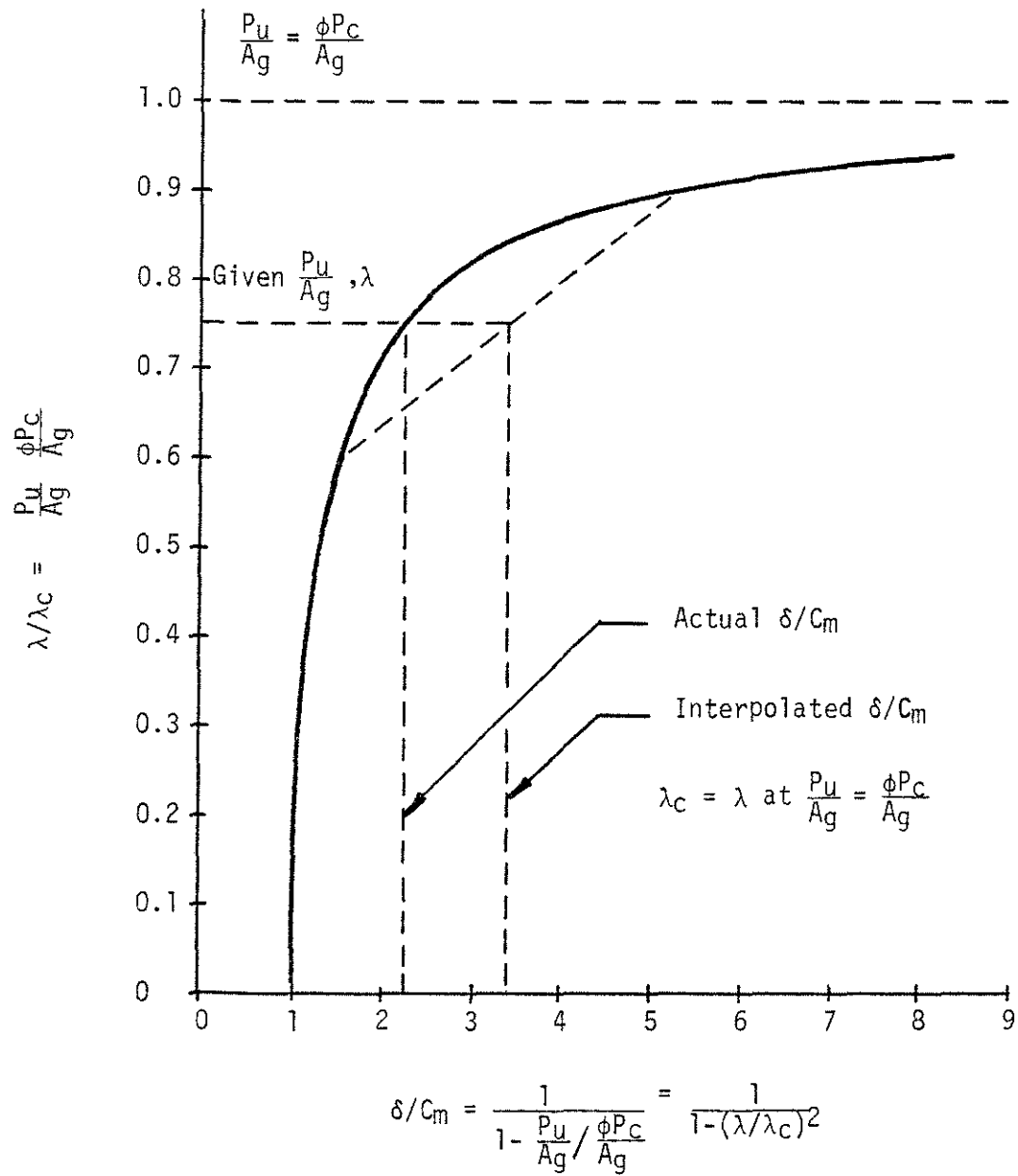


Fig. 3.1 - Modified Slenderness Ratio Versus Moment Magnifier

## APPENDIX A

## NOTATION

$A_g$	= gross area of column cross section
$A_{st}$	= "total area of longitudinal reinforcement," (1)
$C_0$	= intercept constant of Eq. (18)
$C_1$	= change of the intercept divided by the change of the reinforcing ratio, $\rho$ , Eq. (18)
$C_2$	= slope of the interaction lines, Eq. (17)
$C_g$	= characteristic coefficient for calculation of the gross moment of inertia, $I_g$
$C_m$	"a factor relating actual moment diagram to an equivalent uniform moment diagram," (1)
$C_r$	= characteristic coefficient for calculation of the radius of gyration, $r$
$C(\rho)$	= function describing the intercept of the axial load index axis, Eq. (17)
$C_{se}$	= characteristic coefficient for calculation of the reinforcing steel moment of inertia, $I_{se}$
$e$	= eccentricity of axial column load
$E_c$	= "modulus of elasticity of concrete," (1)
$E_s$	= "modulus of elasticity of reinforcement," (1)
$EI$	= "flexural stiffness of compression member," (1)
$f'_c$	= "specified compressive strength of concrete," (1)
$f_y$	= "specified yield strength of nonprestressed reinforcement," (1)
$h$	= dimension of the cross section in the direction perpendicular to the neutral axis of bending

- $i$  = subscript used to denote a specific column
- $I_g$  = "moment of inertia of gross concrete section," (1)
- $I_{se}$  = "moment of inertia of reinforcement," (1)
- $j$  = subscript used to denote terms, Eq. (22)
- $k$  = "effective length factor for compression members," (1)
- $\ell_u$  = "unsupported length of compression member," (1)
- $m$  = subscript used to denote mean parameters of the unbraced frame equivalent column
- $M_c$  = "factored moment to be used for design of compression member," (1)
- $M_u$  = factored moment
- $M_1$  = "value of smaller factored end moment on compression member calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature," (1)
- $M_2$  = "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive," (1)
- $P_c$  = "critical load," (1)
- $P_n$  = "nominal axial load strength at given eccentricity," (1)
- $P_u$  = "factored axial load at given eccentricity  $\leq \phi P_n$ ," (1)
- $r$  = radius of gyration
- $\alpha$  = function combining terms of EI, Eq. (3)
- $\beta_d$  = "ratio of maximum factored dead load moment to maximum factored total load moment, always positive," (1)
- $\gamma$  = geometric index, ratio of the distance between the centroids of the reinforcement in opposite faces parallel to the neutral axis of bending to the column depth,  $h$ , as shown in Fig. B.3



- $\delta$  = "moment magnification factor," (1), or moment magnifier  
 $\lambda$  = modified slenderness ratio =  $\sqrt{1+\beta_d} \frac{k\ell_u}{r}$   
 $\rho$  = total reinforcing ratio =  $\frac{A_{st}}{A_g}$   
 $\phi$  = "strength reduction factor," (1)

## APPENDIX B

### SLENDER REINFORCED CONCRETE COLUMN DESIGN AIDS

The procedures developed for the design of slender reinforced concrete columns are presented in condensed form for convenience in design. For single columns, a logic flow chart is presented in Fig. B.1. For unbraced frames, Fig. B.2 is a logic flow chart and Fig. B.3 is for convenience in calculating the critical load index for individual columns.

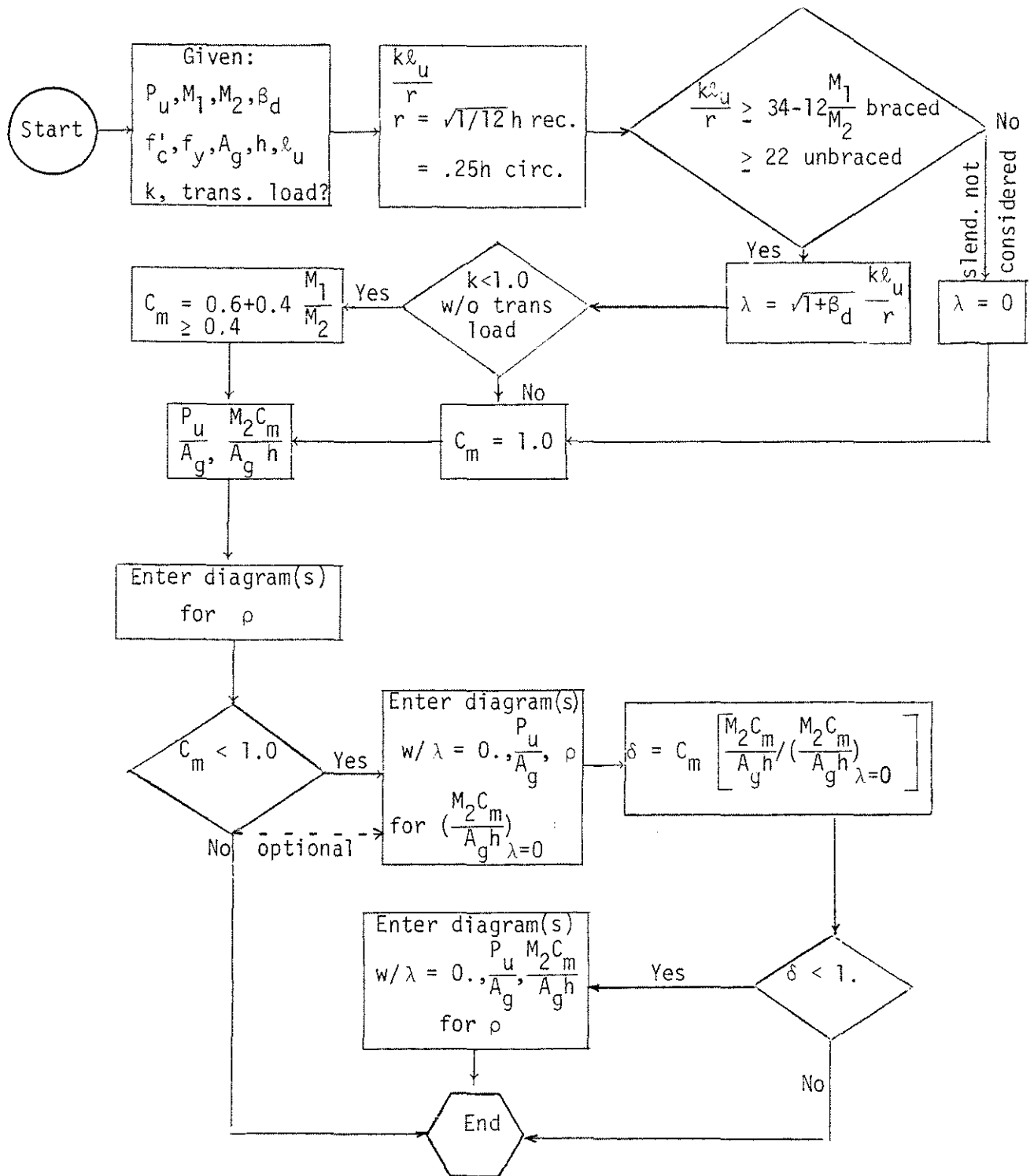


Fig. B.1 Single Column Flow Chart

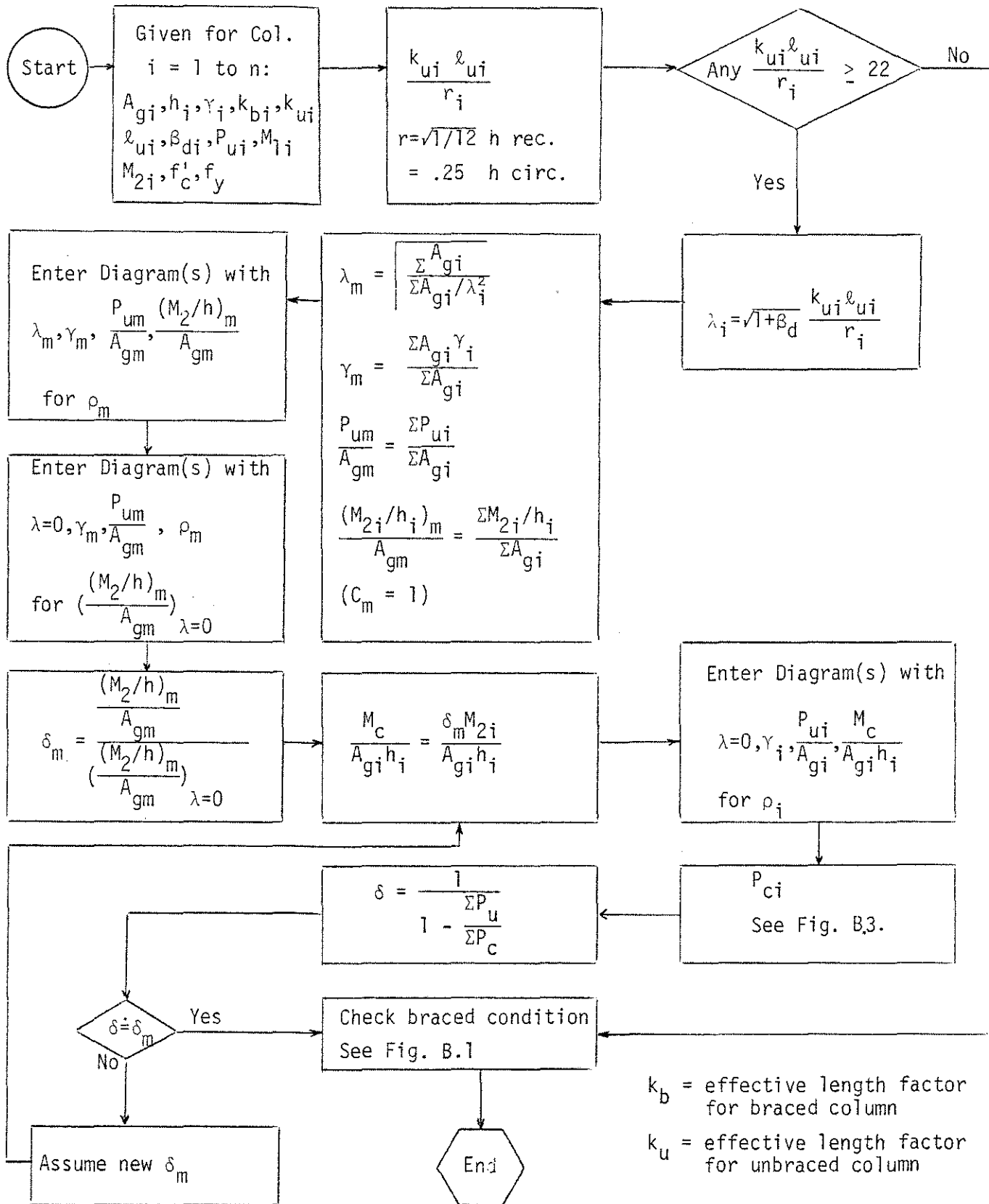


Fig. B.2 Unbraced Frame Flow Chart

## CRITICAL STRESS

$$\frac{P_c}{A_g} = \frac{2K_c}{\left(\sqrt{1+\beta_d} \frac{kl_u}{r}\right)^2}, \text{ ksi for } K_c \geq K_s$$

$$\frac{P_c}{A_g} = \frac{K_c + K_s}{\left(\sqrt{1+\beta_d} \frac{kl_u}{r}\right)^2}, \text{ ksi for } K_c \leq K_s$$

in which  $K_c = \frac{\pi^2 E_c C_g}{5 C_r^2}$  and

$$K_s = \frac{\pi^2 E_s C_{se} \rho \gamma^2}{C_r^2}$$

CONCRETE	
$f'_c$ , ksi	$K_c$ , ksi
4	7116
5	7956
6	8715

REINFORCEMENT	
Column Type	$K_s/\rho\gamma^2$ , ksi
Rectangular	
all faces	572400
end faces	858700
lateral faces	286200
spiral	429300
Circular	572400

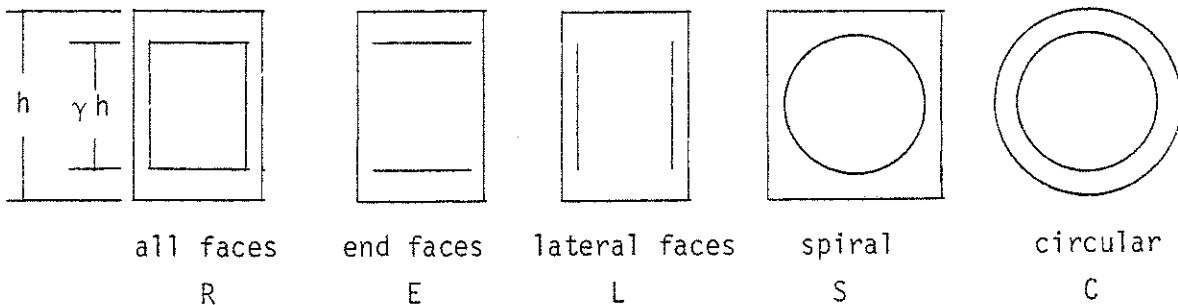


Fig. B.3 Critical Stress Aid

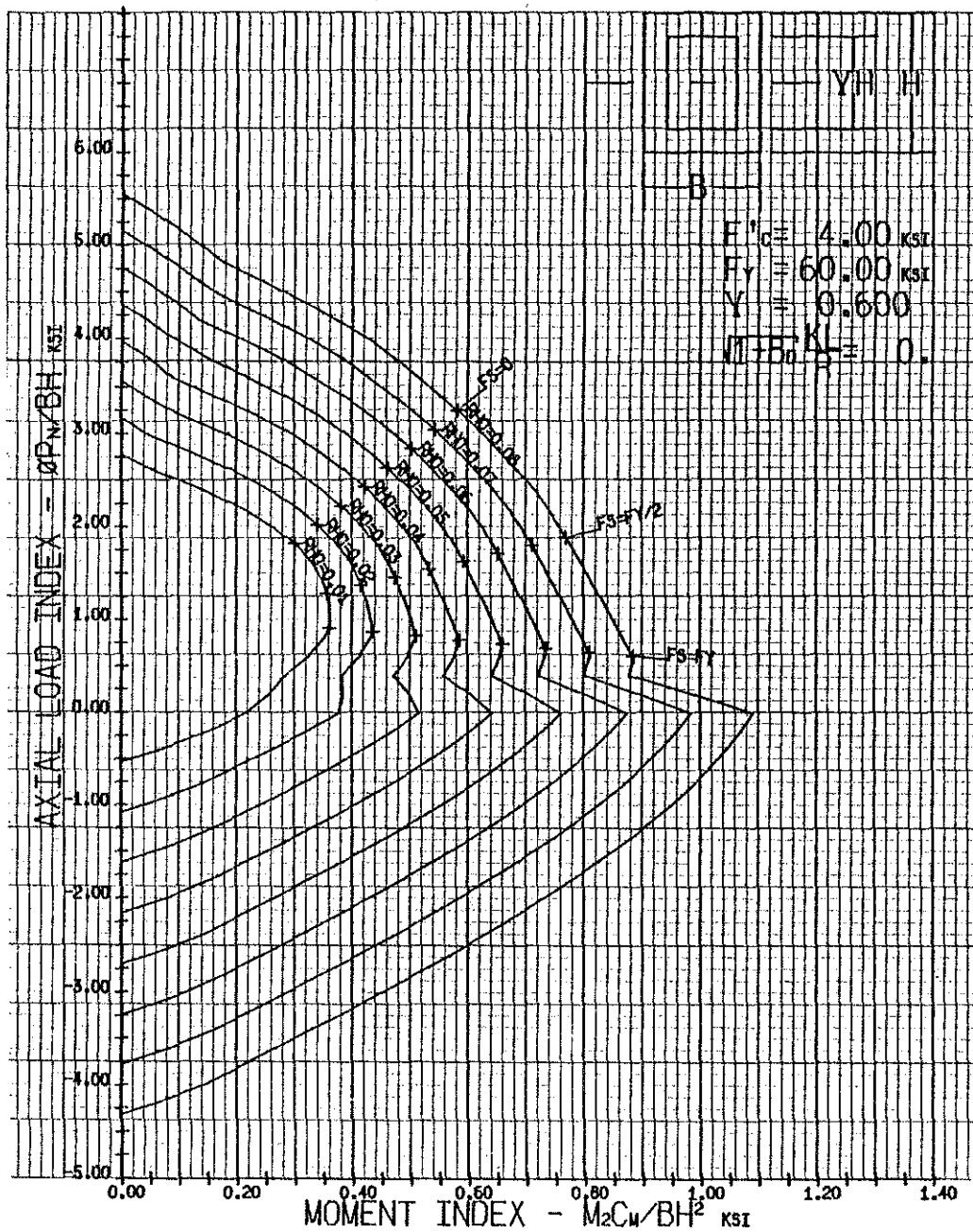


Fig. R4-60.60-0 - Interaction Diagram

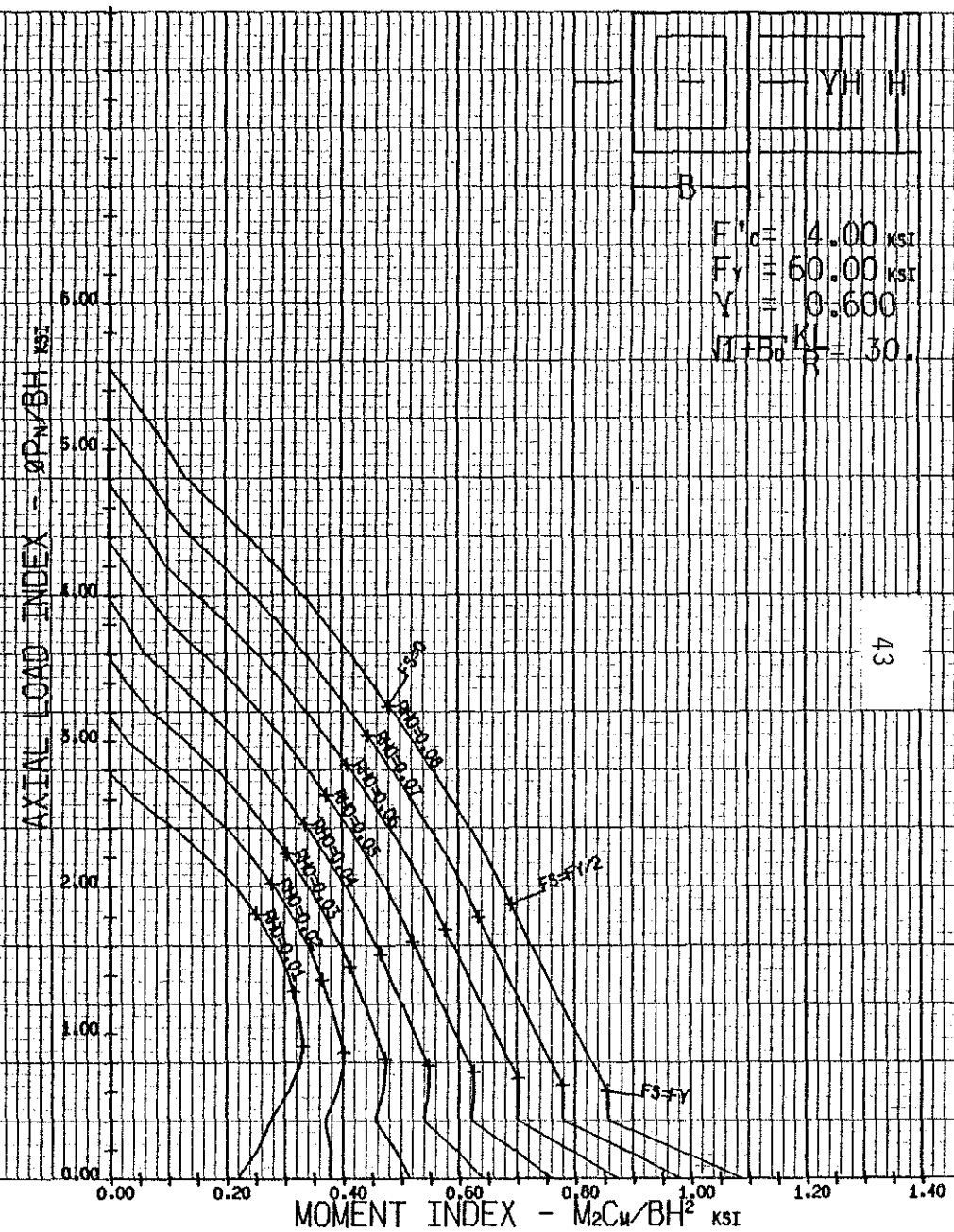


Fig. R4-60.60-30 - Interaction Diagram

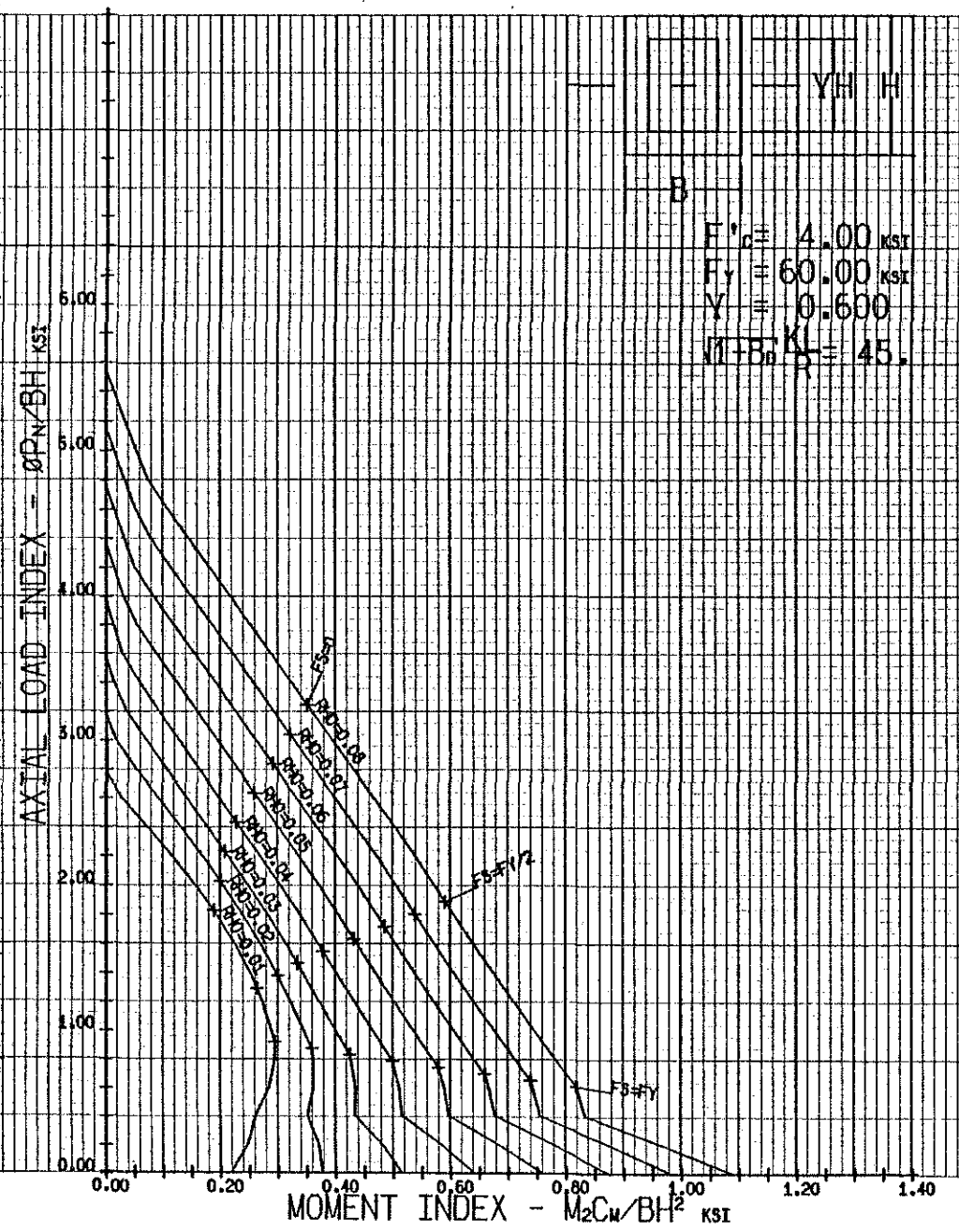


Fig. R4-60.60-45 - Interaction Diagram

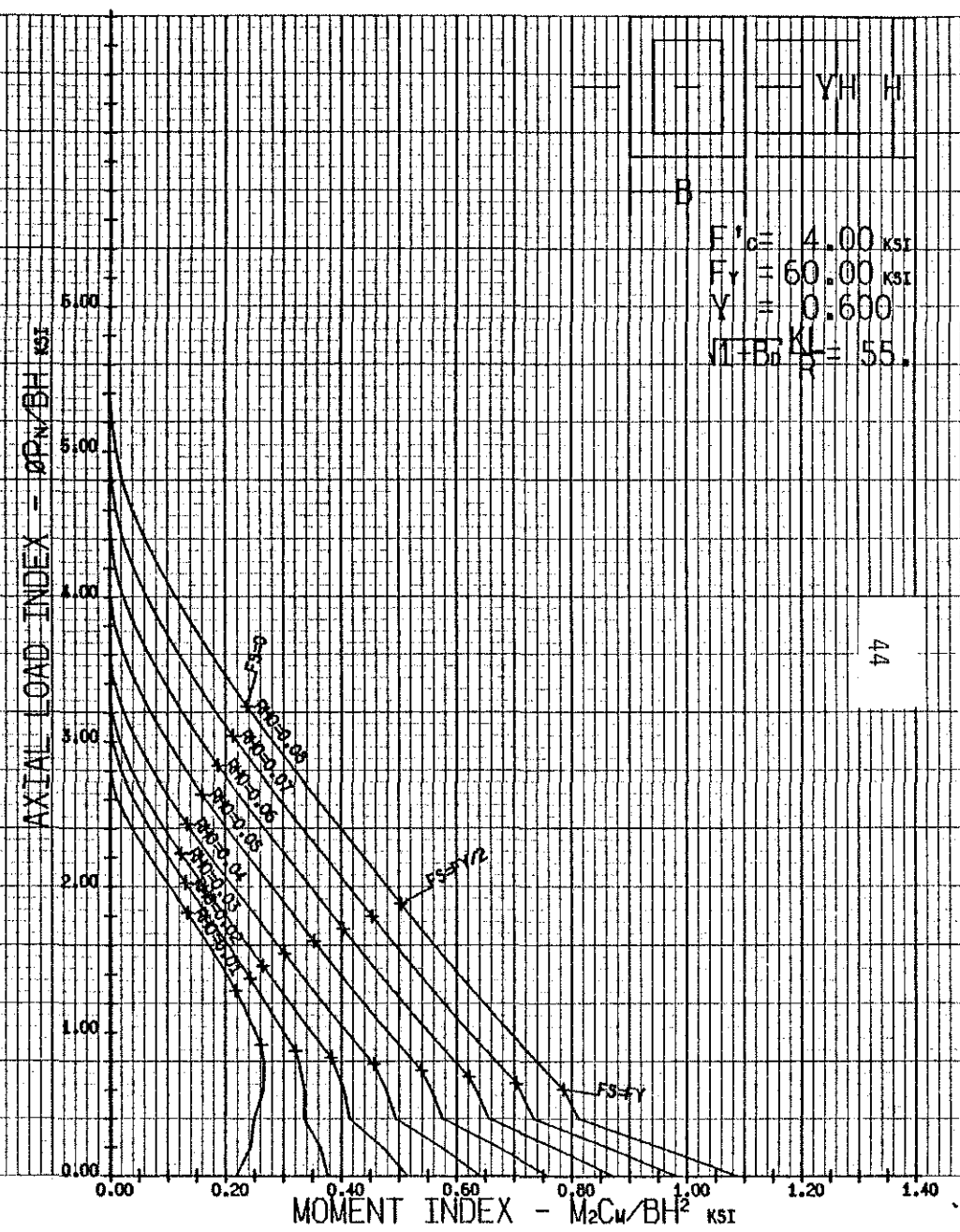


Fig. R4-60.60-55 - Interaction Diagram

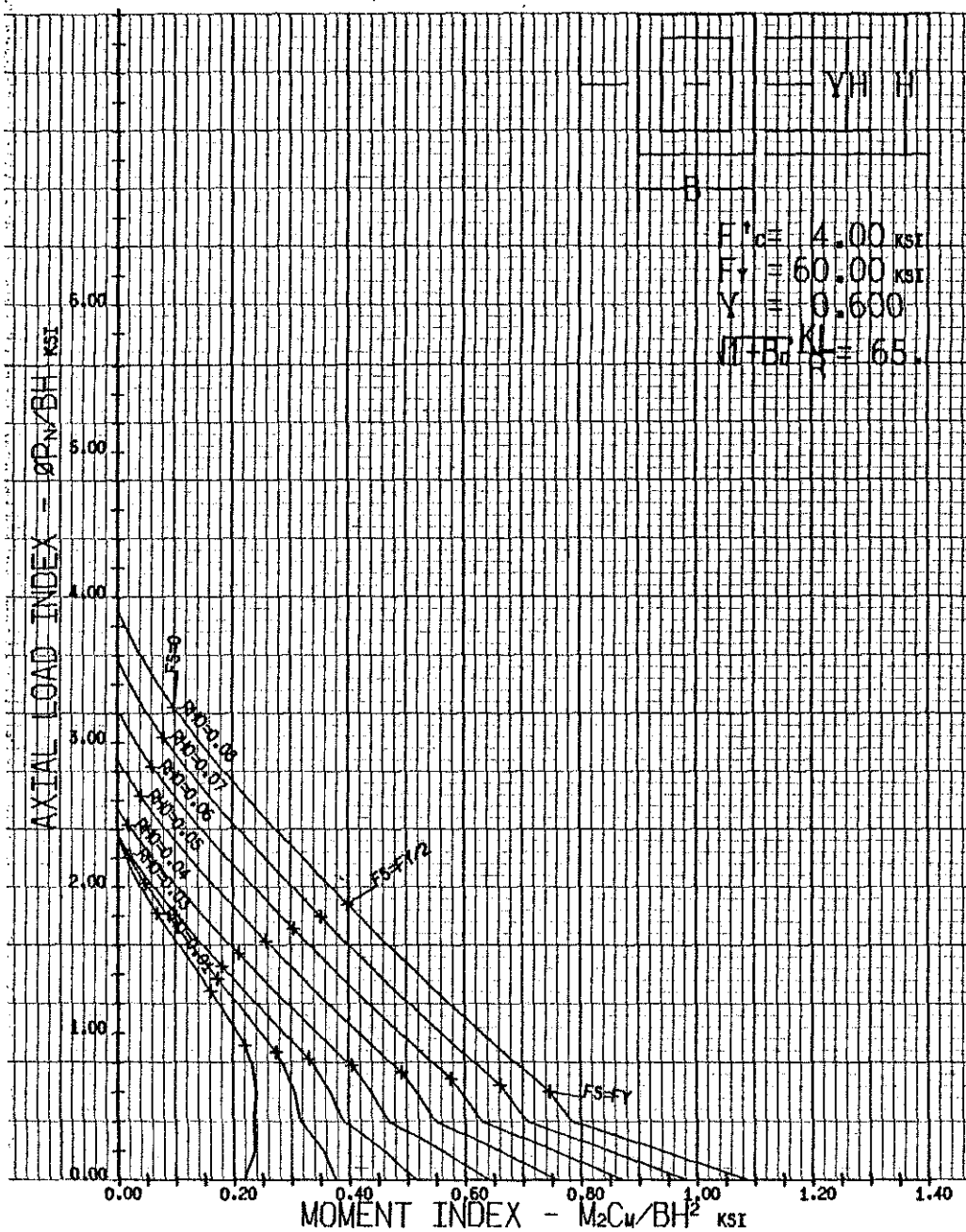


Fig. R4-60.60-65 - Interaction Diagram

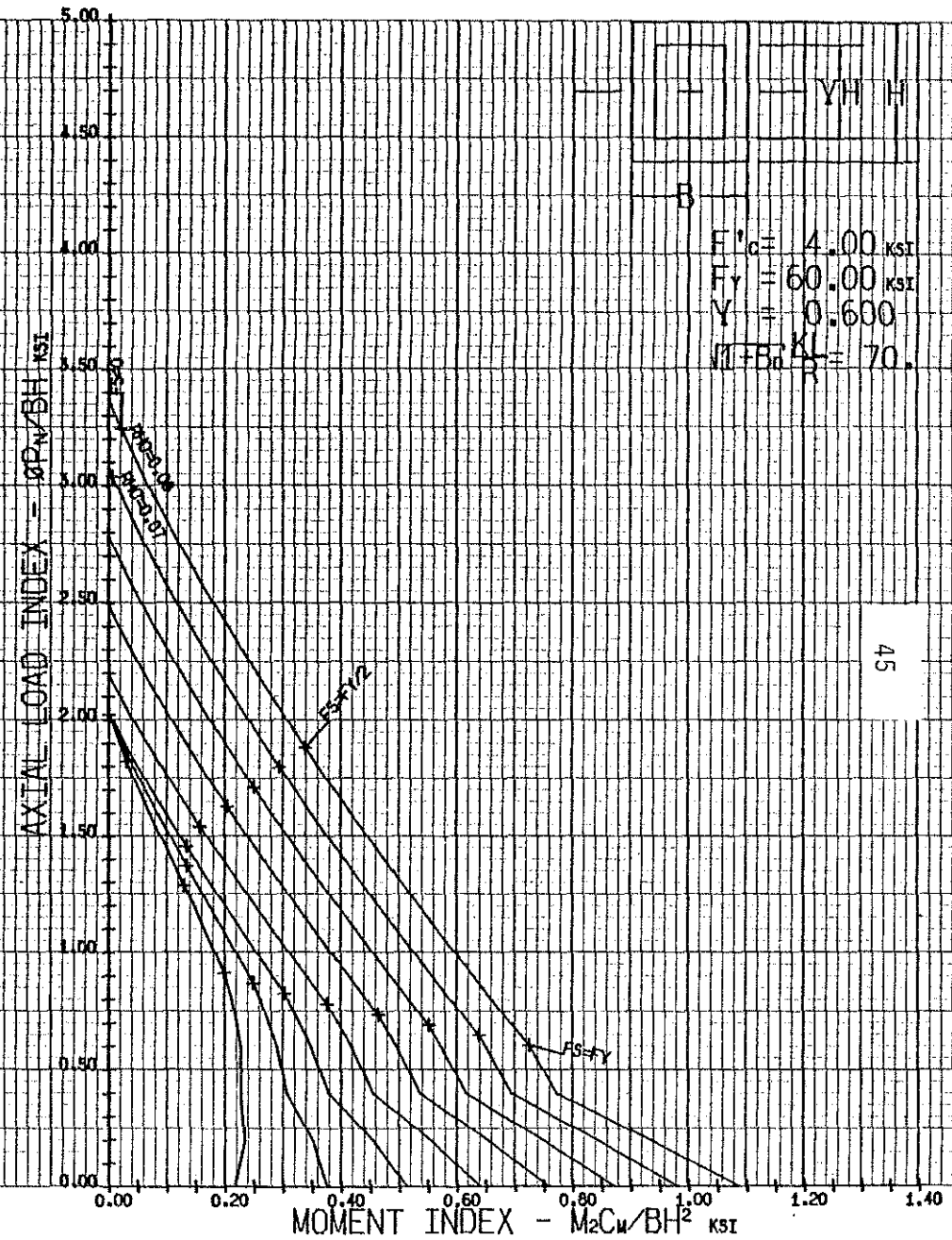


Fig. R4-60.60-70 - Interaction Diagram



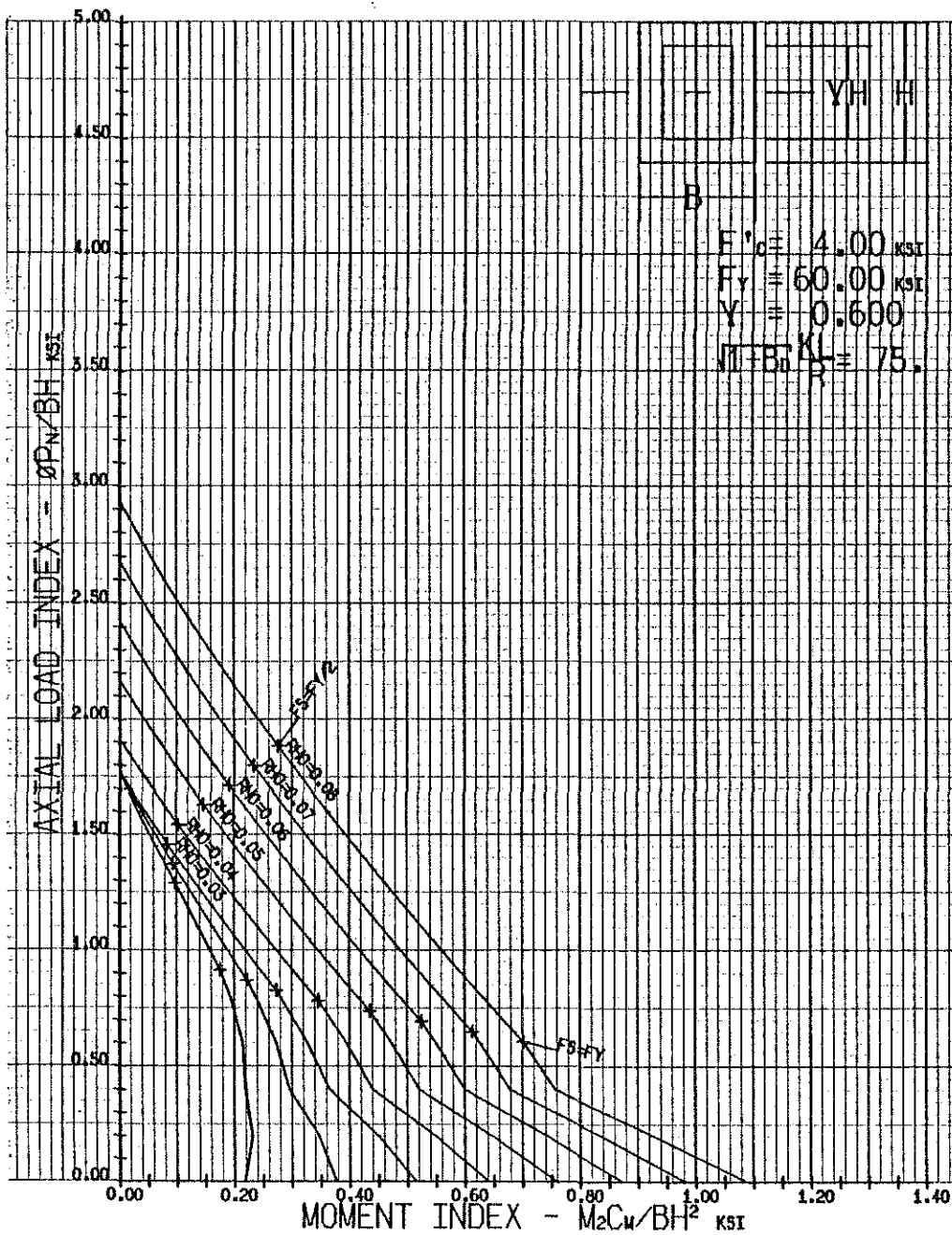


Fig. R4-60.60-75 - Interaction Diagram

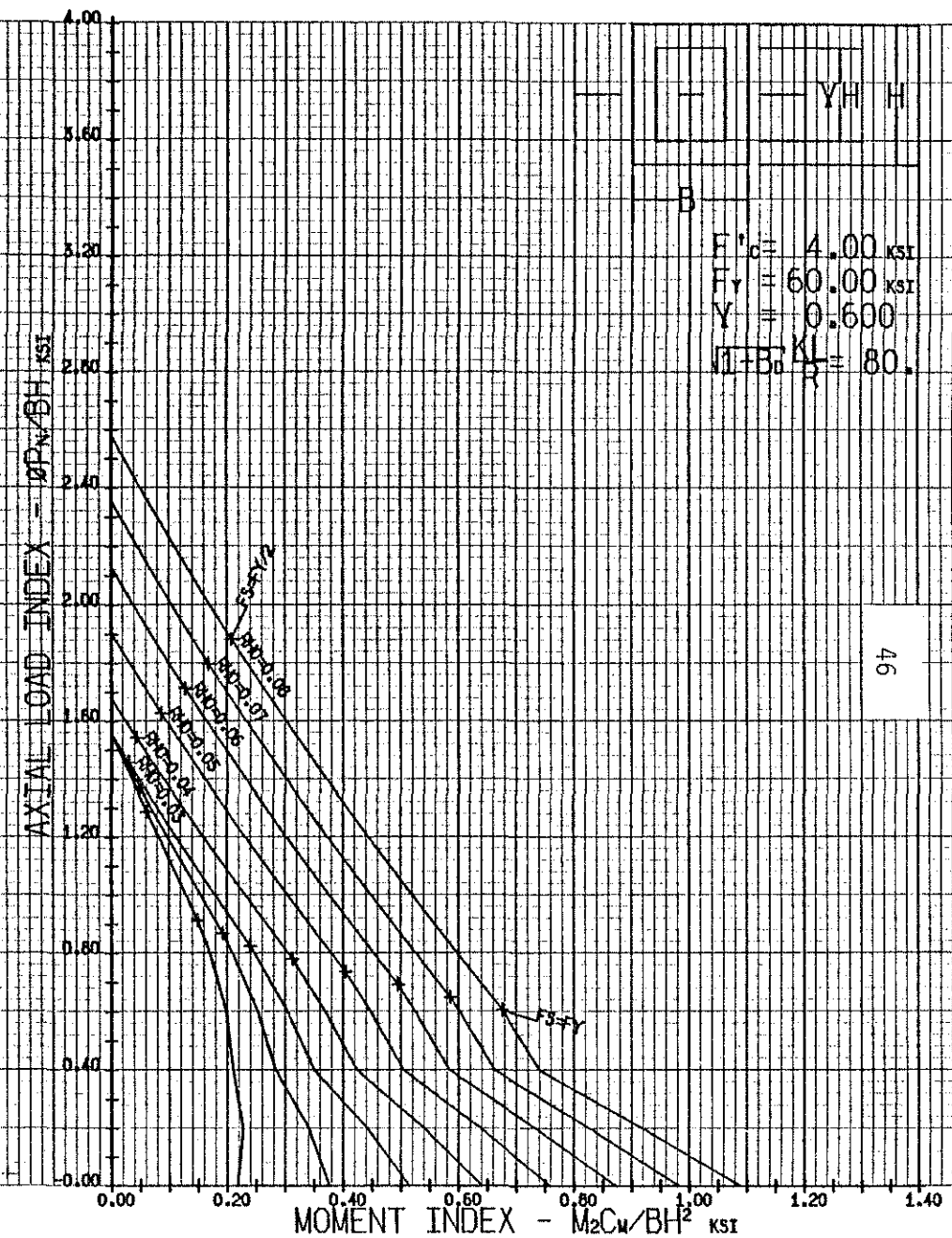


Fig. R4-60.60-80 - Interaction Diagram

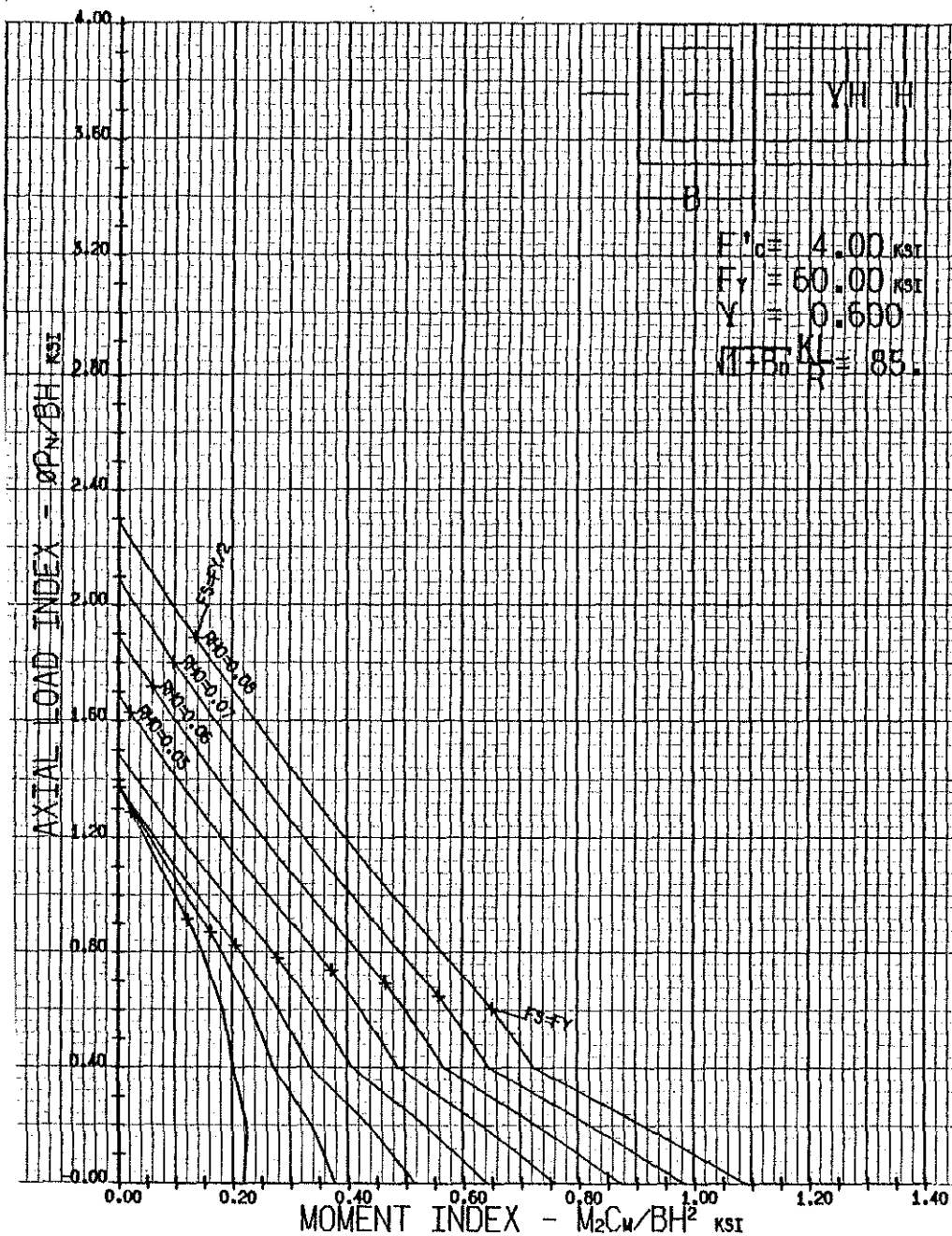


Fig. R4-60.60-85 - Interaction Diagram

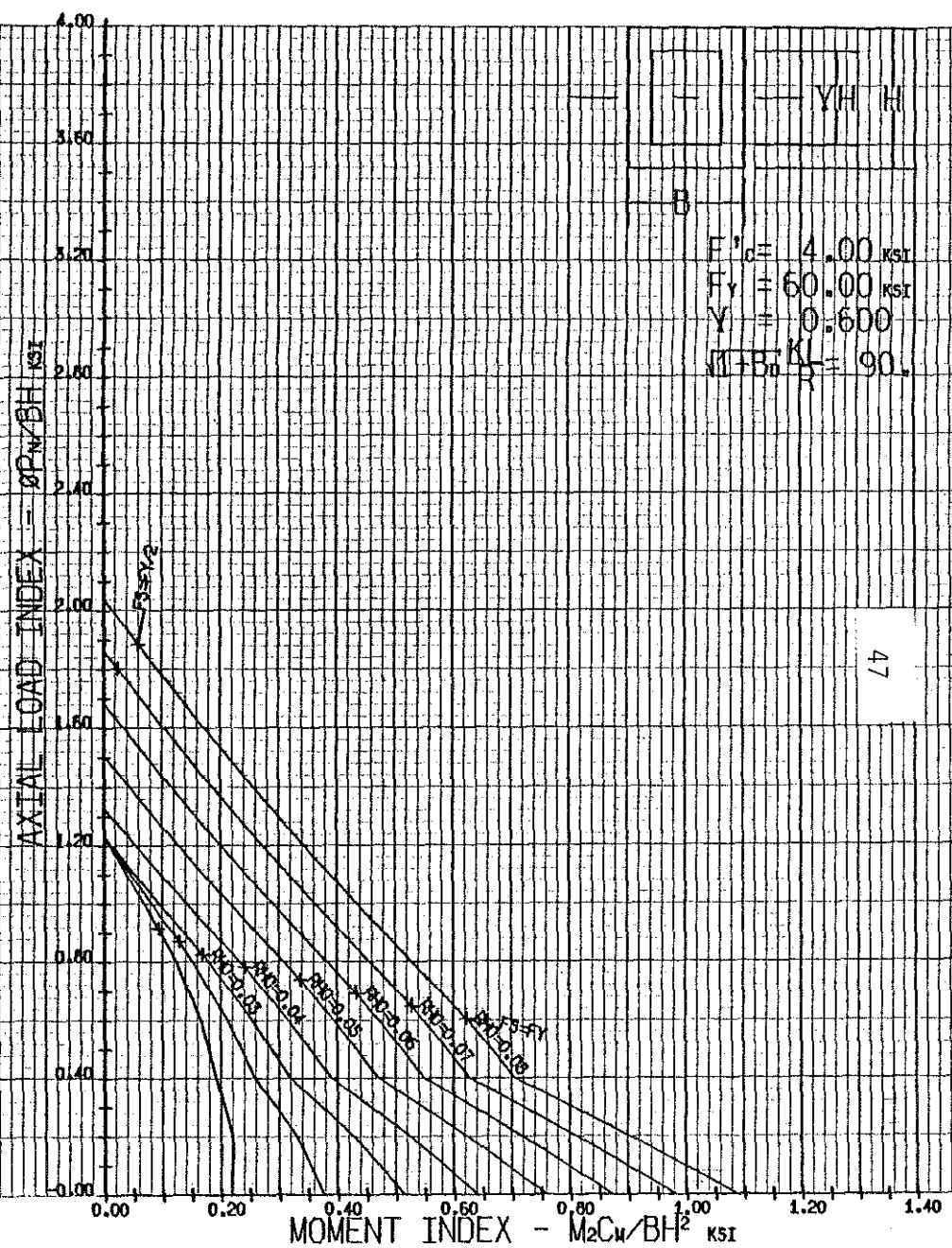


Fig. R4-60.60-90 - Interaction Diagram

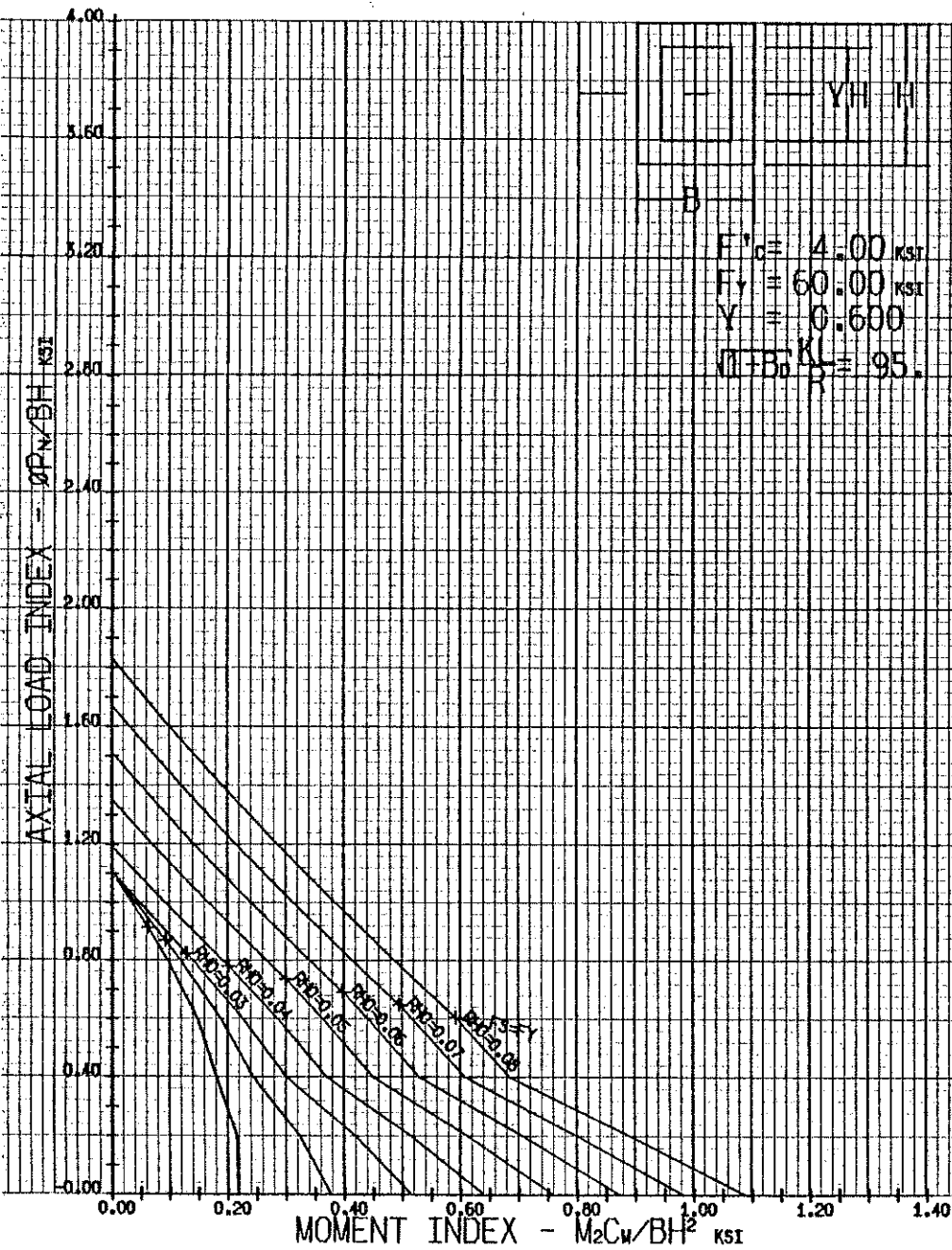


Fig. R4-60.60-95 - Interaction Diagram

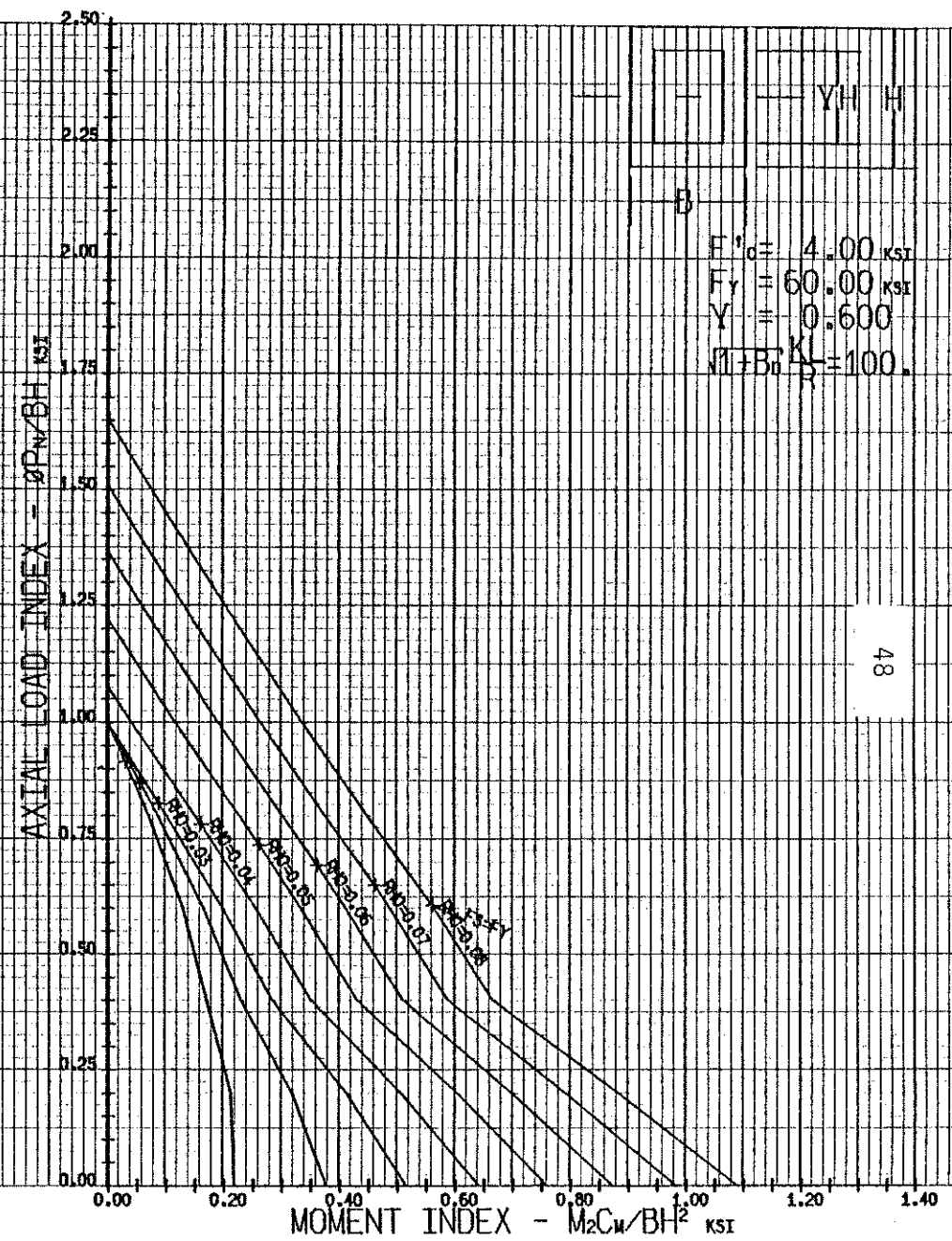


Fig. R4-60.60-100 - Interaction Diagram

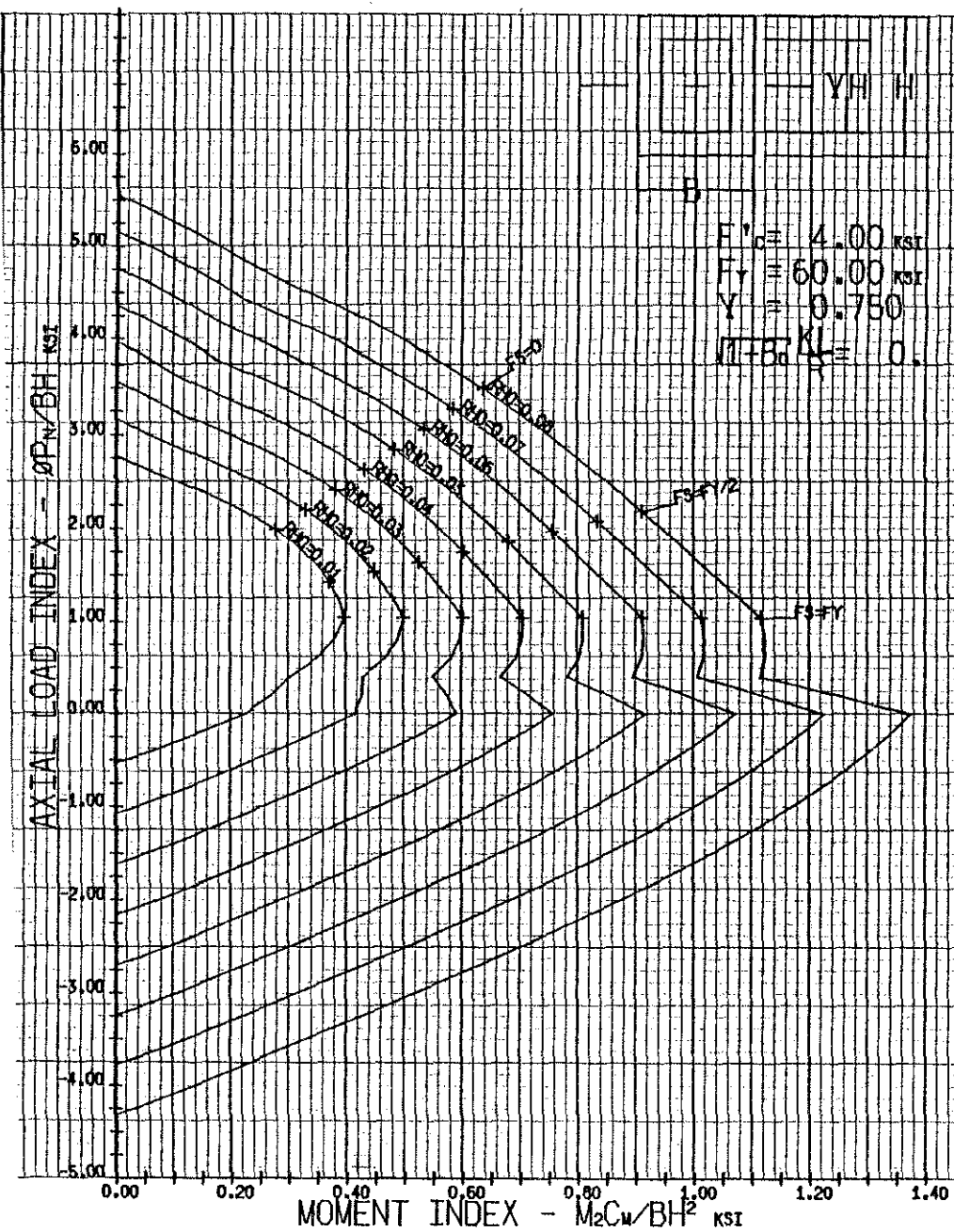


Fig. R4-60.75-0 - Interaction Diagram

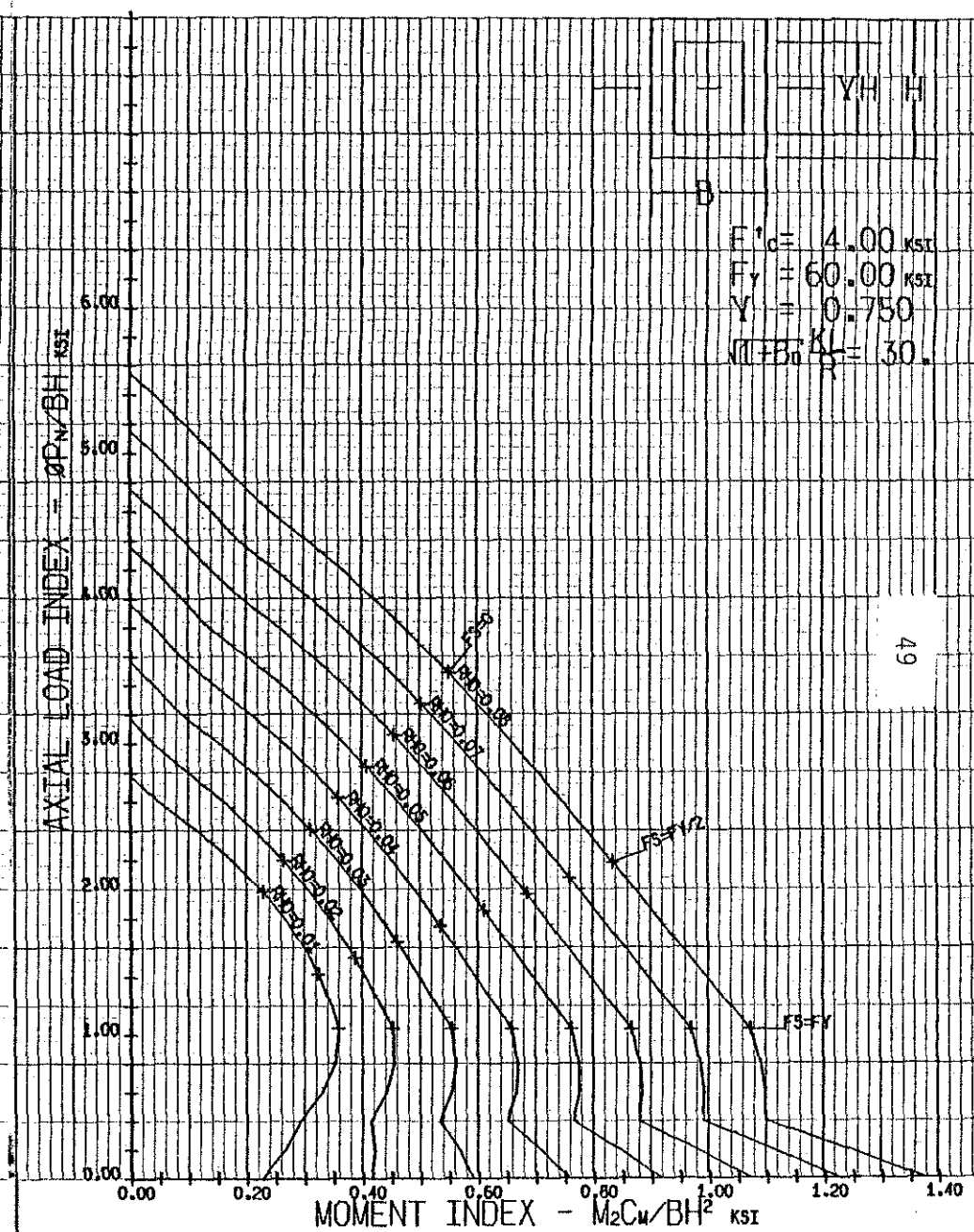


Fig. R4-60.75-30 - Interaction Diagram

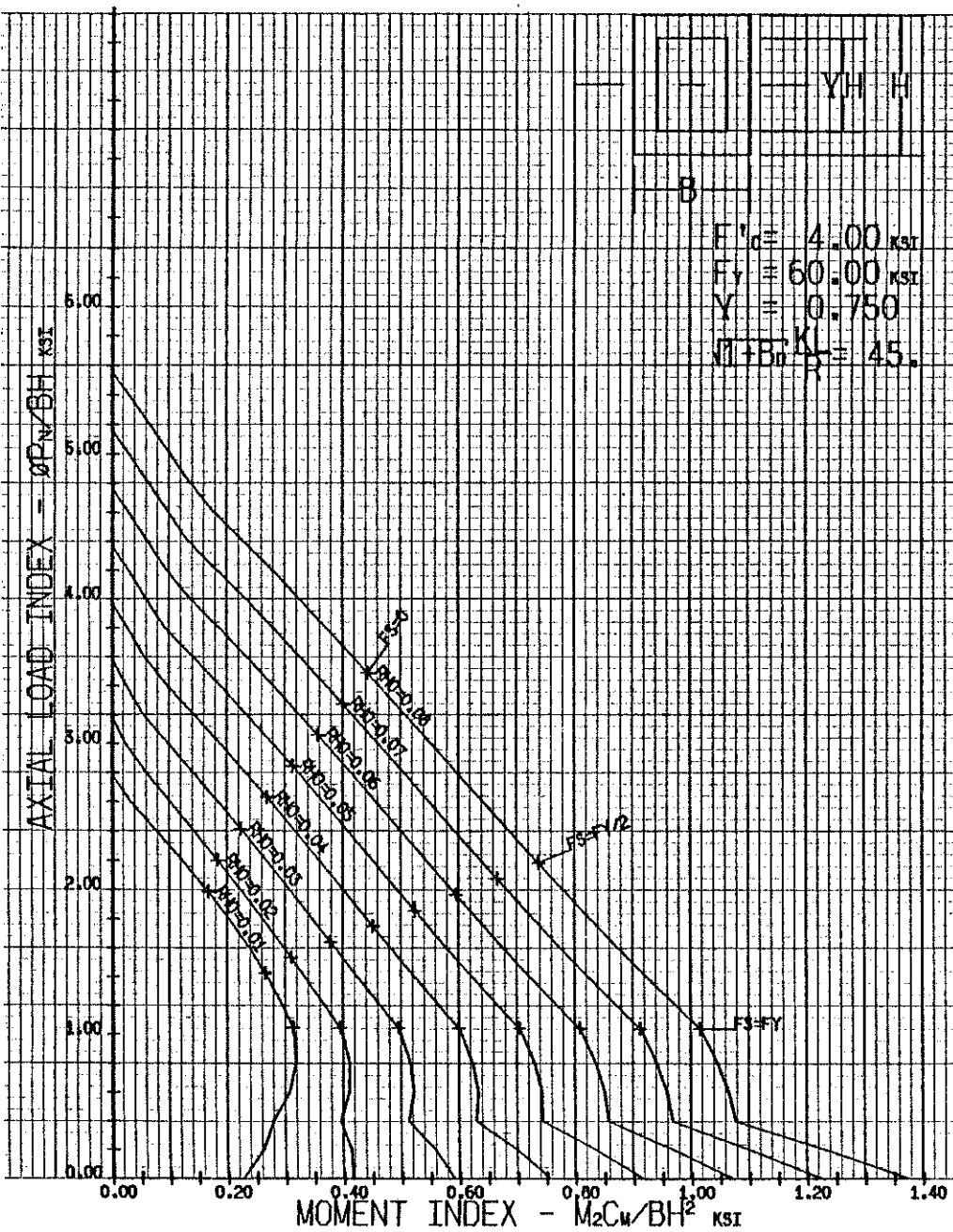


Fig. R4-60.75-45 - Interaction Diagram

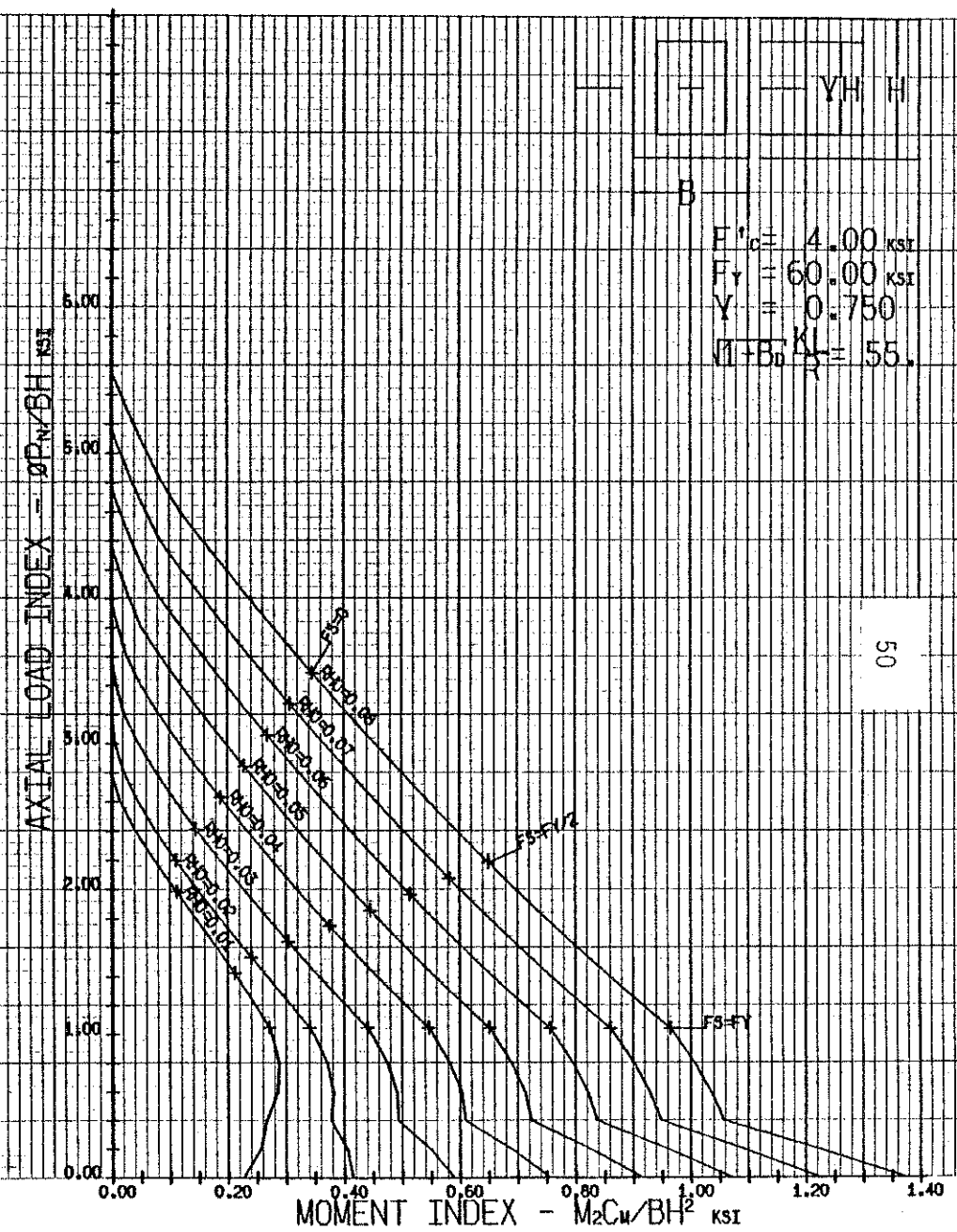


Fig. R4-60.75-55 - Interaction Diagram



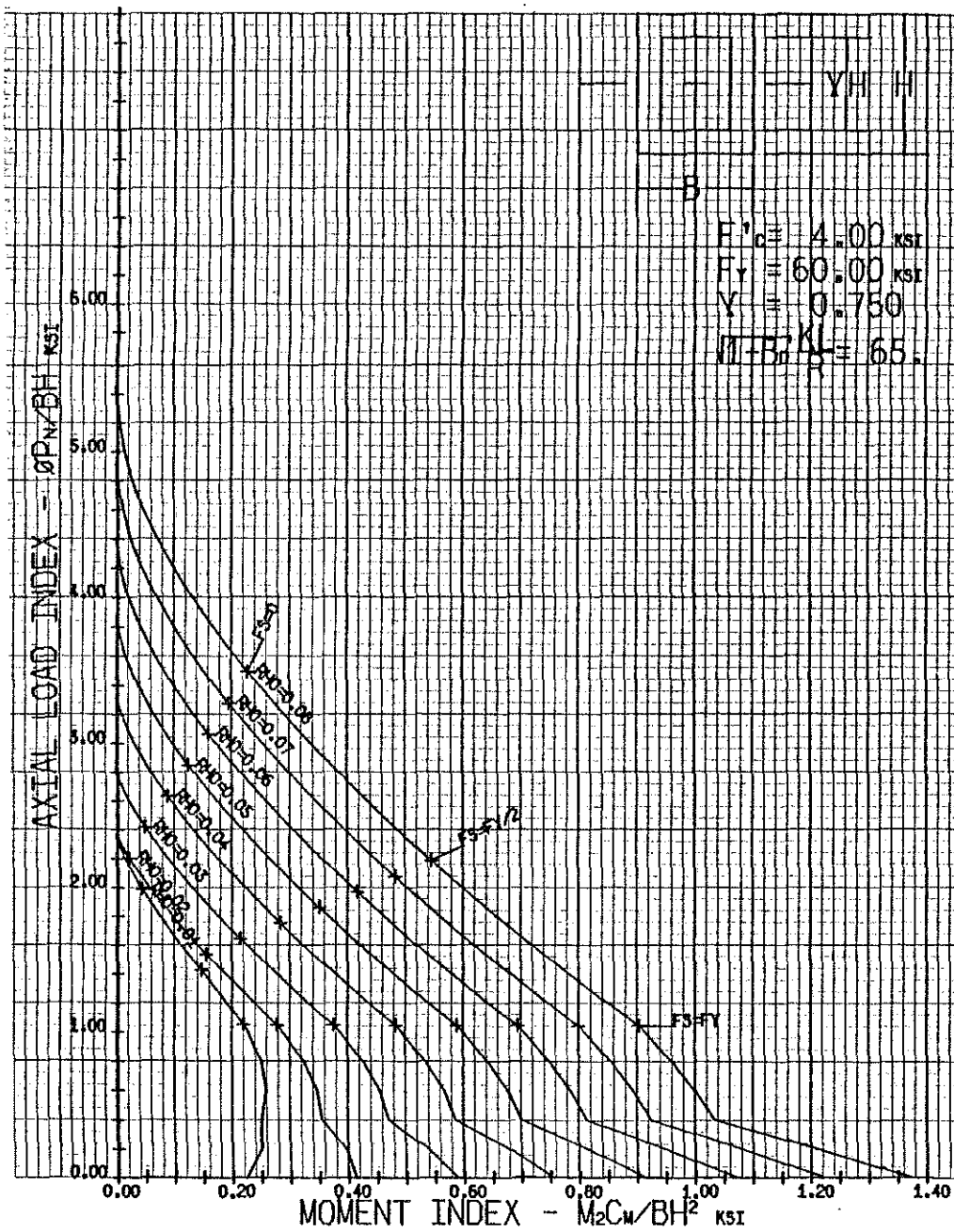


Fig. R4-60.75-65 - Interaction Diagram

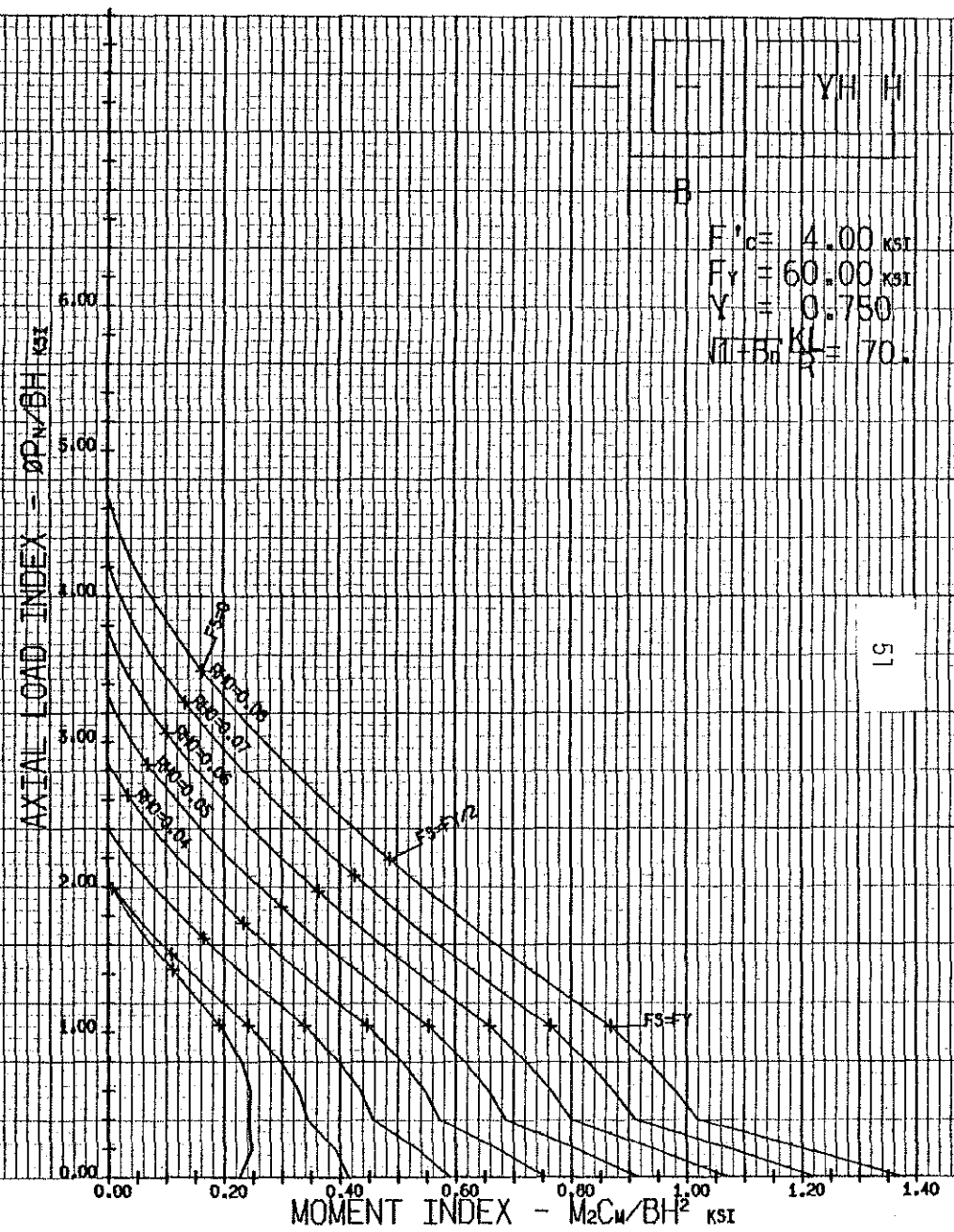


Fig. R4-60.75-70 - Interaction Diagram

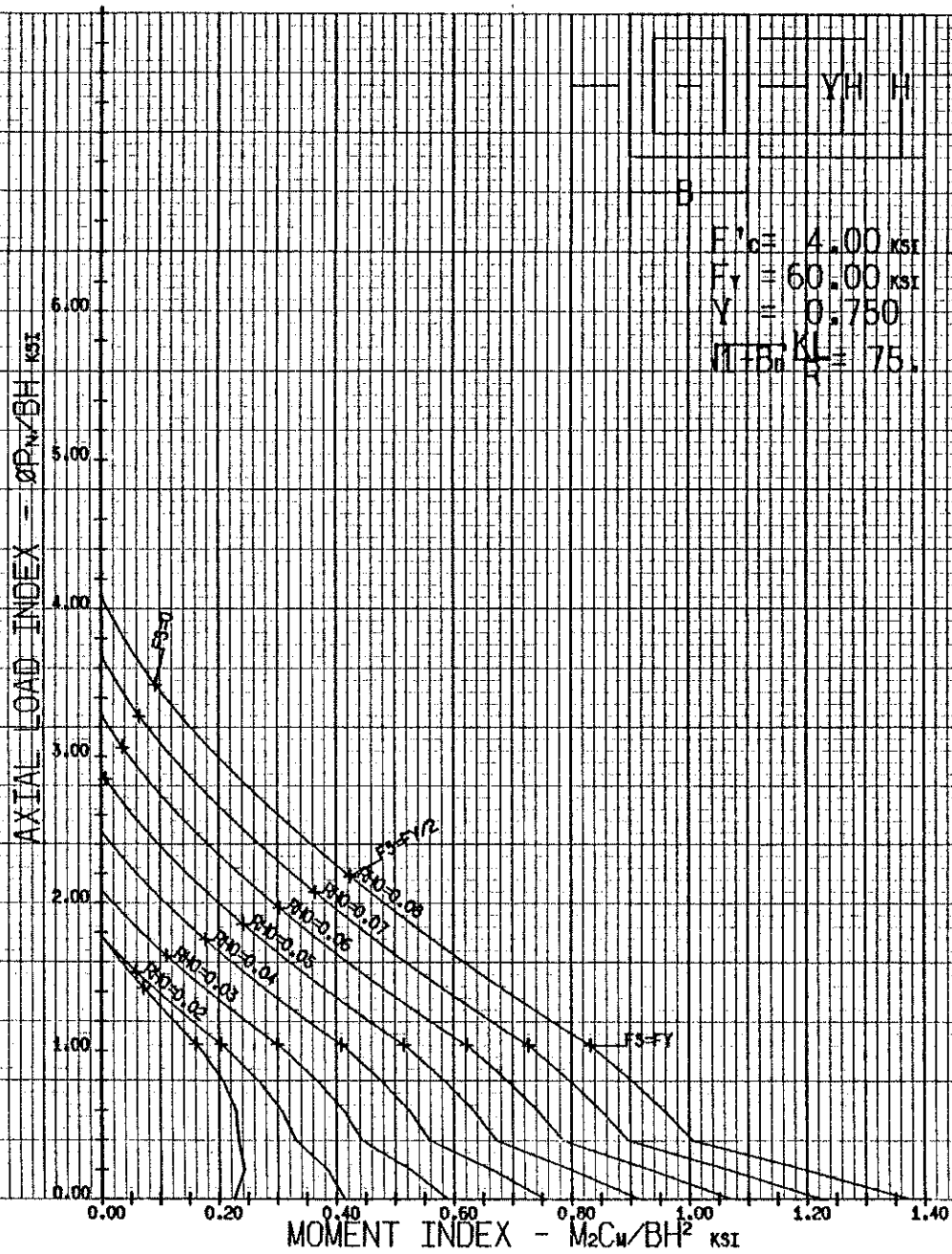


Fig. R4-60.75-75 - Interaction Diagram

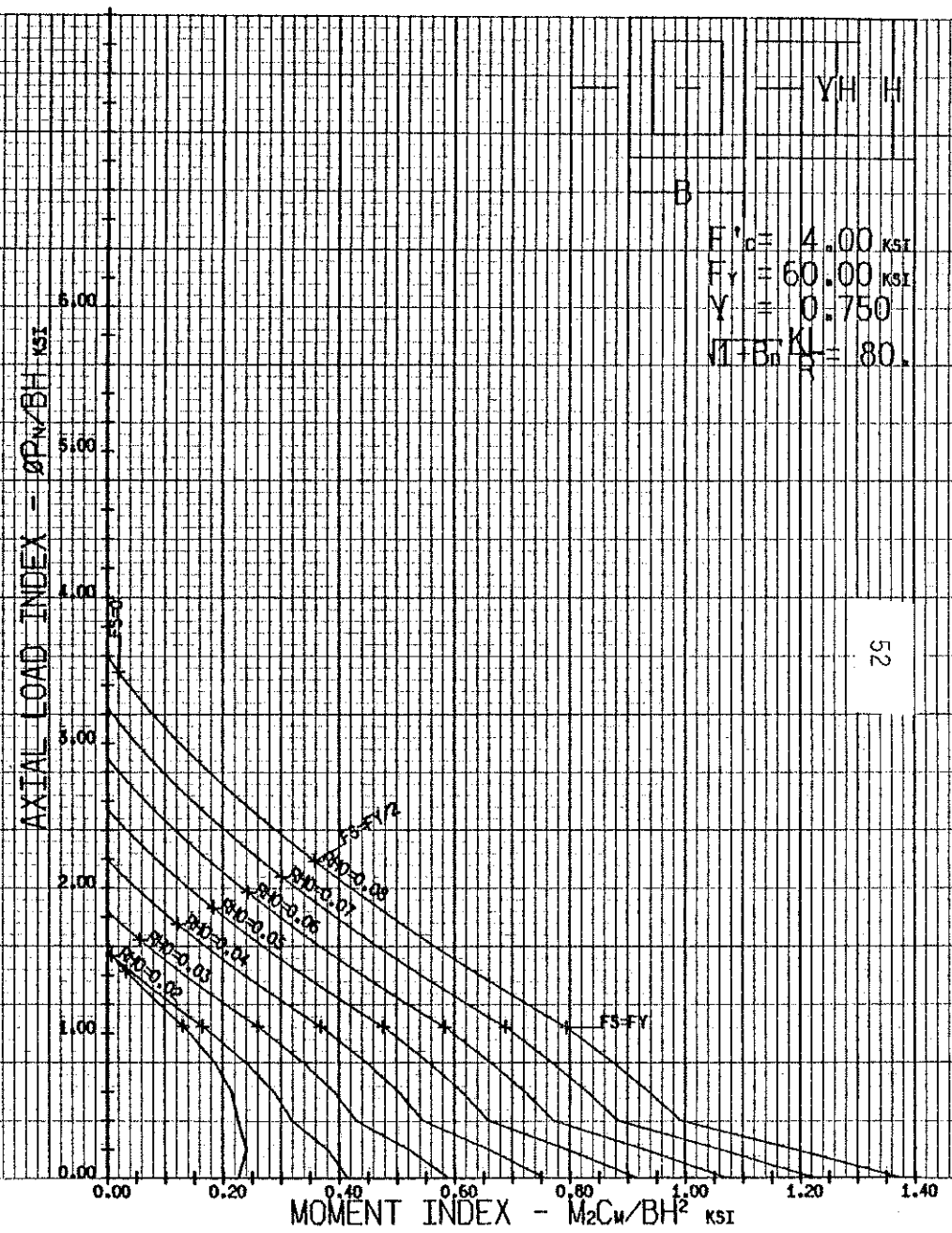


Fig. R4-60.75-80 - Interaction Diagram

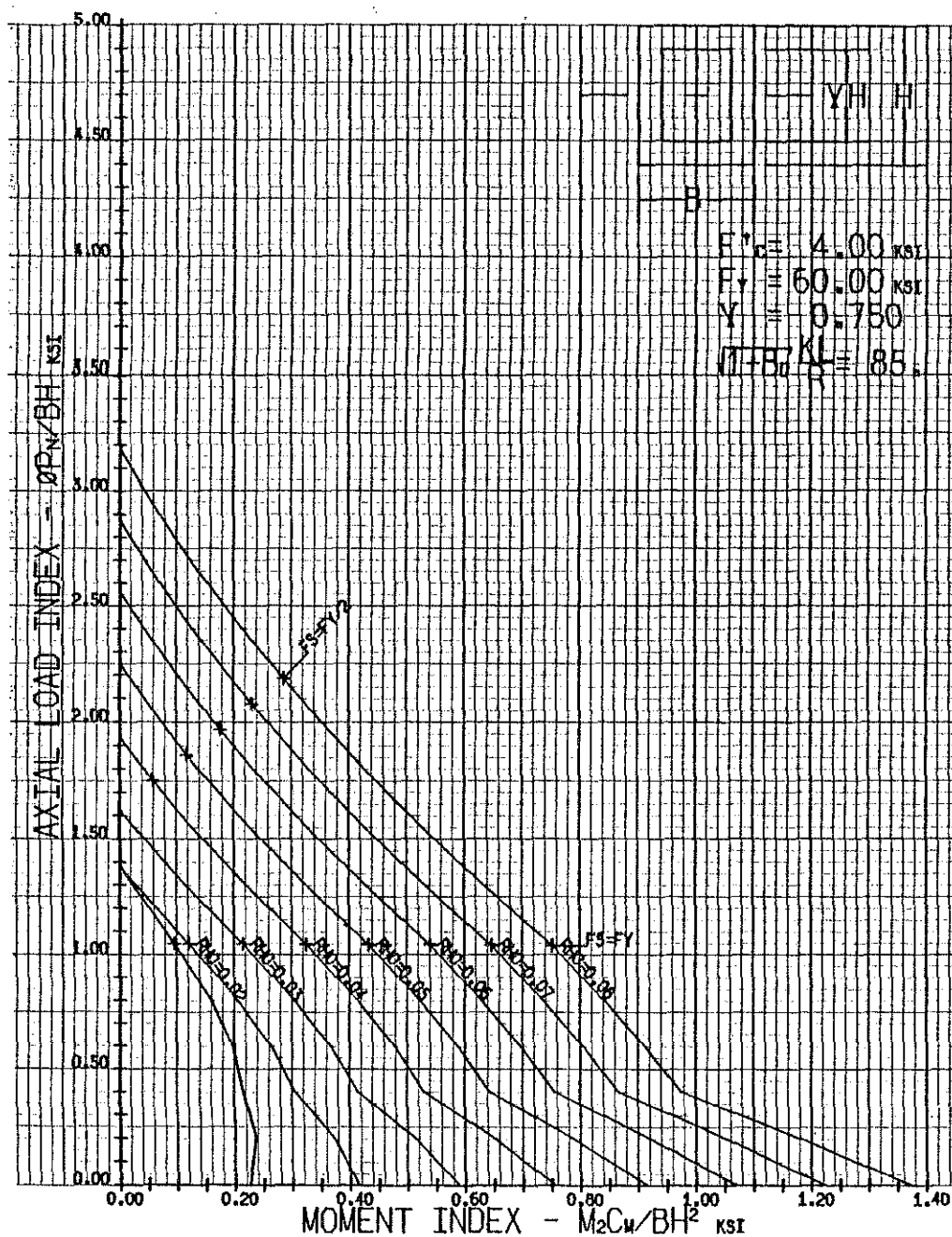


Fig. R4-60.75-85 - Interaction Diagram

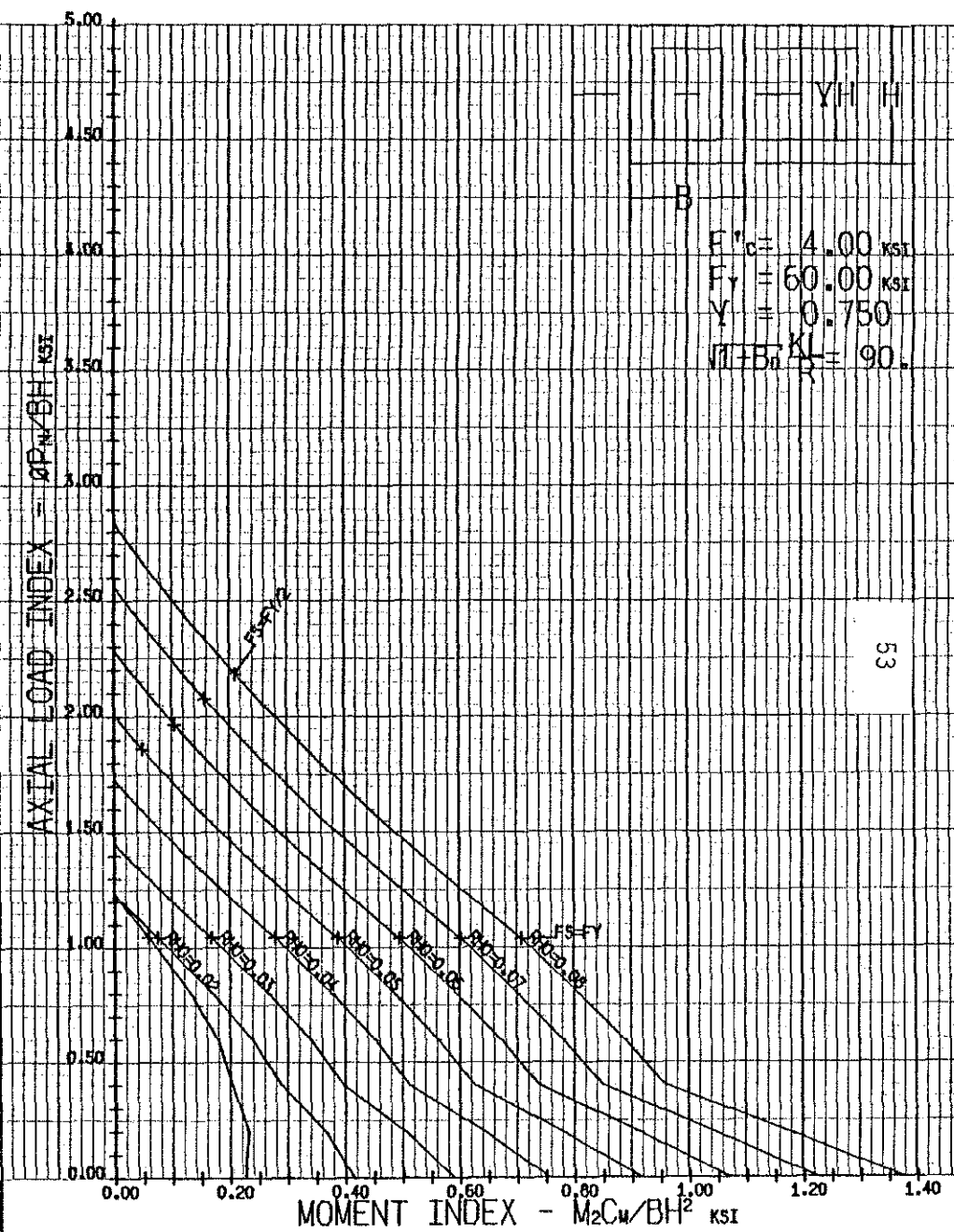


Fig. R4-60.75-90 - Interaction Diagram



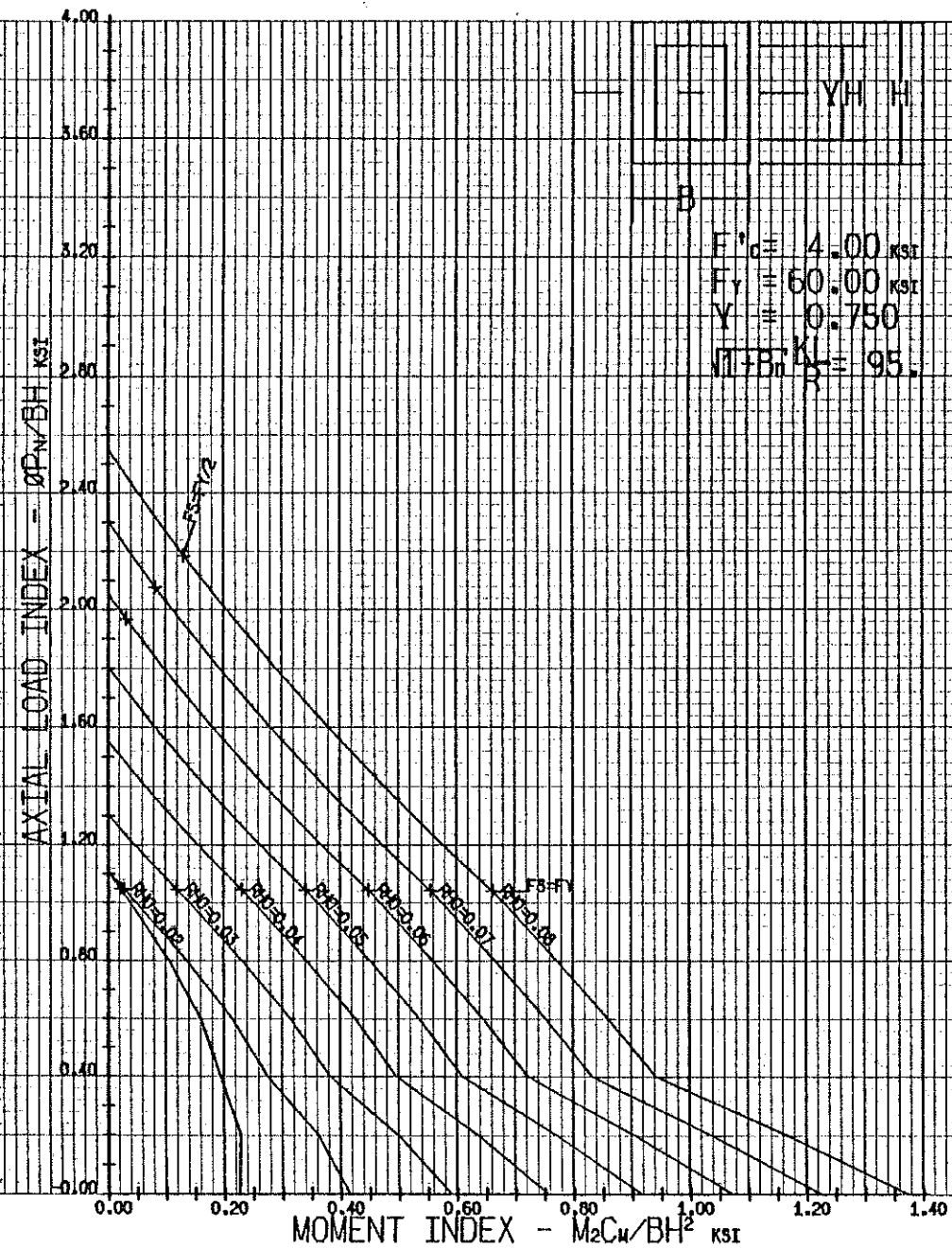


Fig. R4-60.75-95 - Interaction Diagram

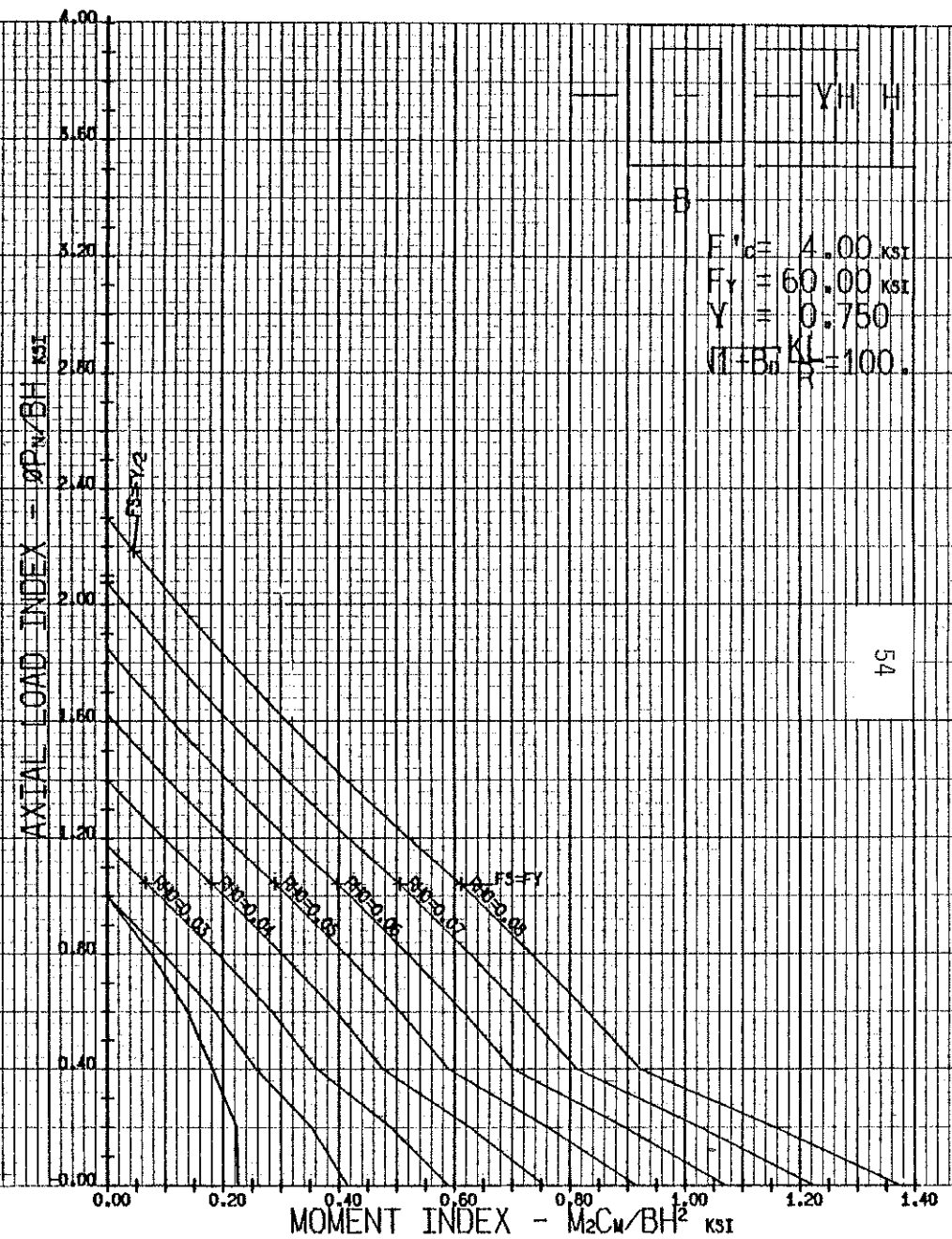


Fig. R4-60.75-100 - Interaction Diagram



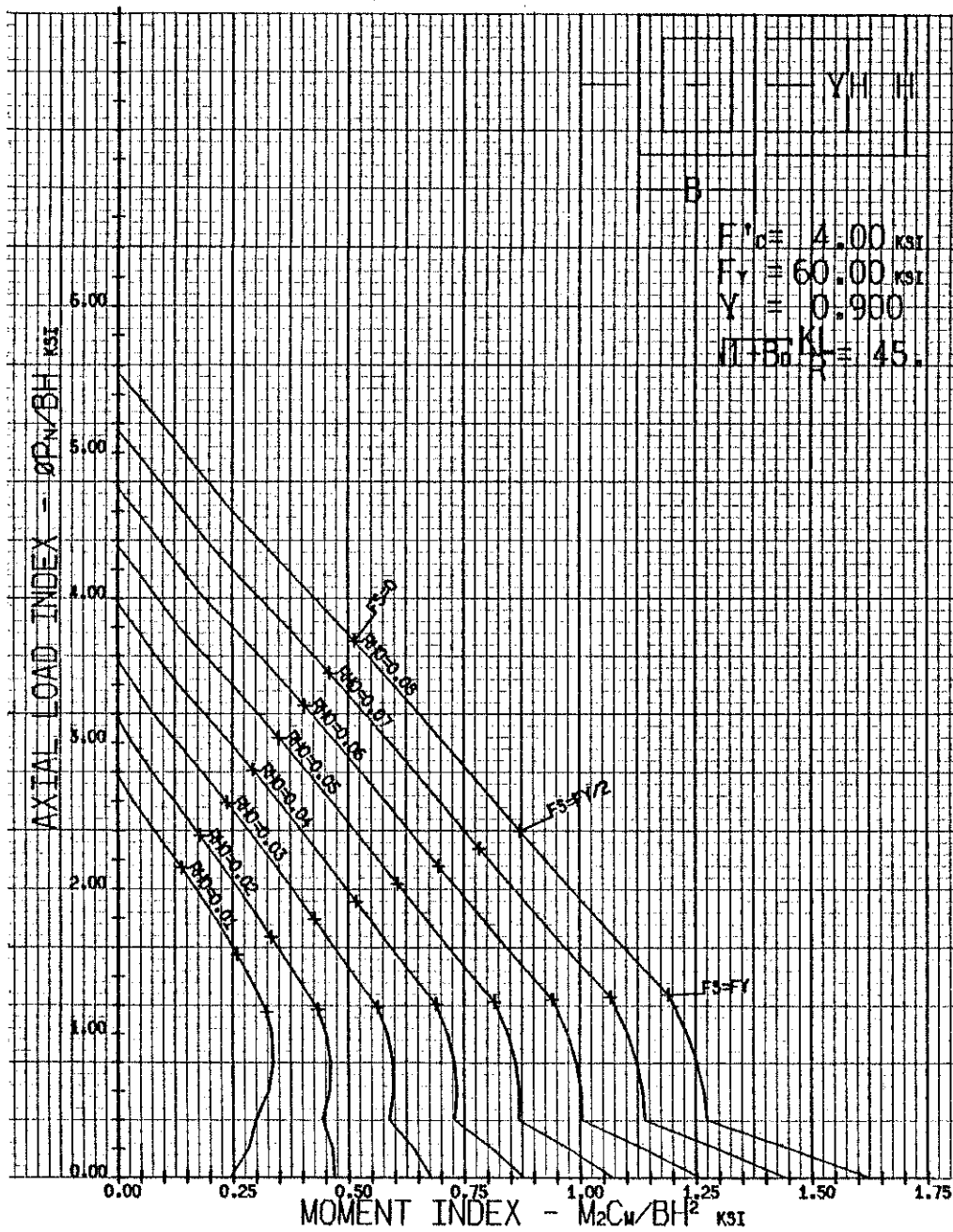


Fig. R4-60.90-45 - Interaction Diagram

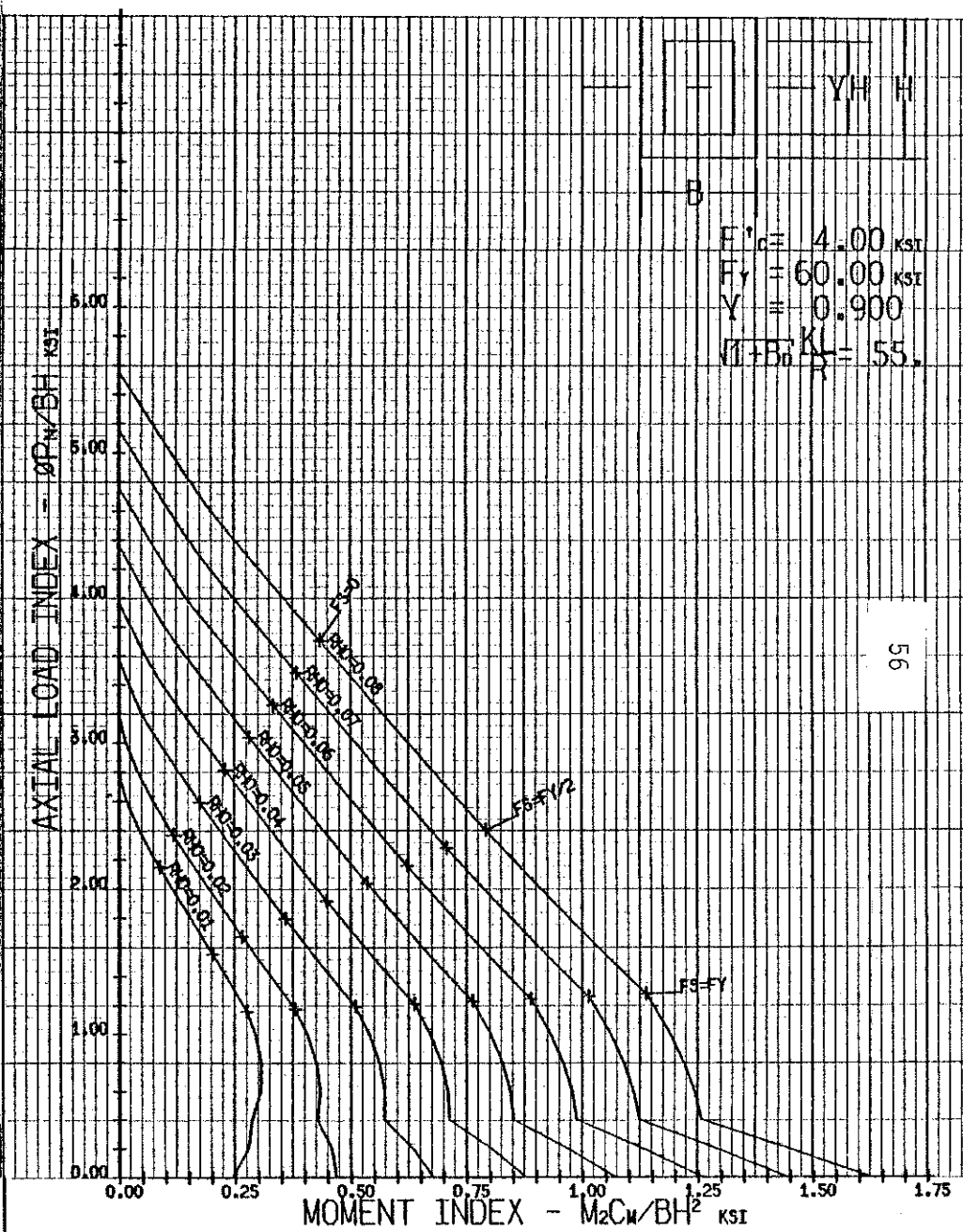


Fig. R4-60.90-55 - Interaction Diagram

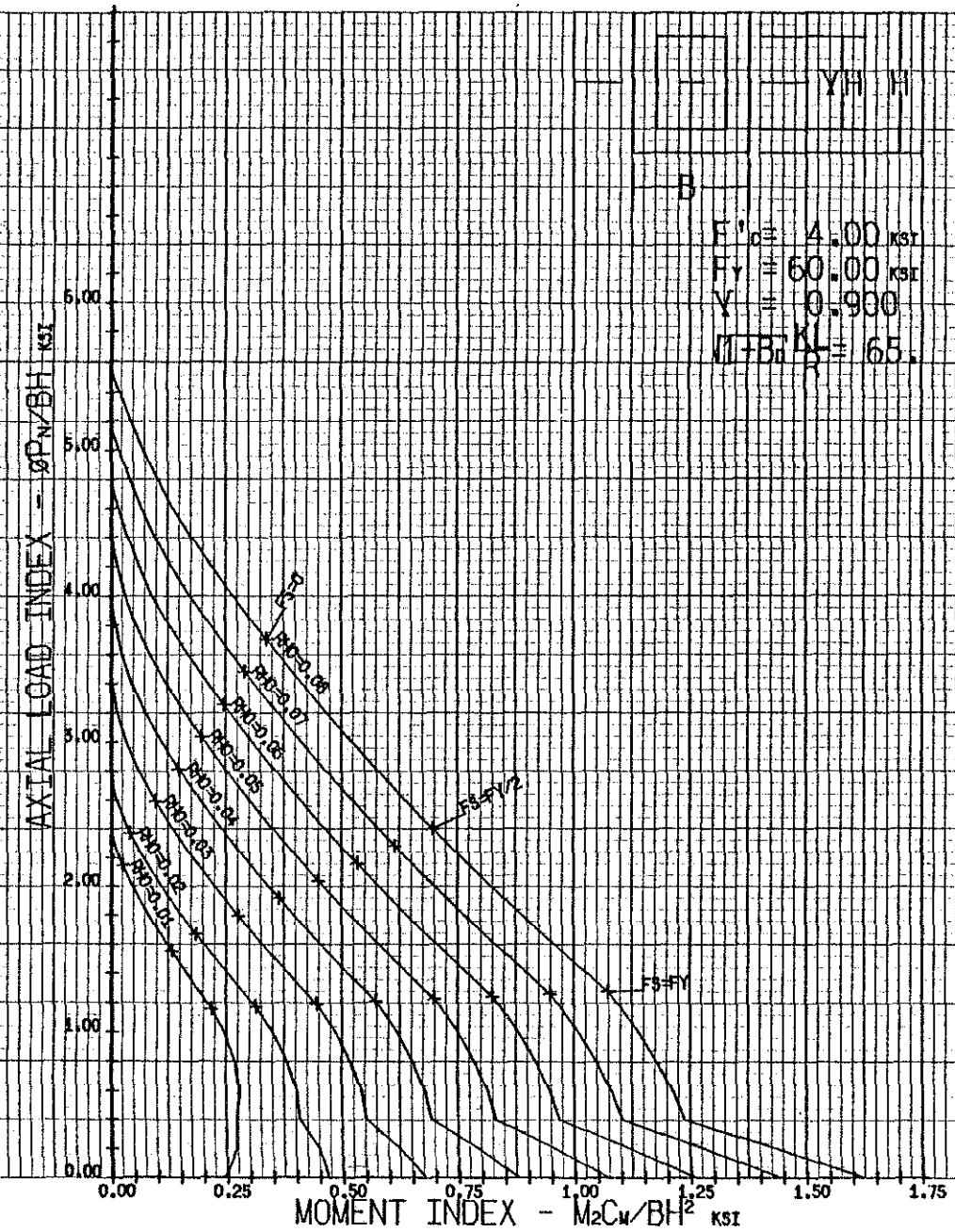


Fig. R4-60.90-65 - Interaction Diagram

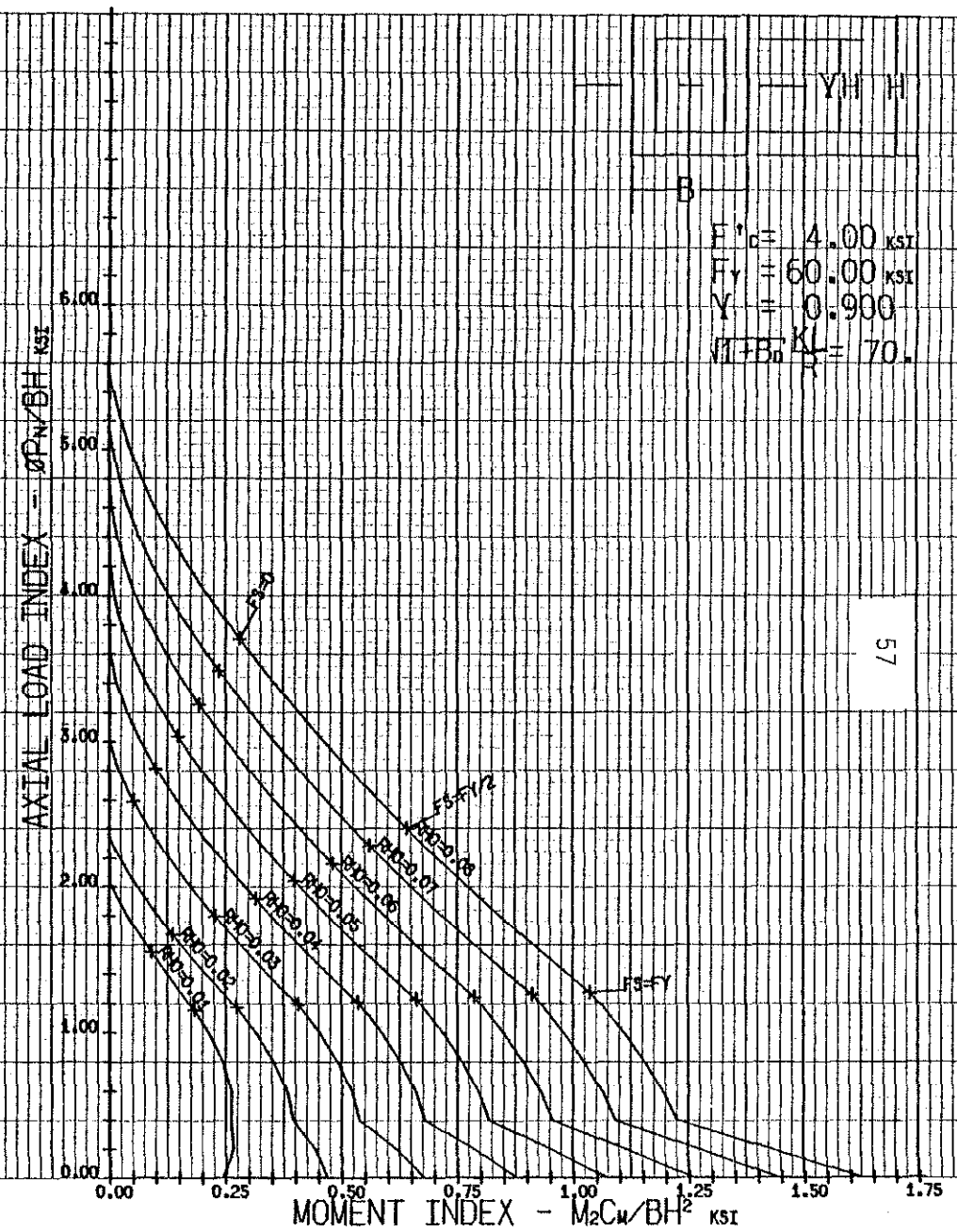


Fig. R4-60.90-70 - Interaction Diagram

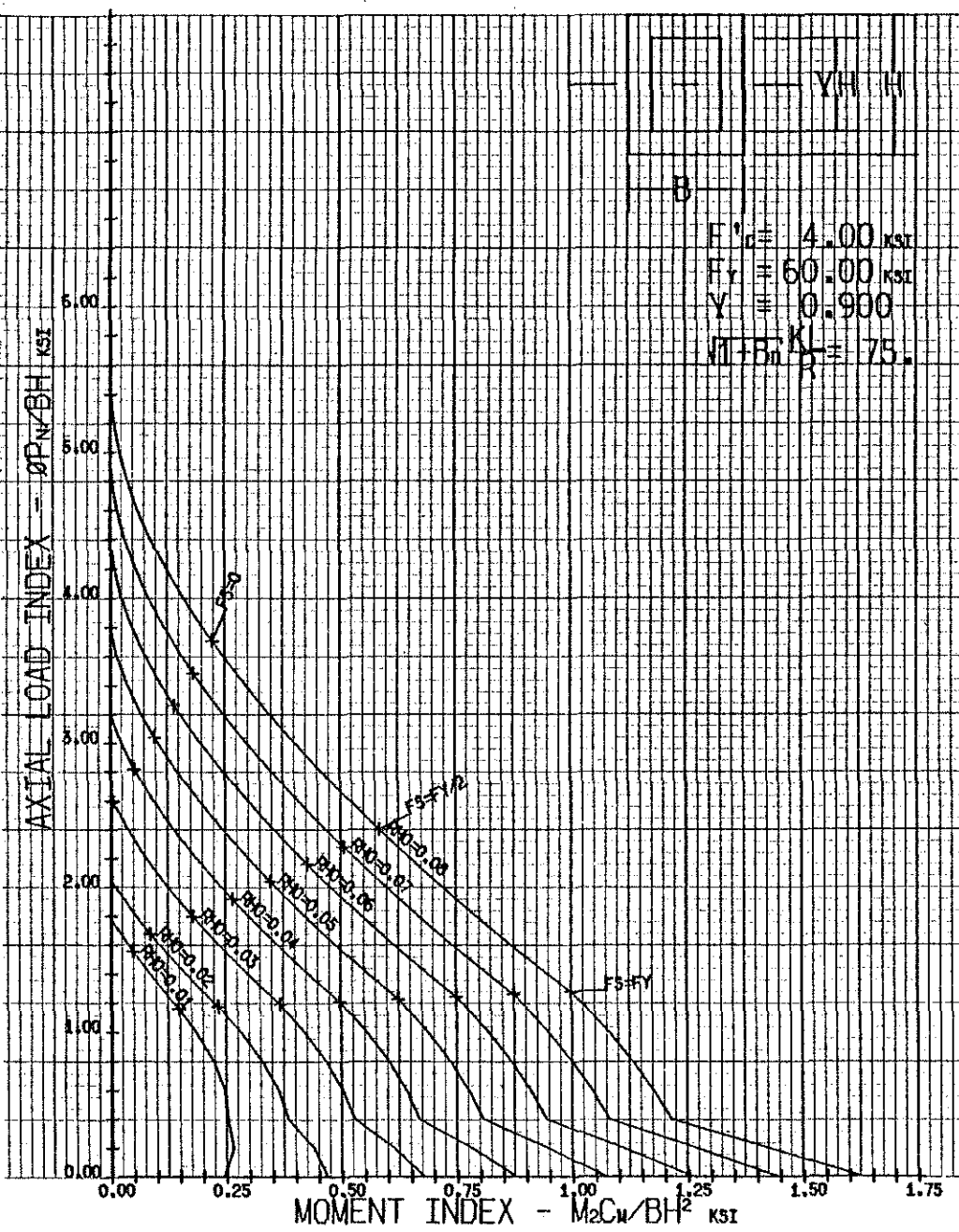


Fig. R4-60.90-75 - Interaction Diagram

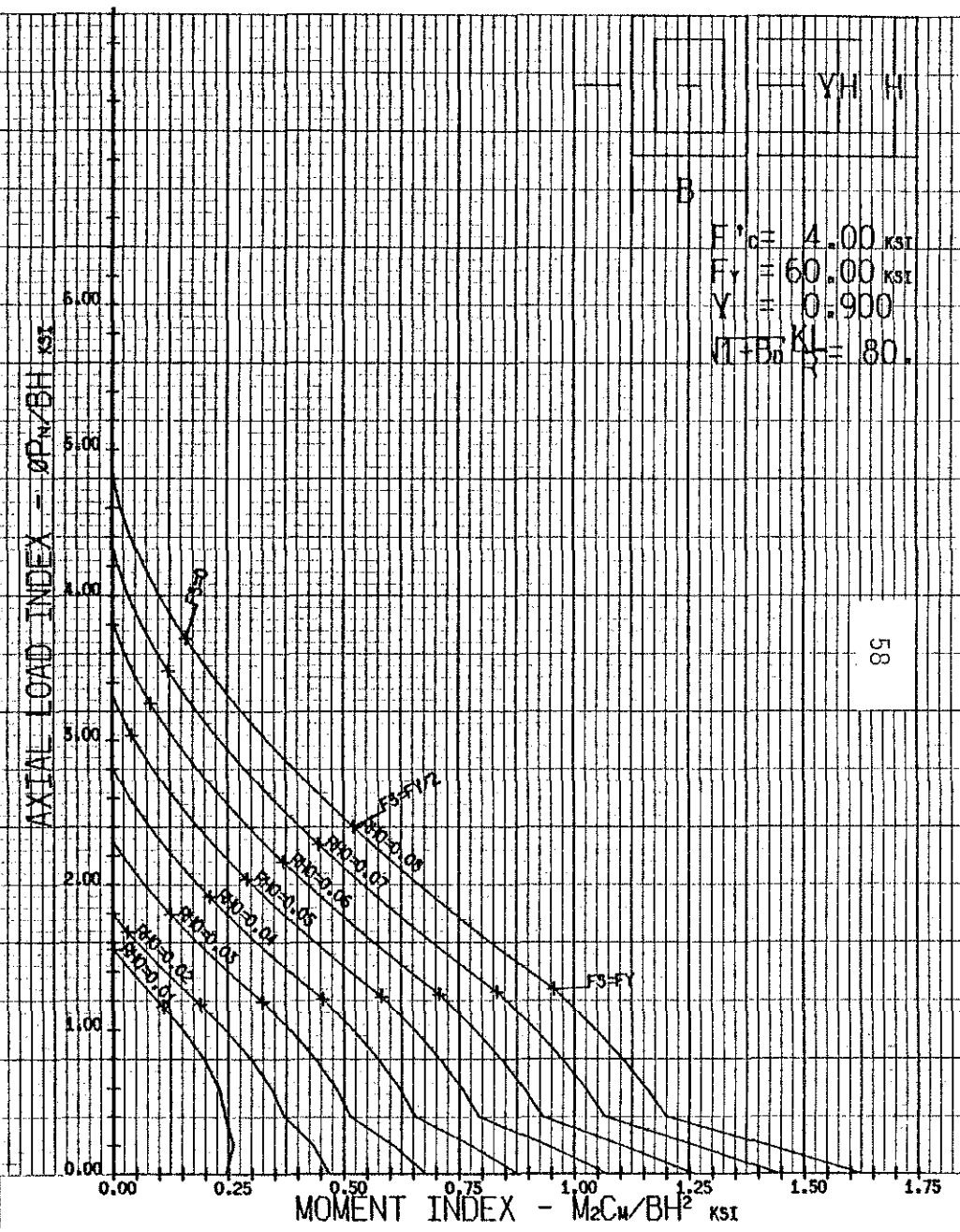
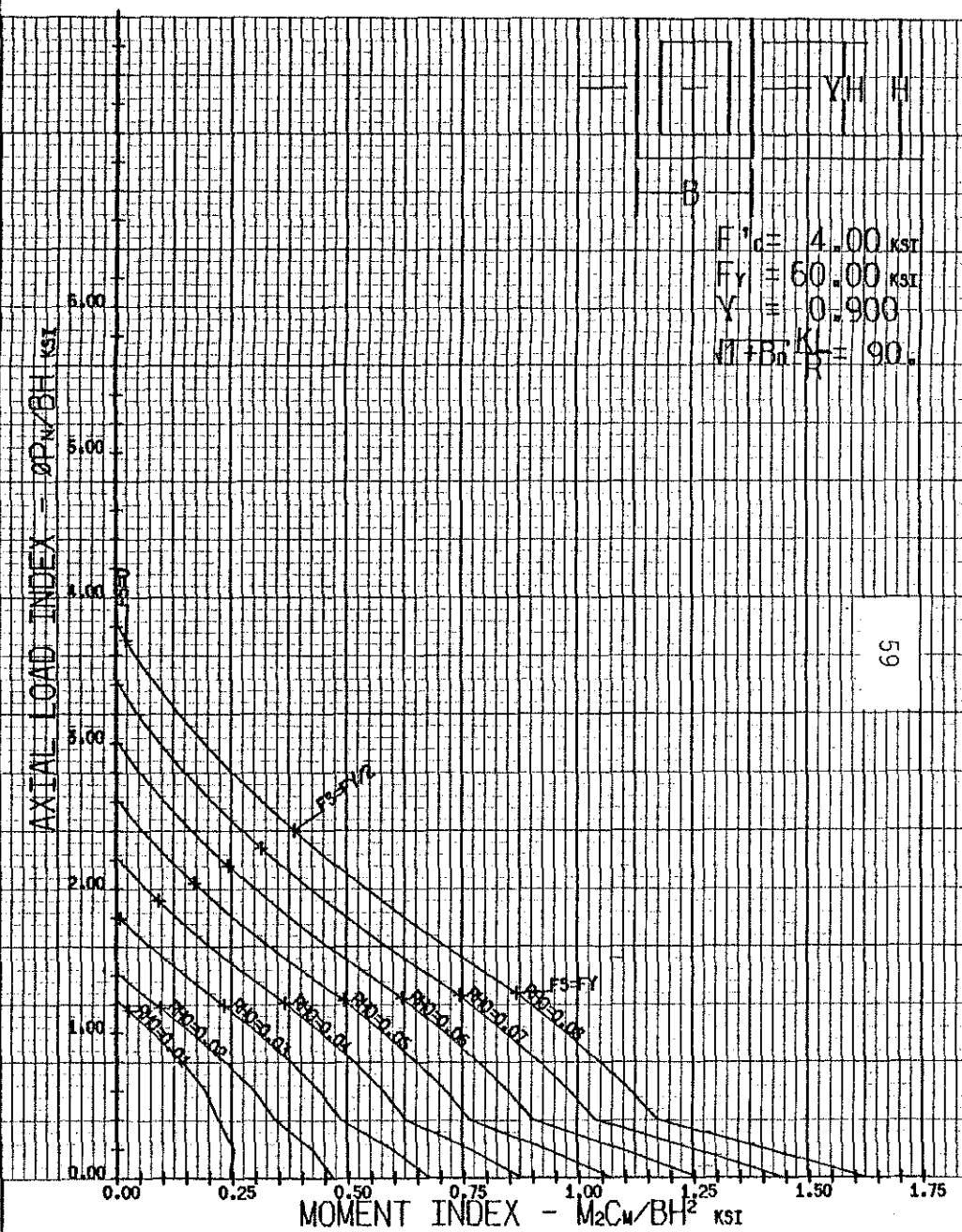
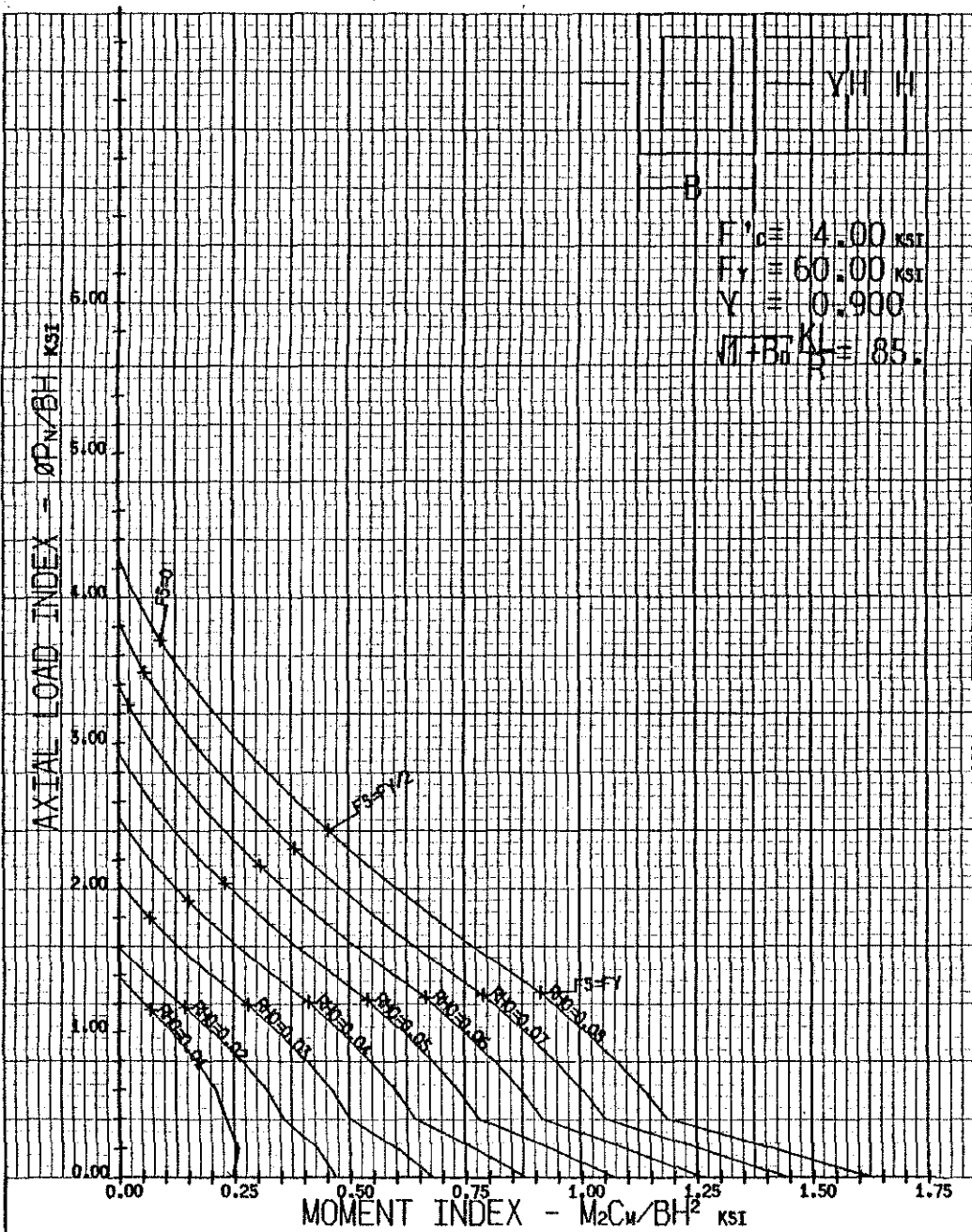


Fig. R4-60.90-80 - Interaction Diagram





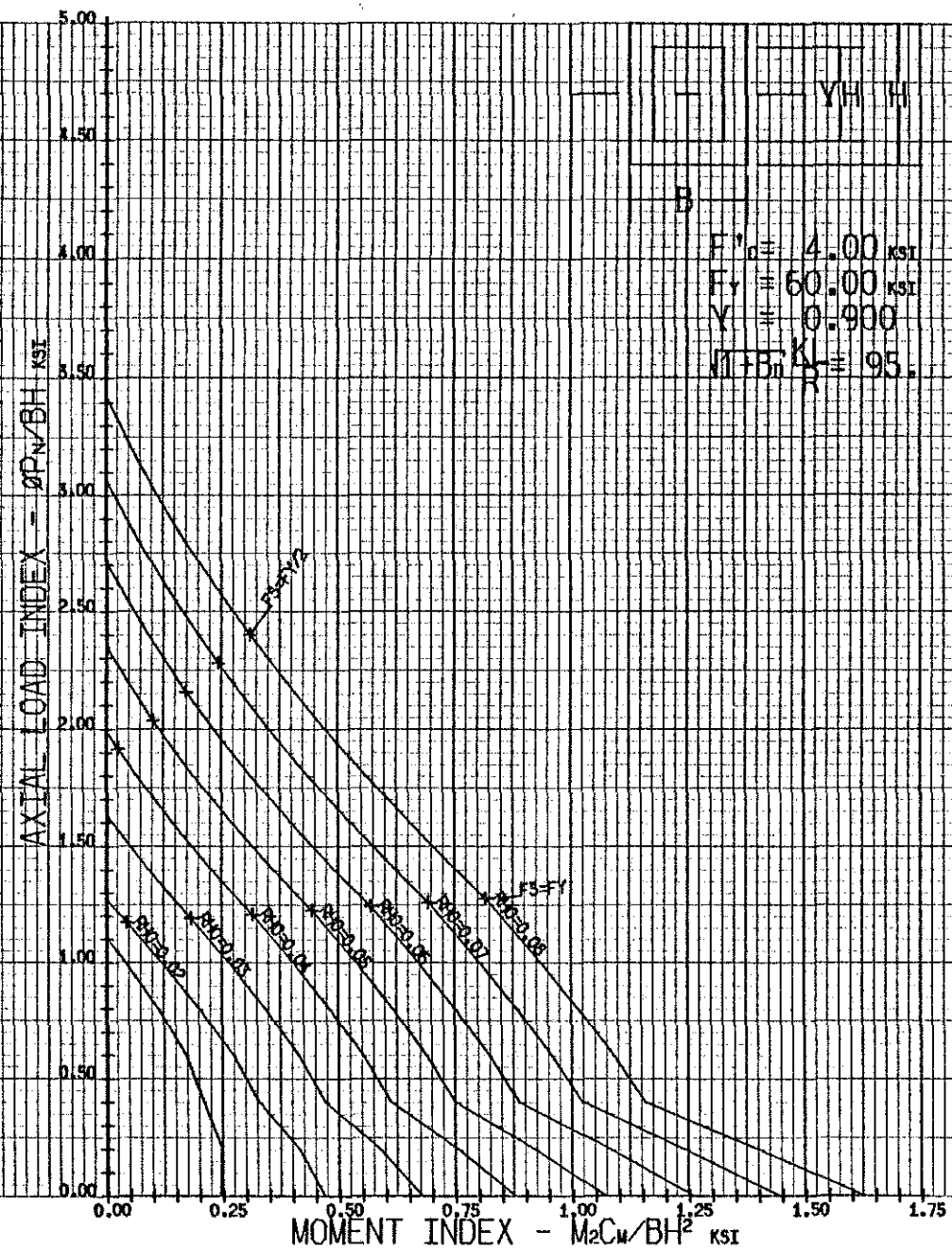


Fig. R4-60.90-95 - Interaction Diagram

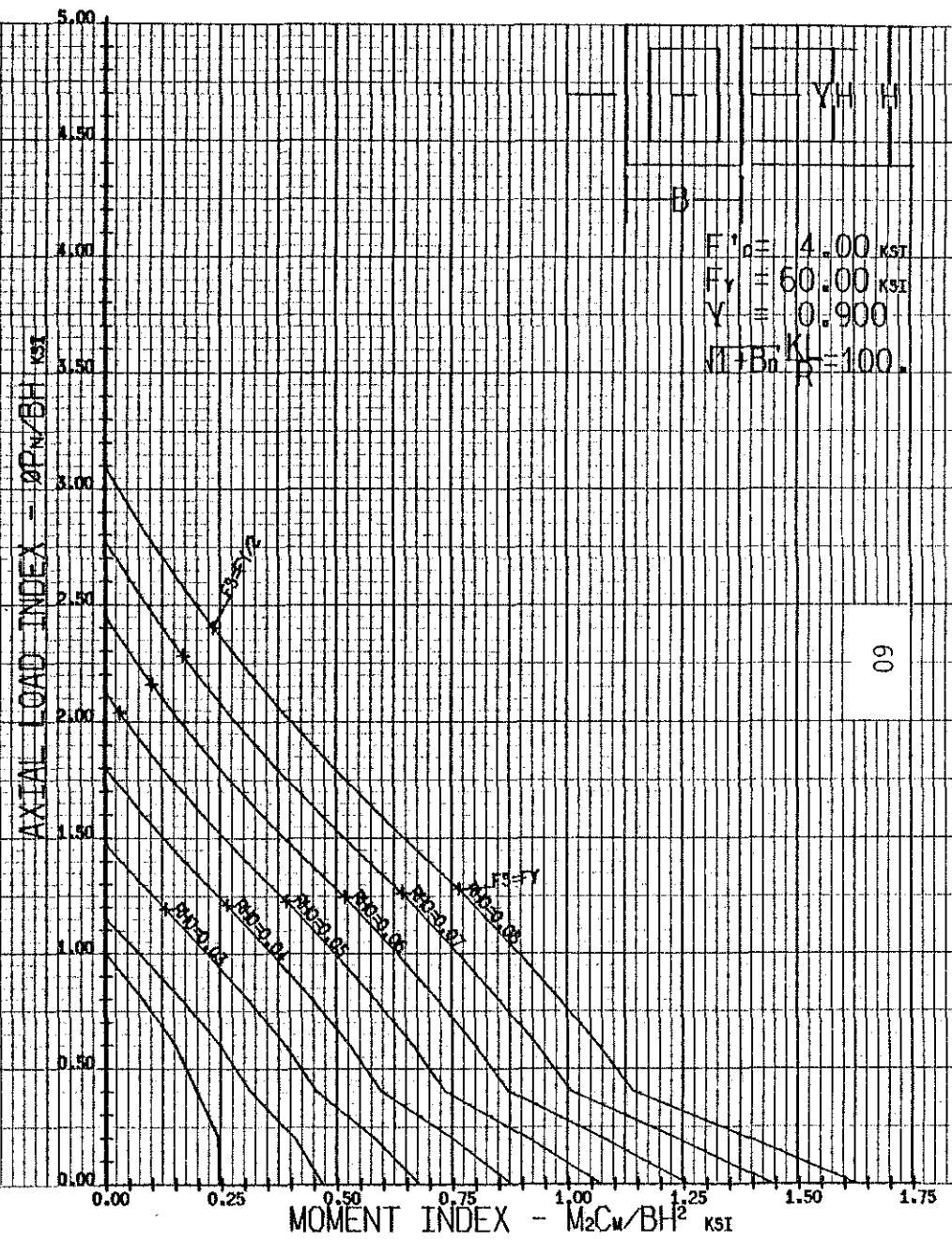


Fig. R4-60.90-100 - Interaction Diagram

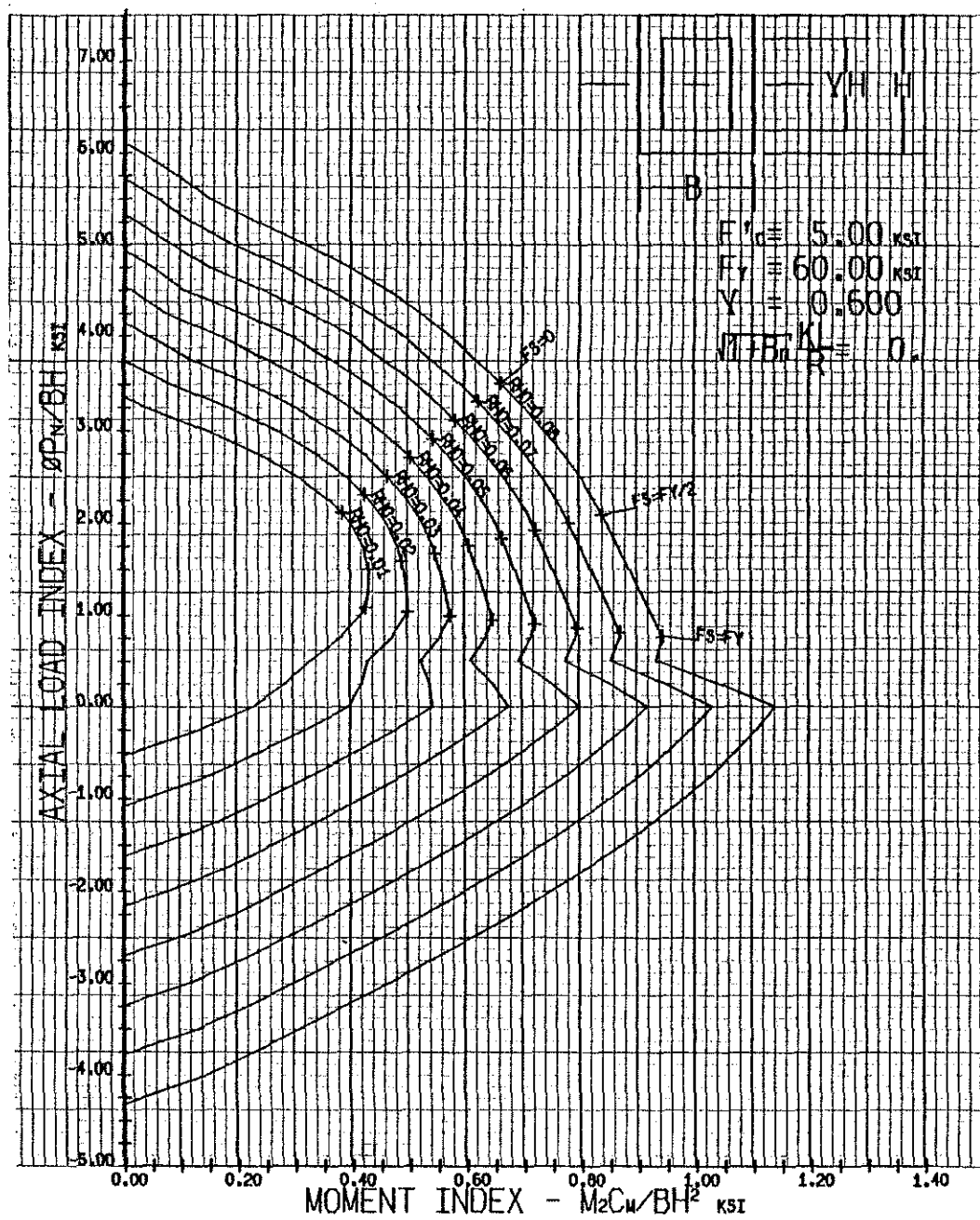


Fig. R5-60.60-0 - Interaction Diagram

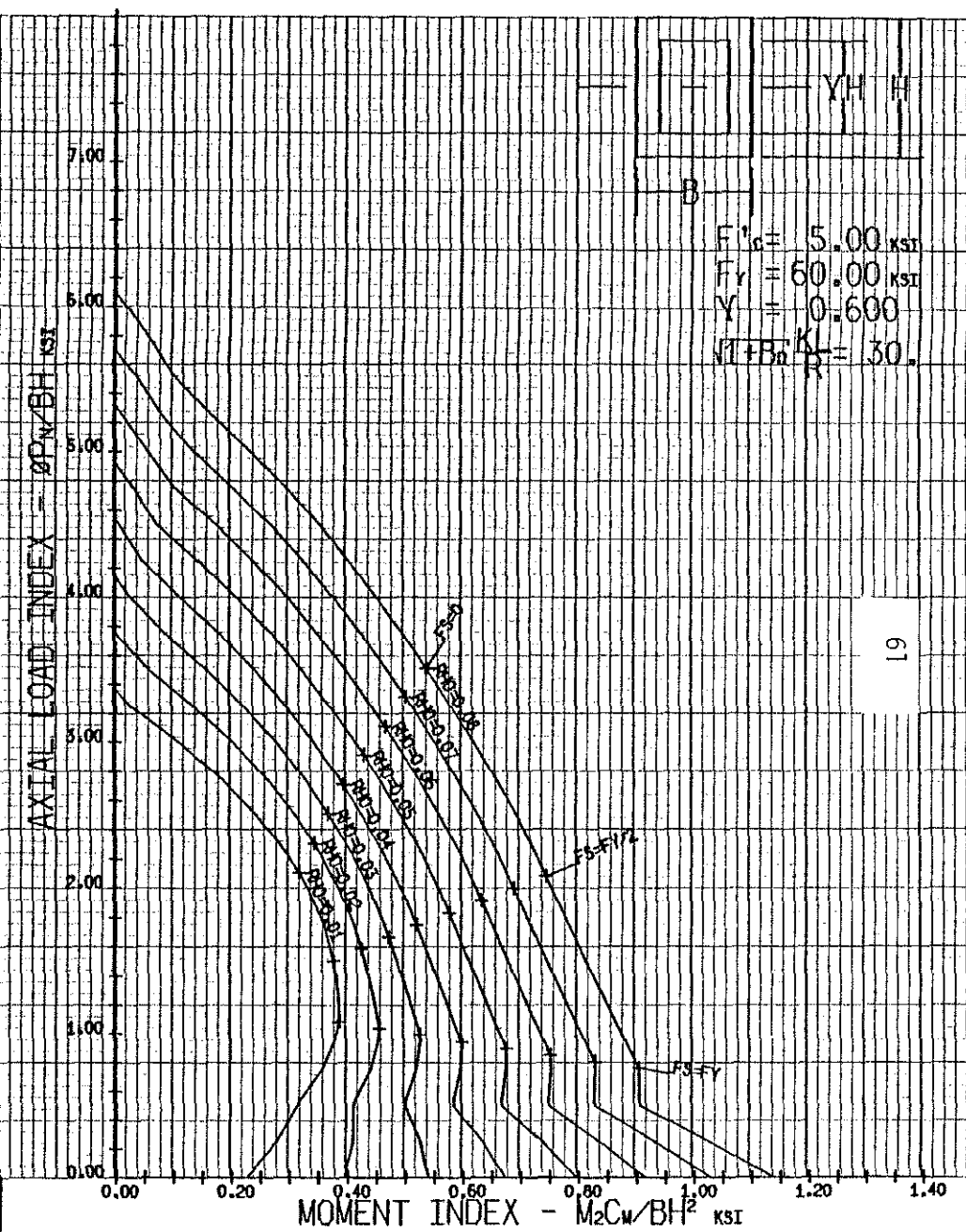


Fig. R5-60.60-30 - Interaction Diagram



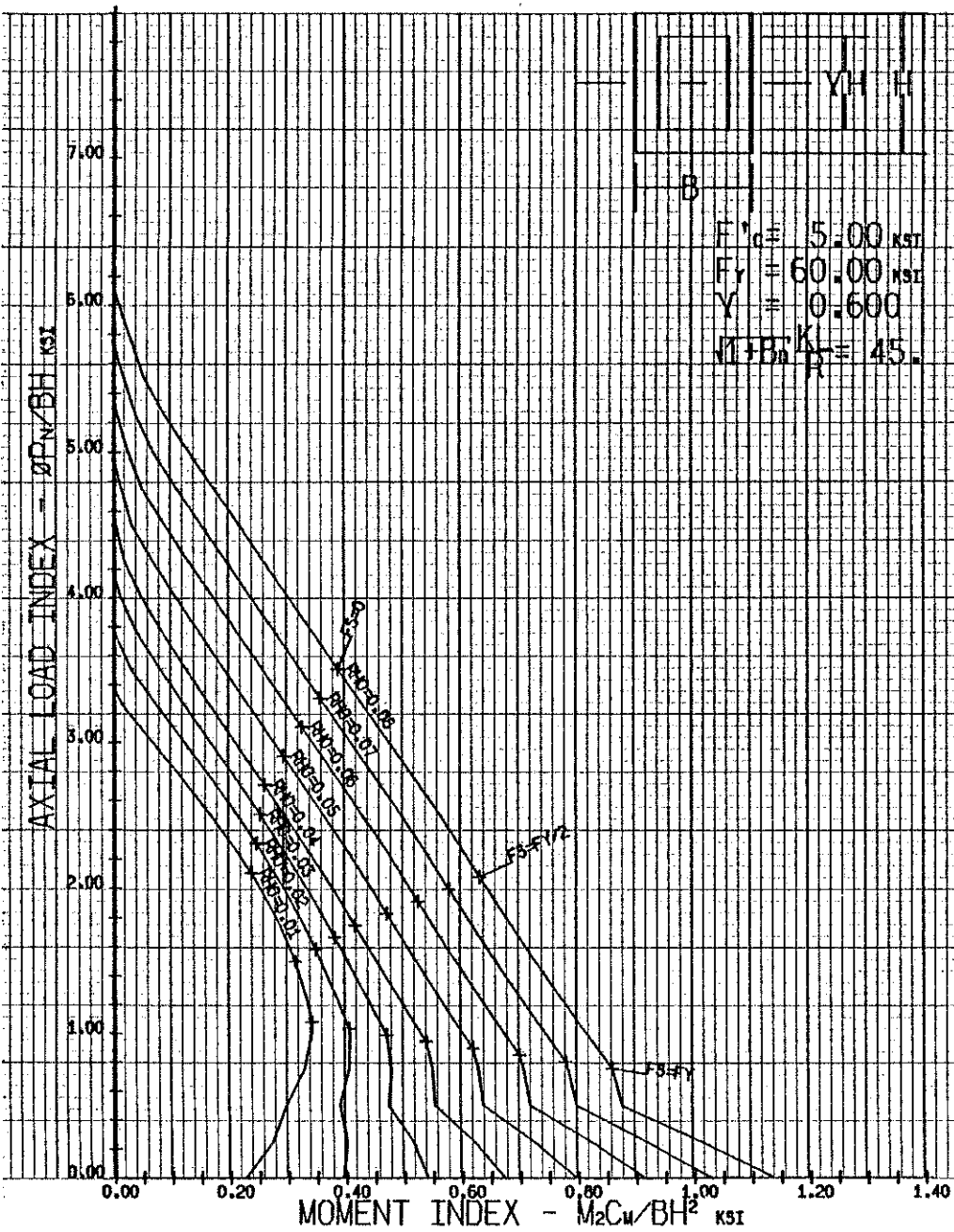


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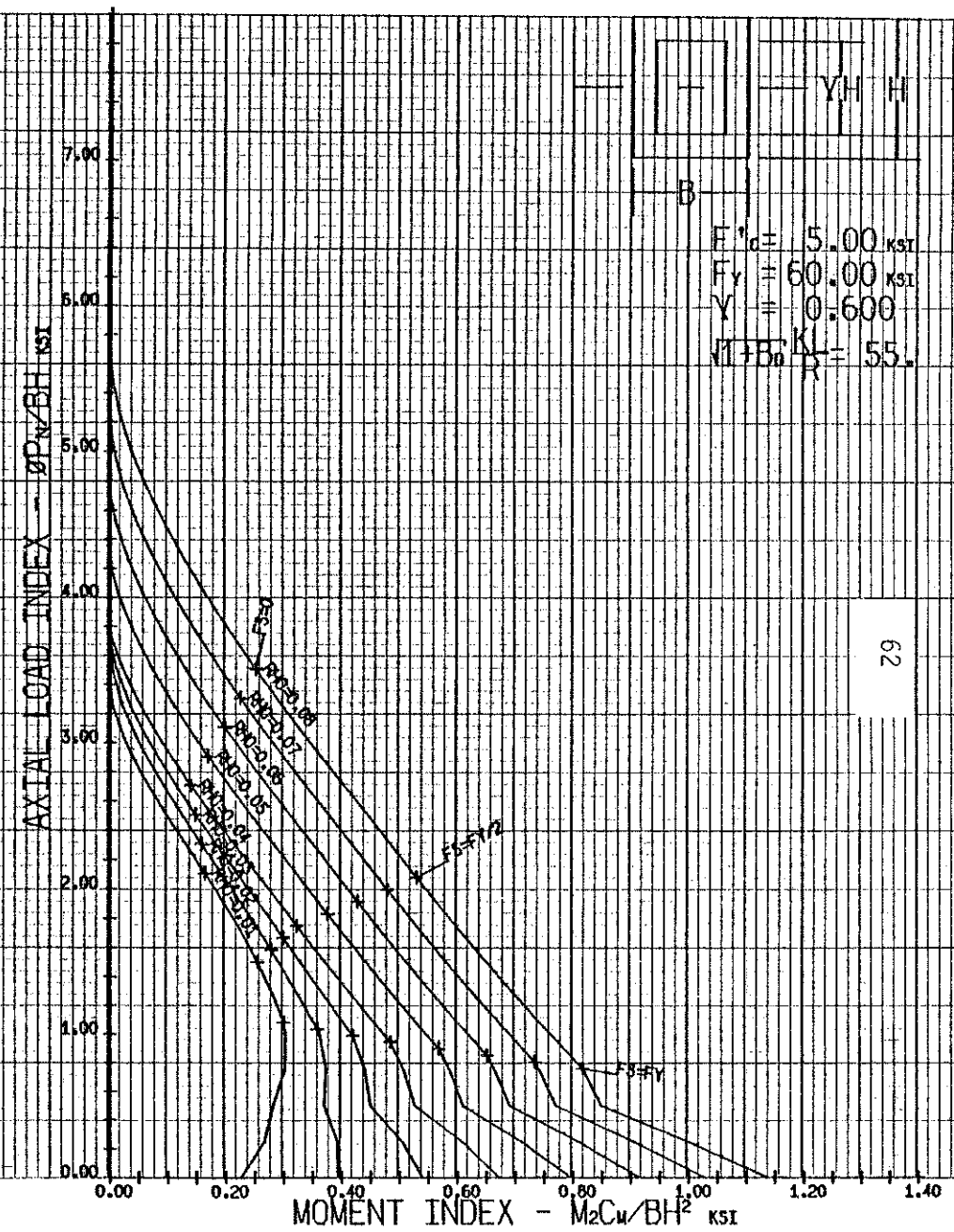


Fig. R5-60.60-55 - Interaction Diagram

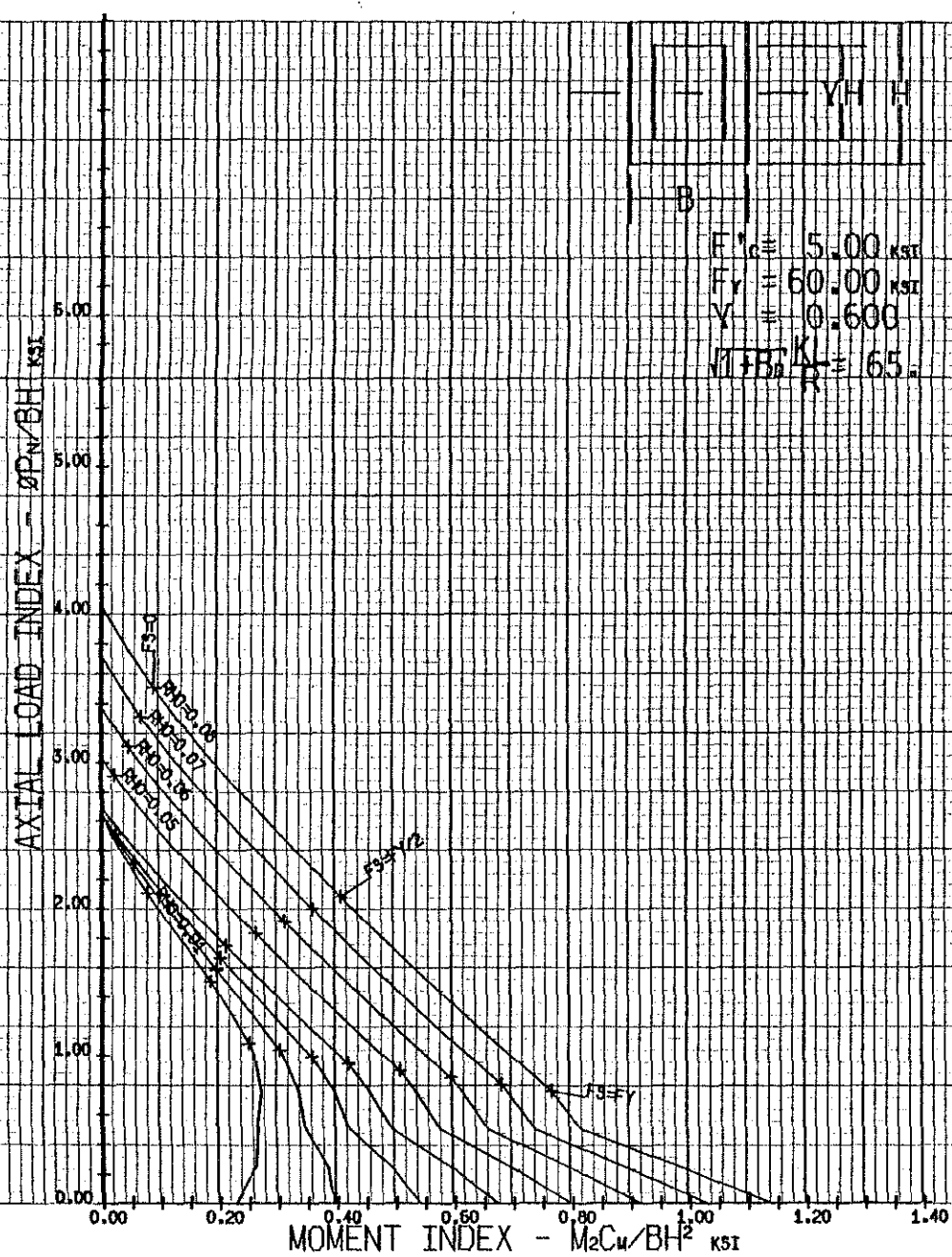


Fig. R5-60.60-65 - Interaction Diagram

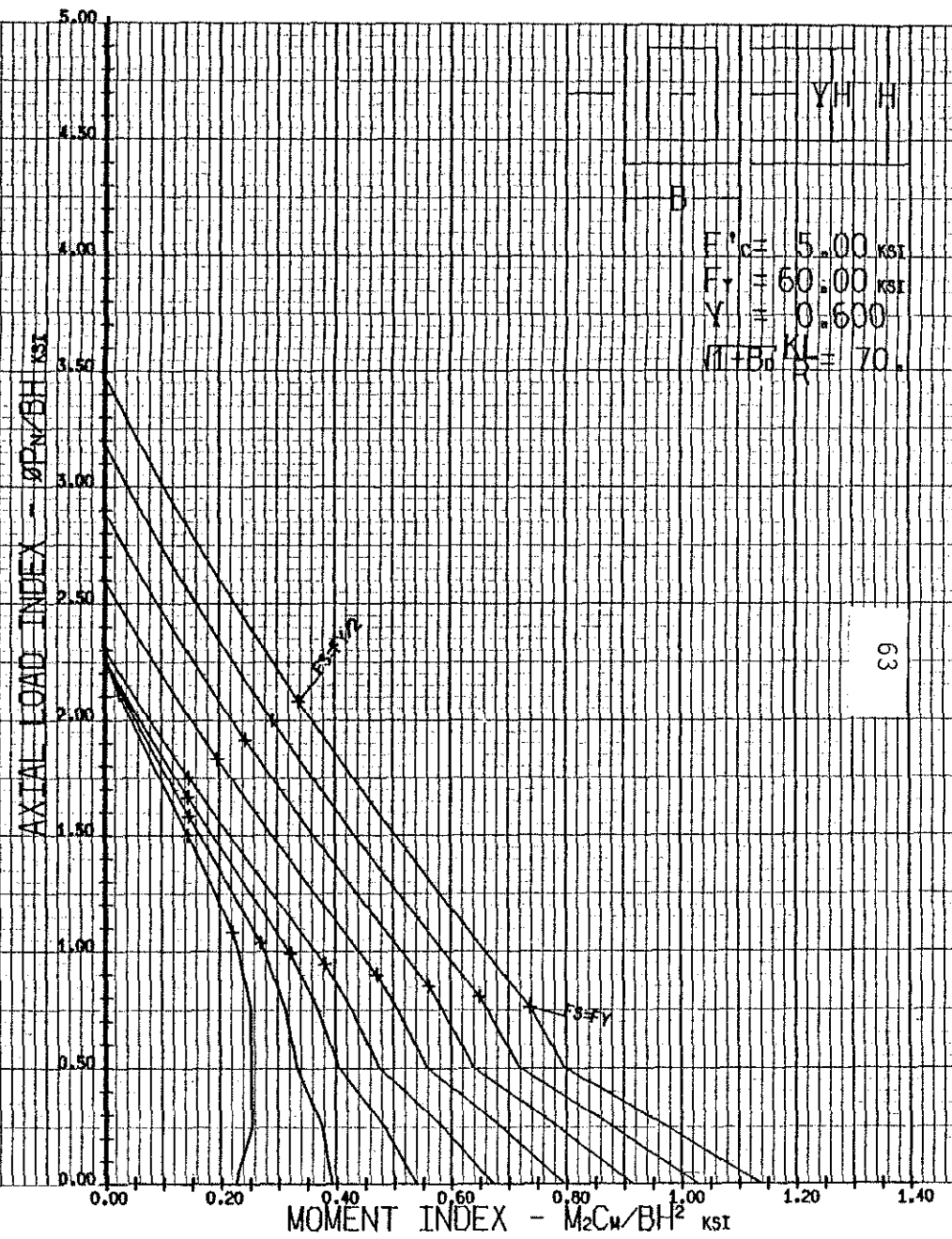


Fig. R5-60.60-70 - Interaction Diagram

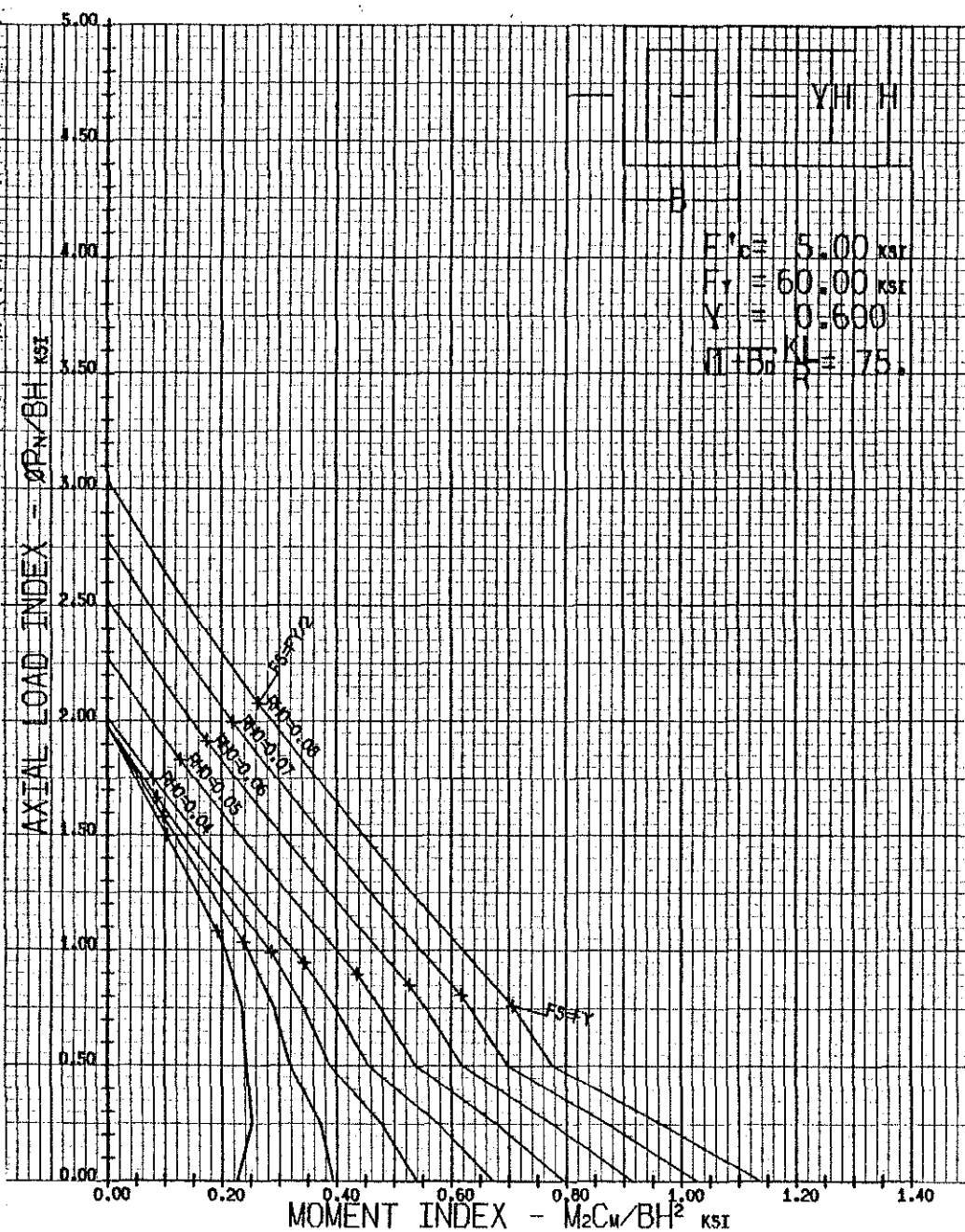


Fig. R5-60.60-75 - Interaction Diagram

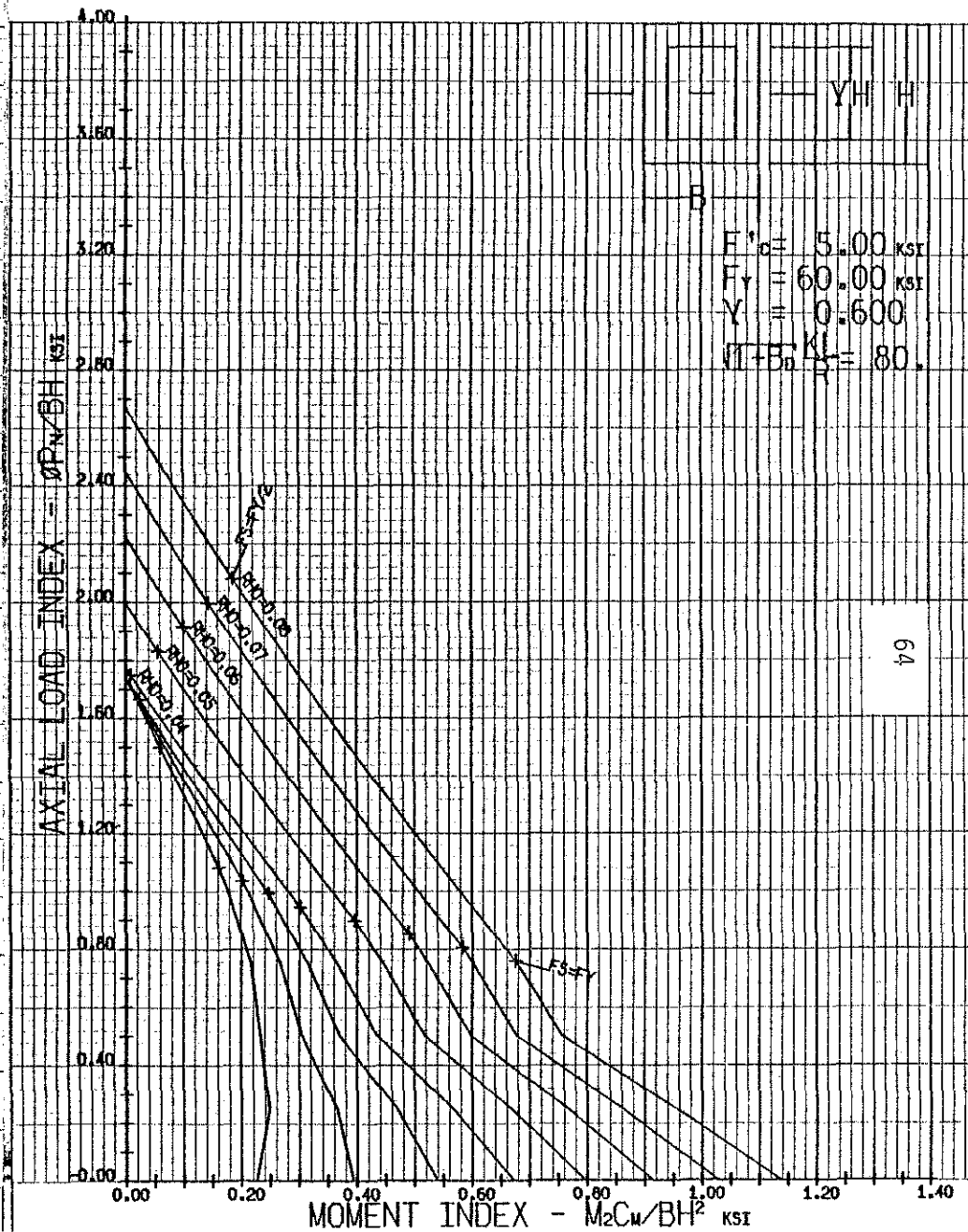


Fig. R5-60.60-80 - Interaction Diagram

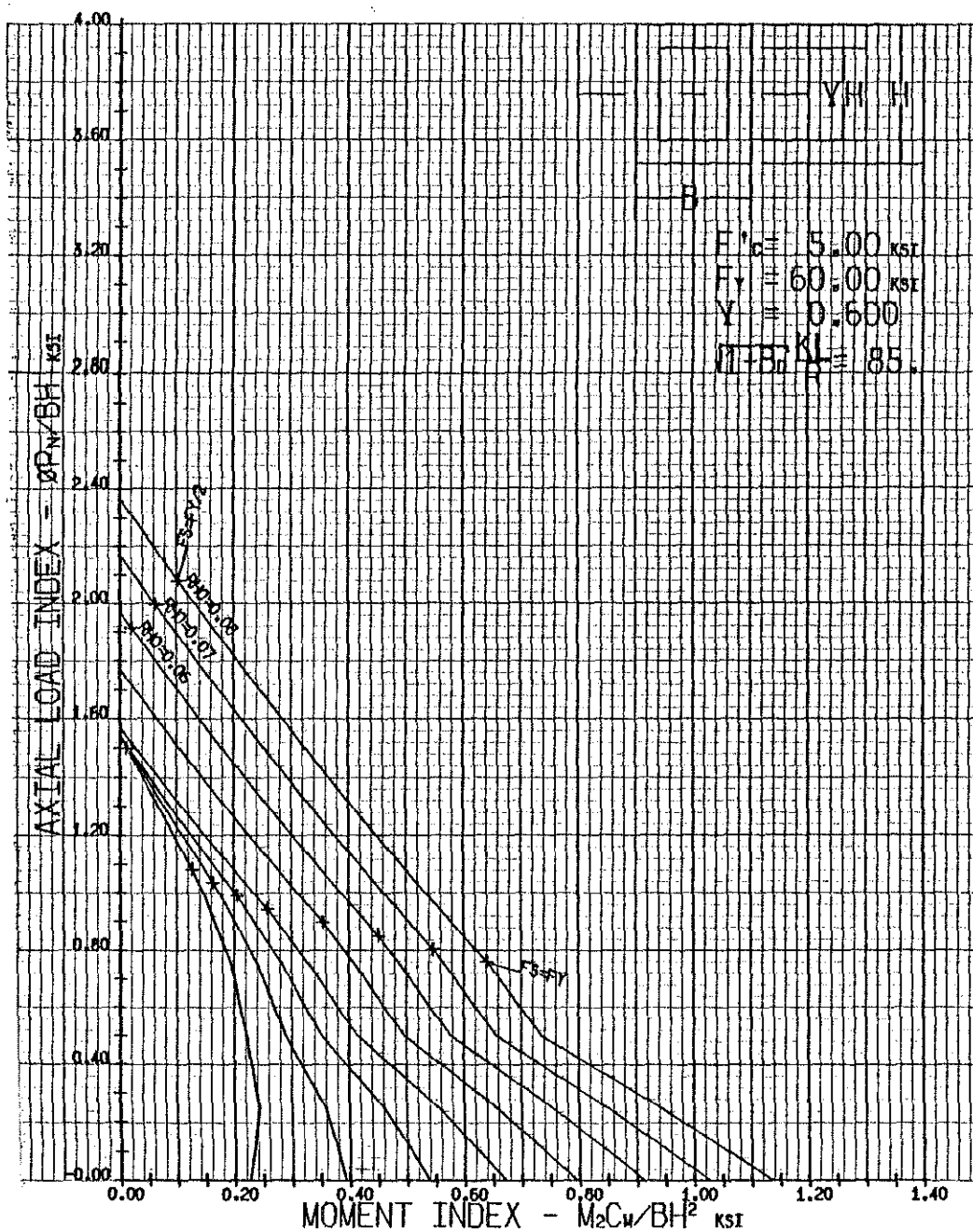


Fig. R5-60.60-85 - Interaction Diagram

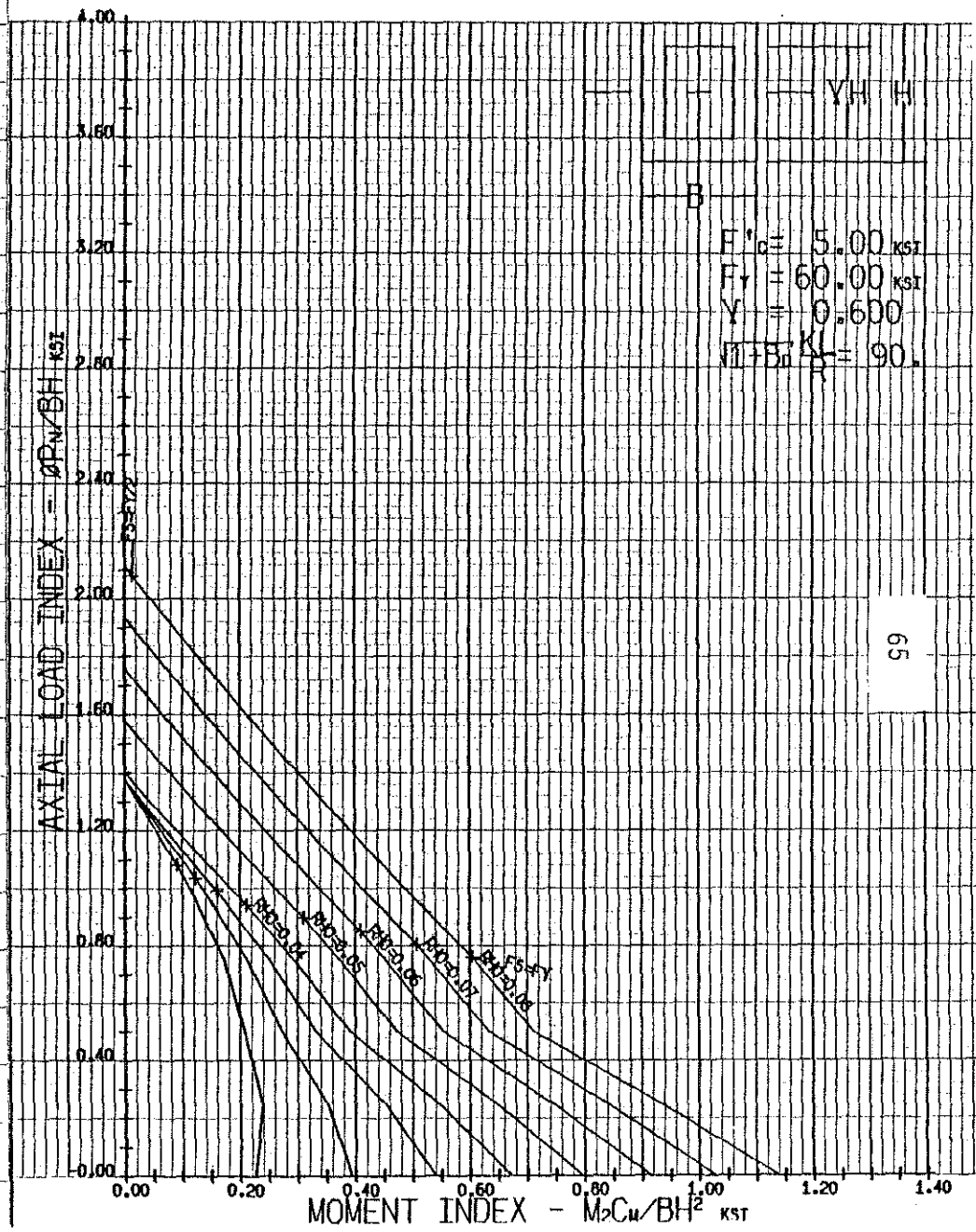


Fig. R5-60.60-90 - Interaction Diagram

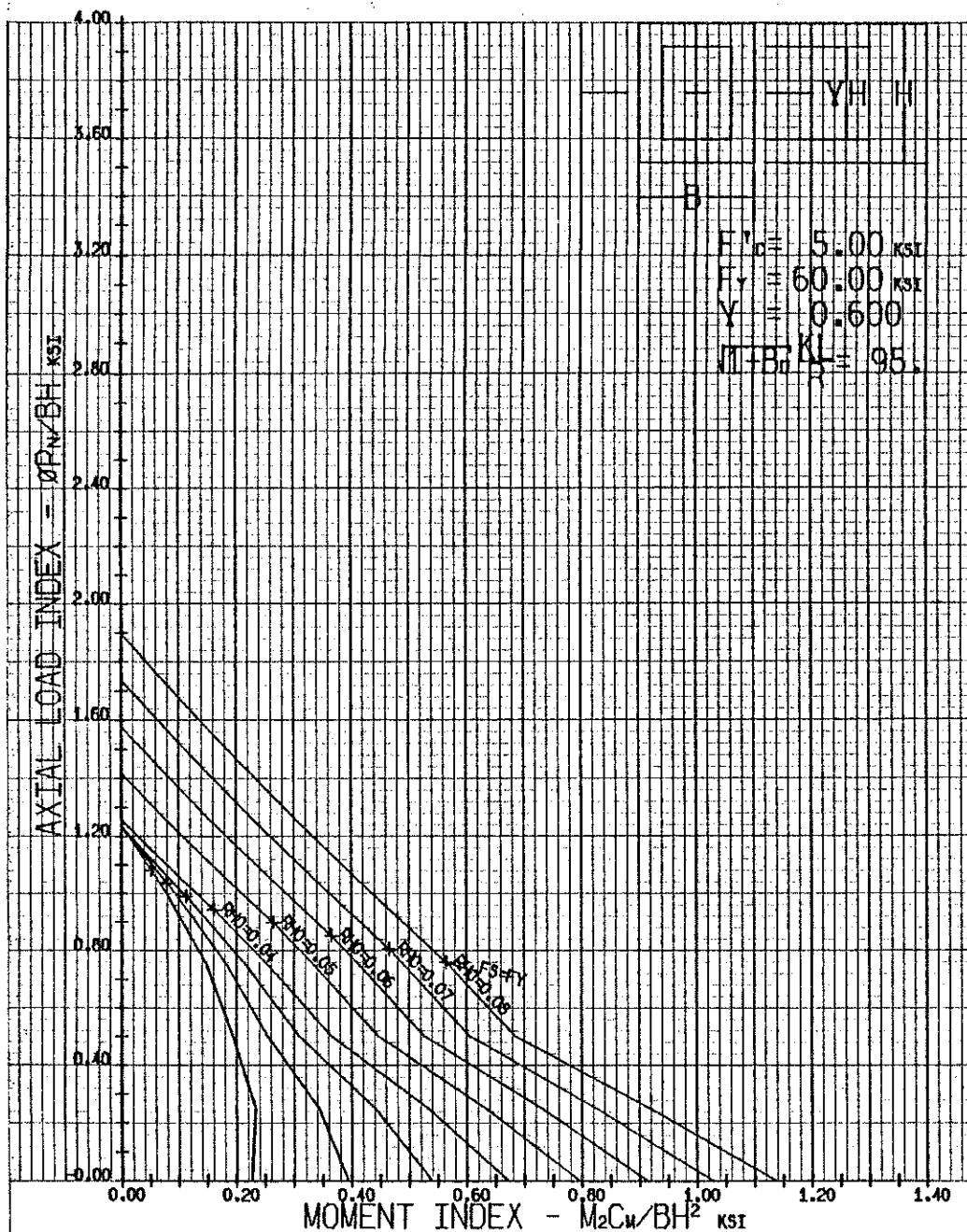


Fig. R5-60.60-95 - Interaction Diagram

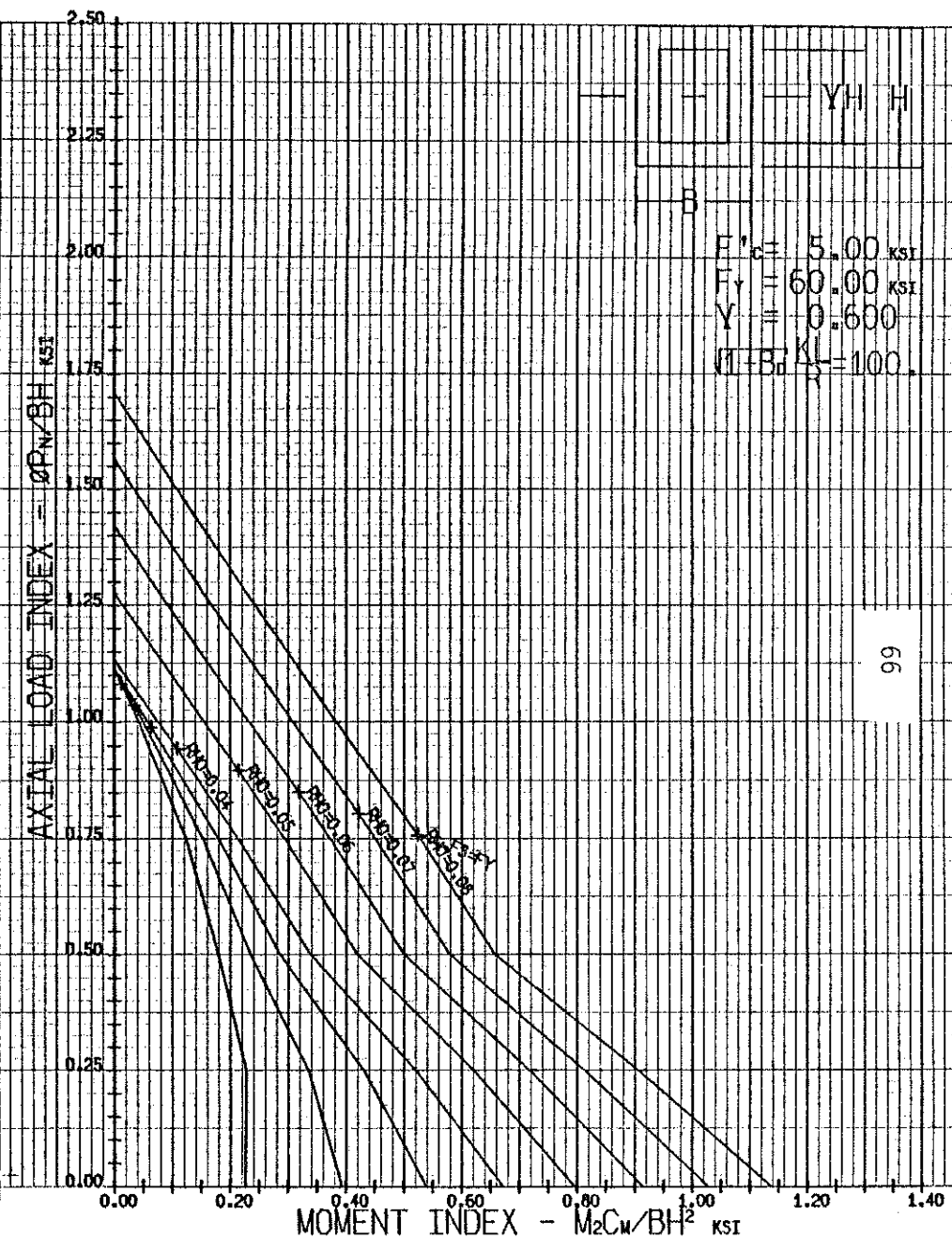
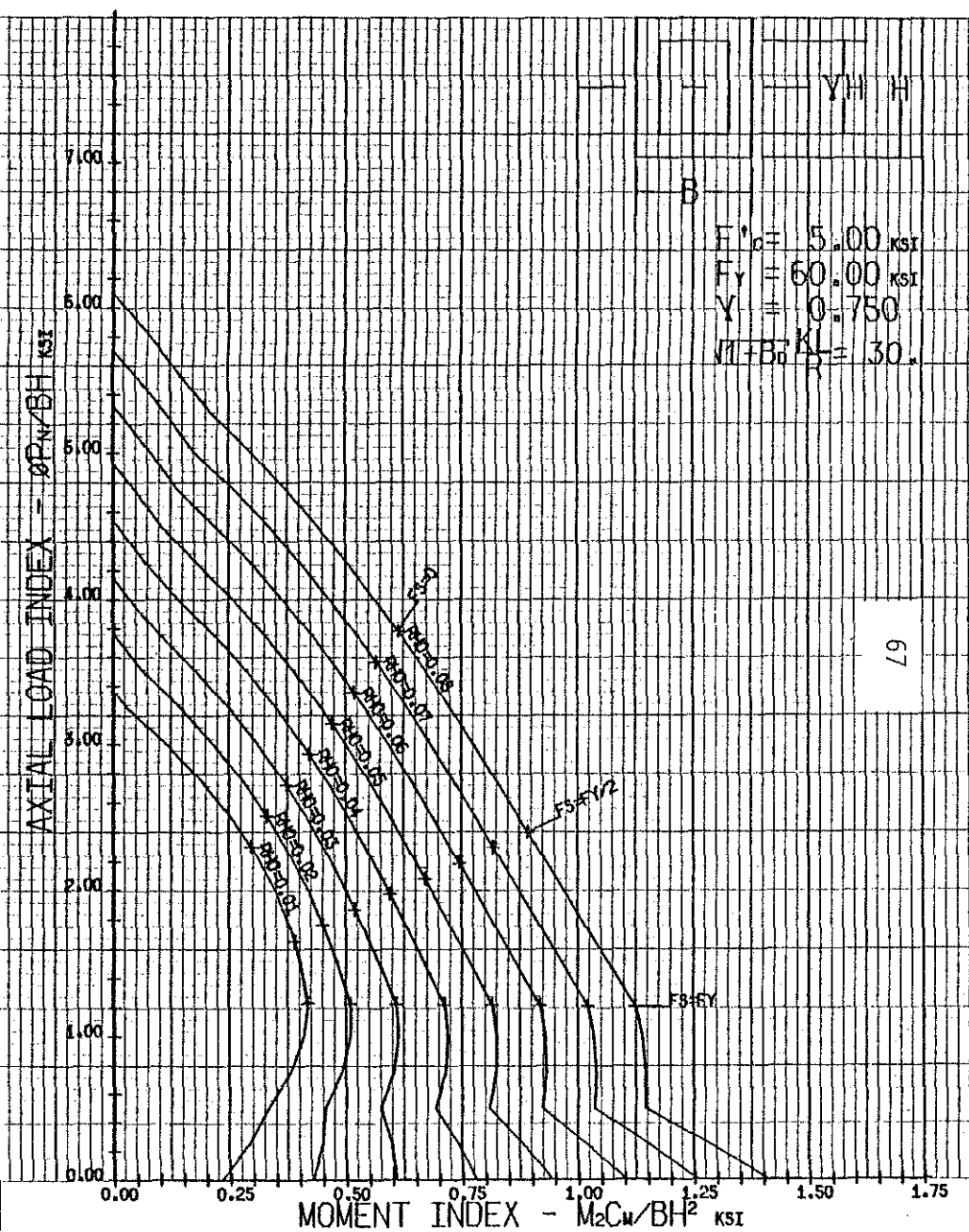
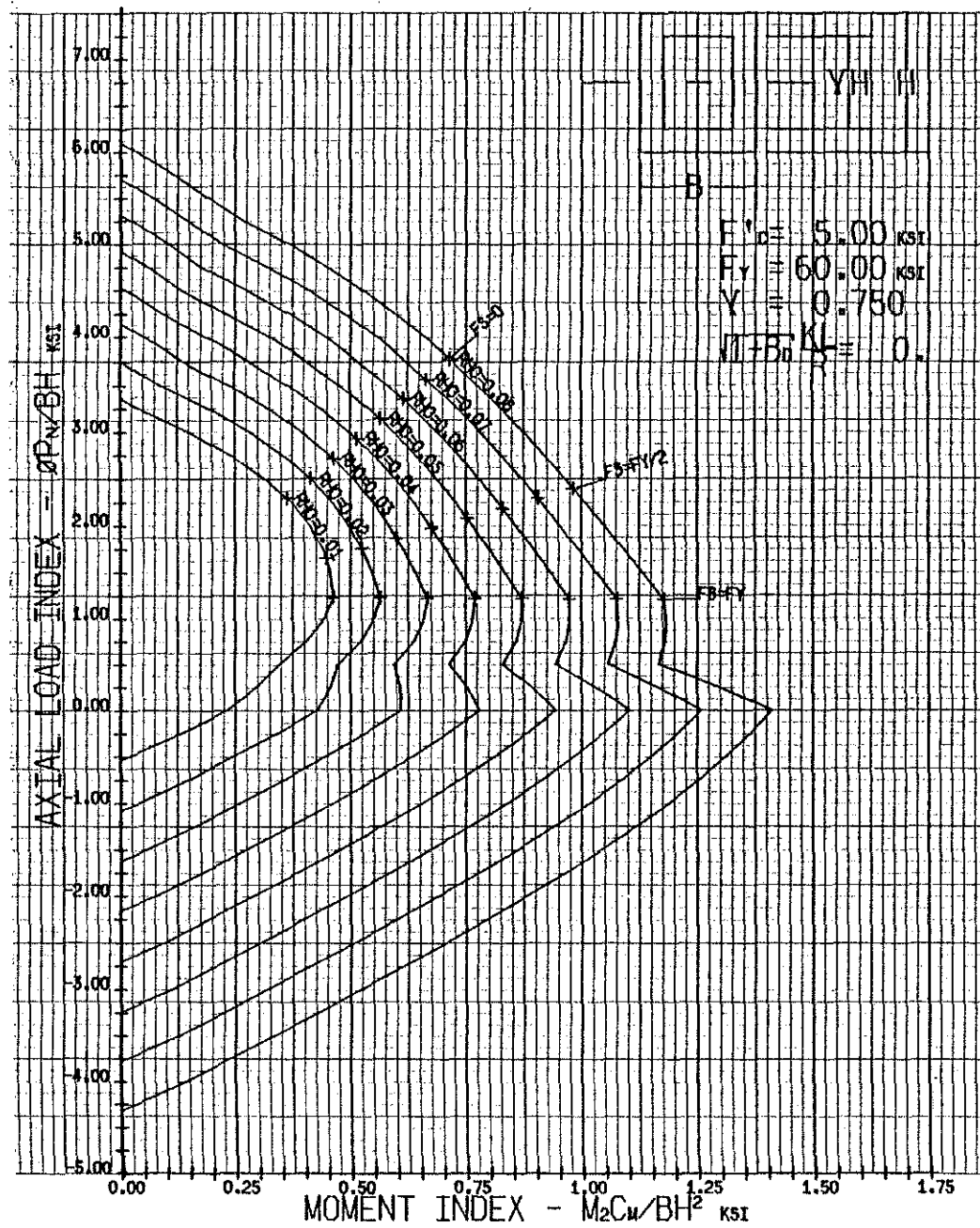


Fig. R5-60.60-100 - Interaction Diagram





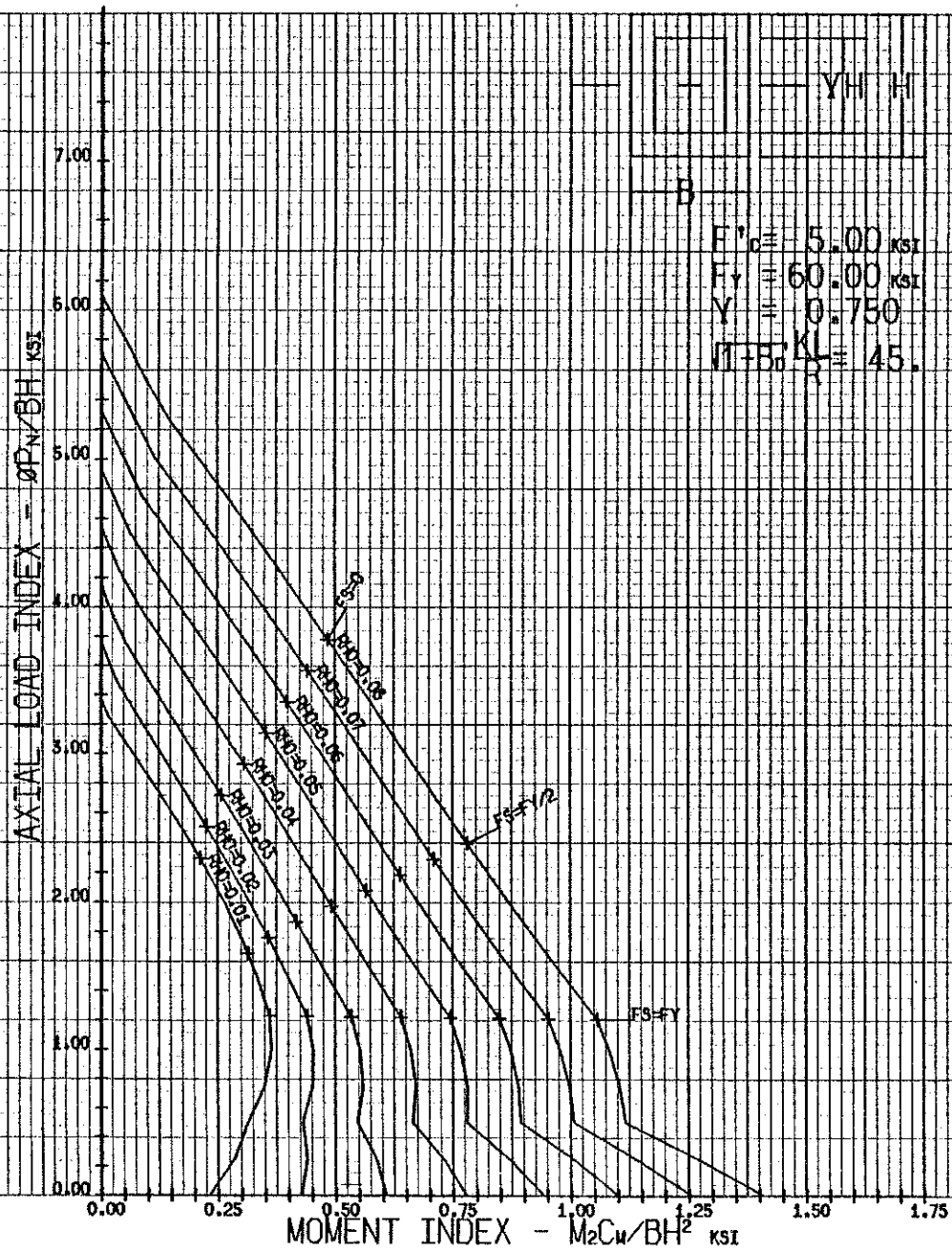


Fig. R5-60.75-45 - Interaction Diagram

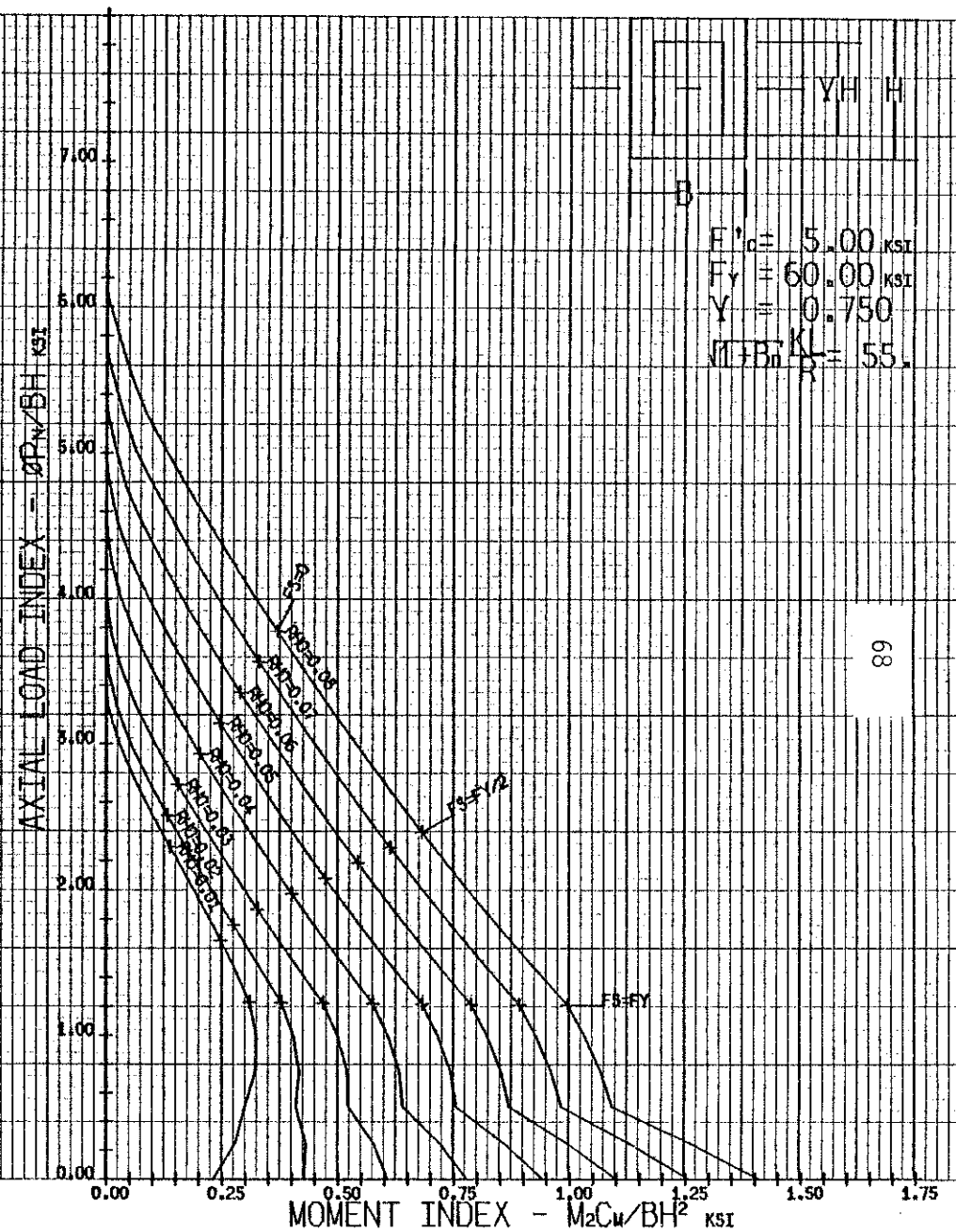


Fig. R5-60.75-55 - Interaction Diagram

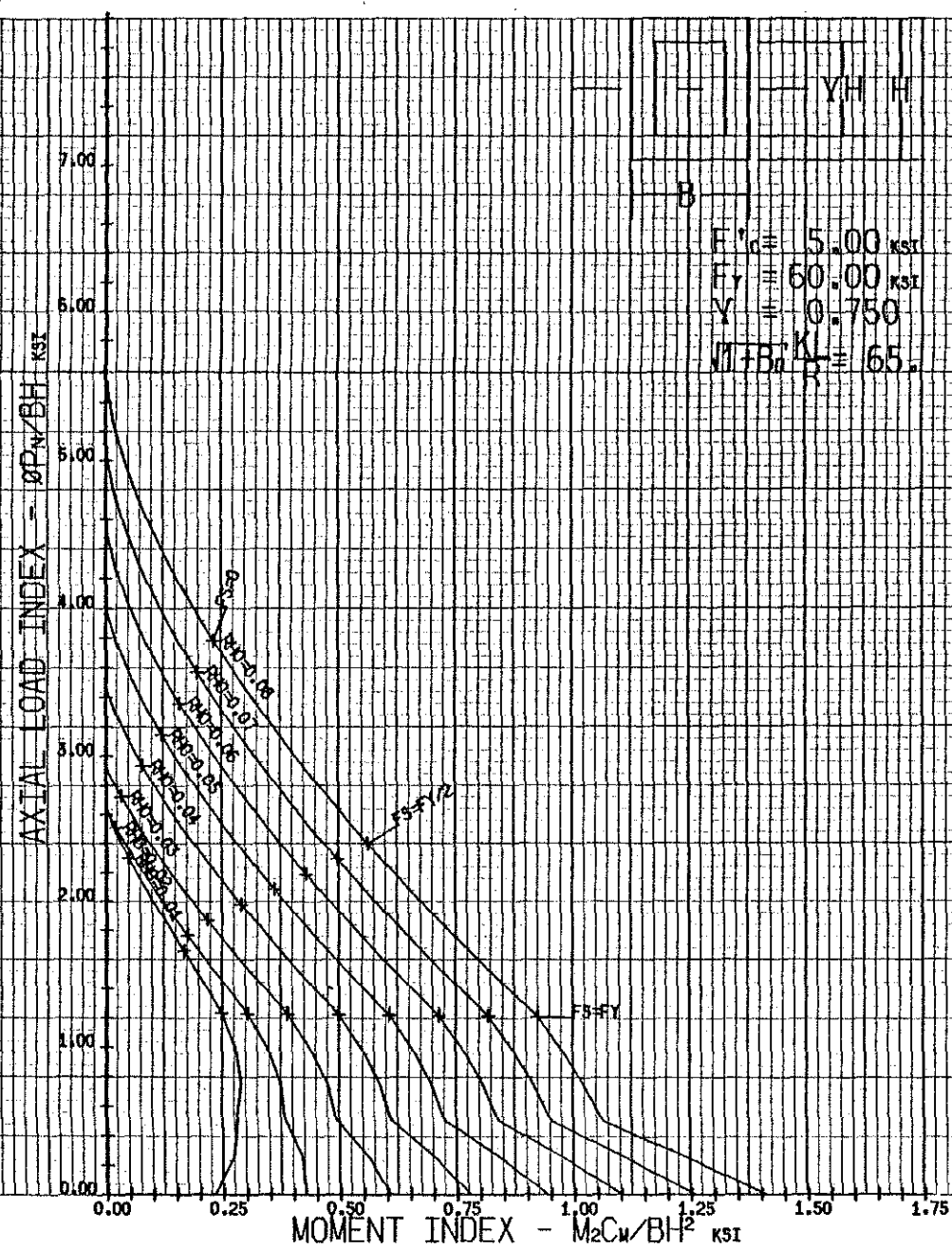


Fig. R5-60.75-65 - Interaction Diagram

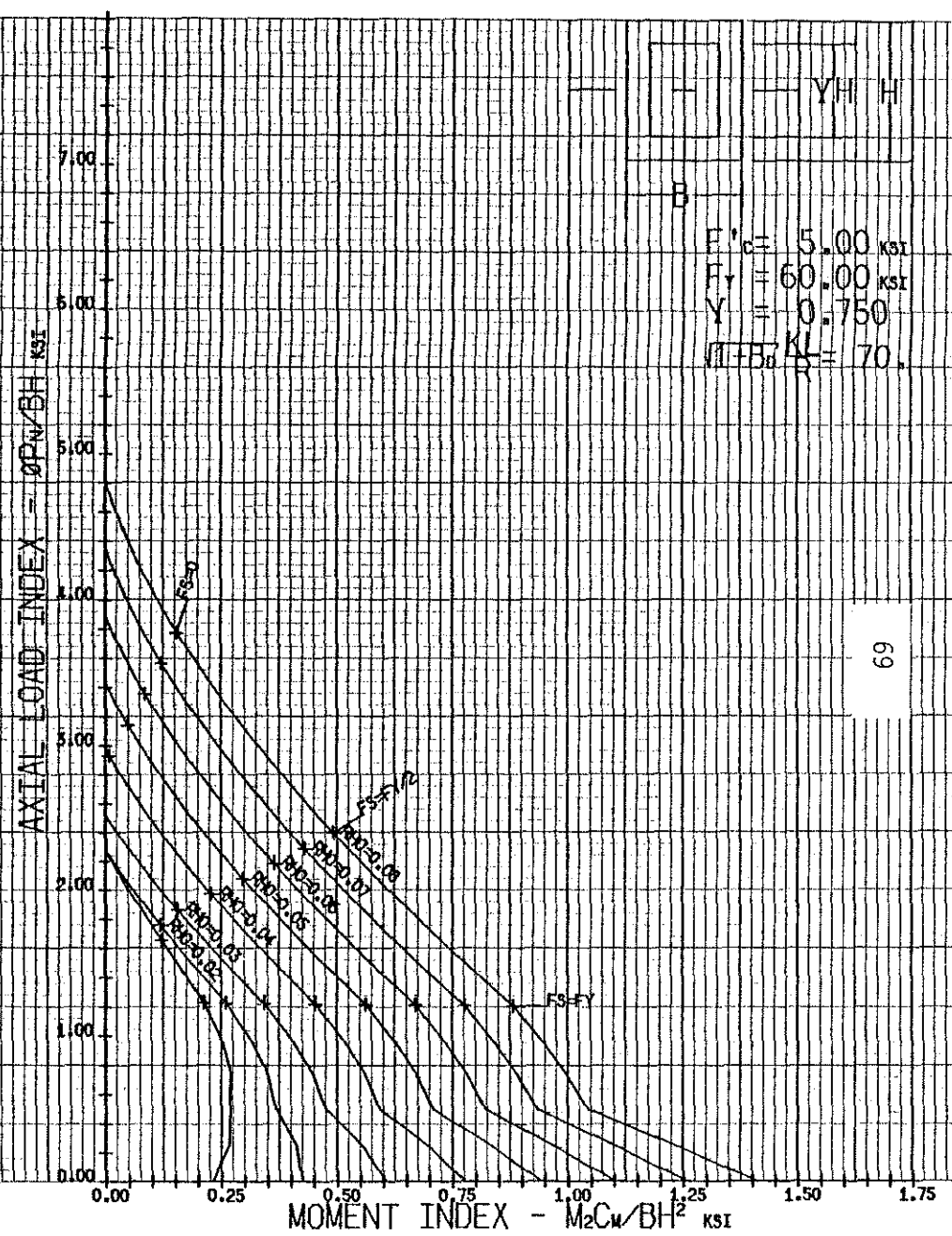


Fig. R5-60.75-70 - Interaction Diagram



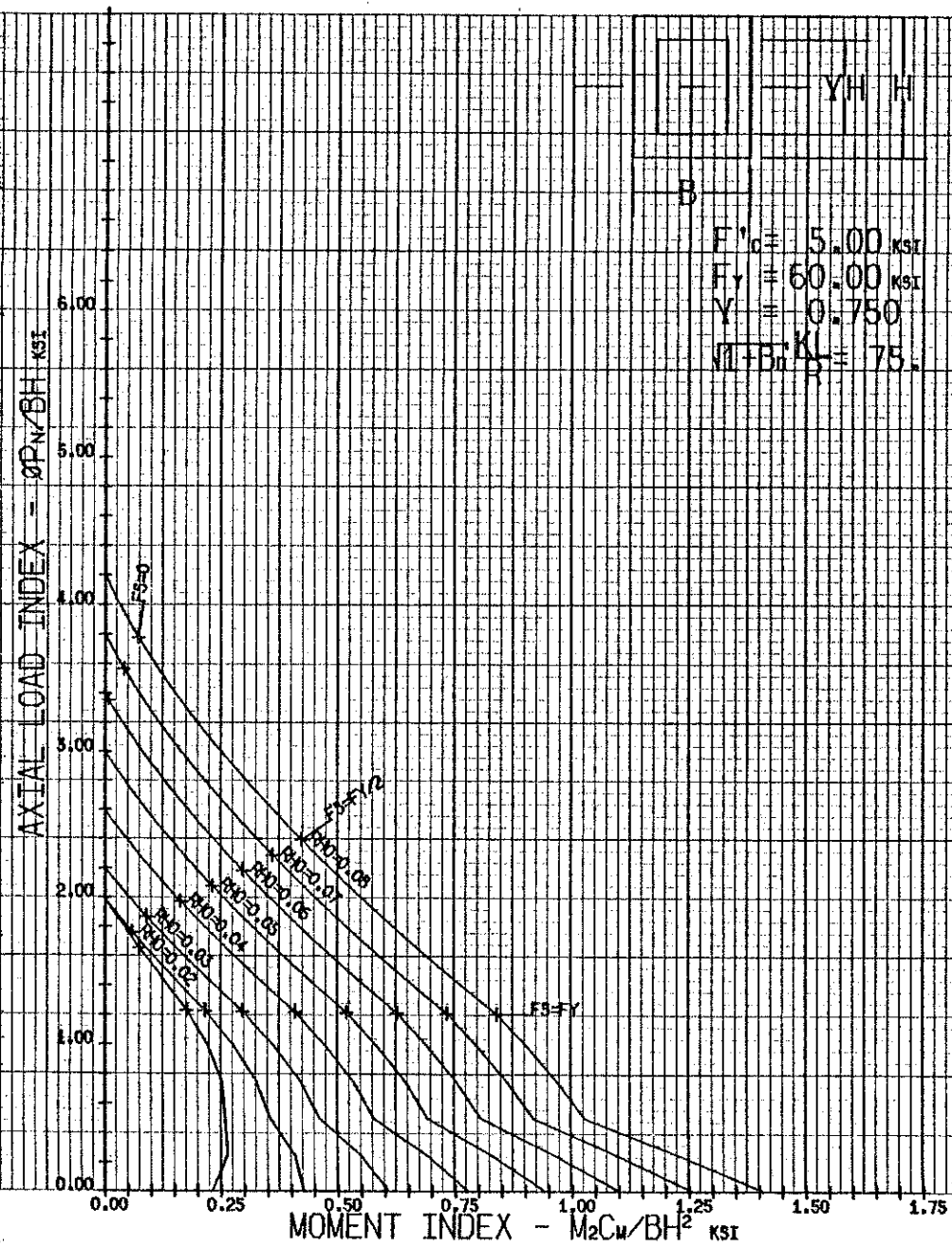


Fig. R5-60.75-75 - Interaction Diagram

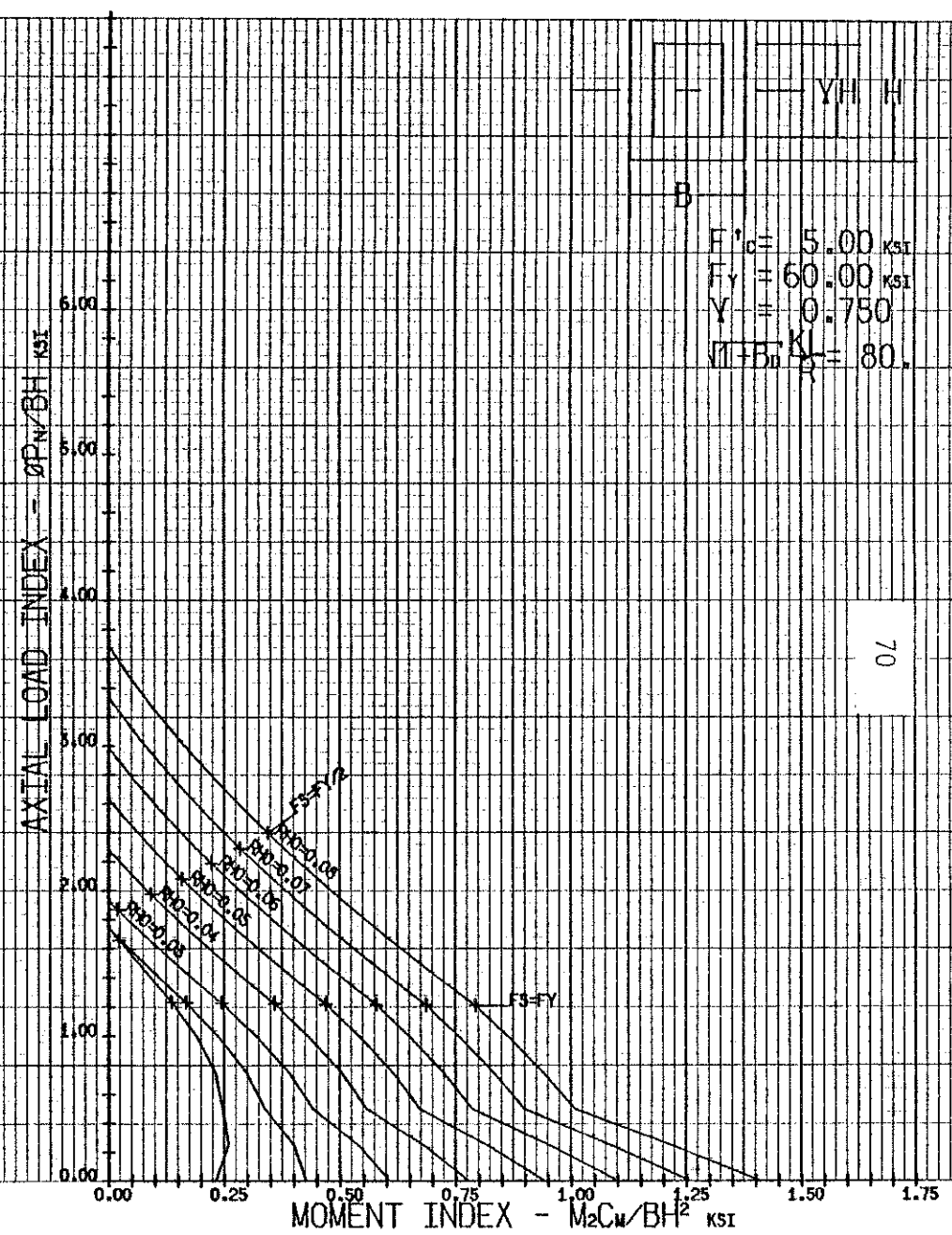


Fig. R5-60.75-80 - Interaction Diagram

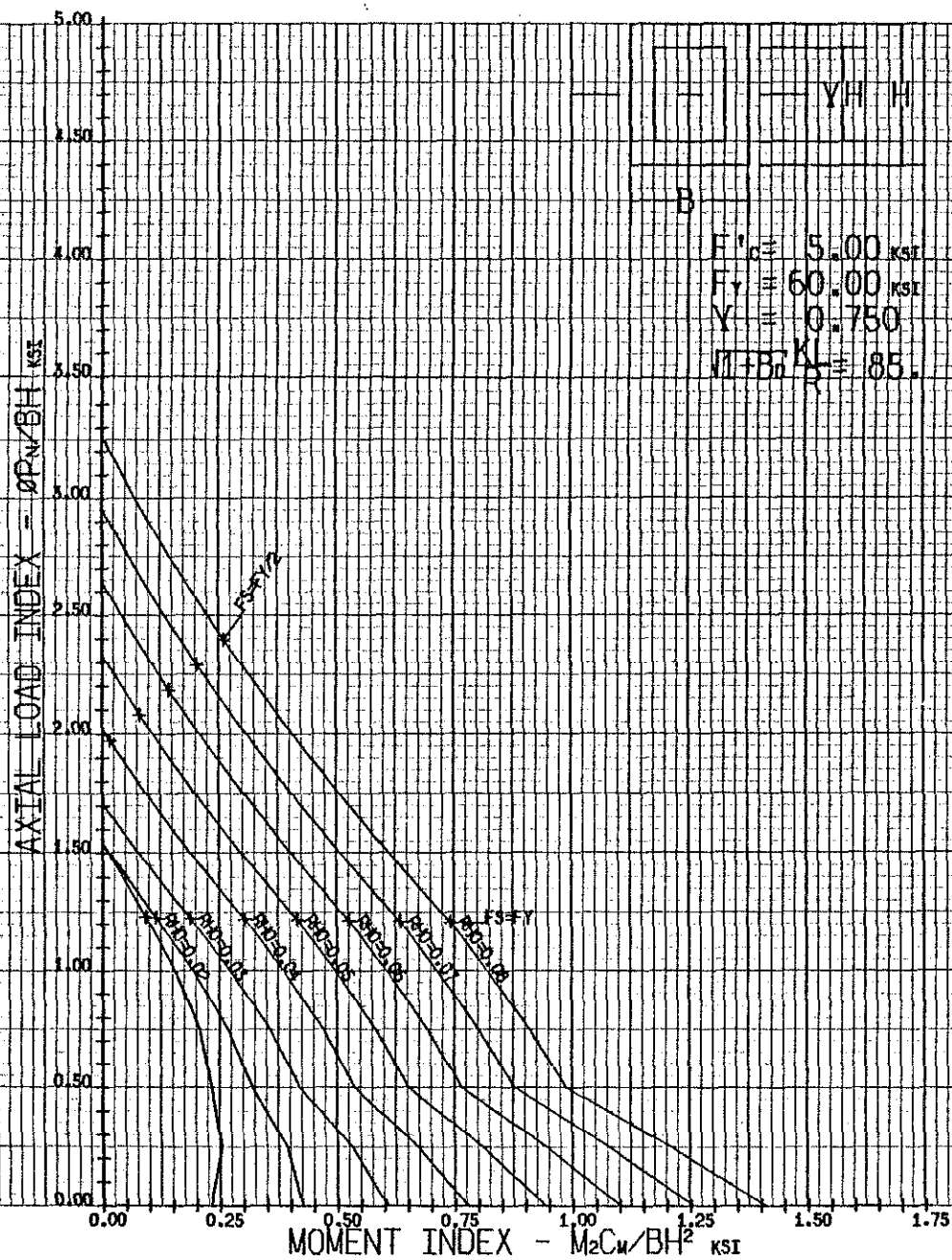


Fig. R5-60.75-85 - Interaction Diagram

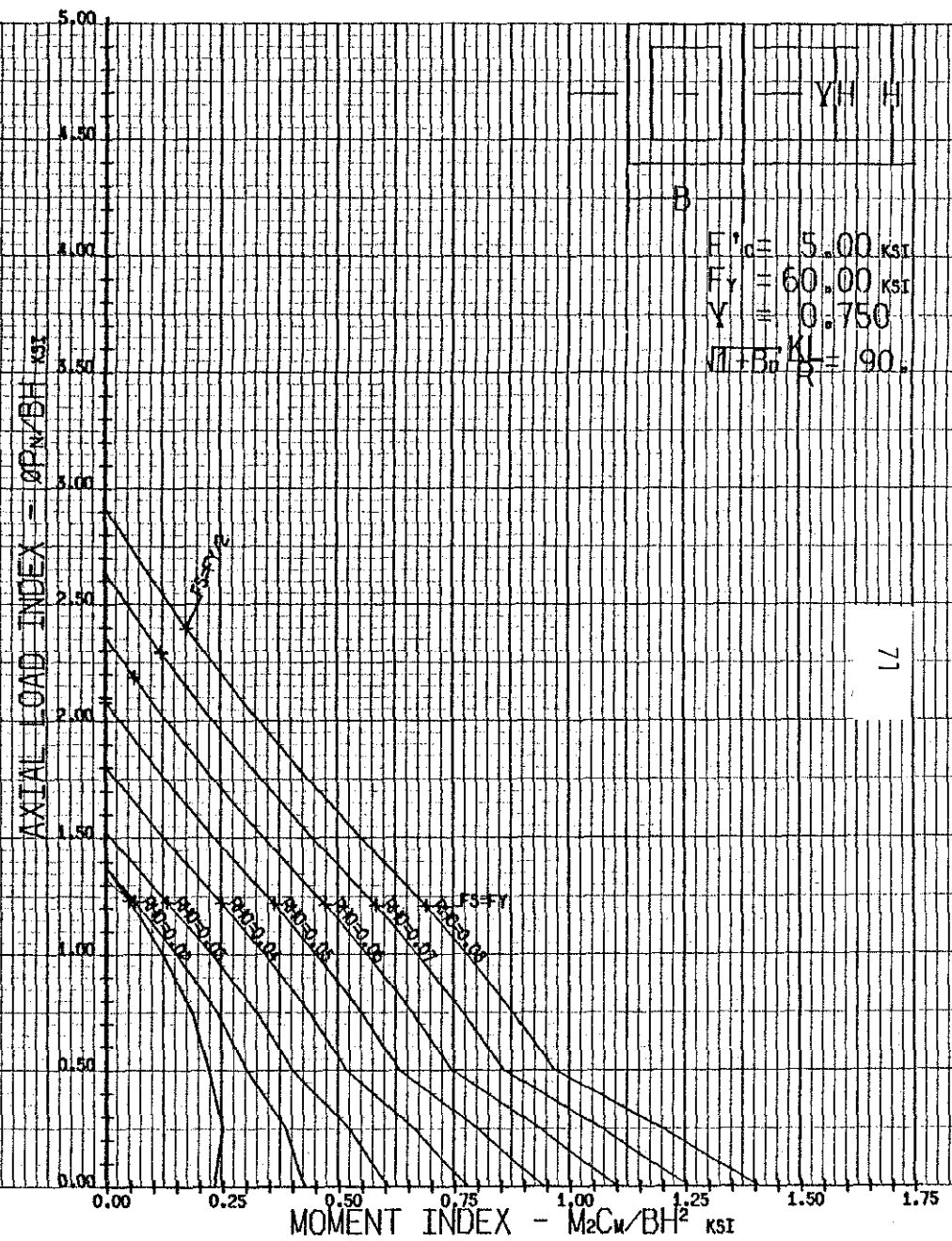


Fig. R5-60.75-90 - Interaction Diagram

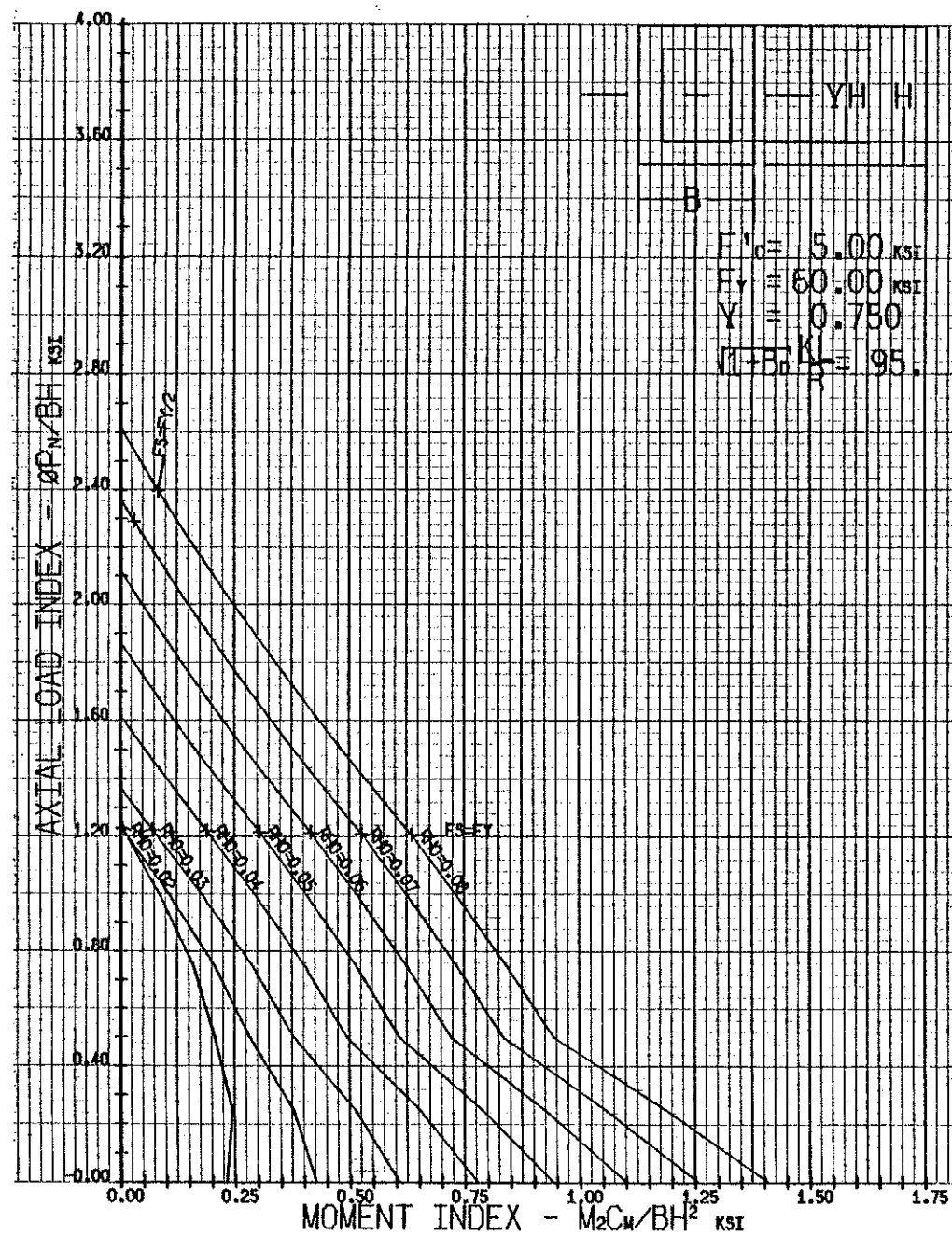


Fig. R5-60.75-95 - Interaction Diagram

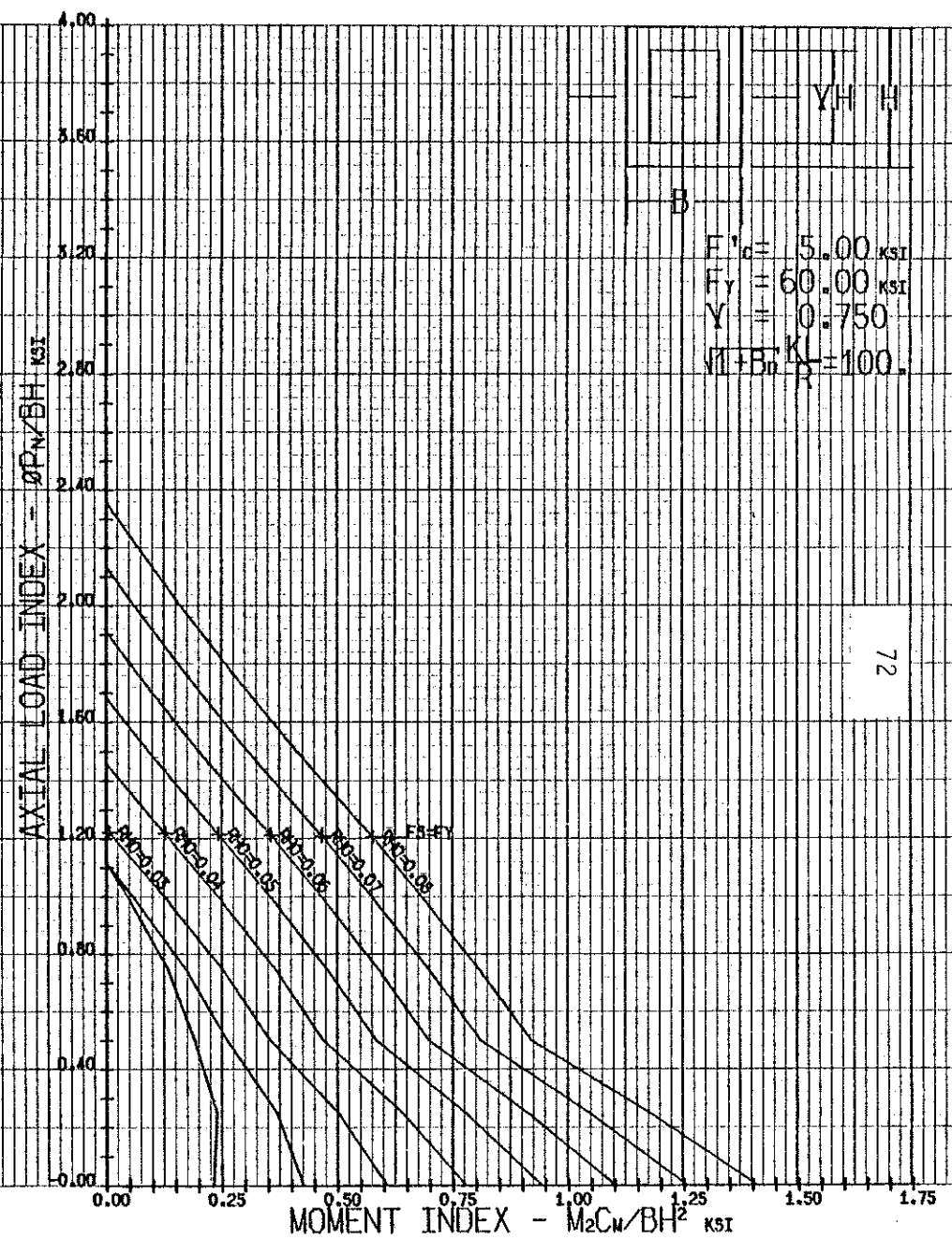


Fig. R5-60.75-100 - Interaction Diagram

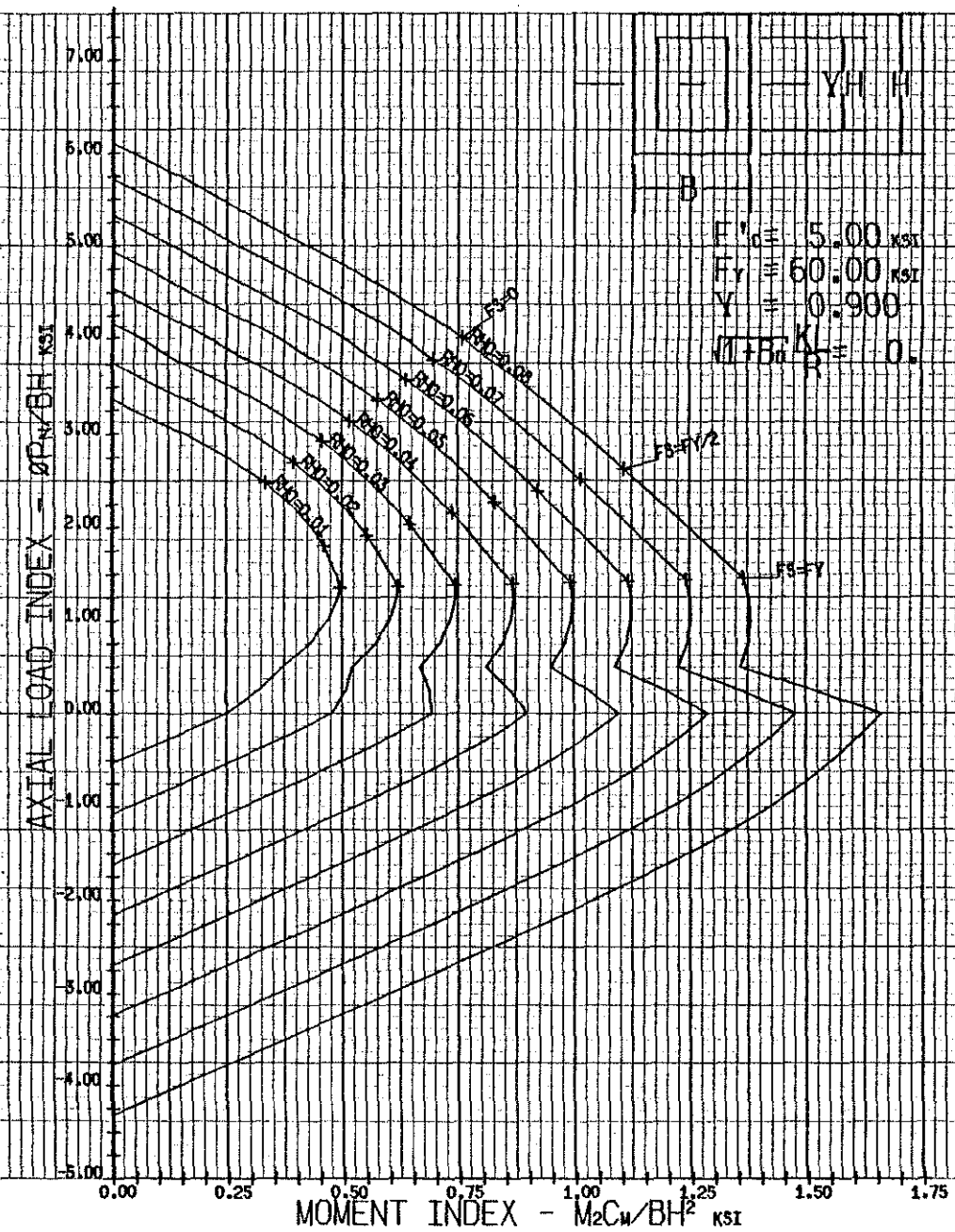


Fig. R5-60.90-0 - Interaction Diagram

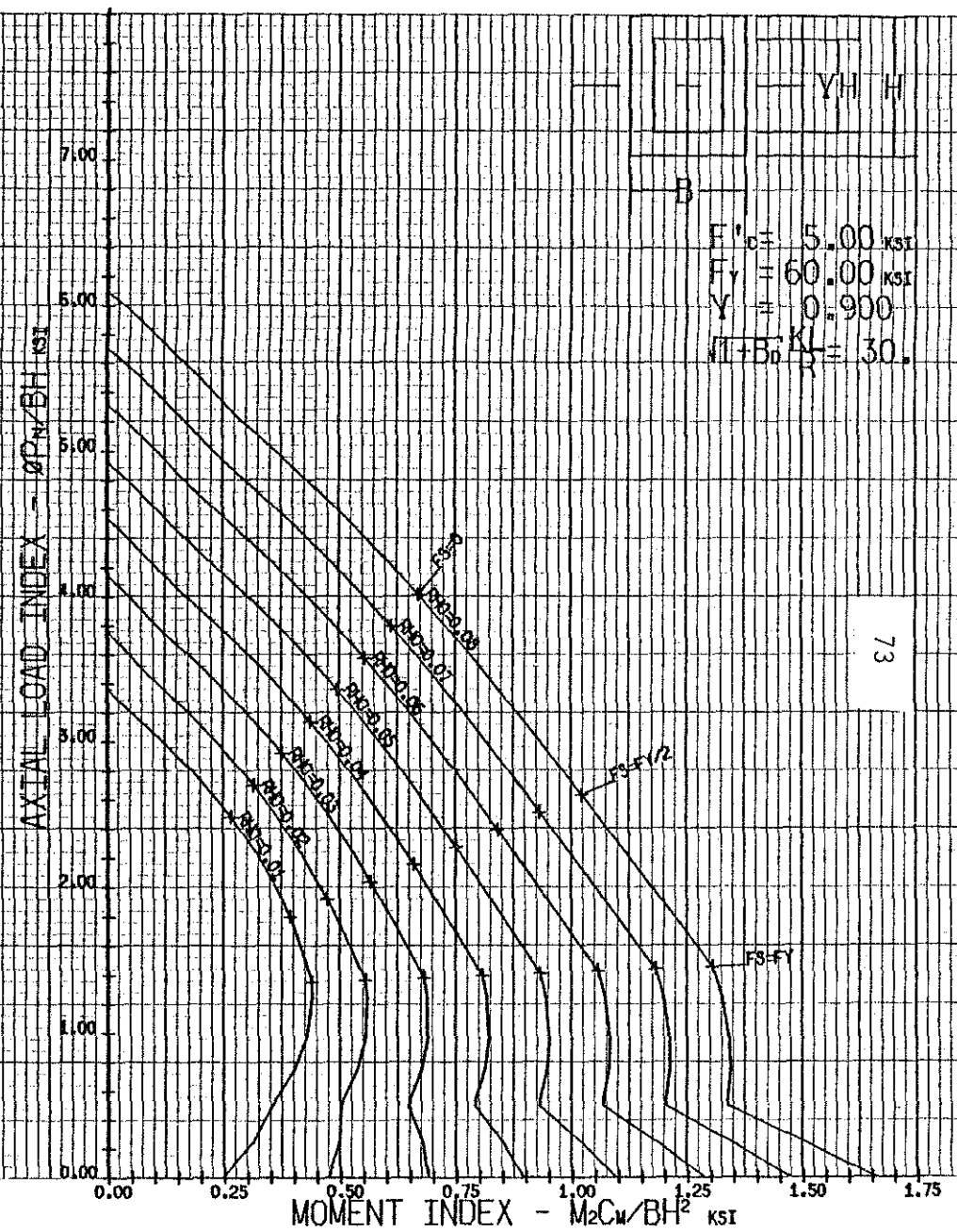


Fig. R5-60.90-30 - Interaction Diagram

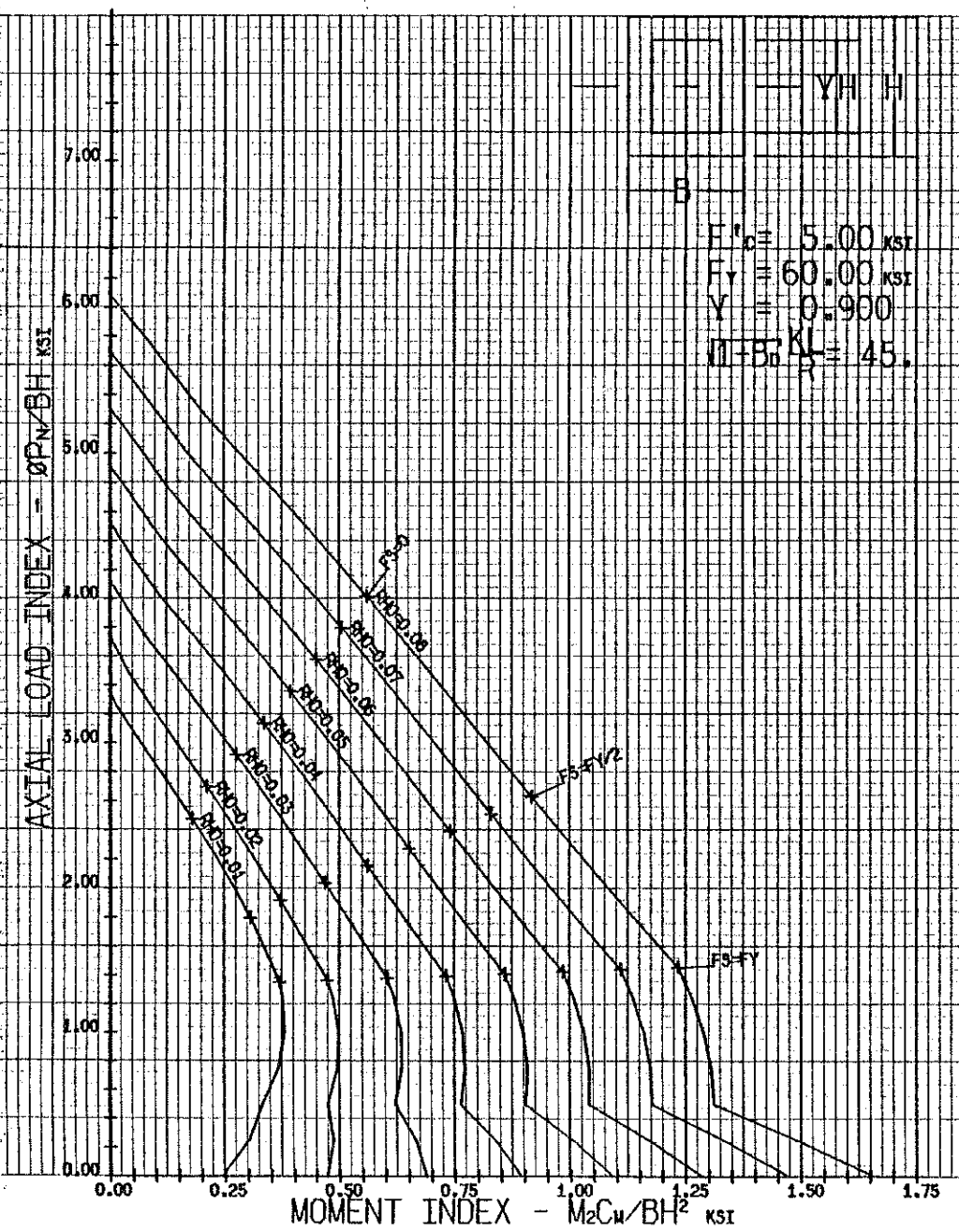


Fig. R5-60.90-45 - Interaction Diagram

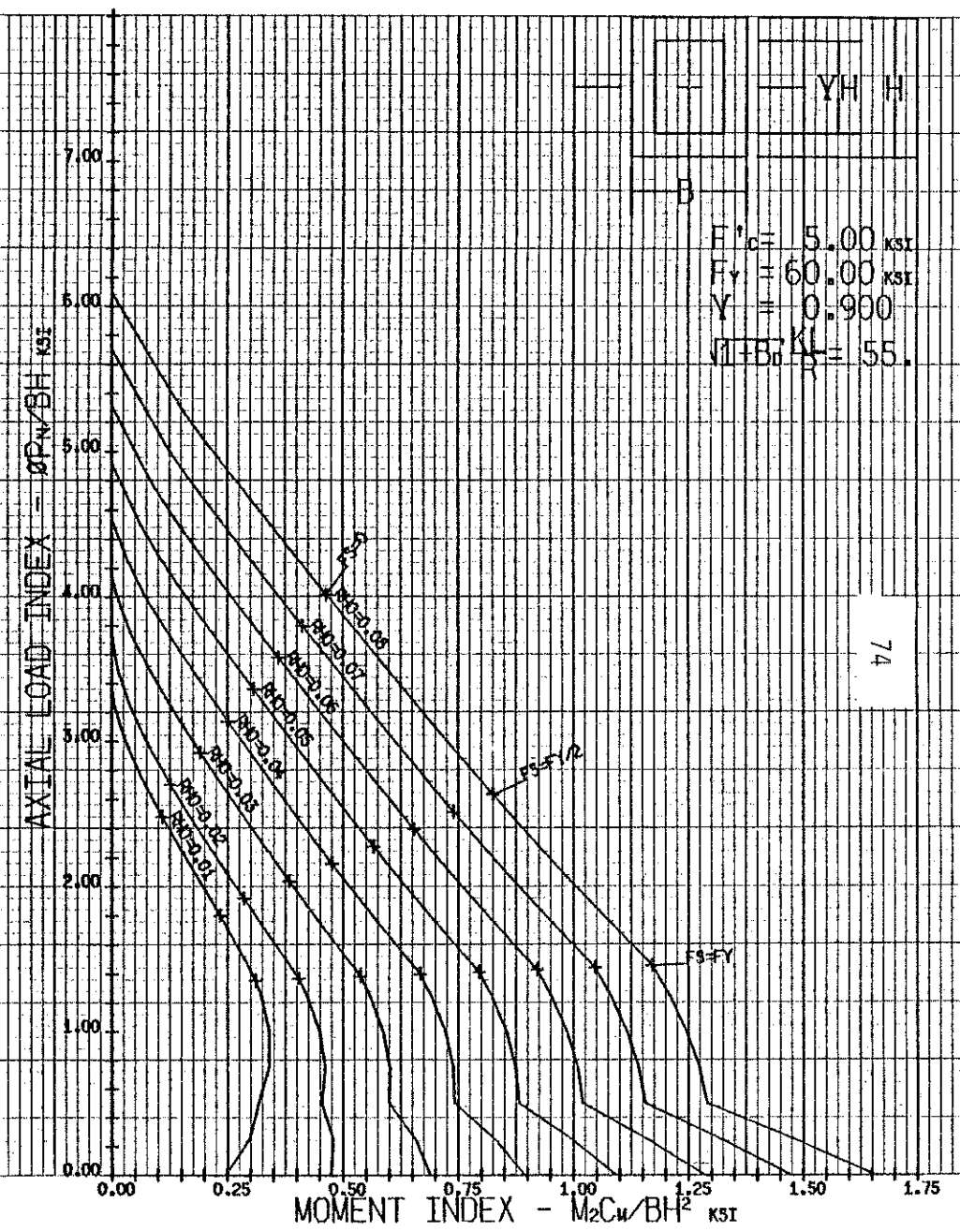


Fig. R5-60.90-55 - Interaction Diagram



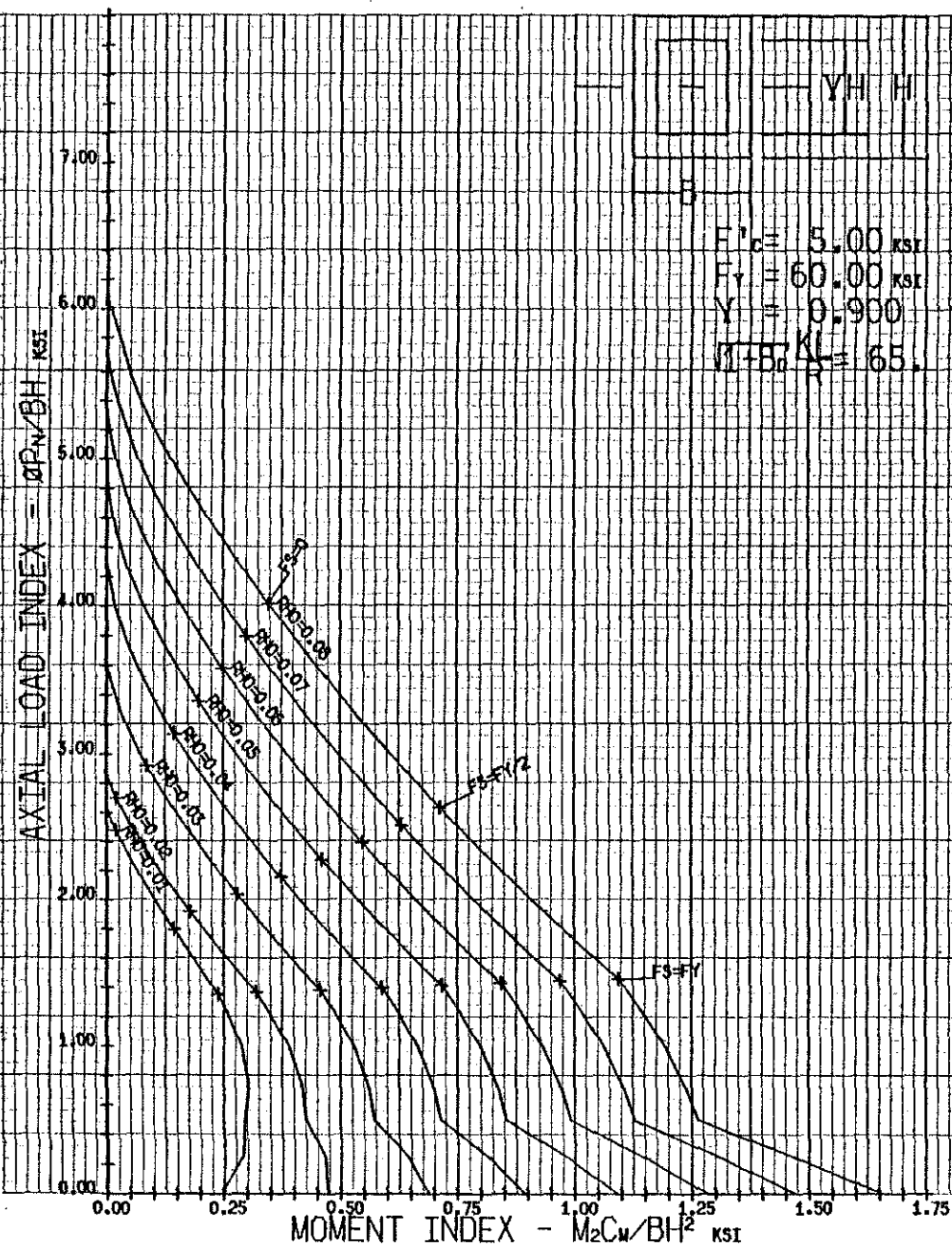


Fig. R5-60.90-65 - Interaction Diagram

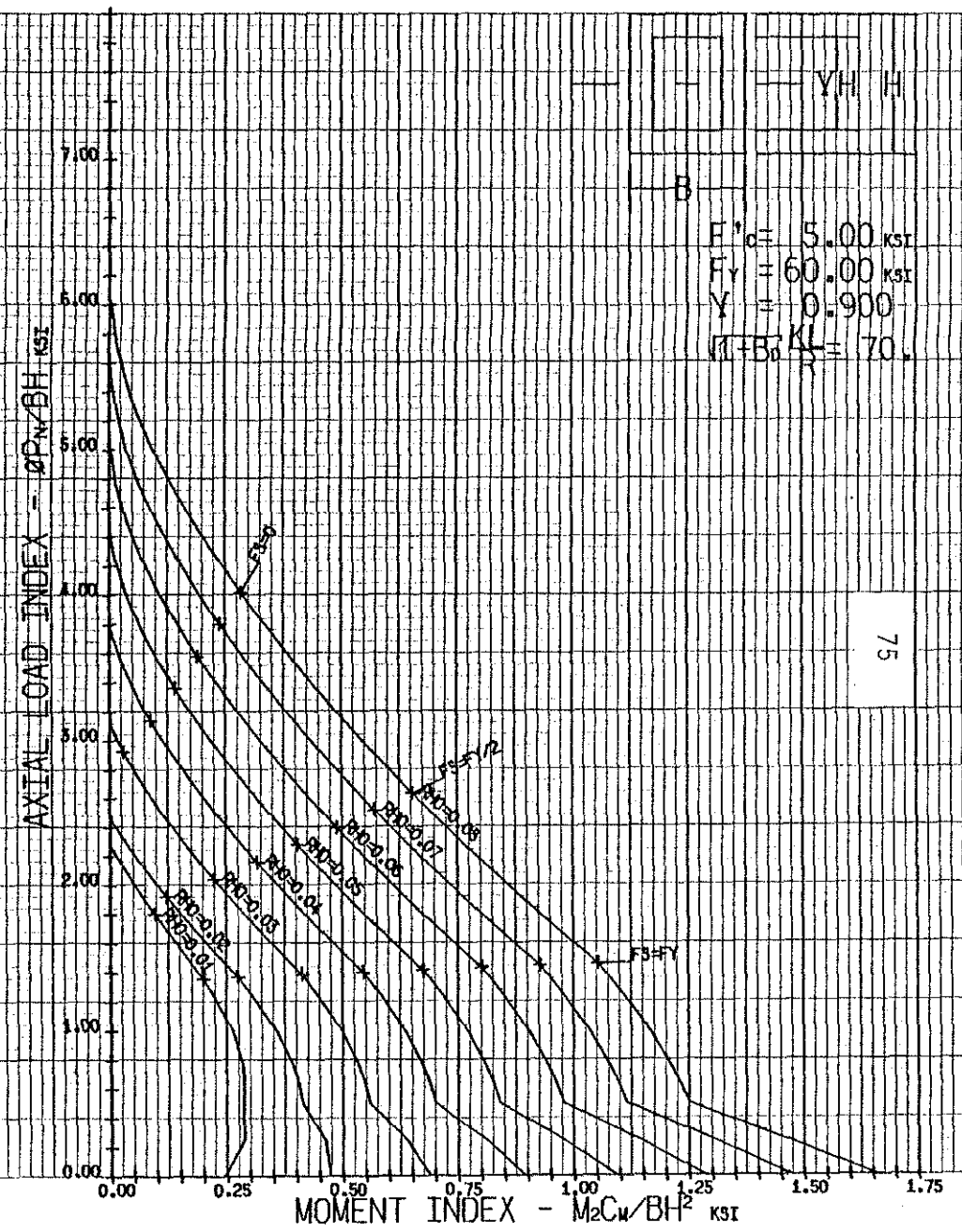


Fig. R5-60.90-70 - Interaction Diagram

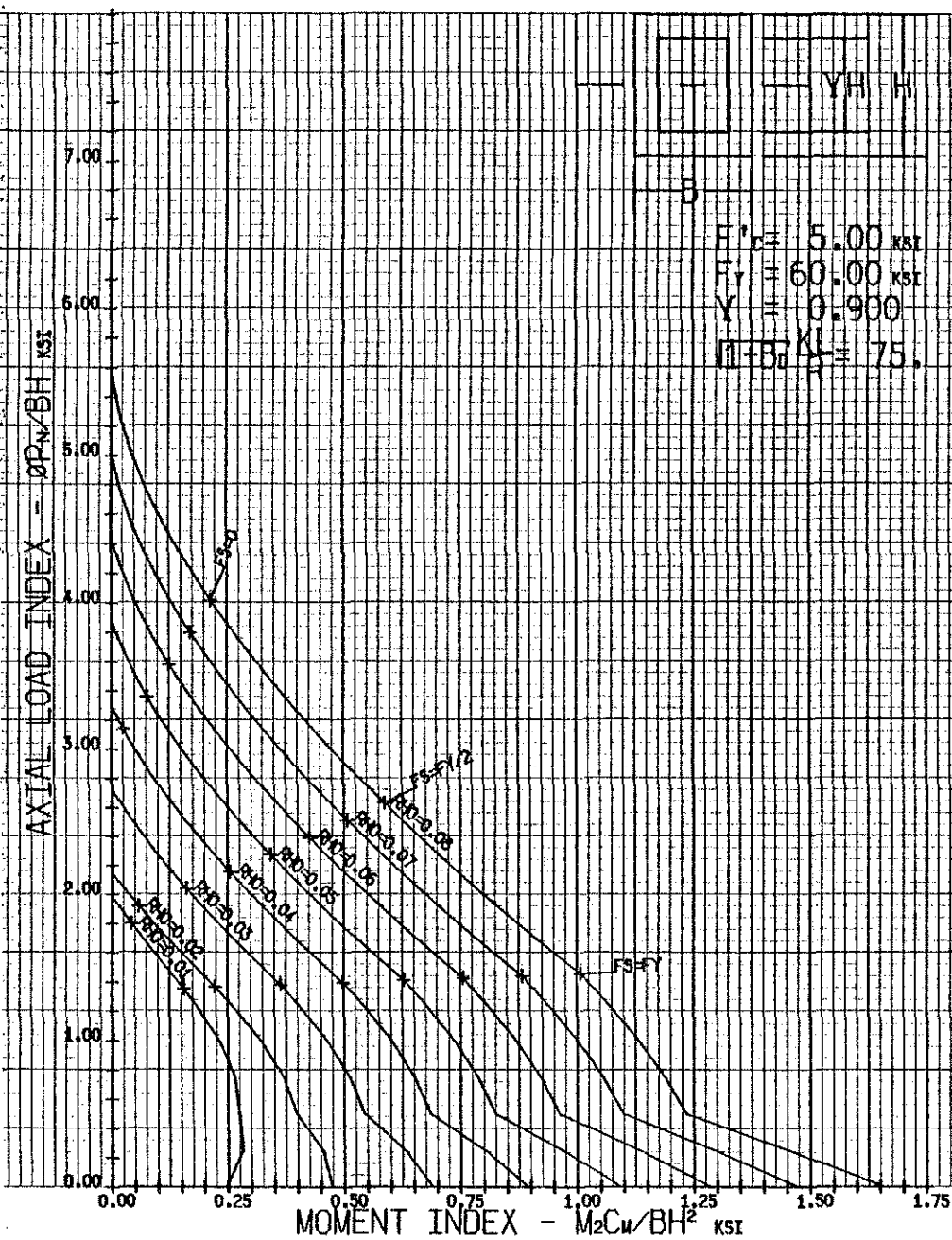


Fig. R5-60.90-75 - Interaction Diagram

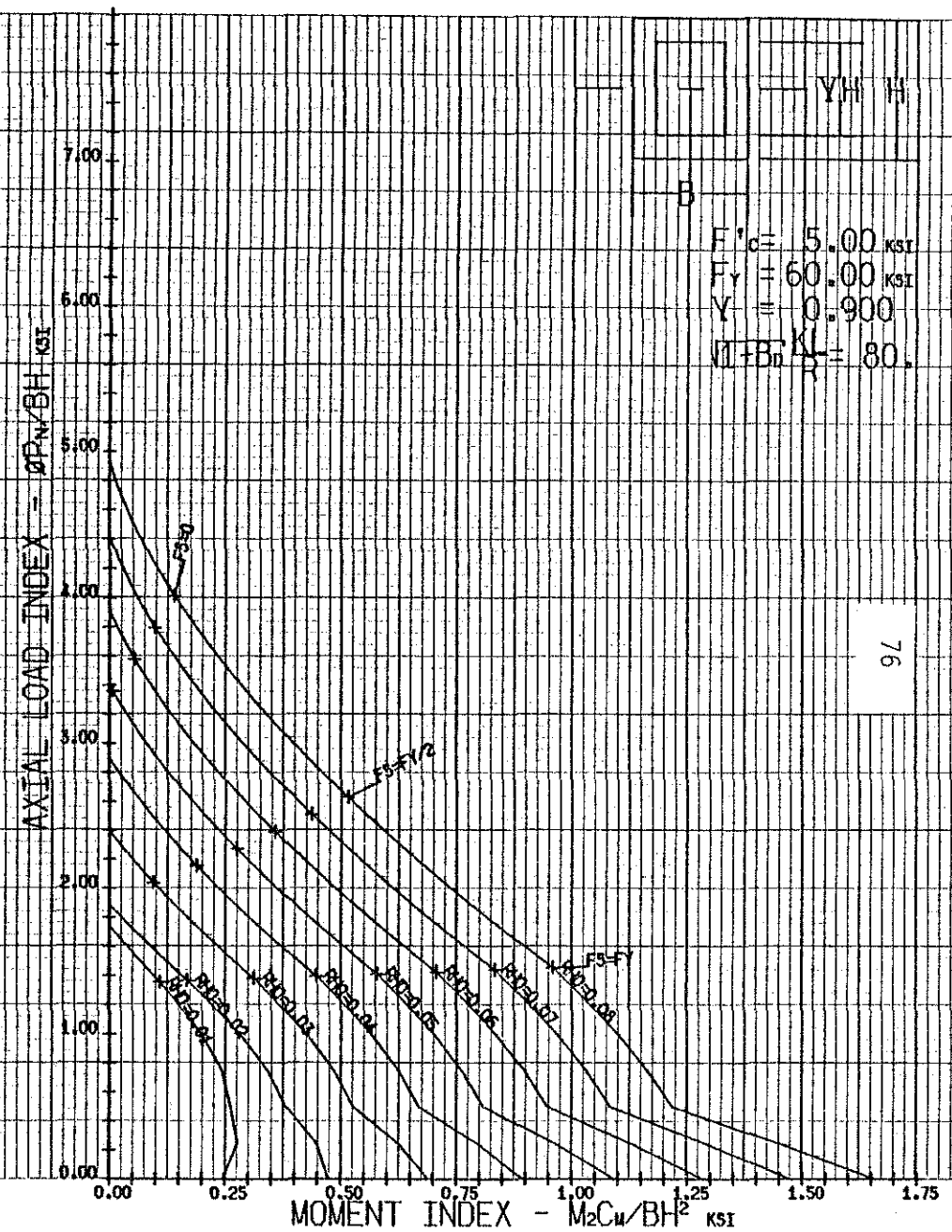


Fig. R5-60.90-80 - Interaction Diagram





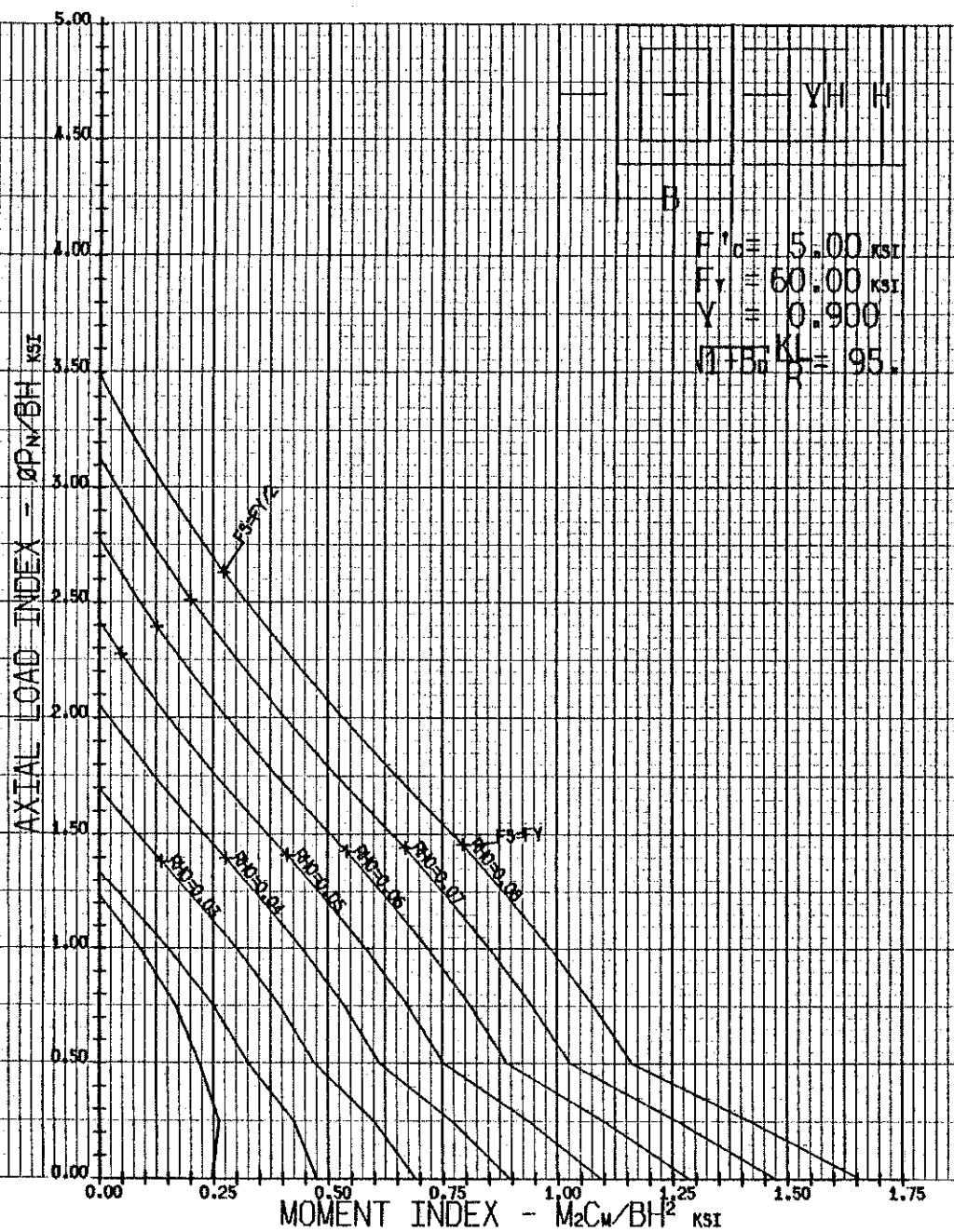


Fig. R5-60.90-95 - Interaction Diagram

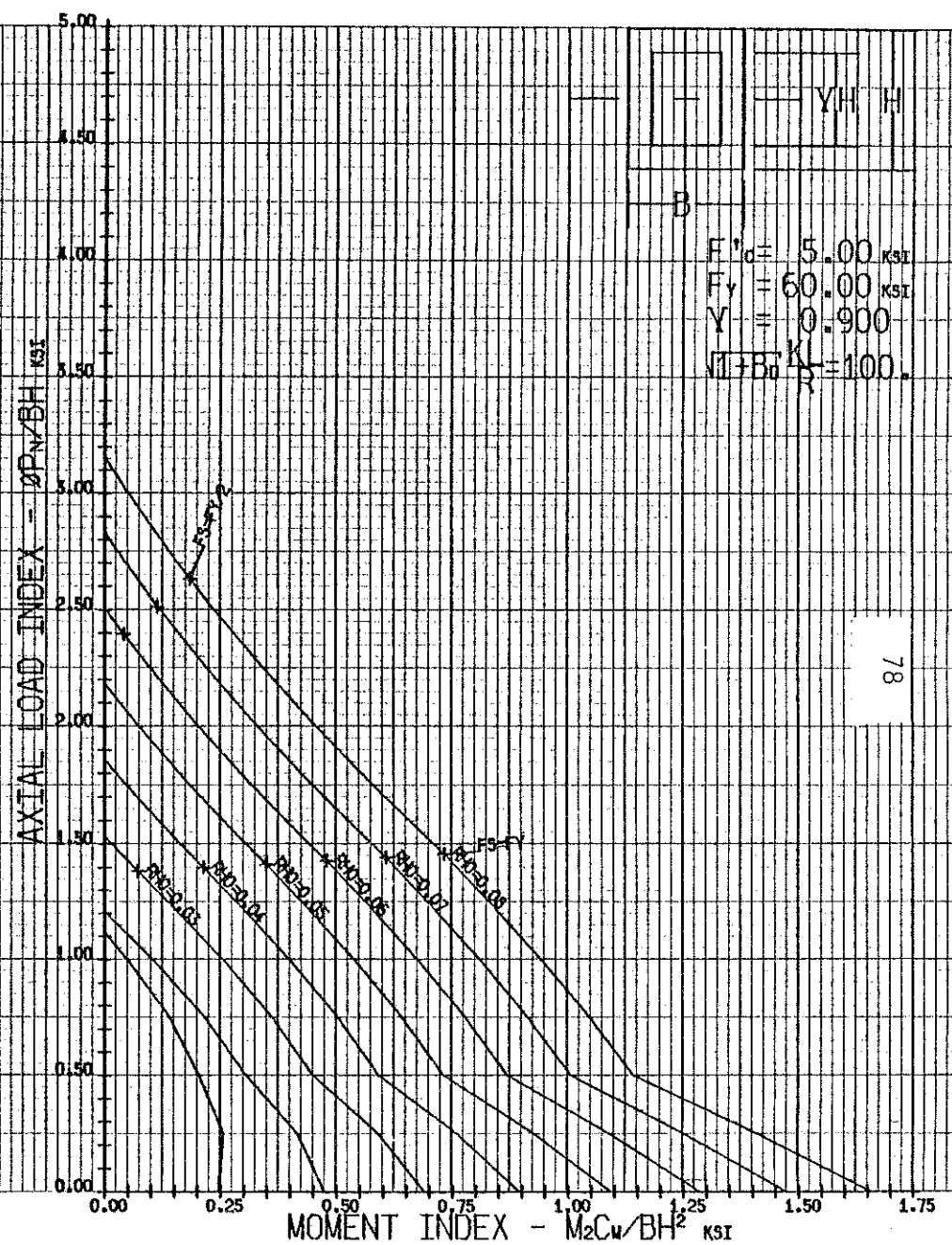


Fig. R5-60.90-100 - Interaction Diagram

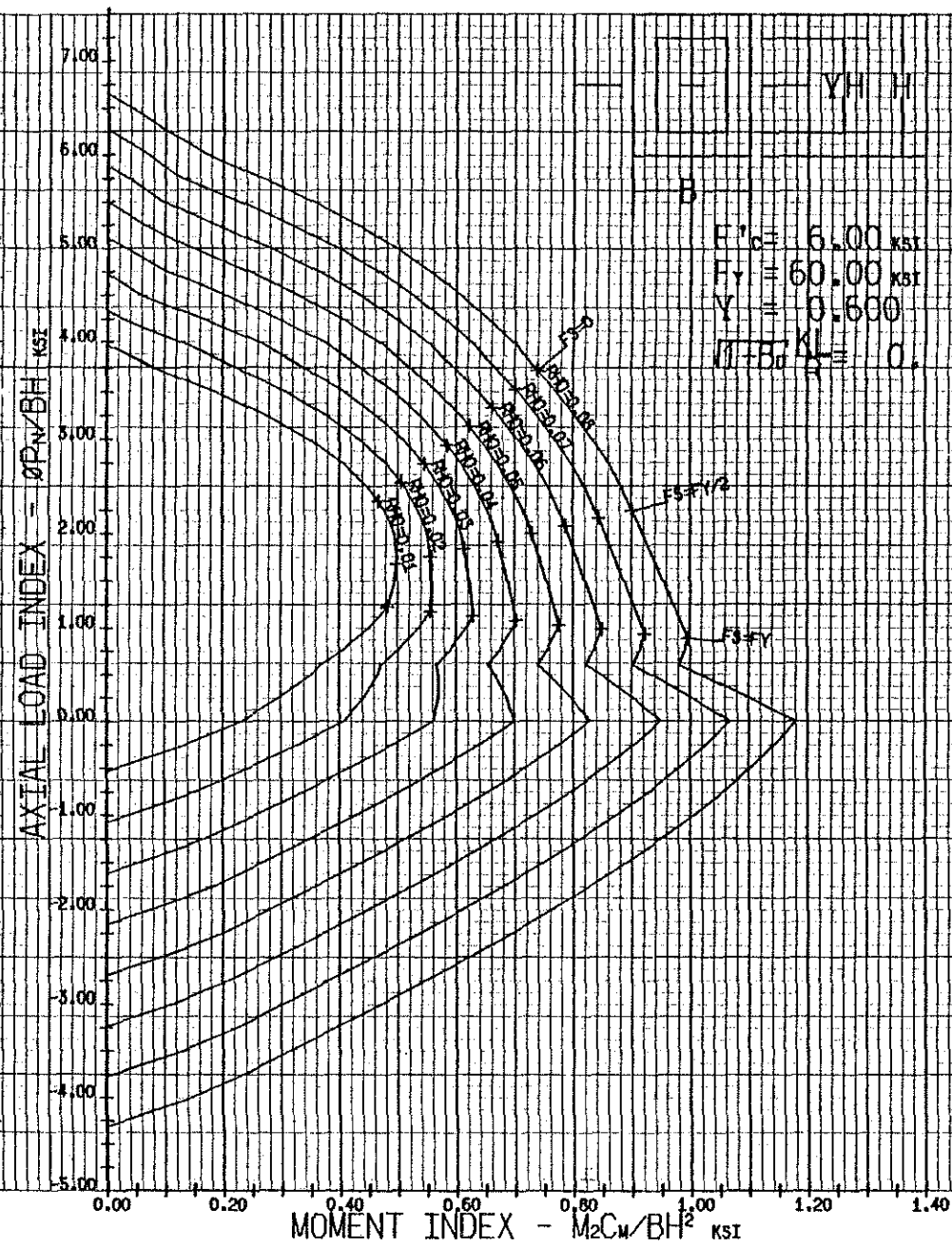


Fig. R6-60.60-0 - Interaction Diagram

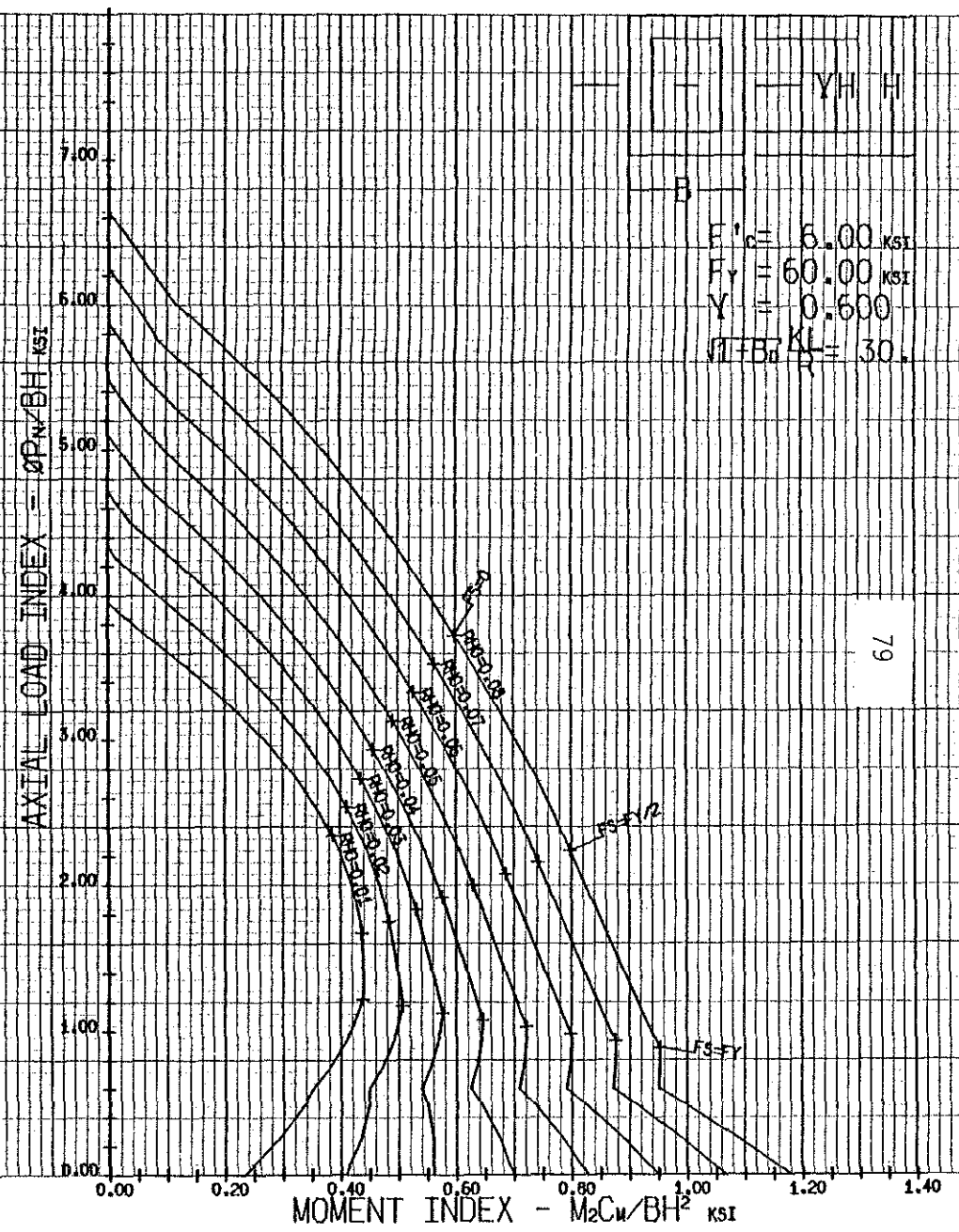


Fig. R6-60.60-30 - Interaction Diagram

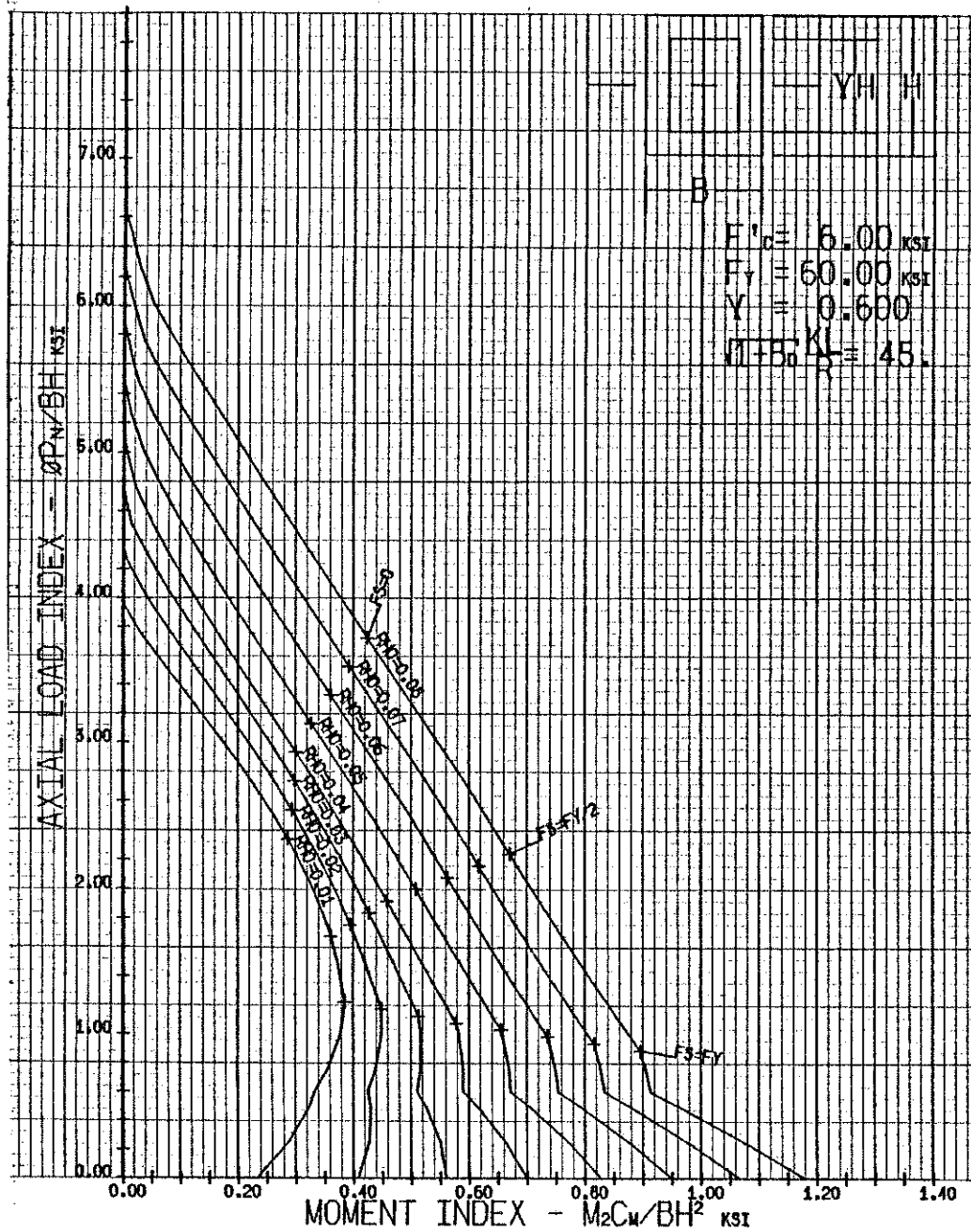


Fig. R6-60.60-45 - Interaction Diagram

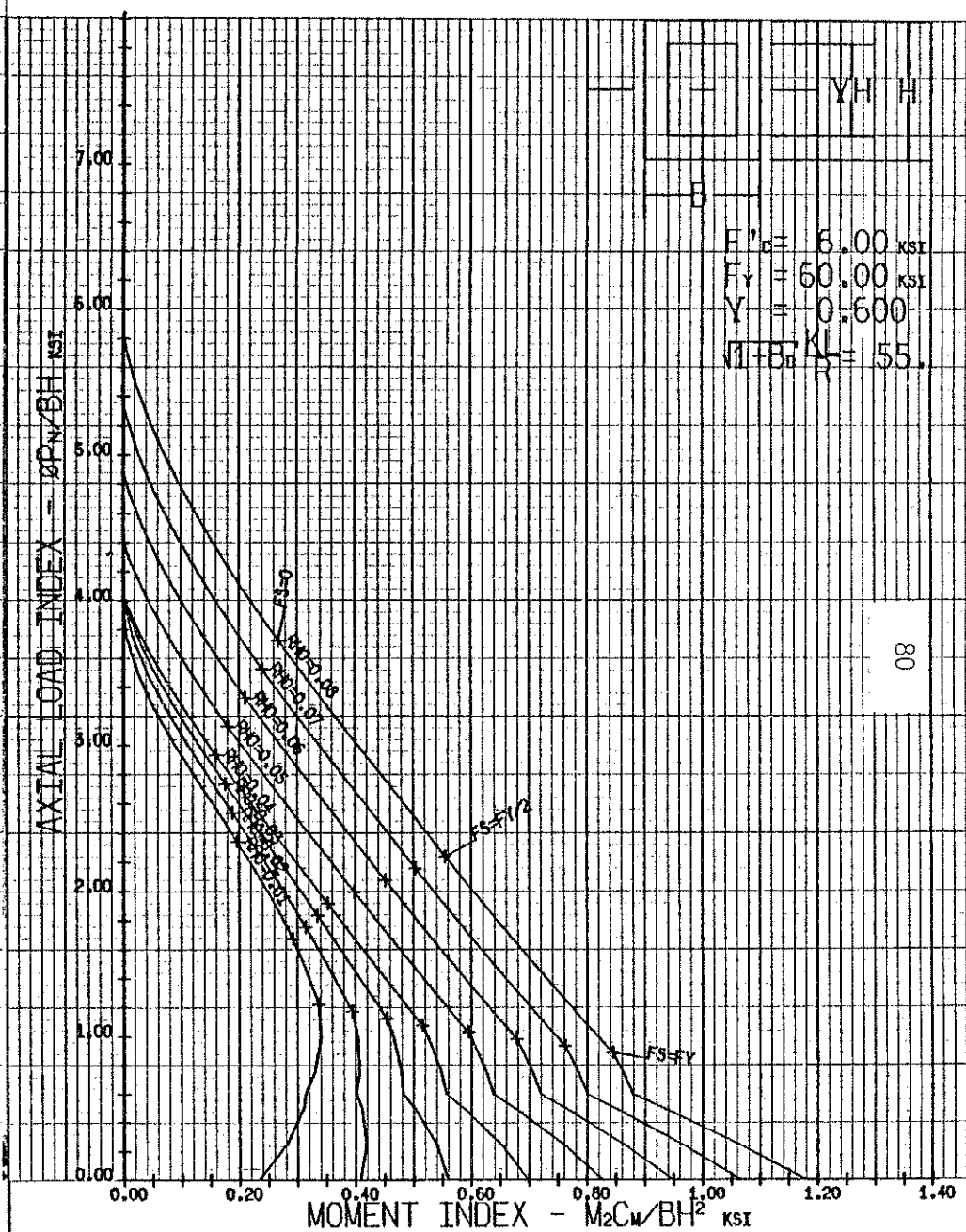


Fig. R6-60.60-55 - Interaction Diagram

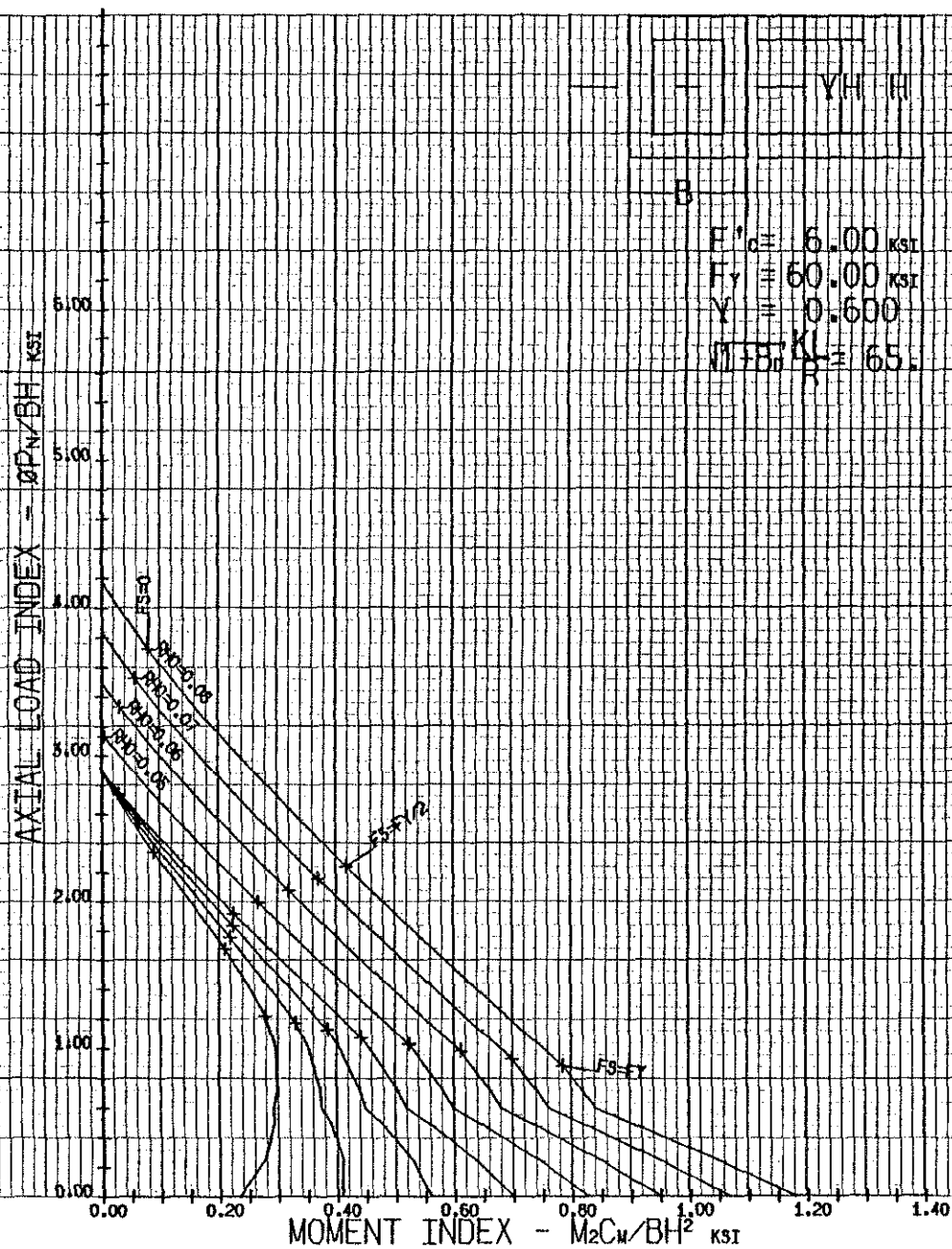


Fig. R6-60.60-65 - Interaction Diagram

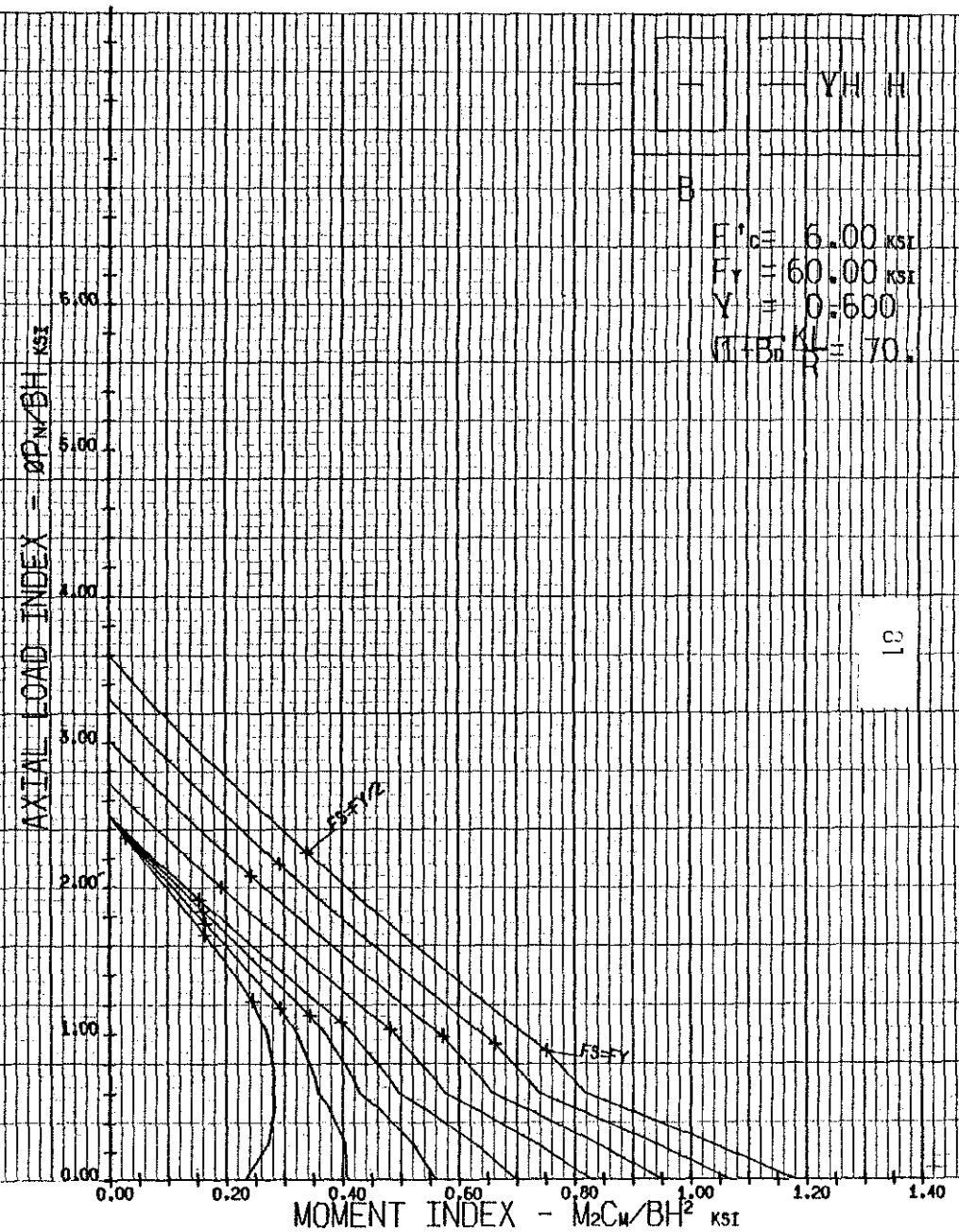


Fig. R6-60.60-70 - Interaction Diagram

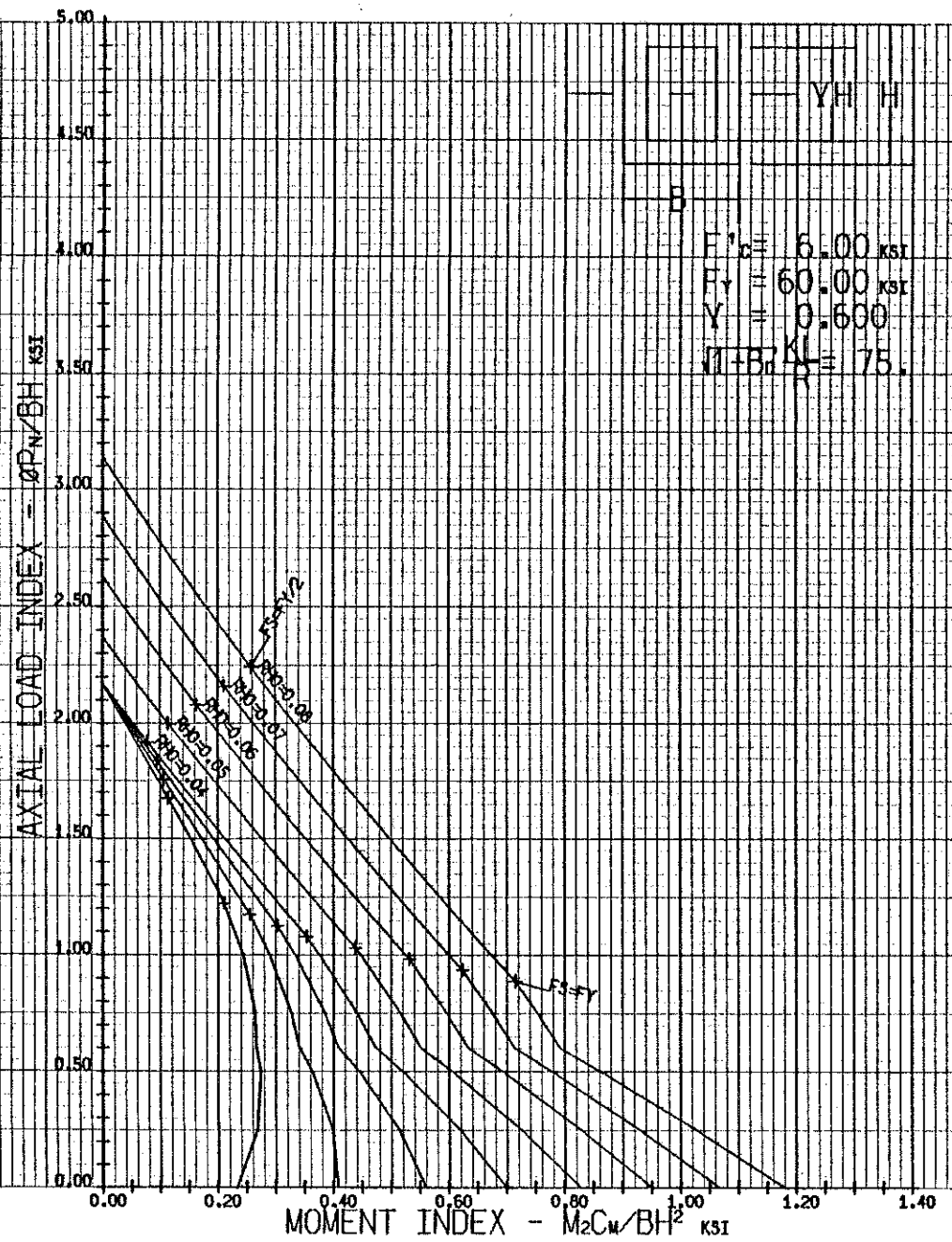


Fig. R6-60.60-75 - Interaction Diagram

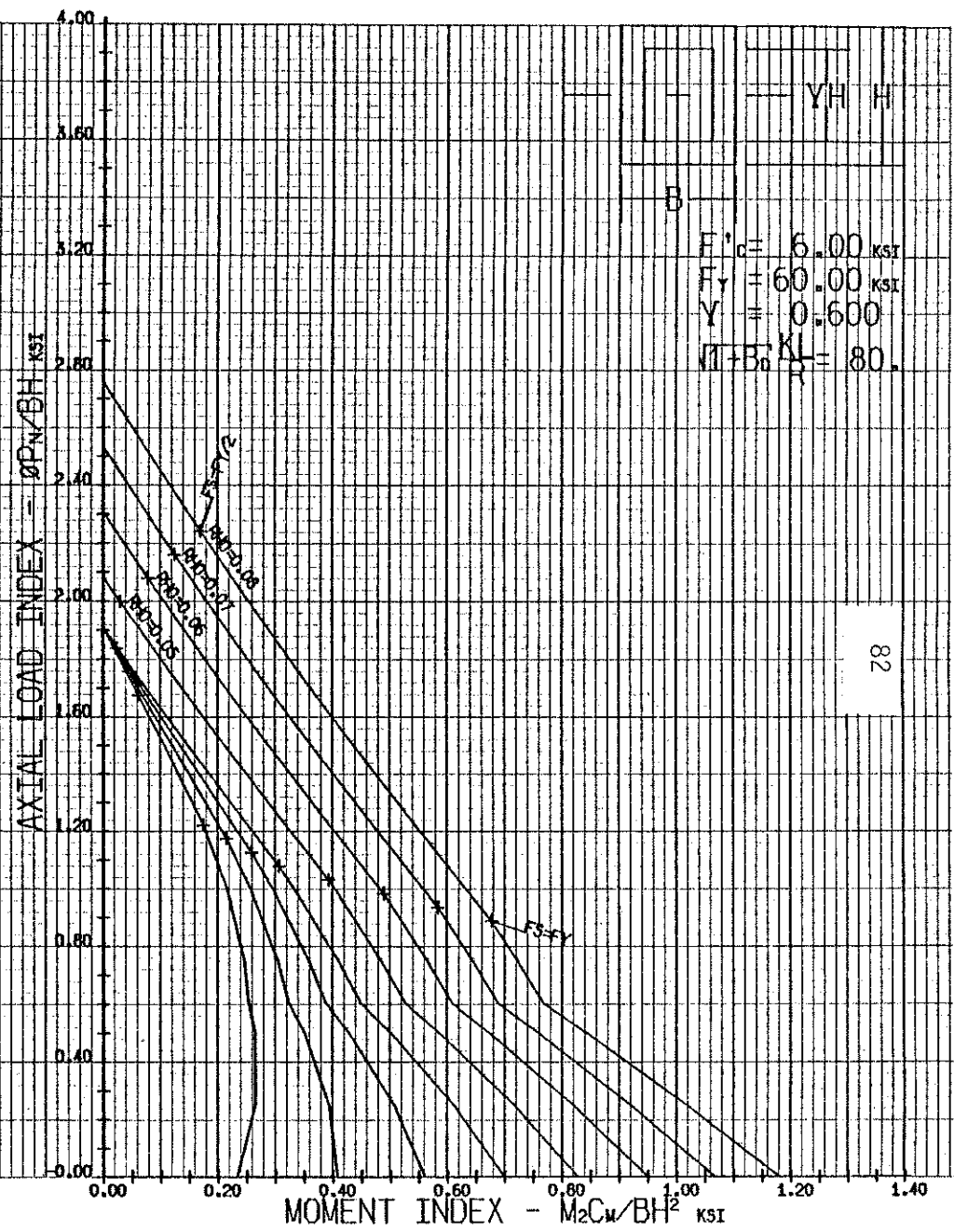


Fig. R6-60.60-80 - Interaction Diagram



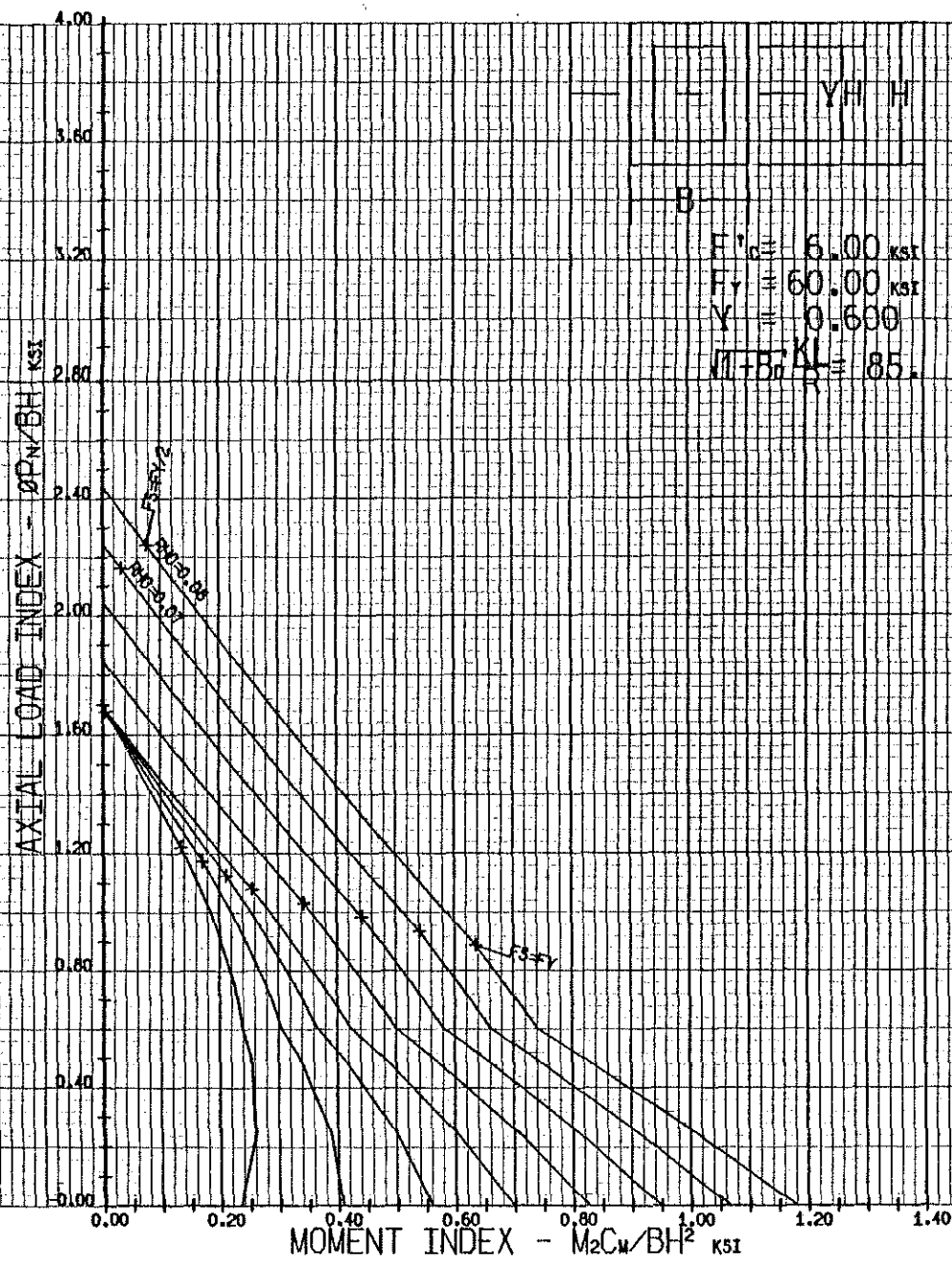


Fig. R6-60.60-85 - Interaction Diagram

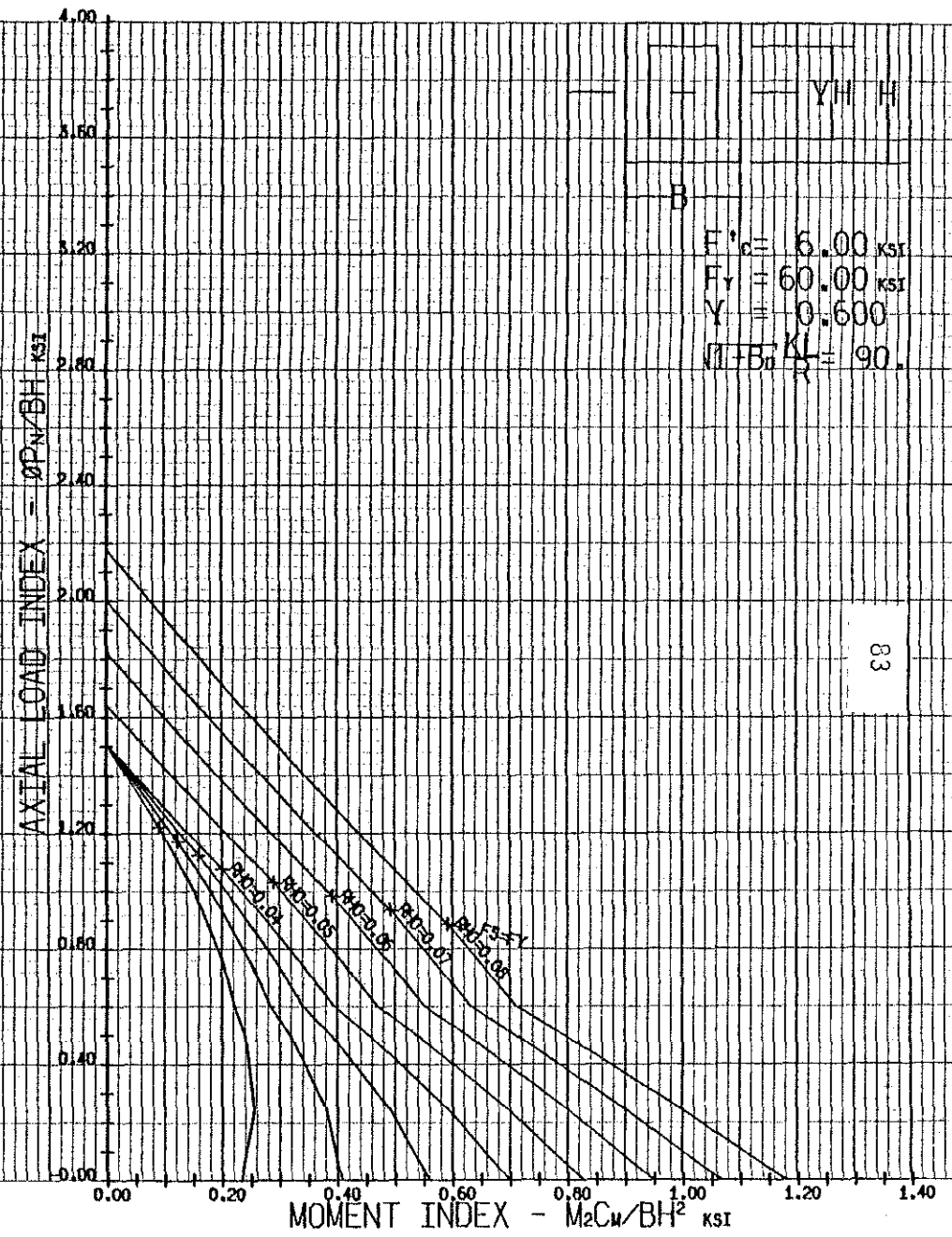


Fig. R6-60.60-90 - Interaction Diagram

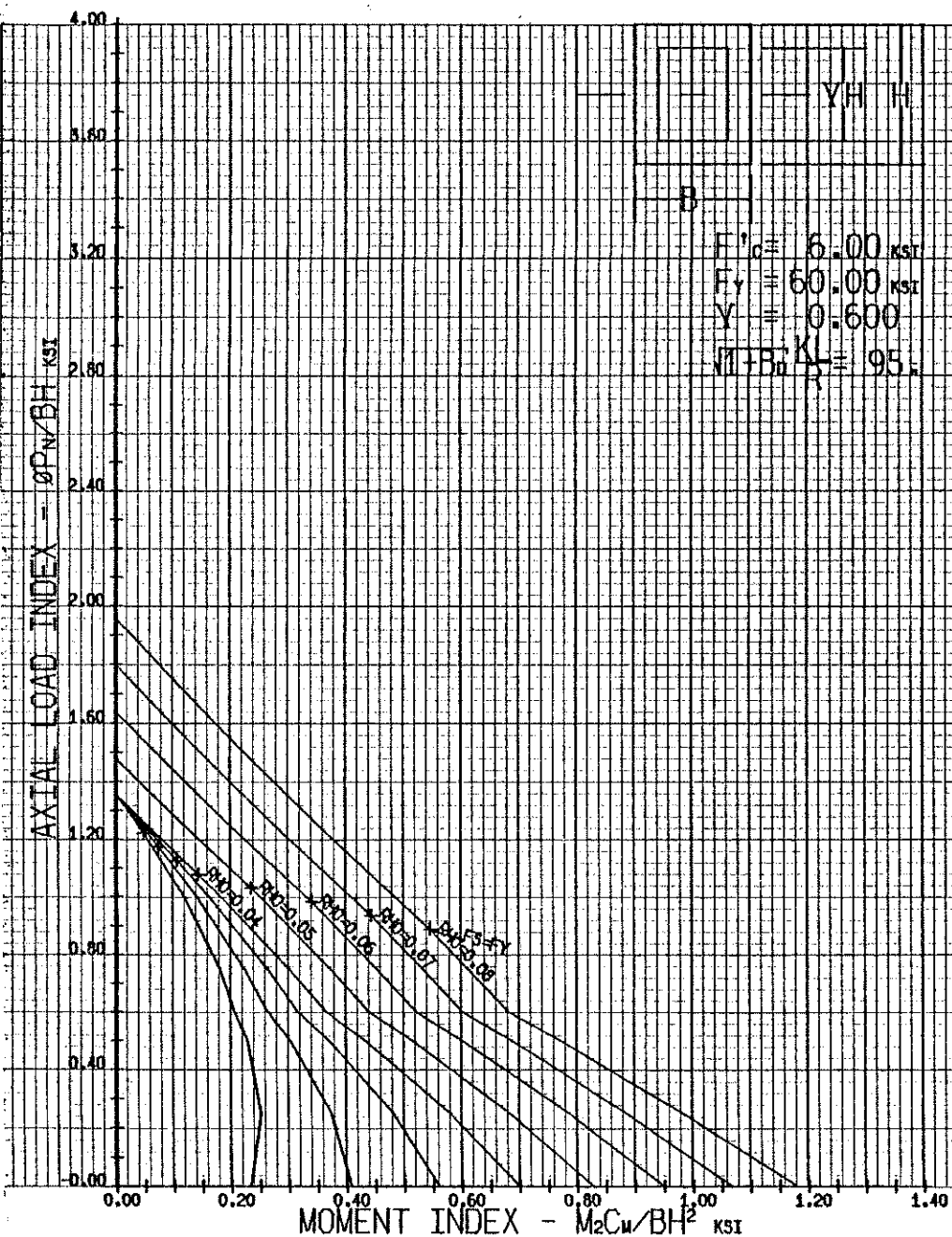


Fig. R6-60.60-95 - Interaction Diagram

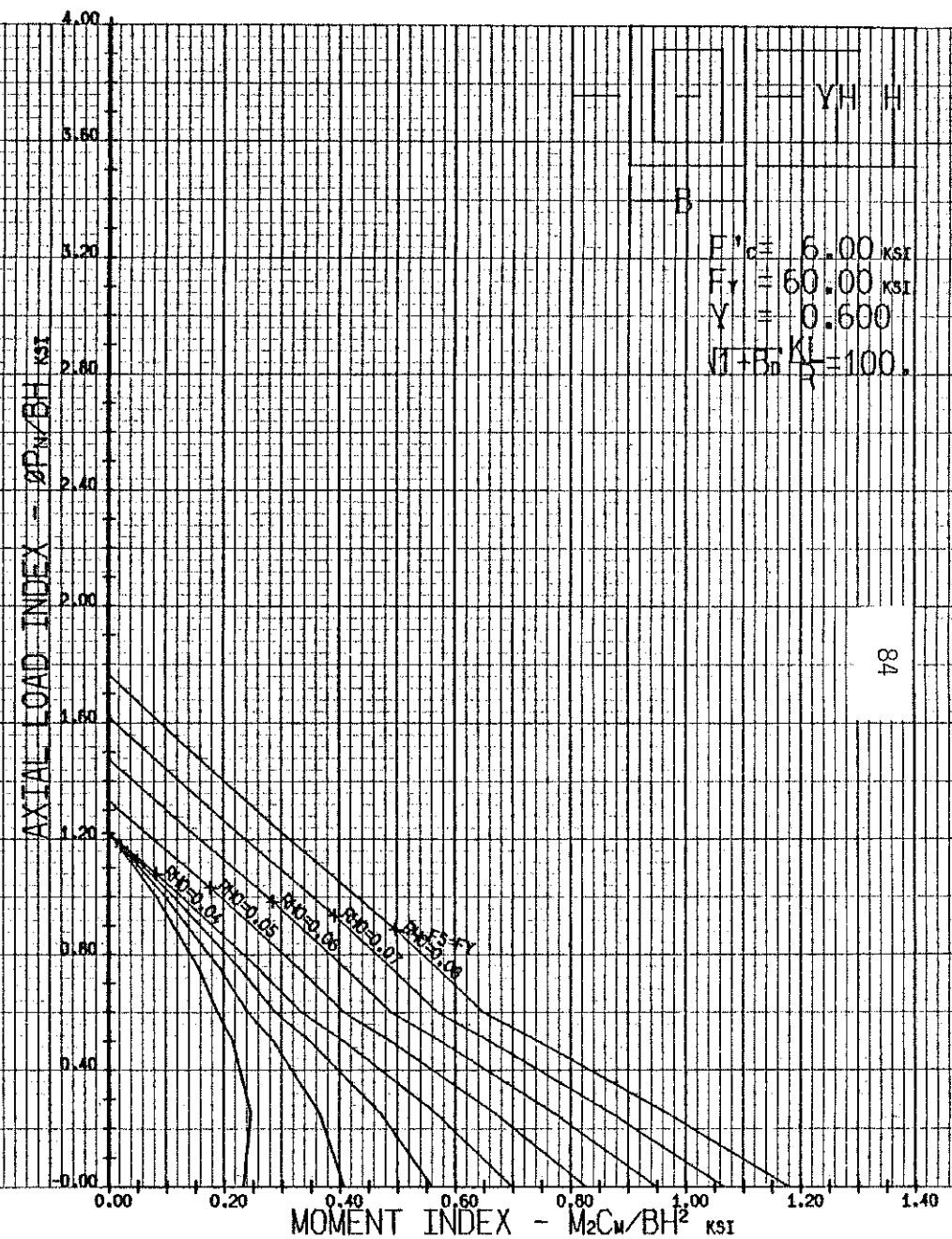


Fig. R6-60.60-100 - Interaction Diagram

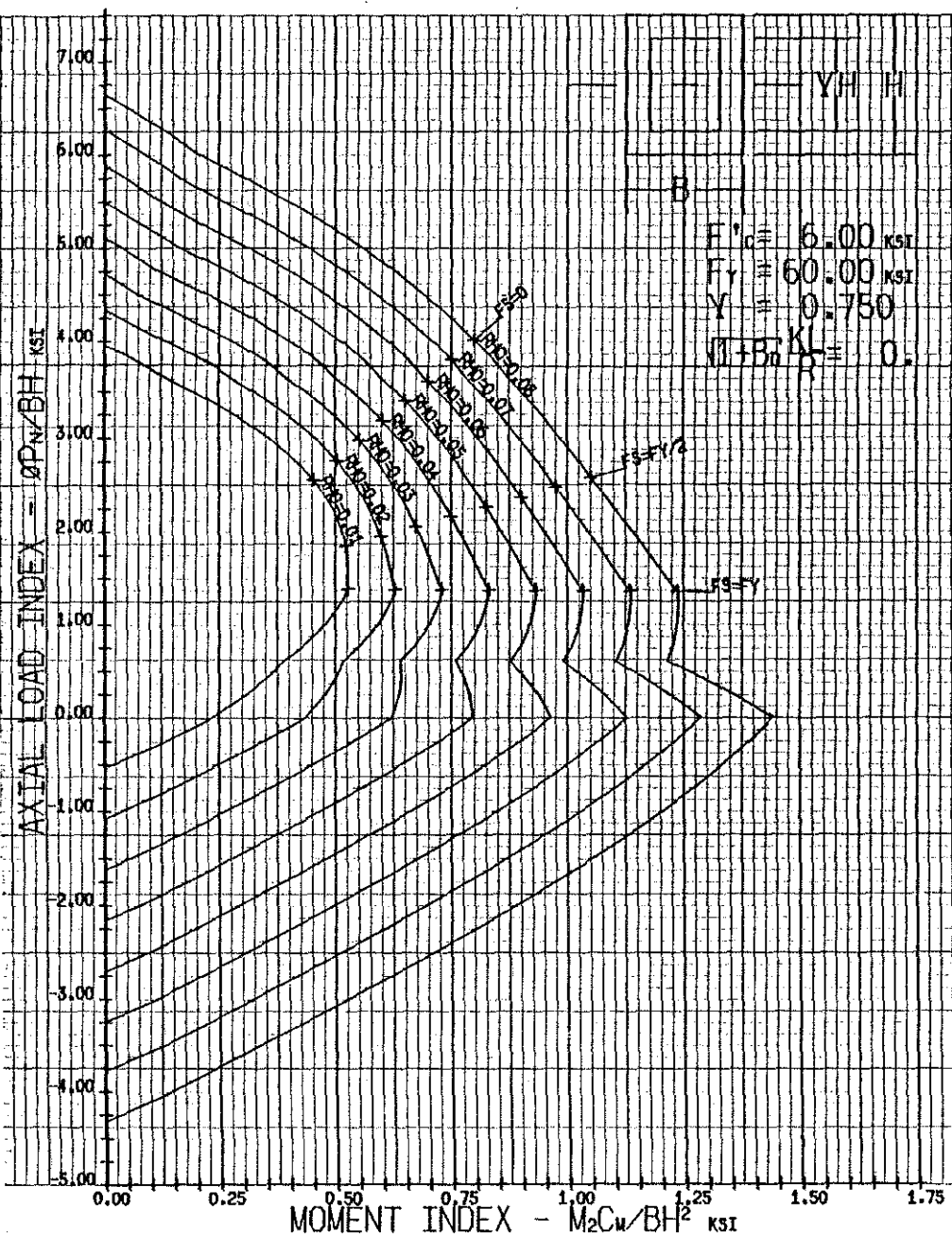


Fig. R6-60.75-0 - Interaction Diagram

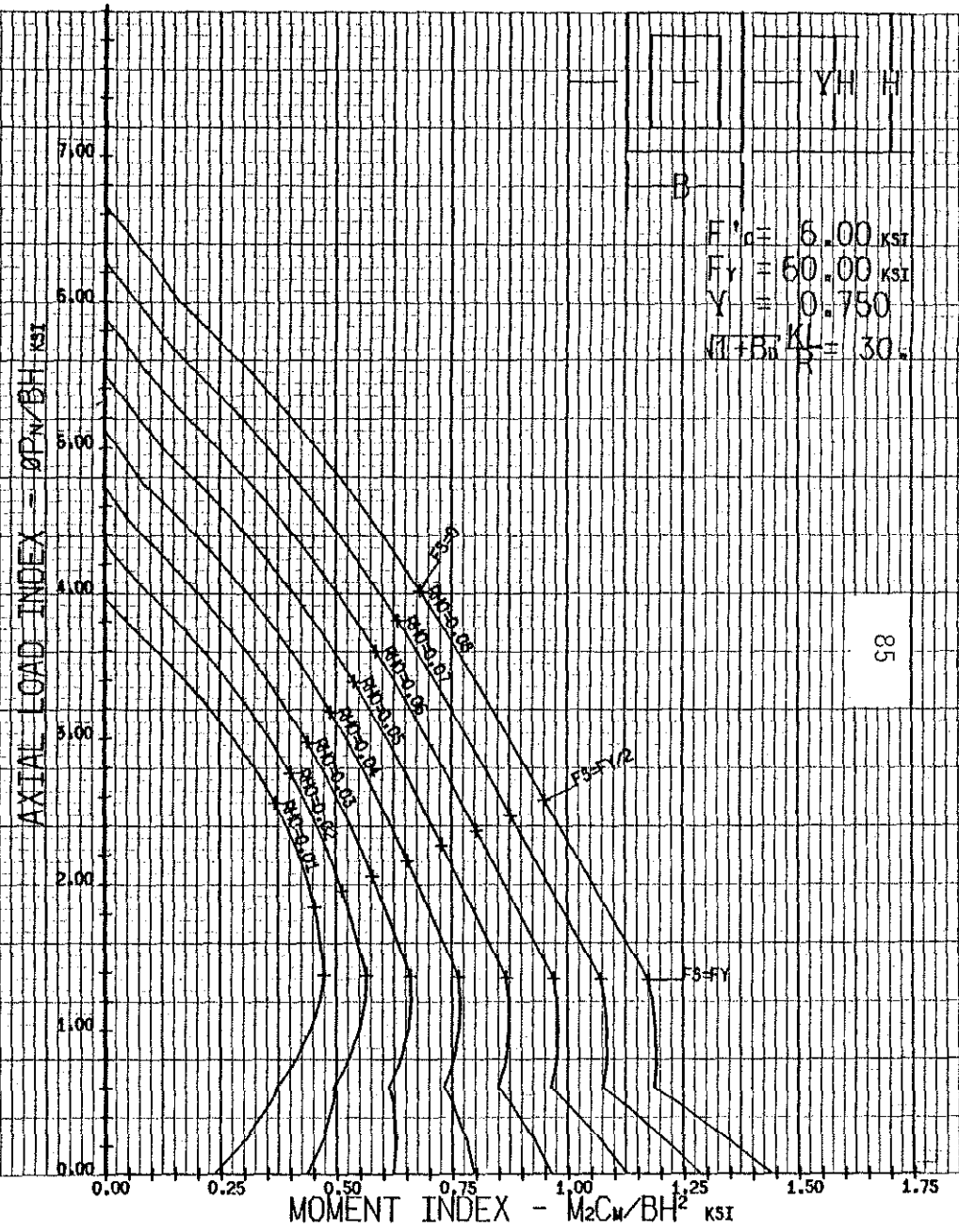


Fig. R6-60.75-30 - Interaction Diagram



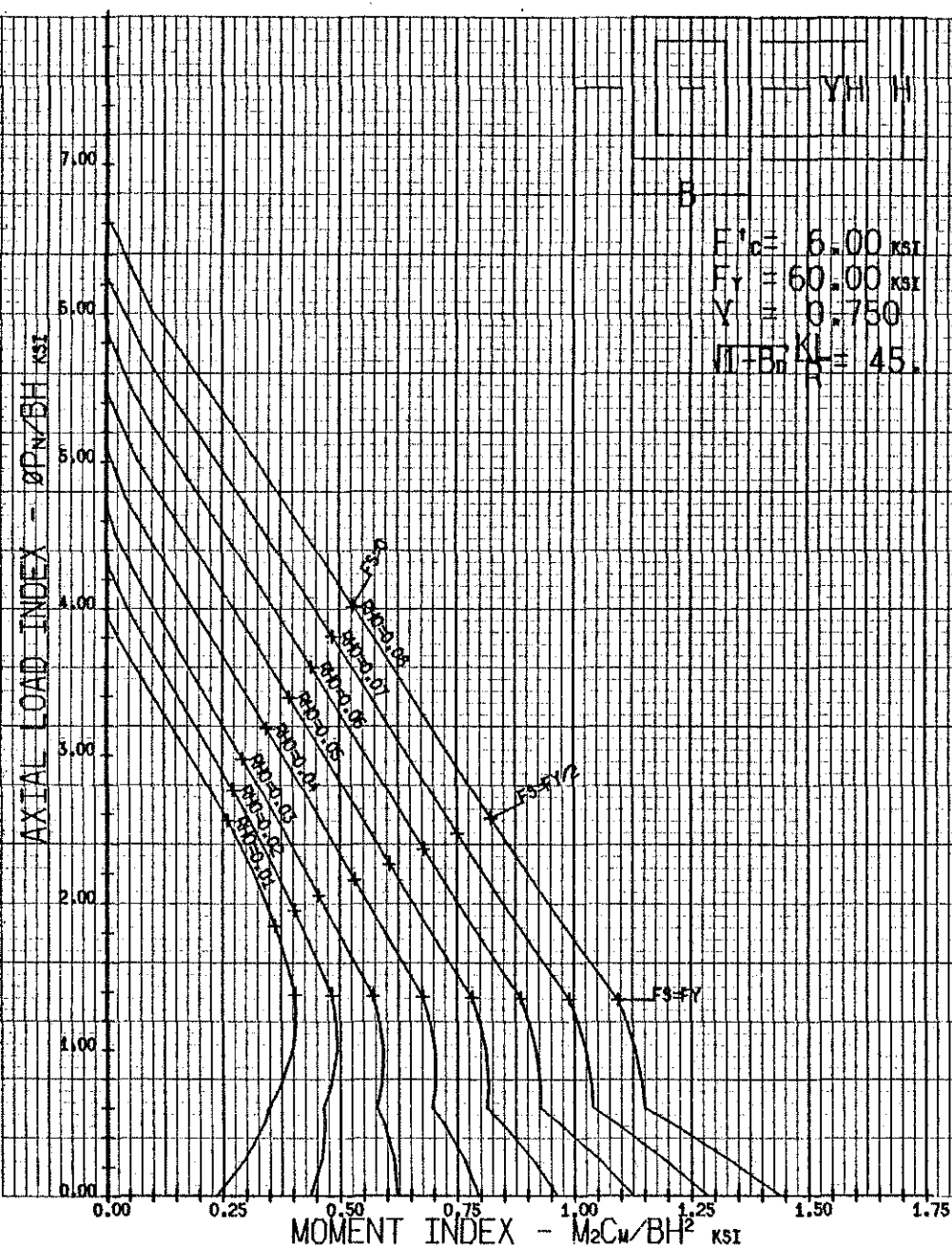


Fig. R6-60.75-45 - Interaction Diagram

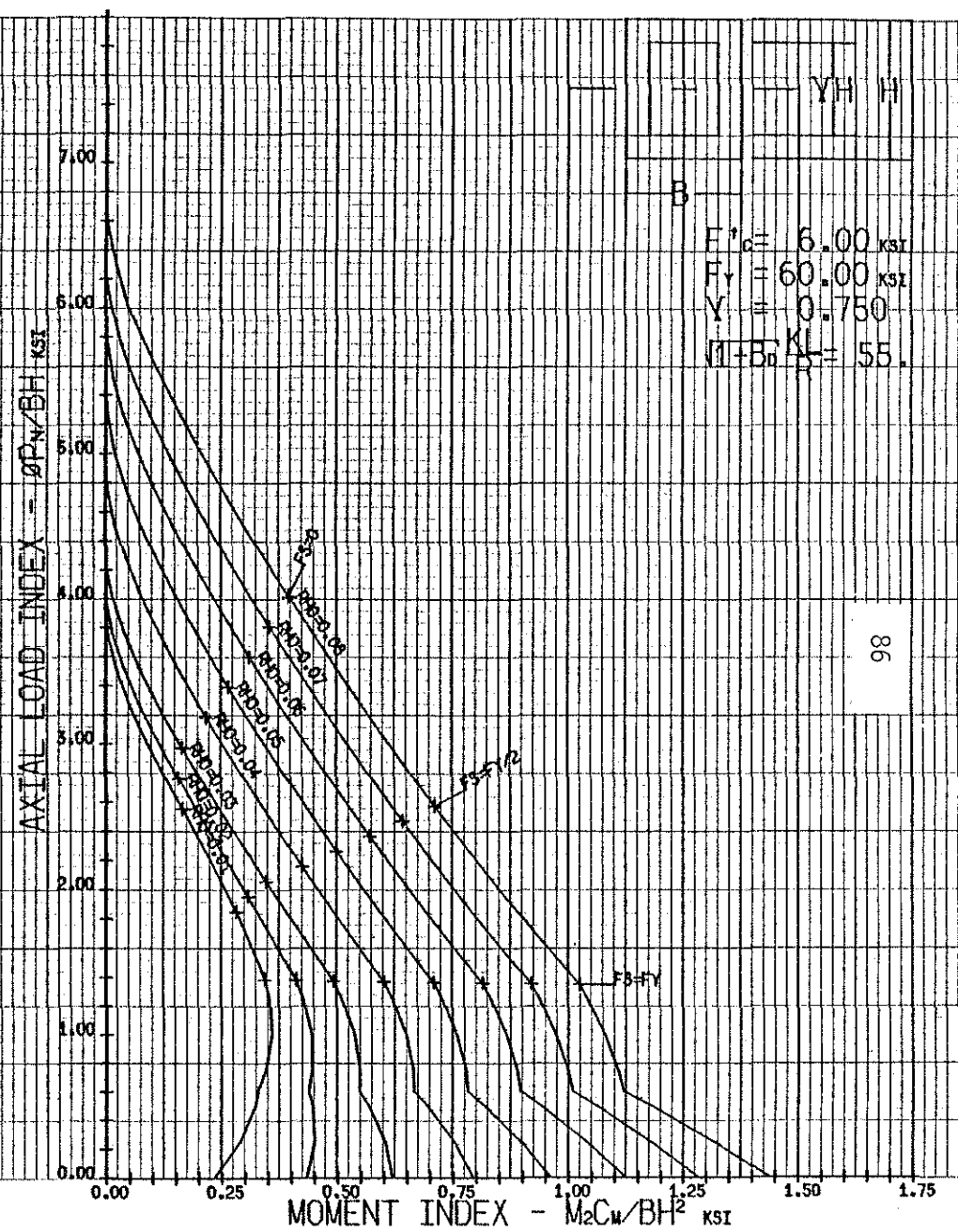


Fig. R6-60.75-55 - Interaction Diagram

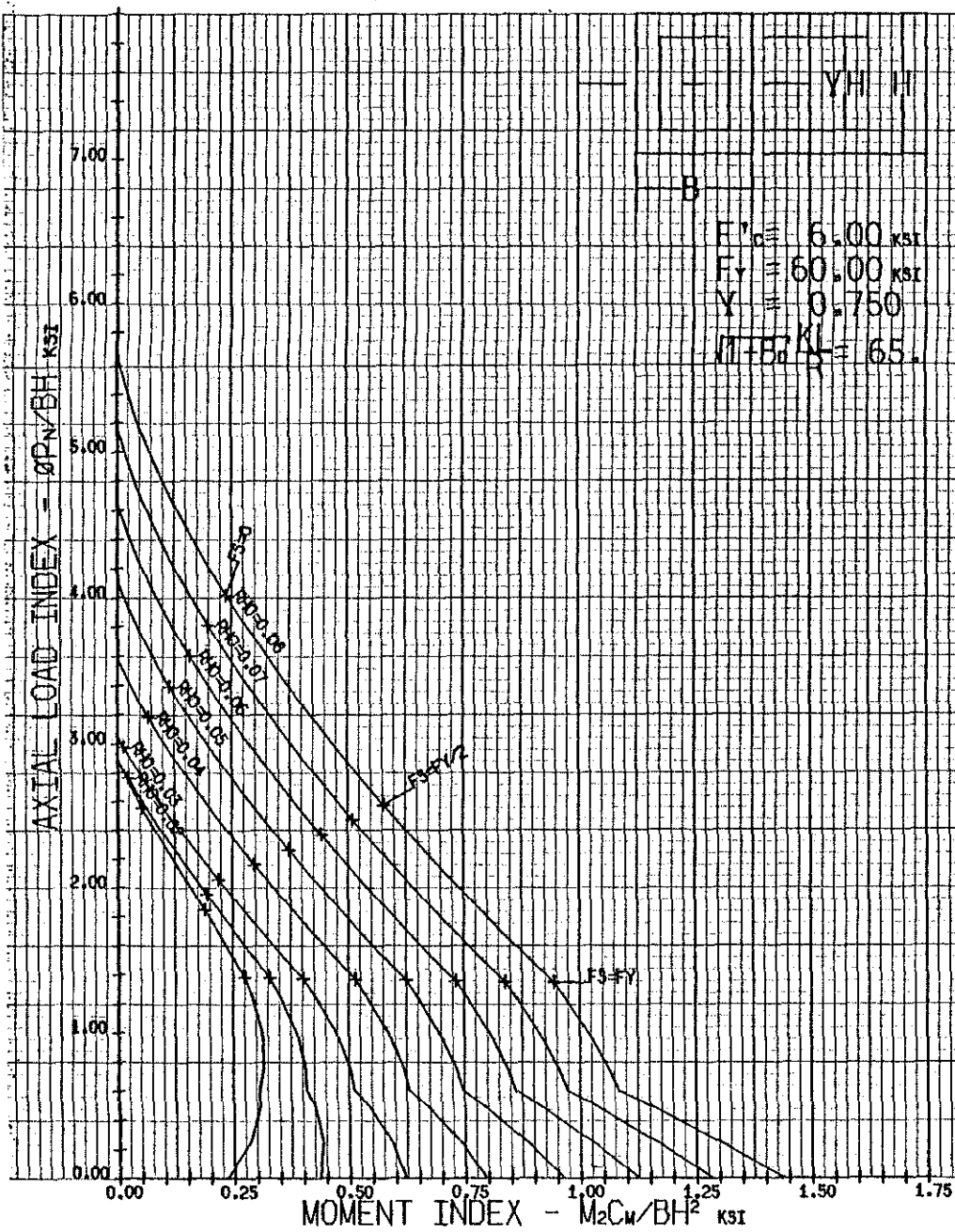


Fig. R6-60.75-65 - Interaction Diagram

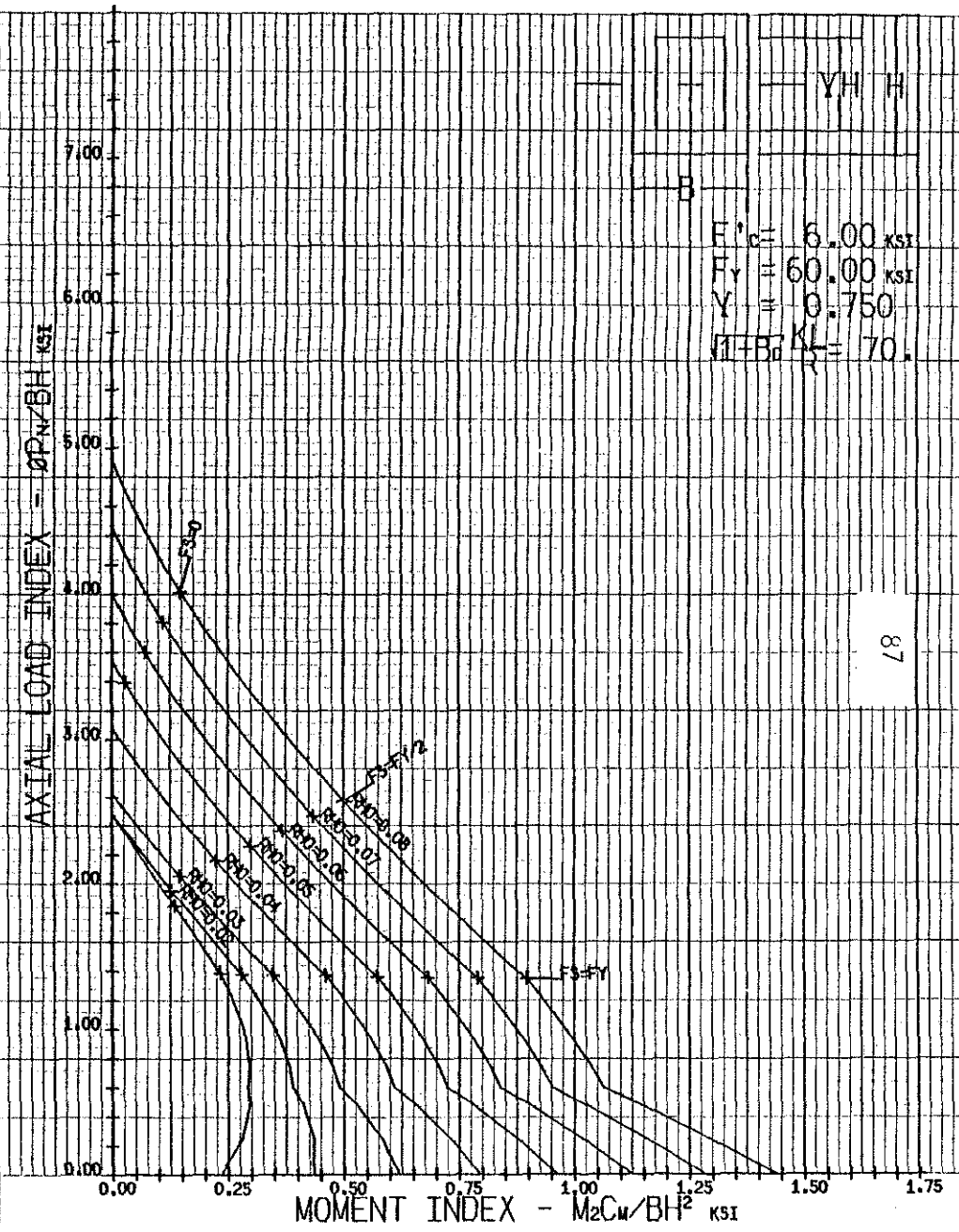


Fig. R6-60.75-70 - Interaction Diagram

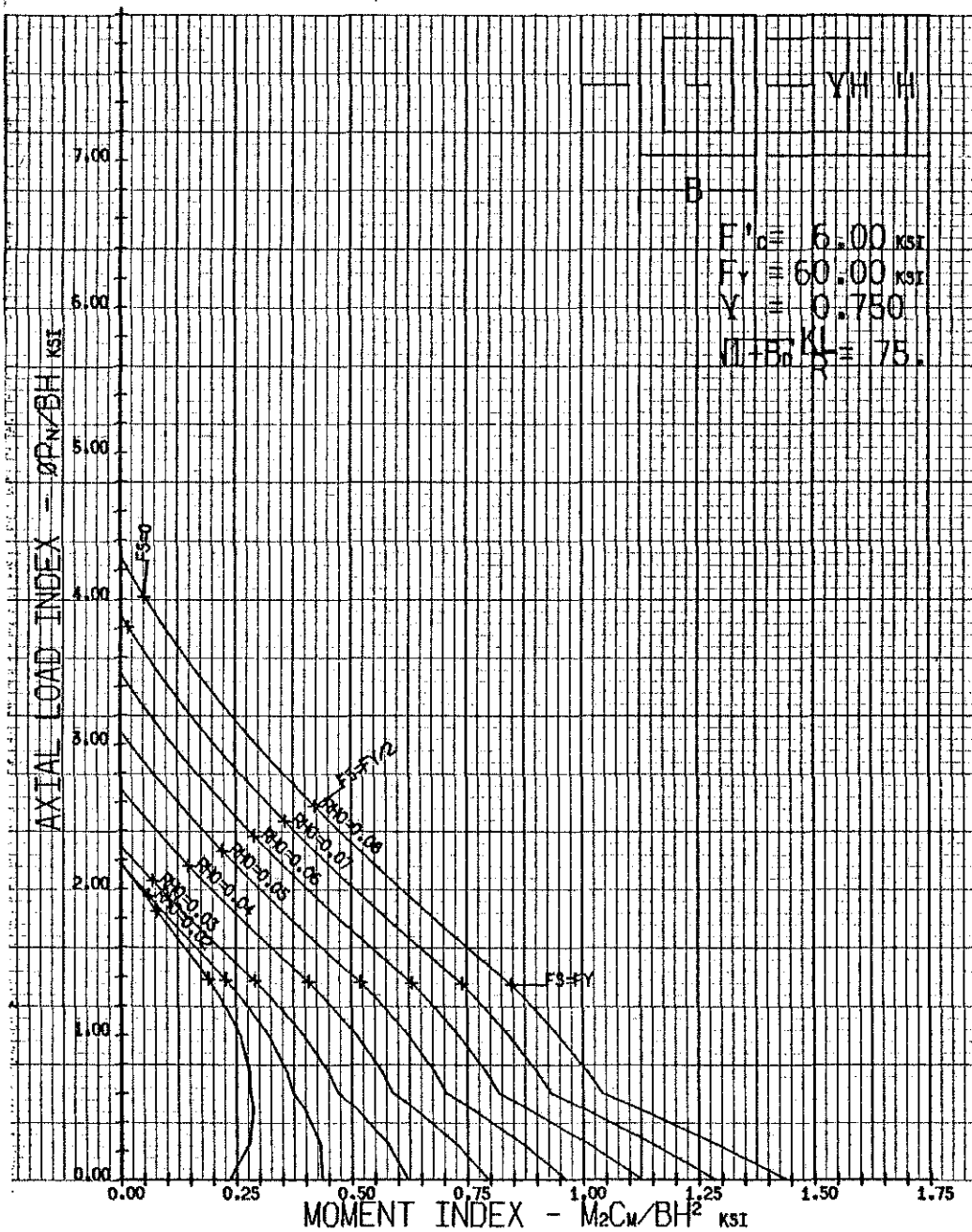


Fig. R6-60.75-75 - Interaction Diagram

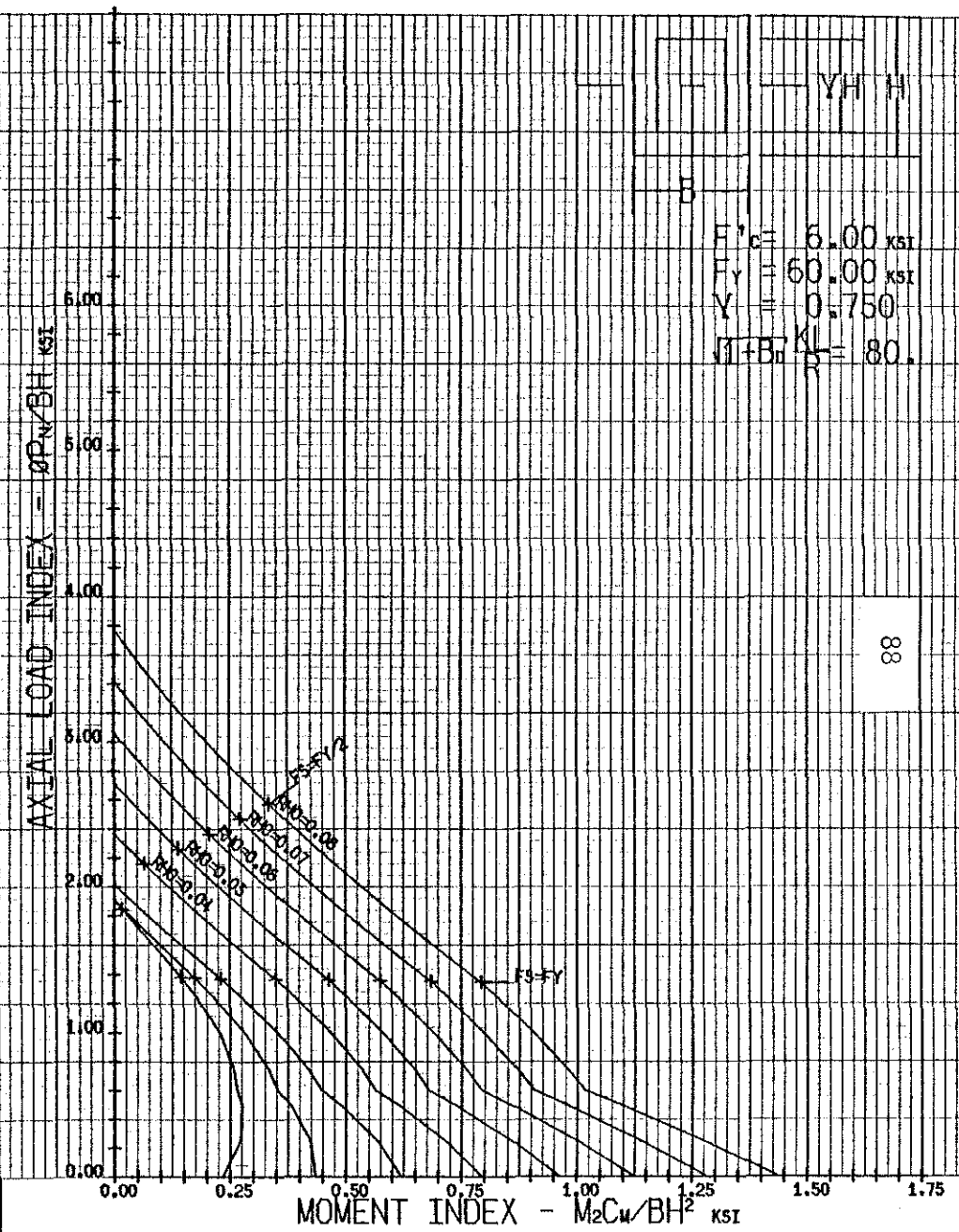


Fig. R6-60.75-80 - Interaction Diagram

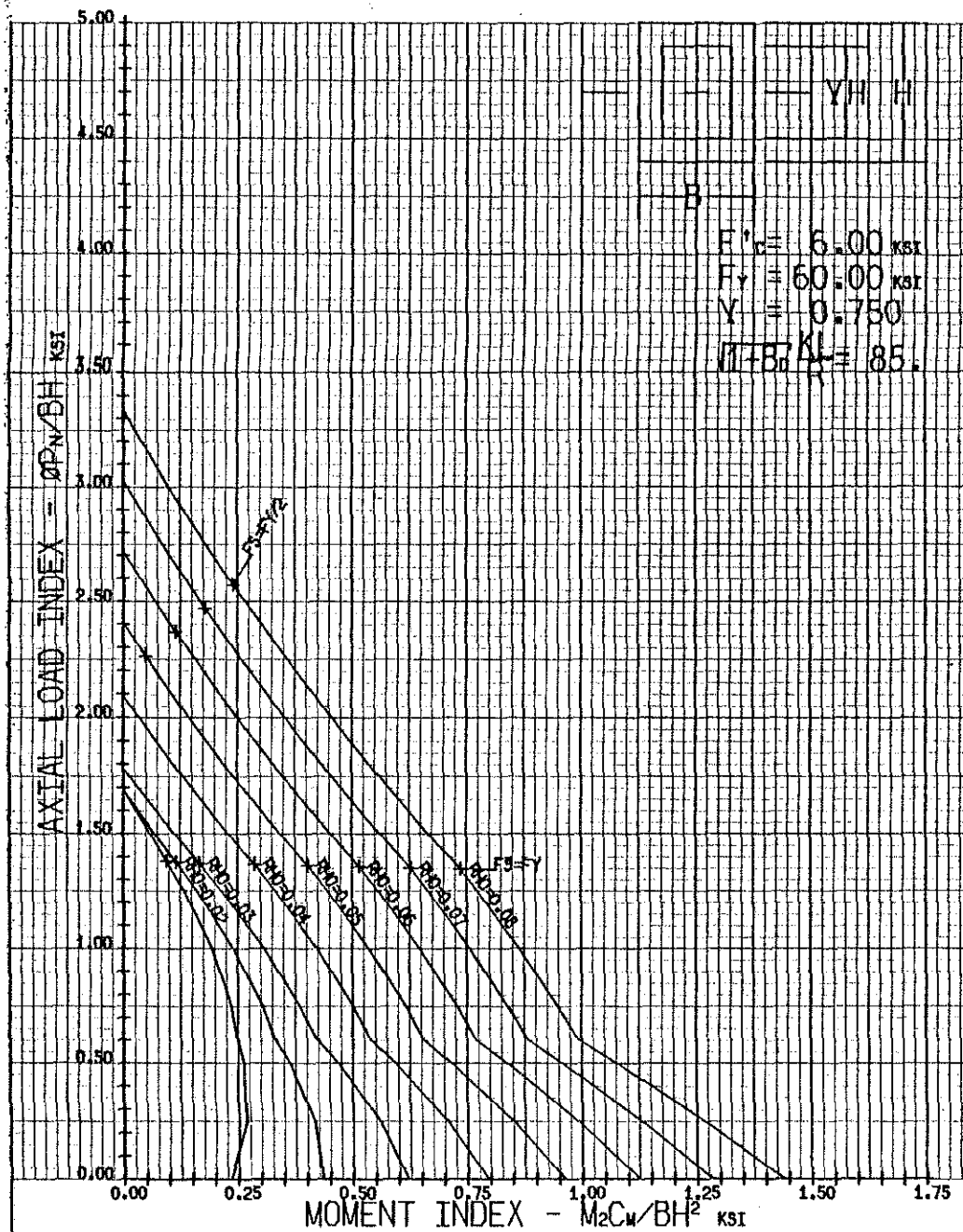


Fig. R6-60.75-85 - Interaction Diagram

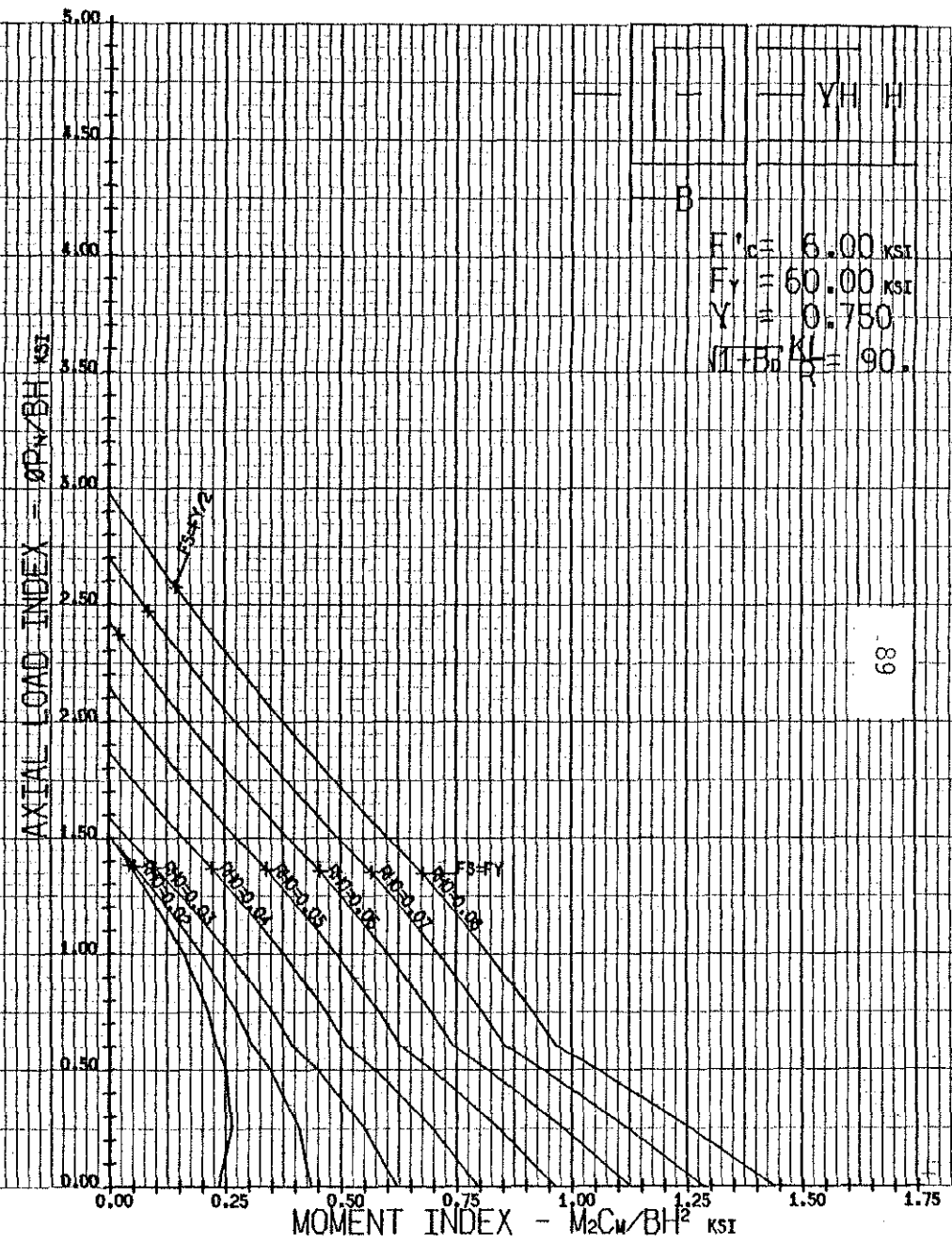


Fig. R6-60.75-90 - Interaction Diagram

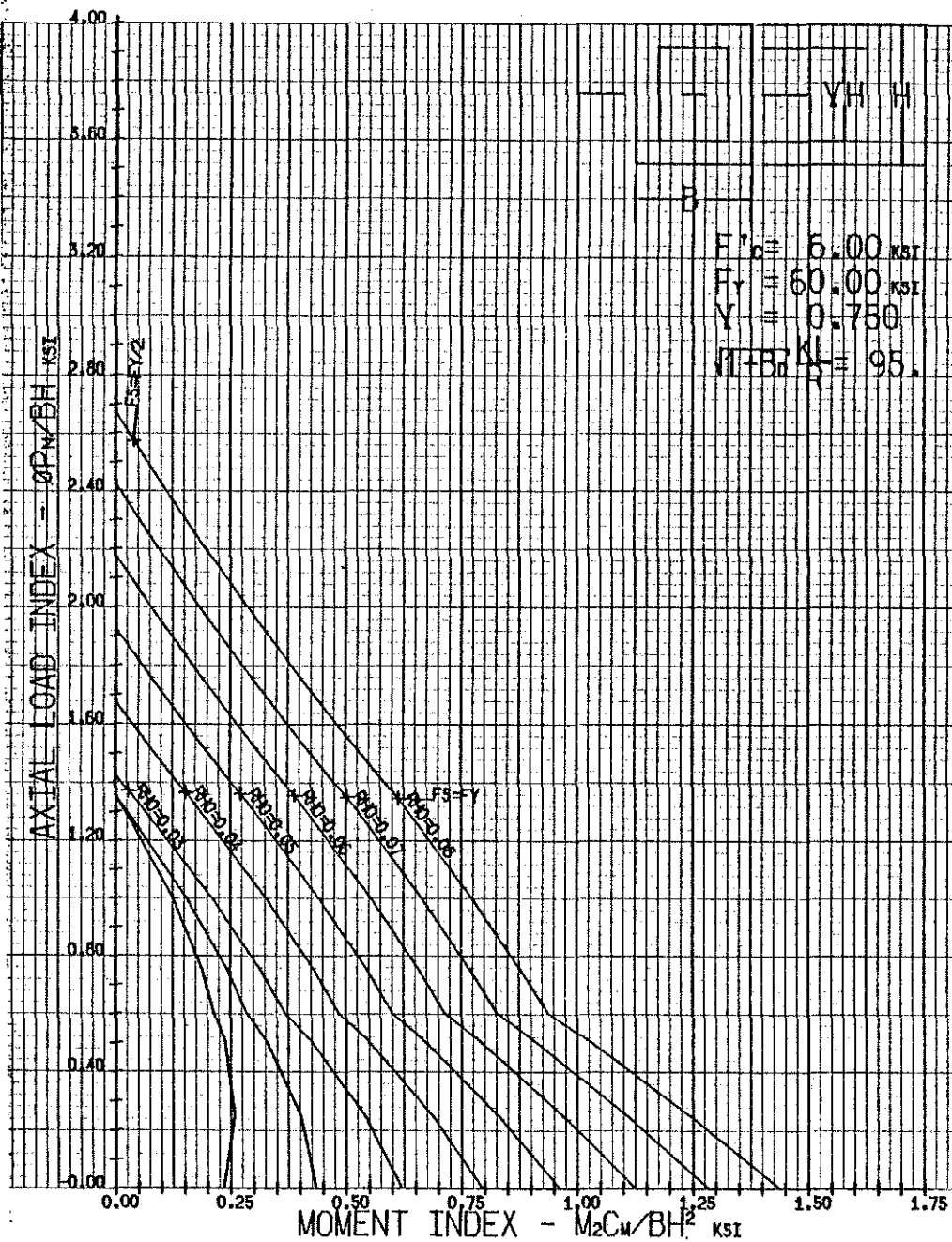


Fig. R6-60.75-95 - Interaction Diagram

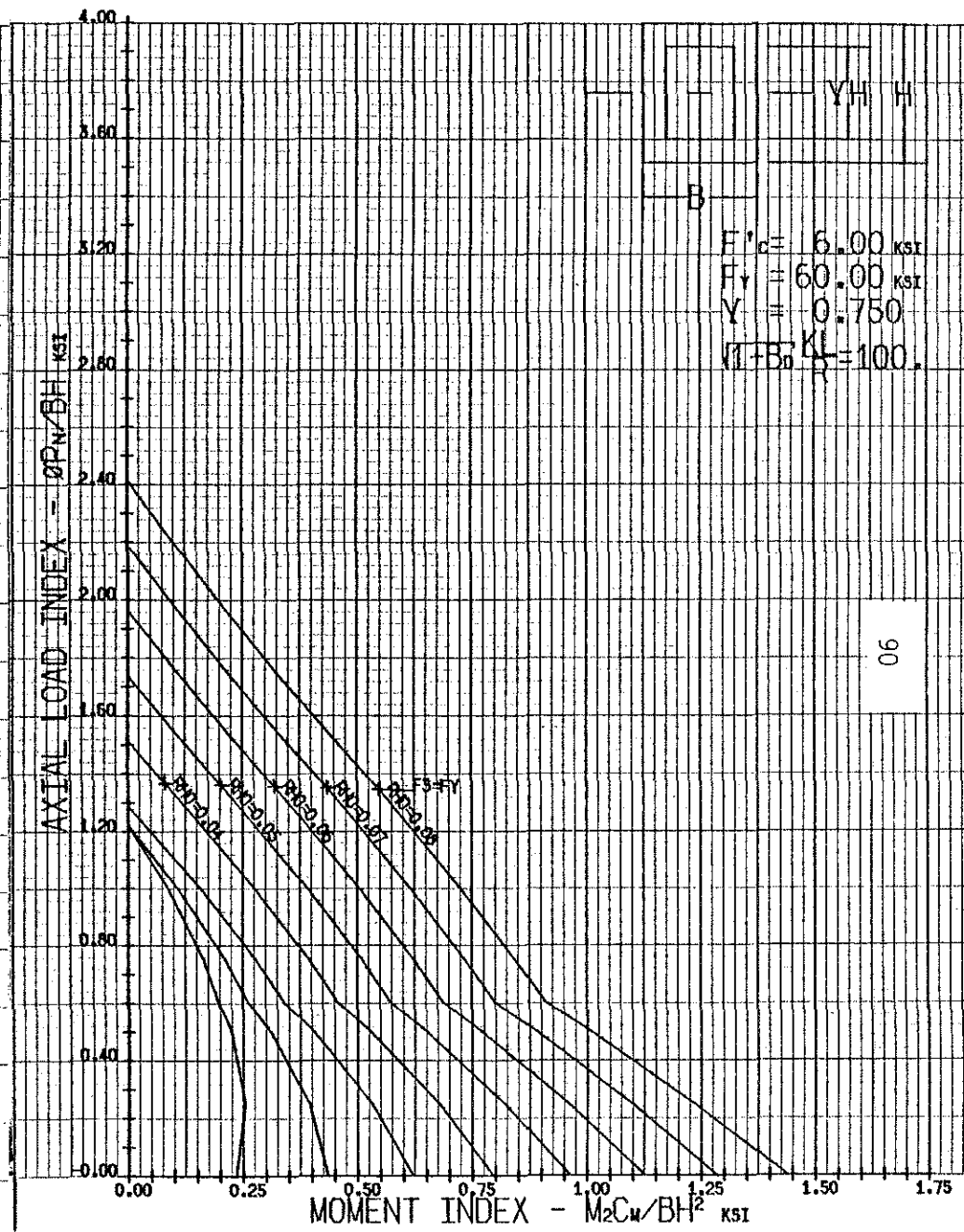


Fig. R6-60.75-100 - Interaction Diagram



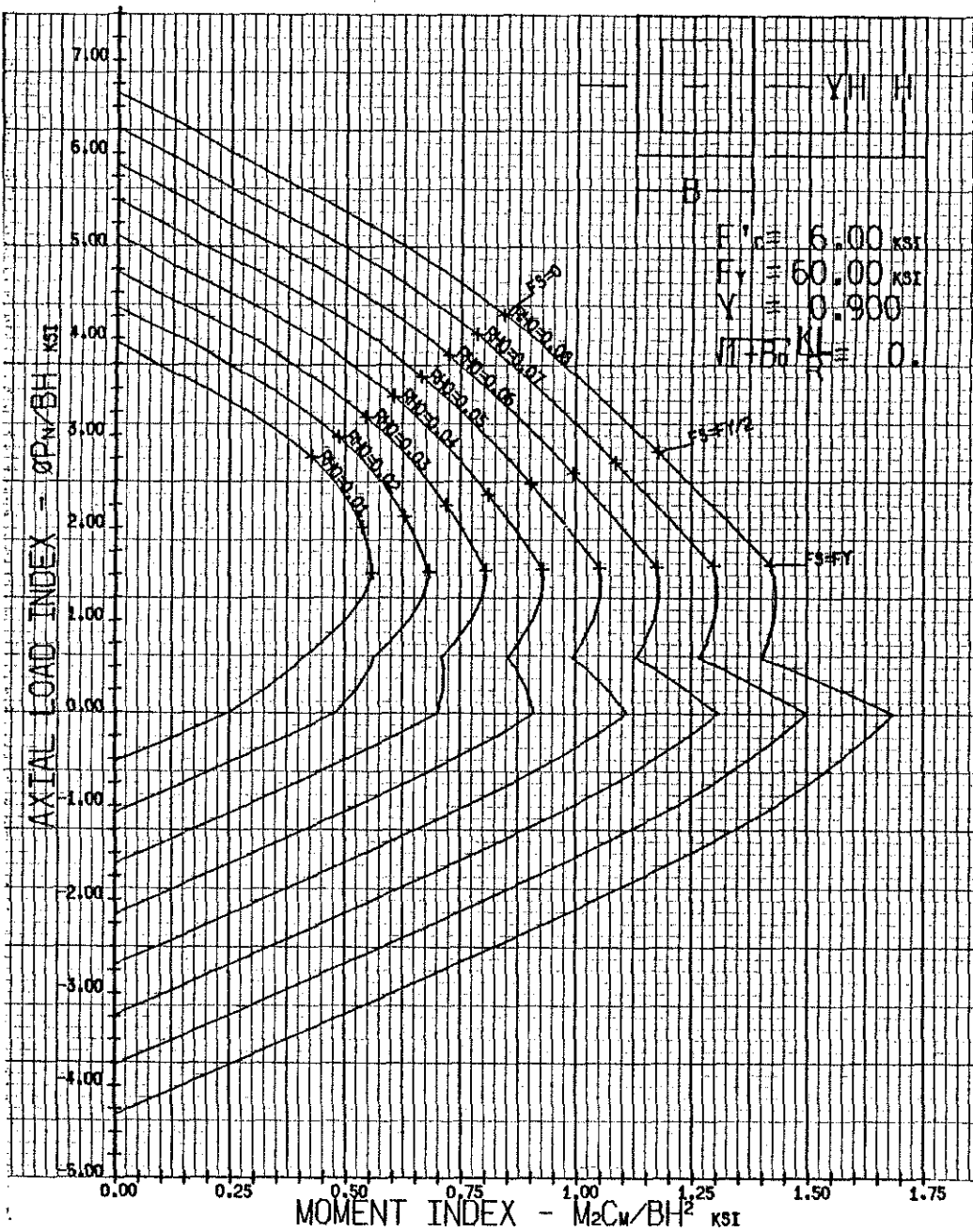


Fig. R6-60.90-0 - Interaction Diagram

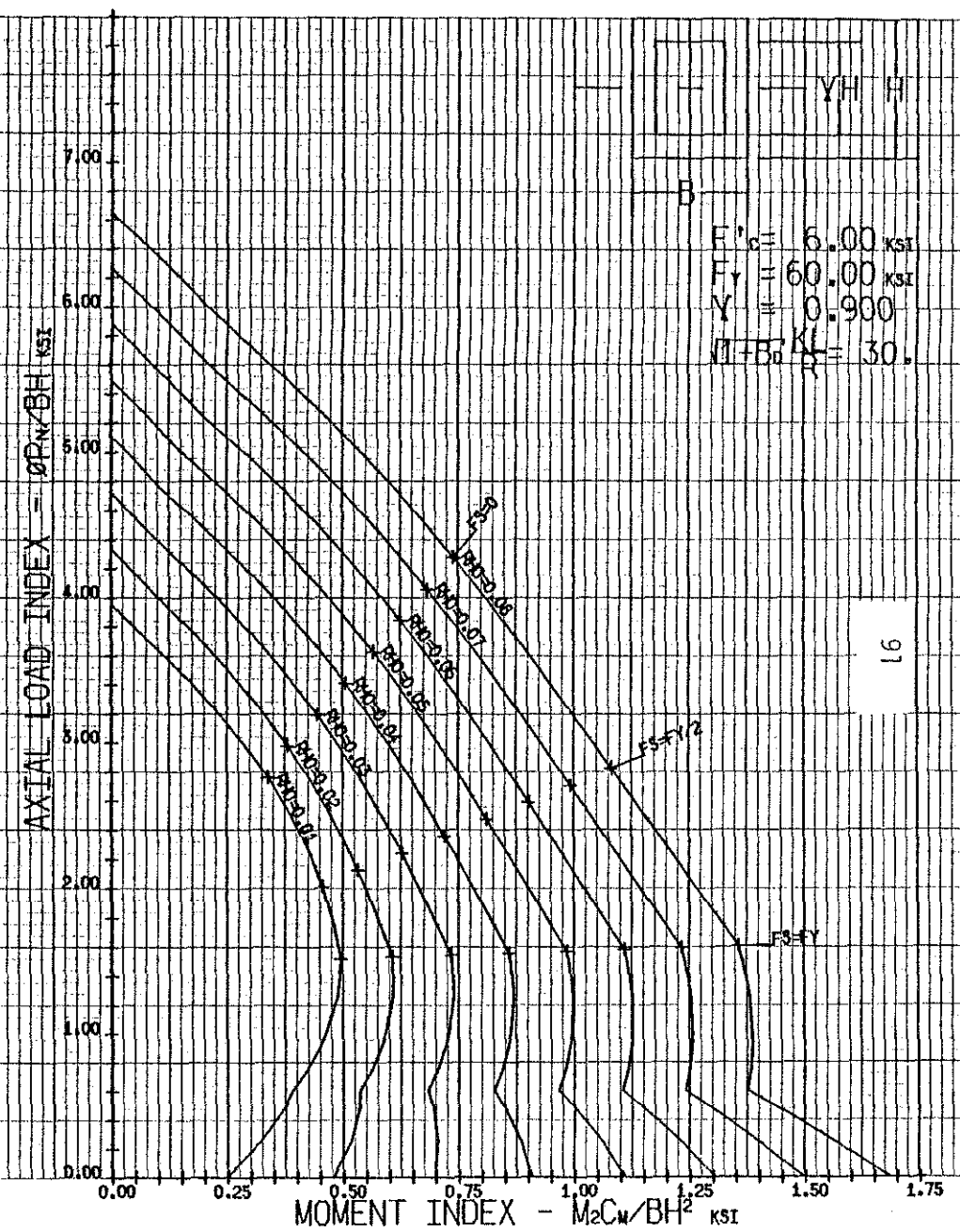


Fig. R6-60.90-30 - Interaction Diagram

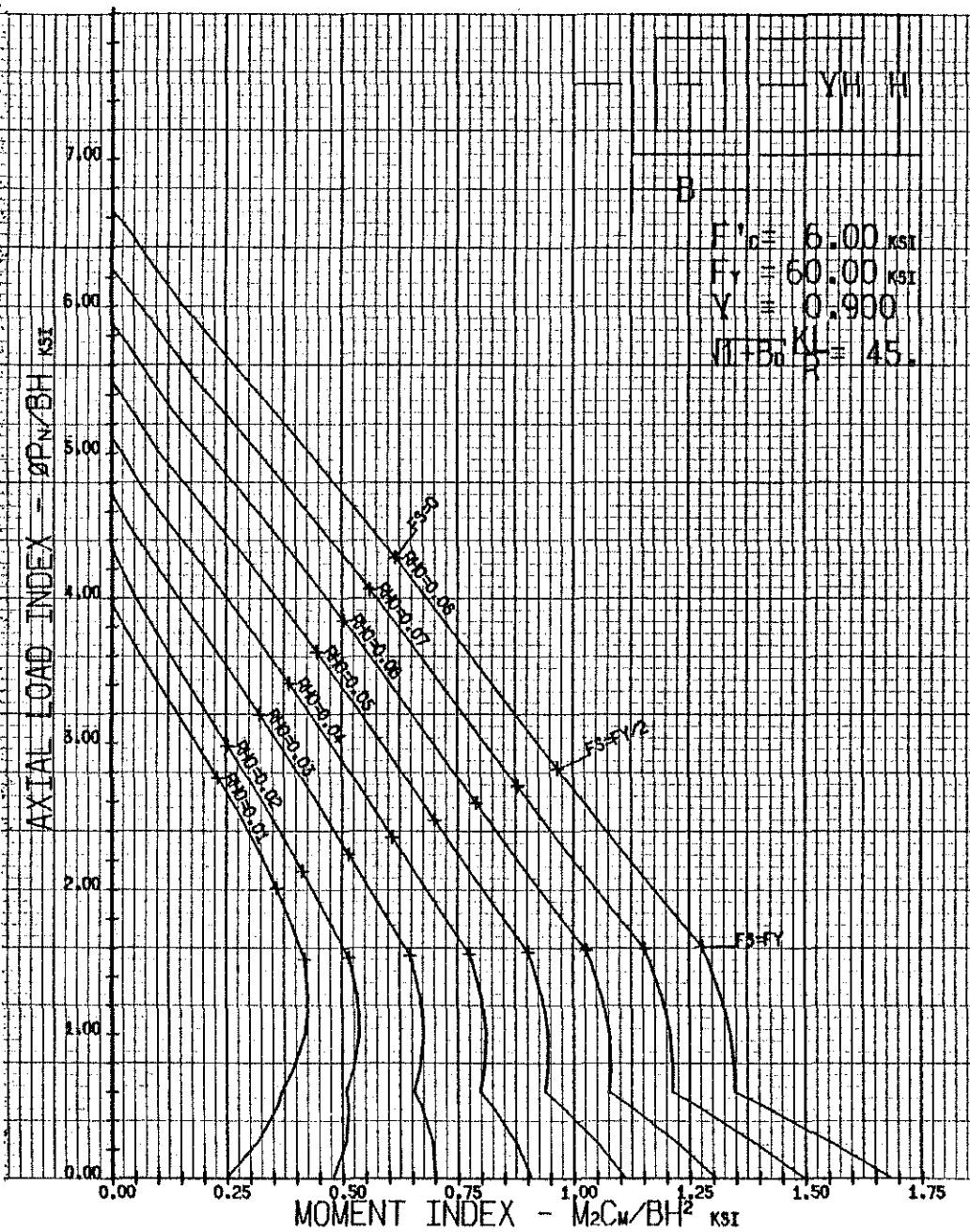


Fig. R6-60.90-45 - Interaction Diagram

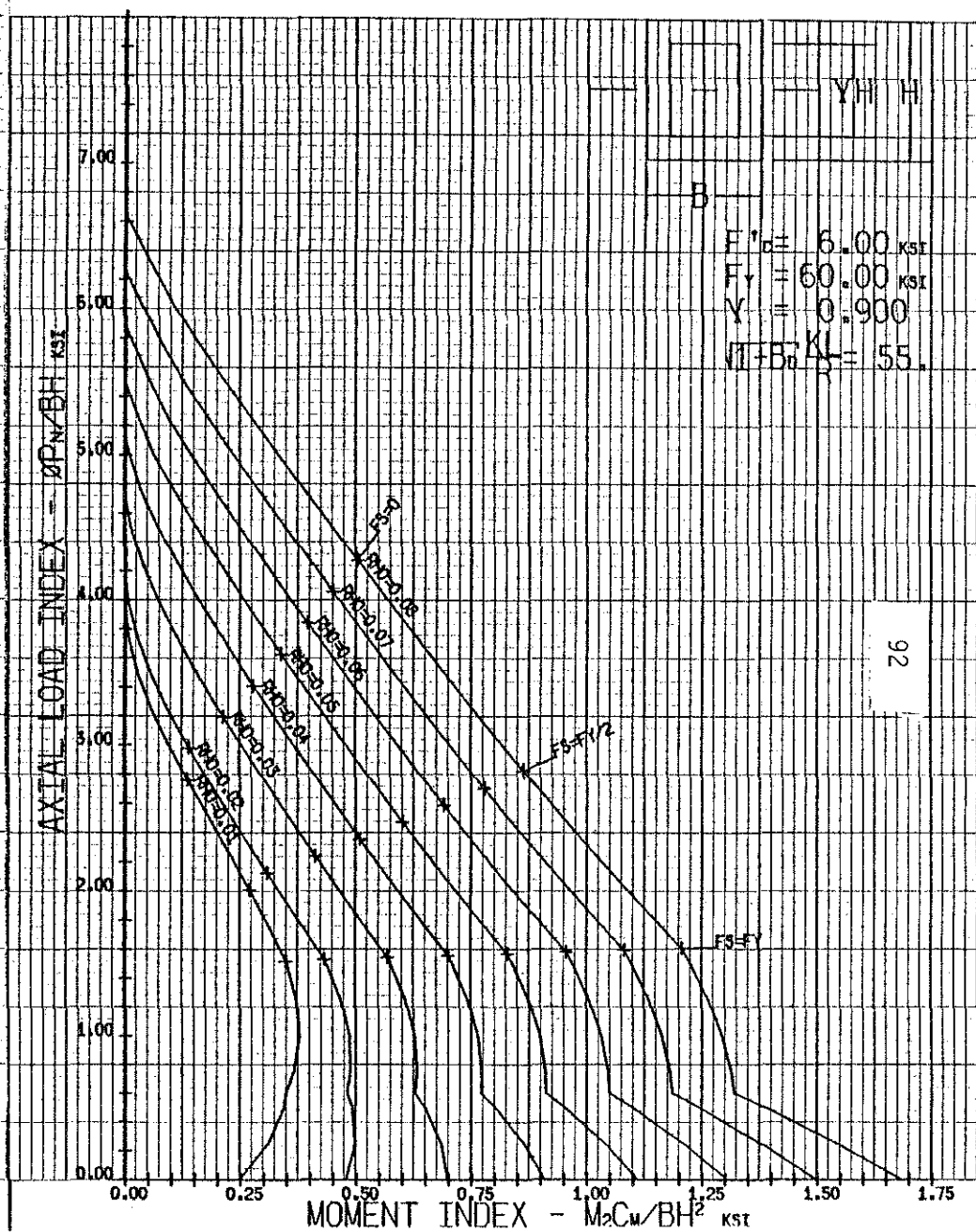


Fig. R6-60.90-55 - Interaction Diagram

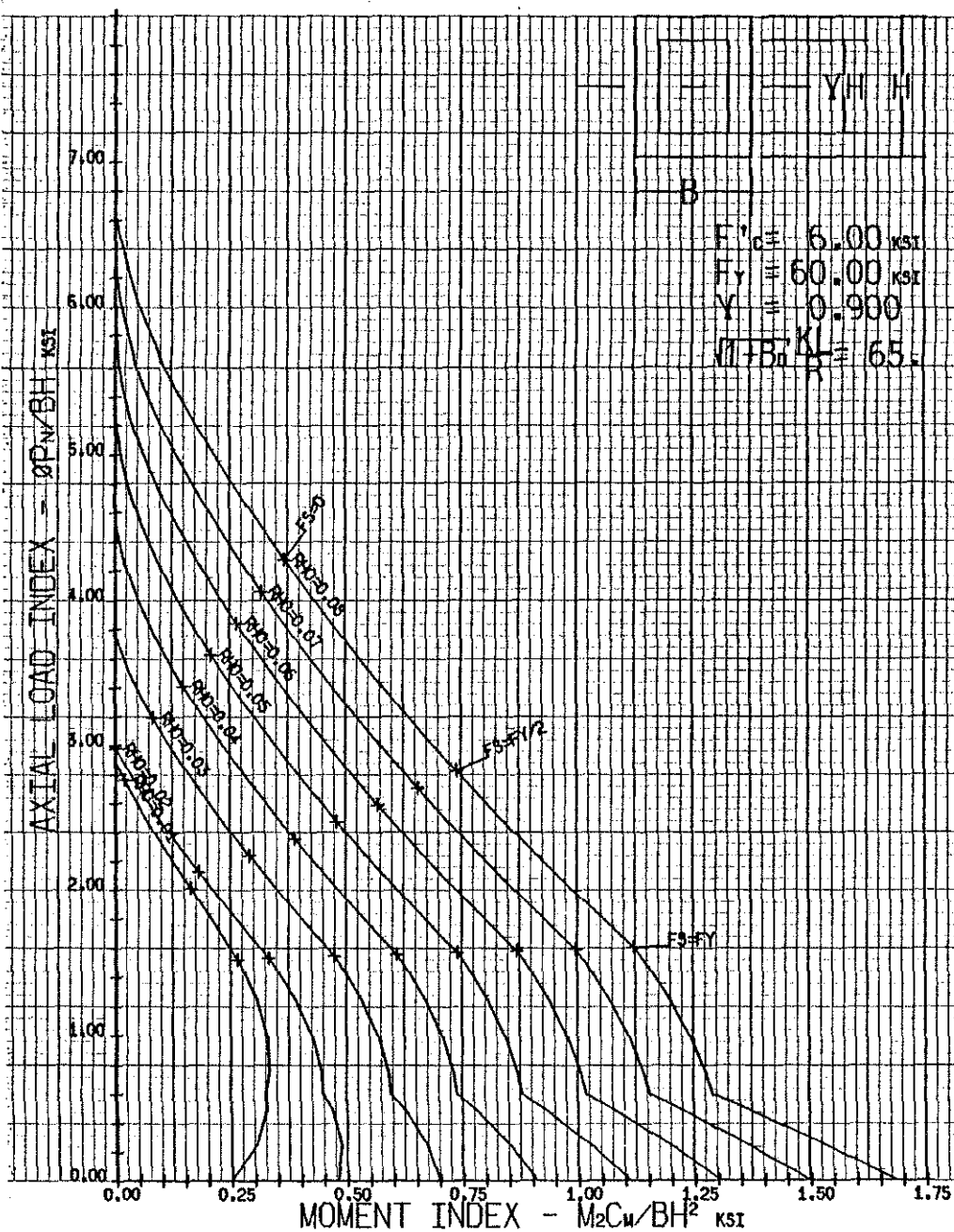


Fig. R6-60.90-65 - Interaction Diagram

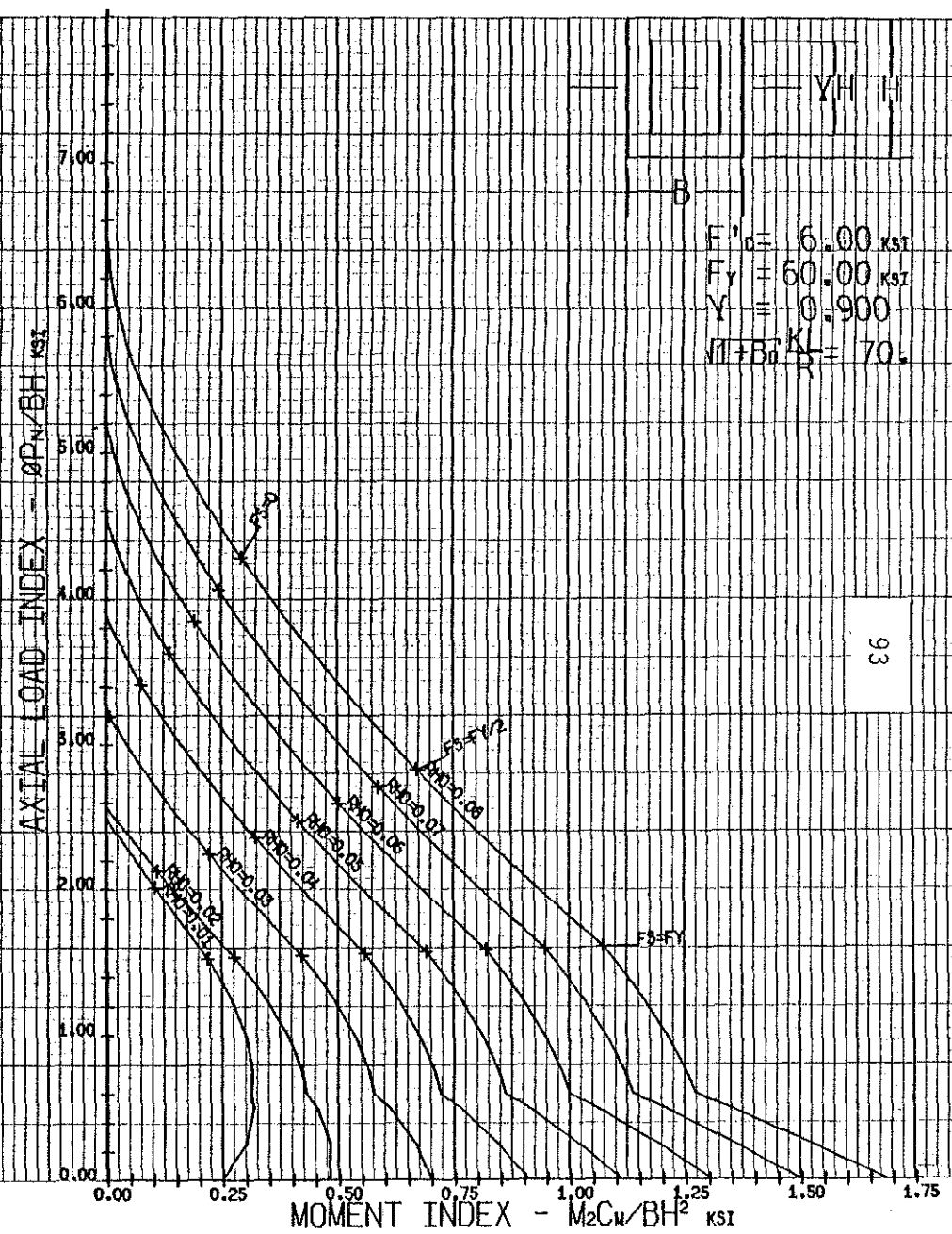


Fig. R6-60.90-70 - Interaction Diagram



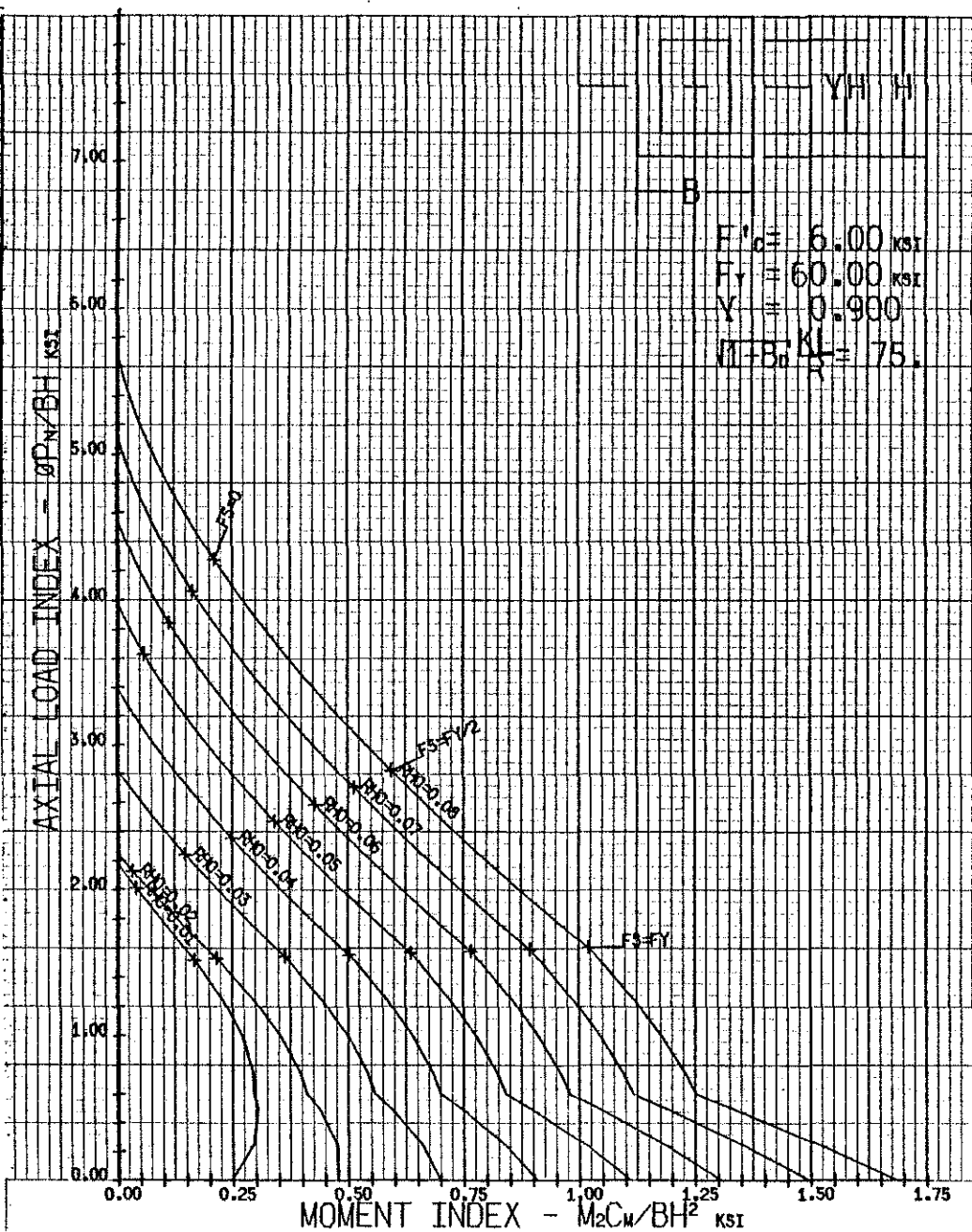


Fig. R6-60.90-75 - Interaction Diagram

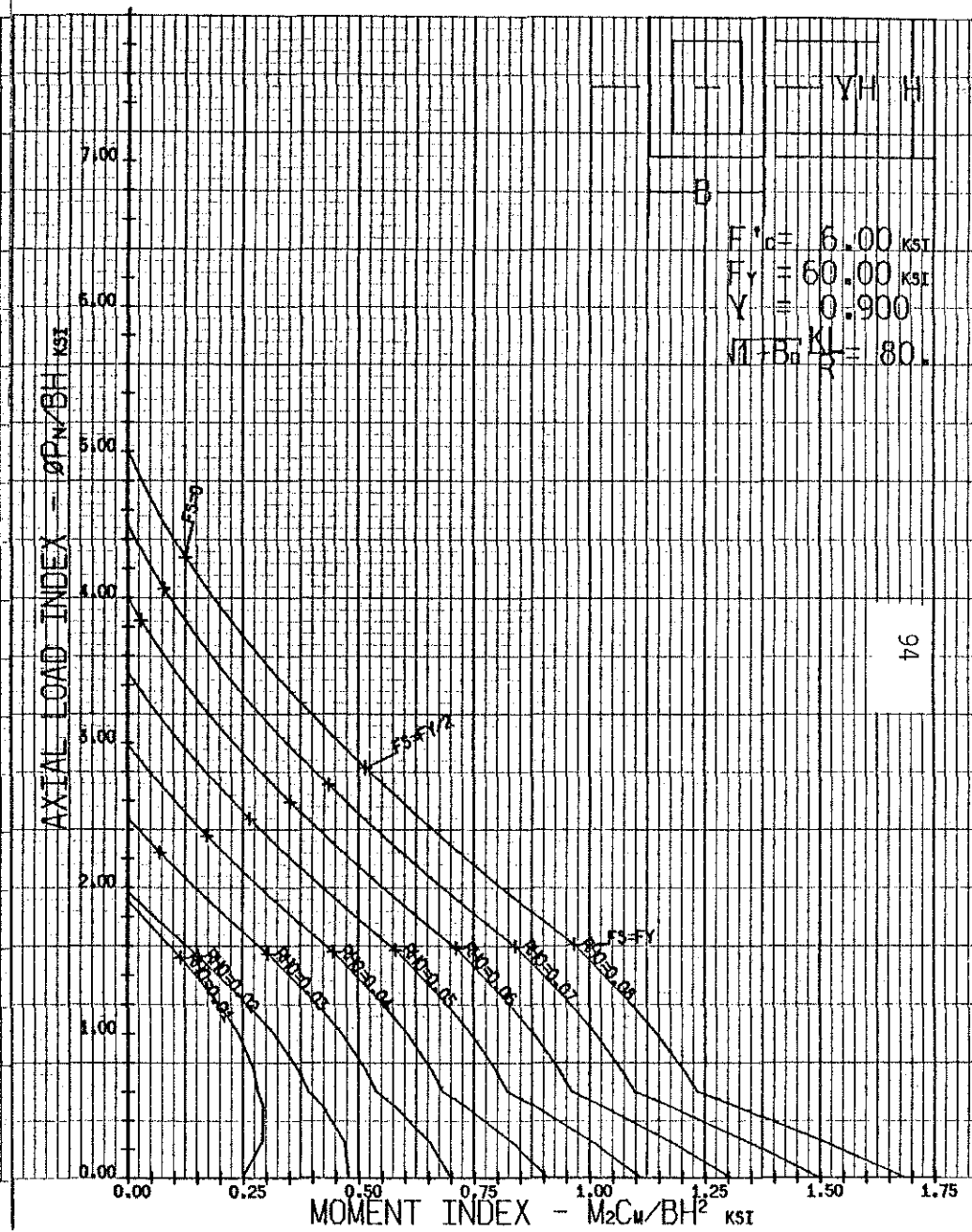


Fig. R6-60.90-80 - Interaction Diagram

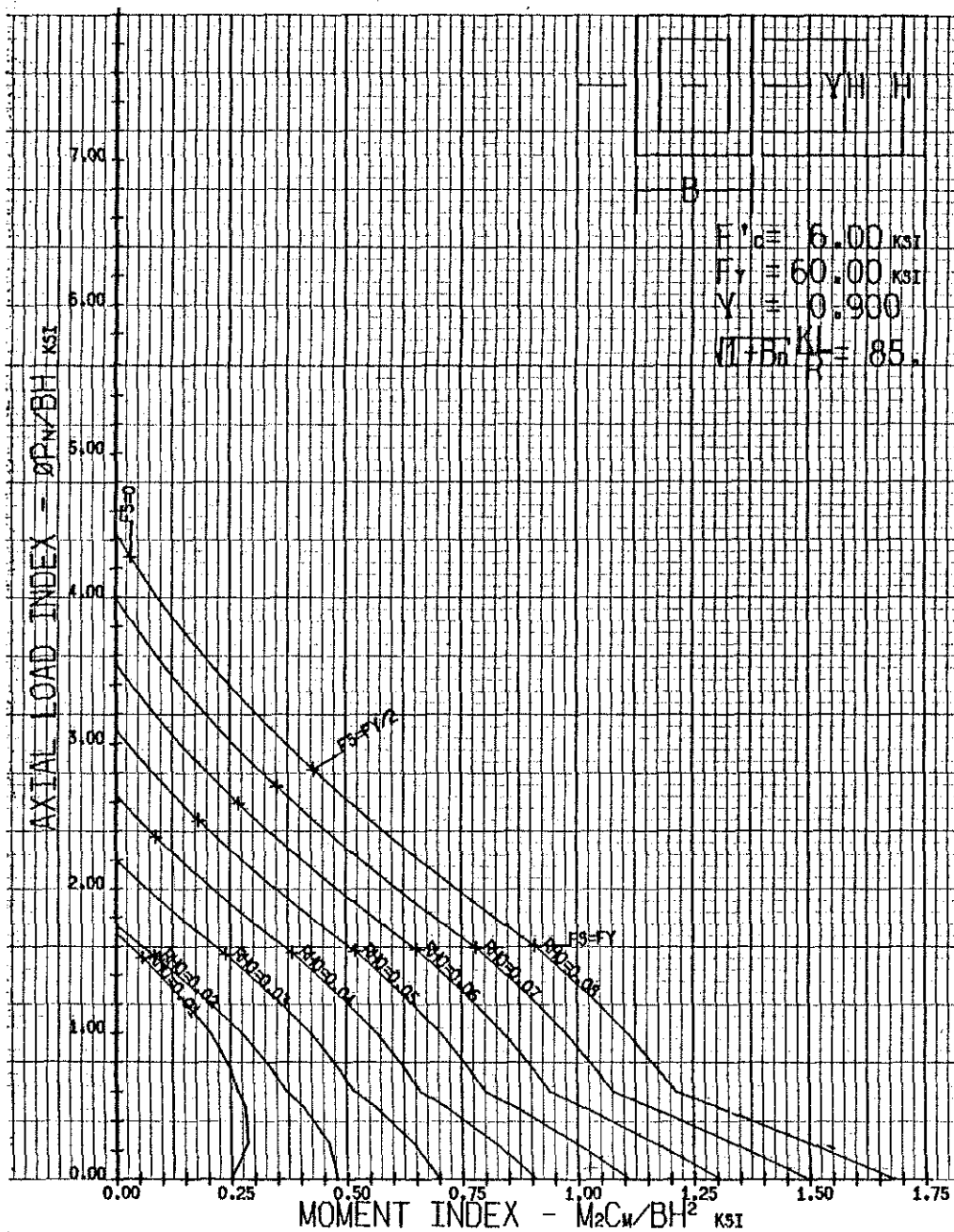


Fig. R6-60.90-85 - Interaction Diagram

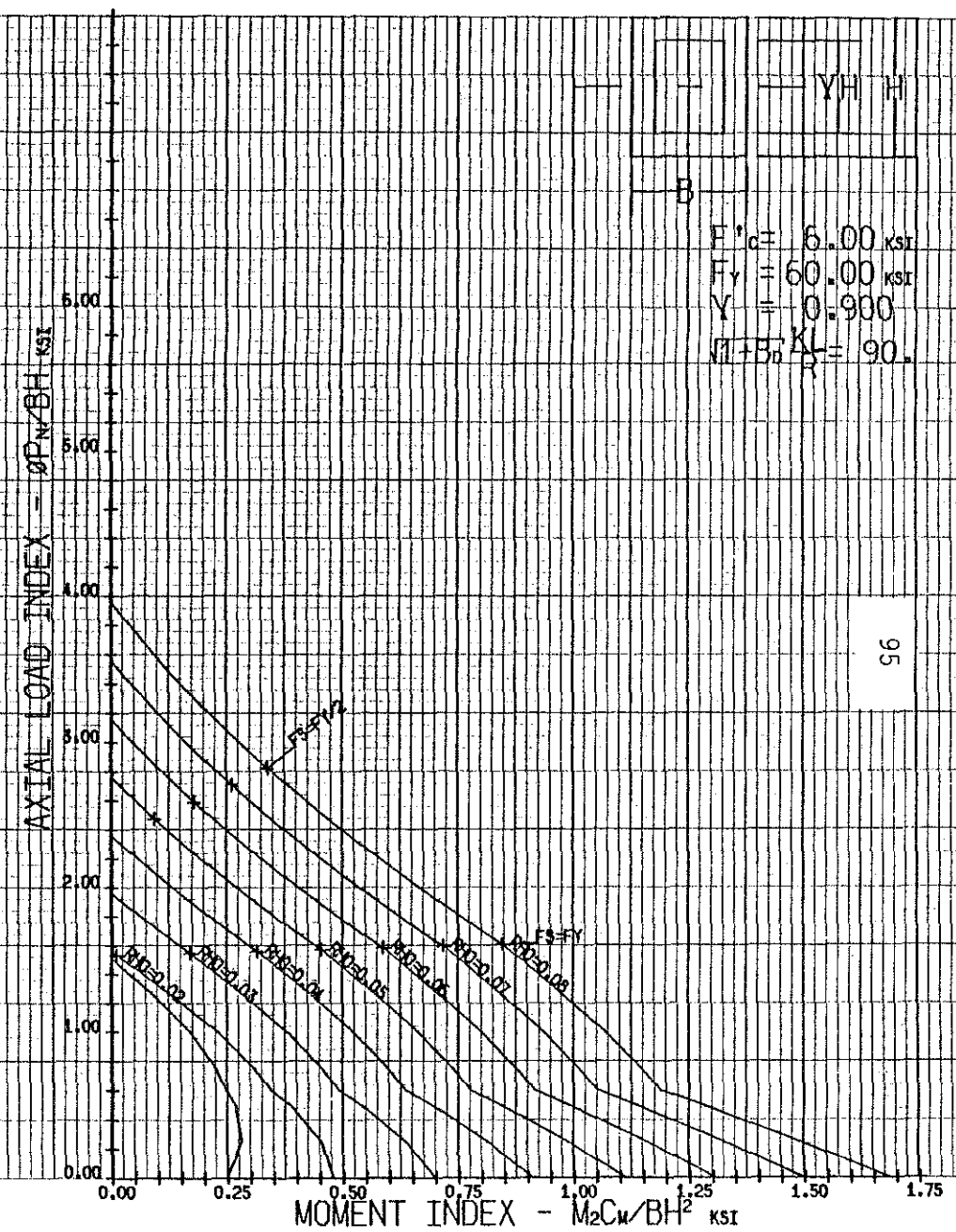


Fig. R6-60.90-90 - Interaction Diagram

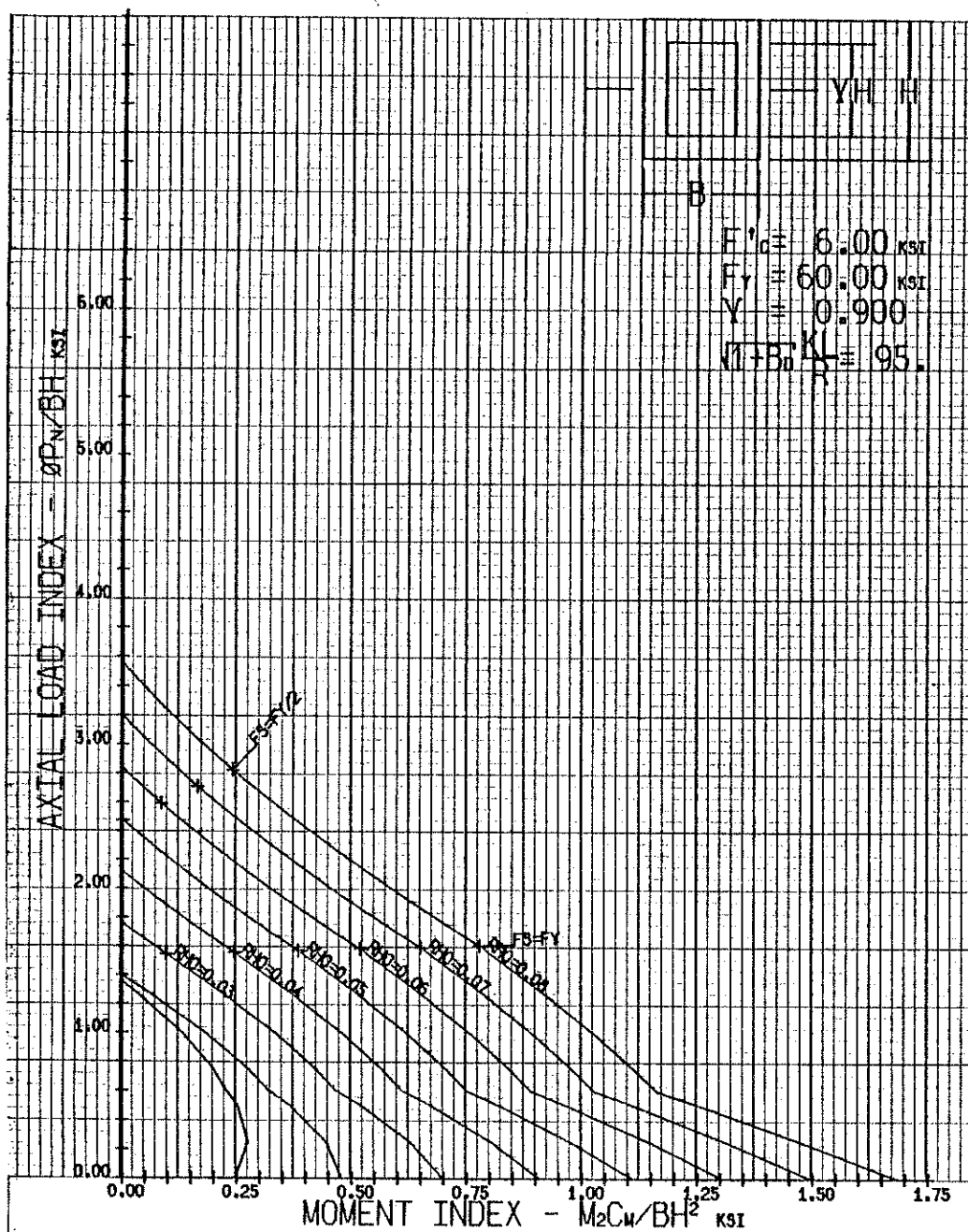


Fig. R6-60.90-95 - Interaction Diagram

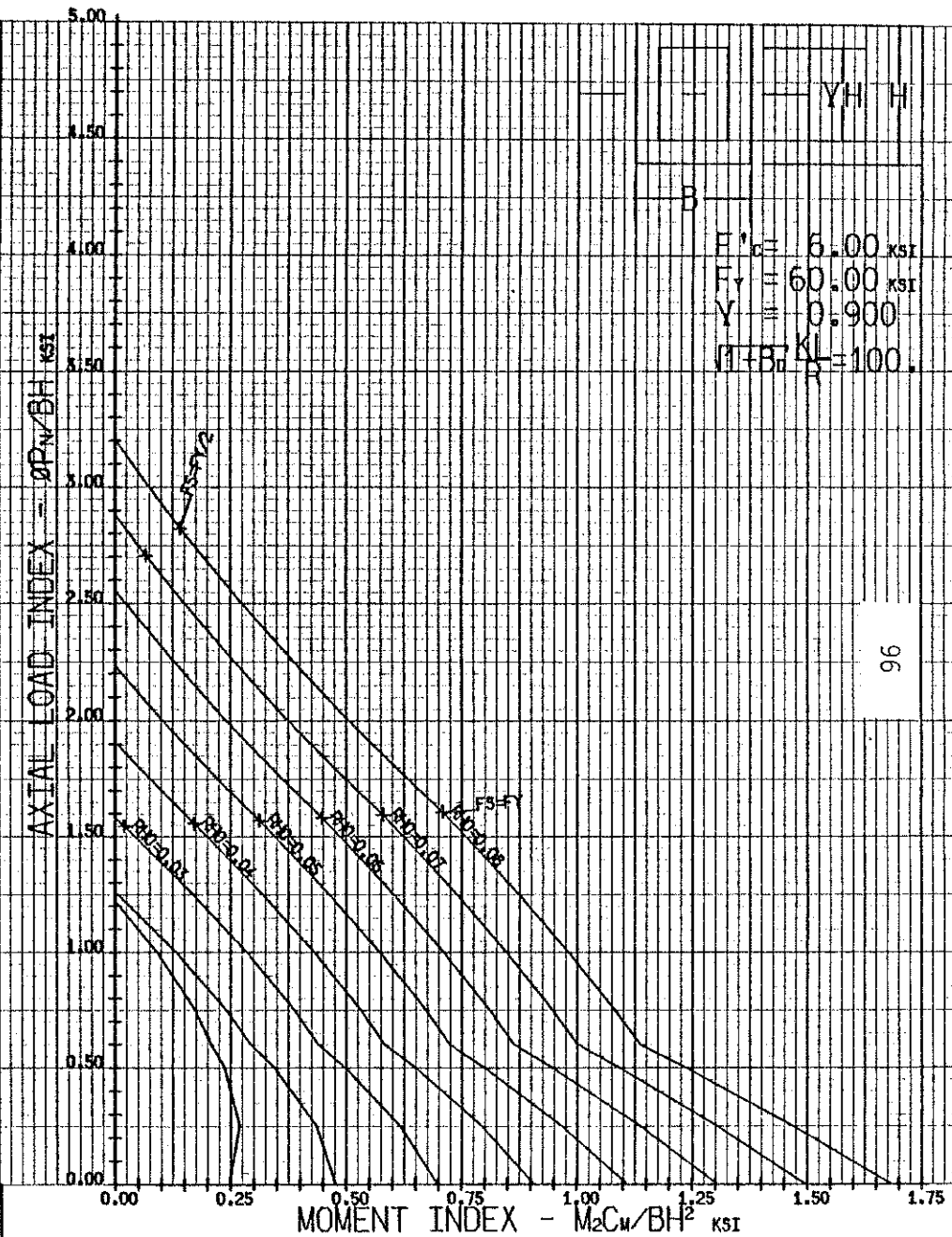


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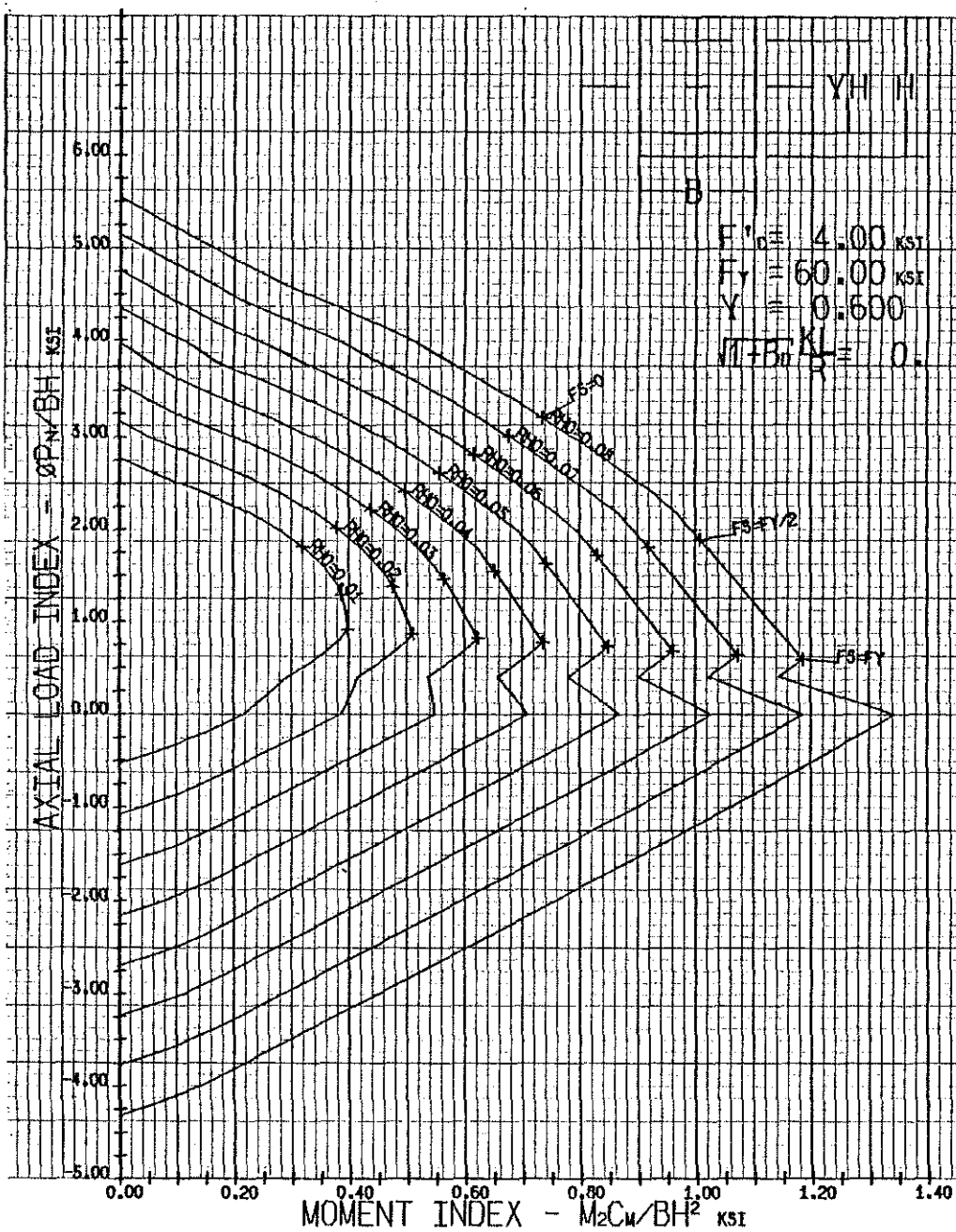


Fig. E4-60.60-0 - Interaction Diagram

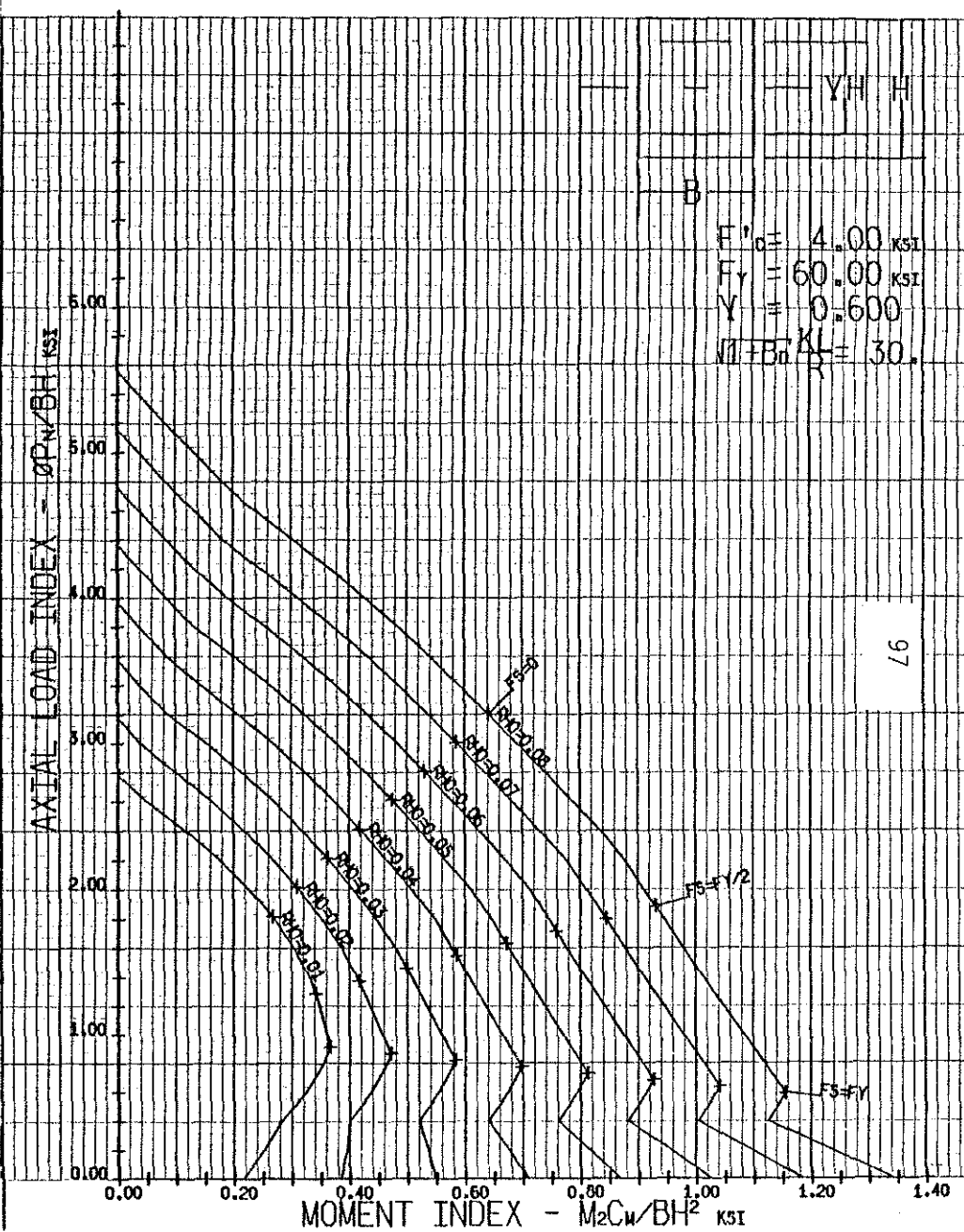


Fig. E4-60.60-30 - Interaction Diagram

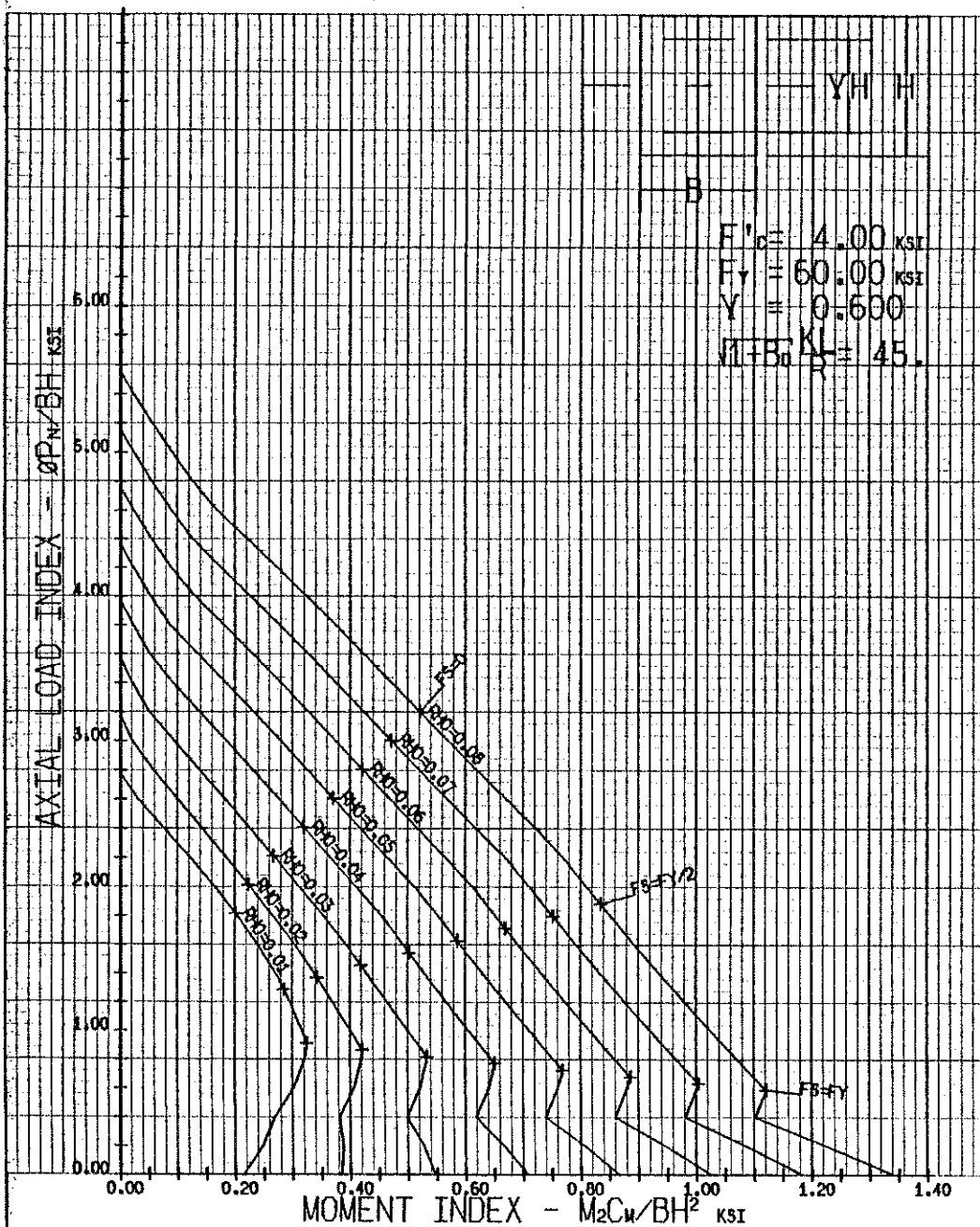


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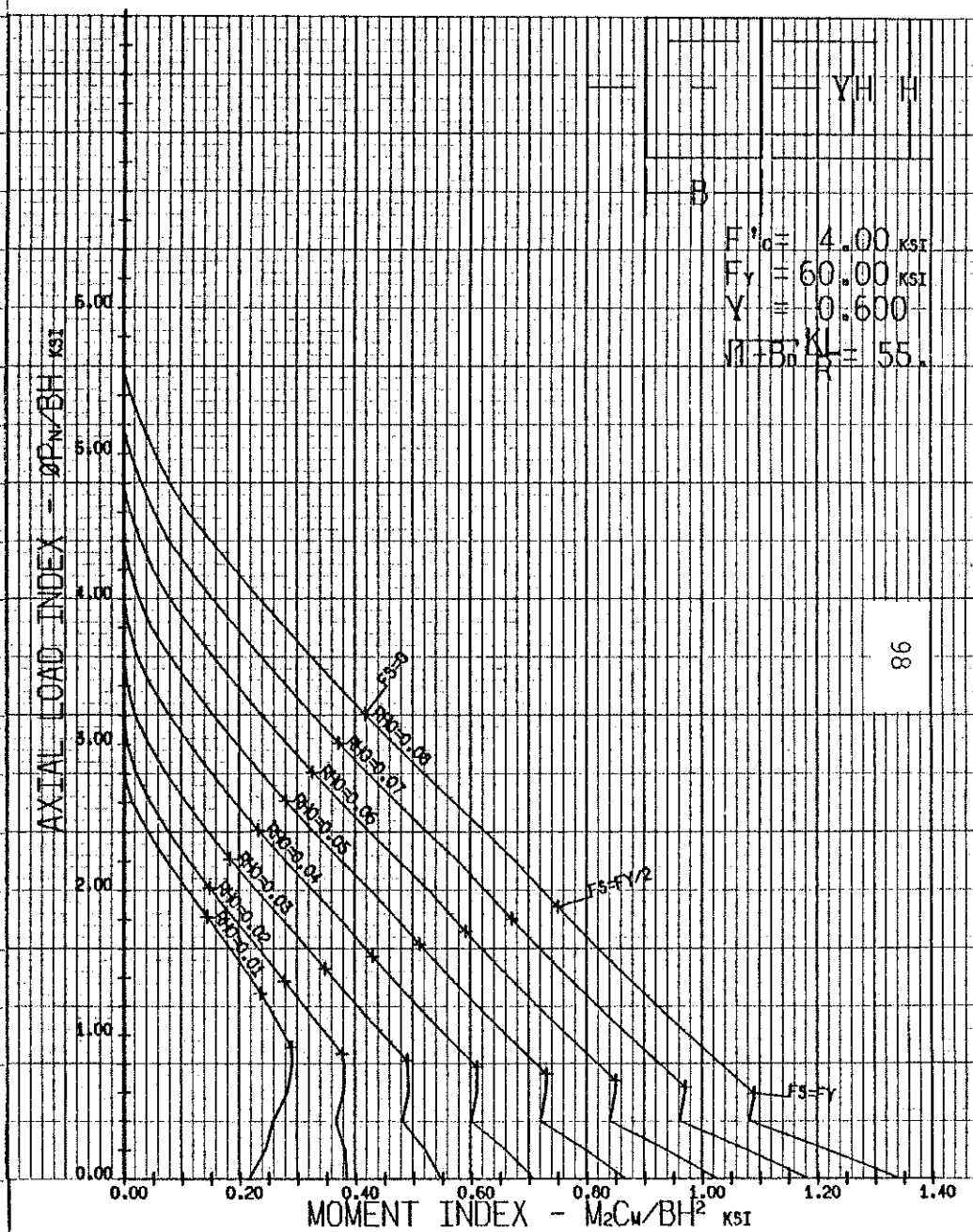


Fig. E4-60.60-55 - Interaction Diagram



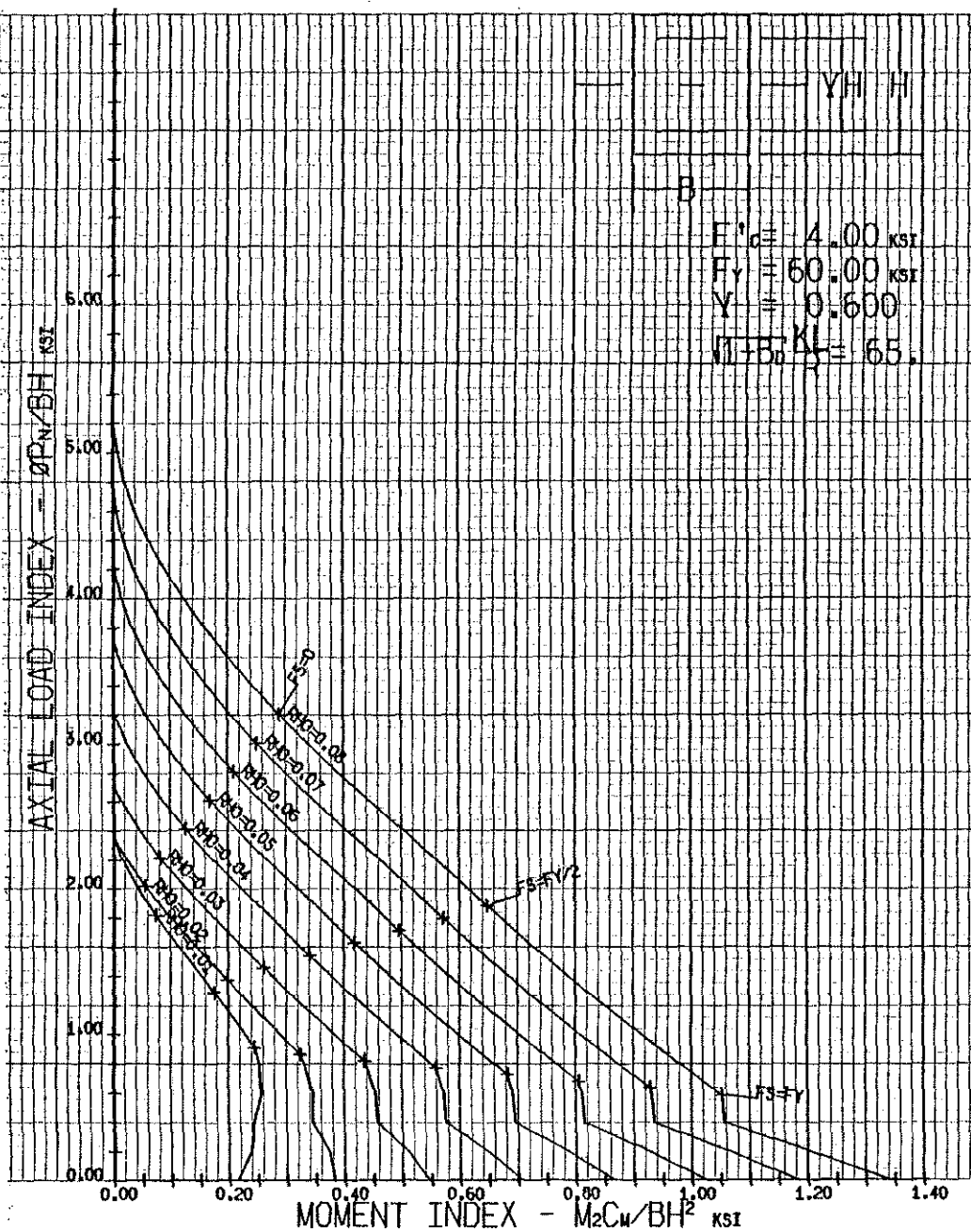


Fig. E4-60.60-65 - Interaction Diagram

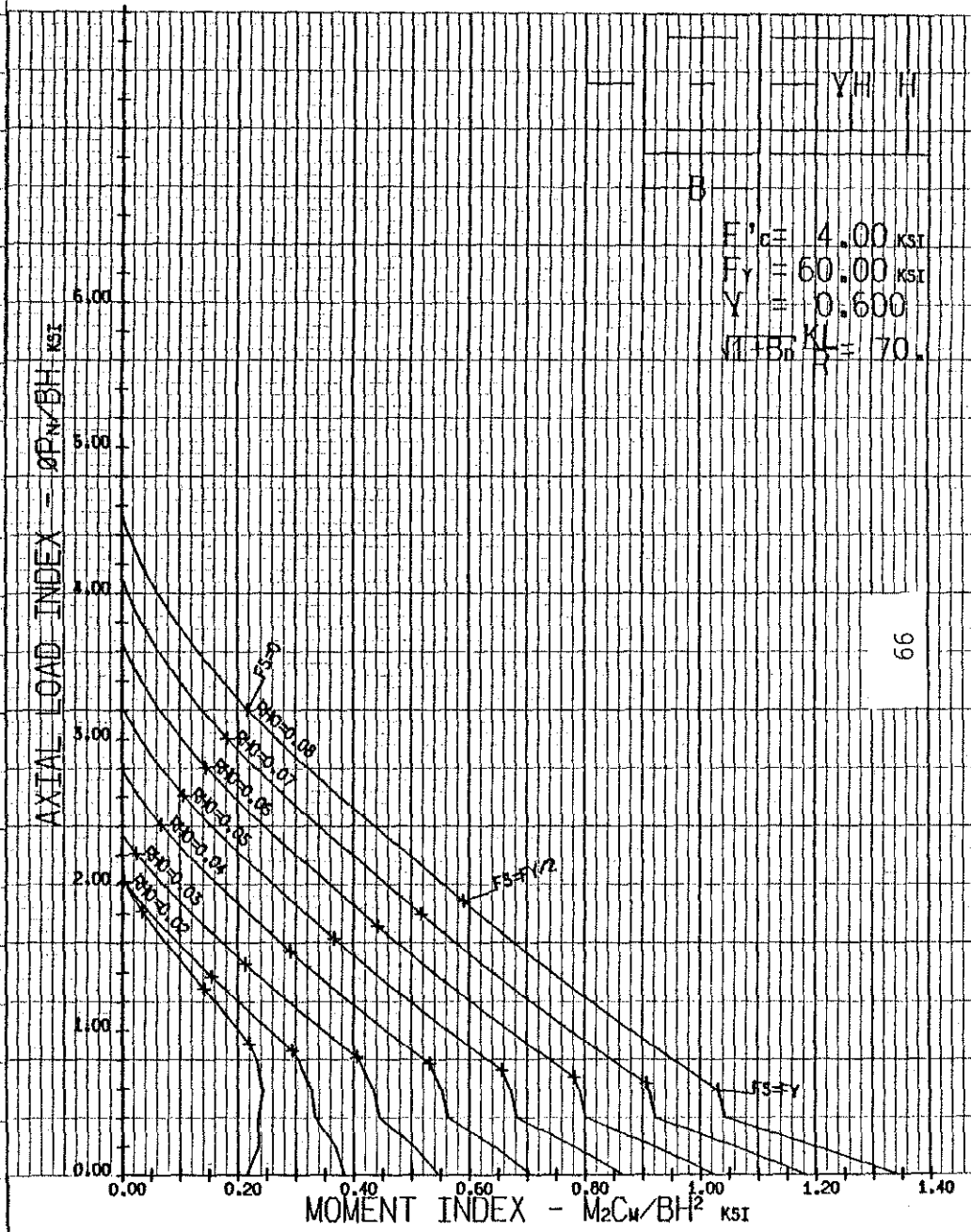


Fig. E4-60.60-70 - Interaction Diagram

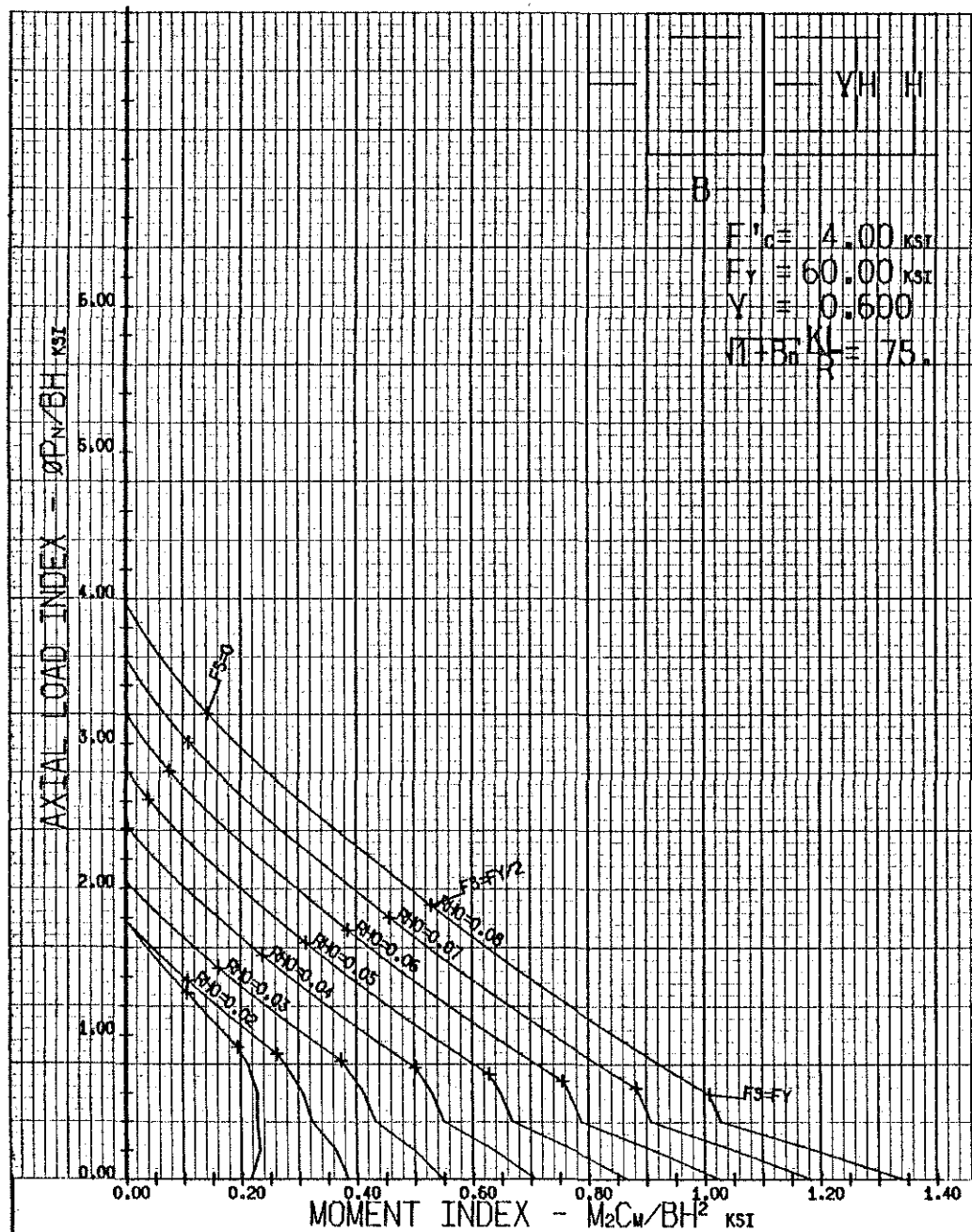


Fig. E4-60.60-75 - Interaction Diagram

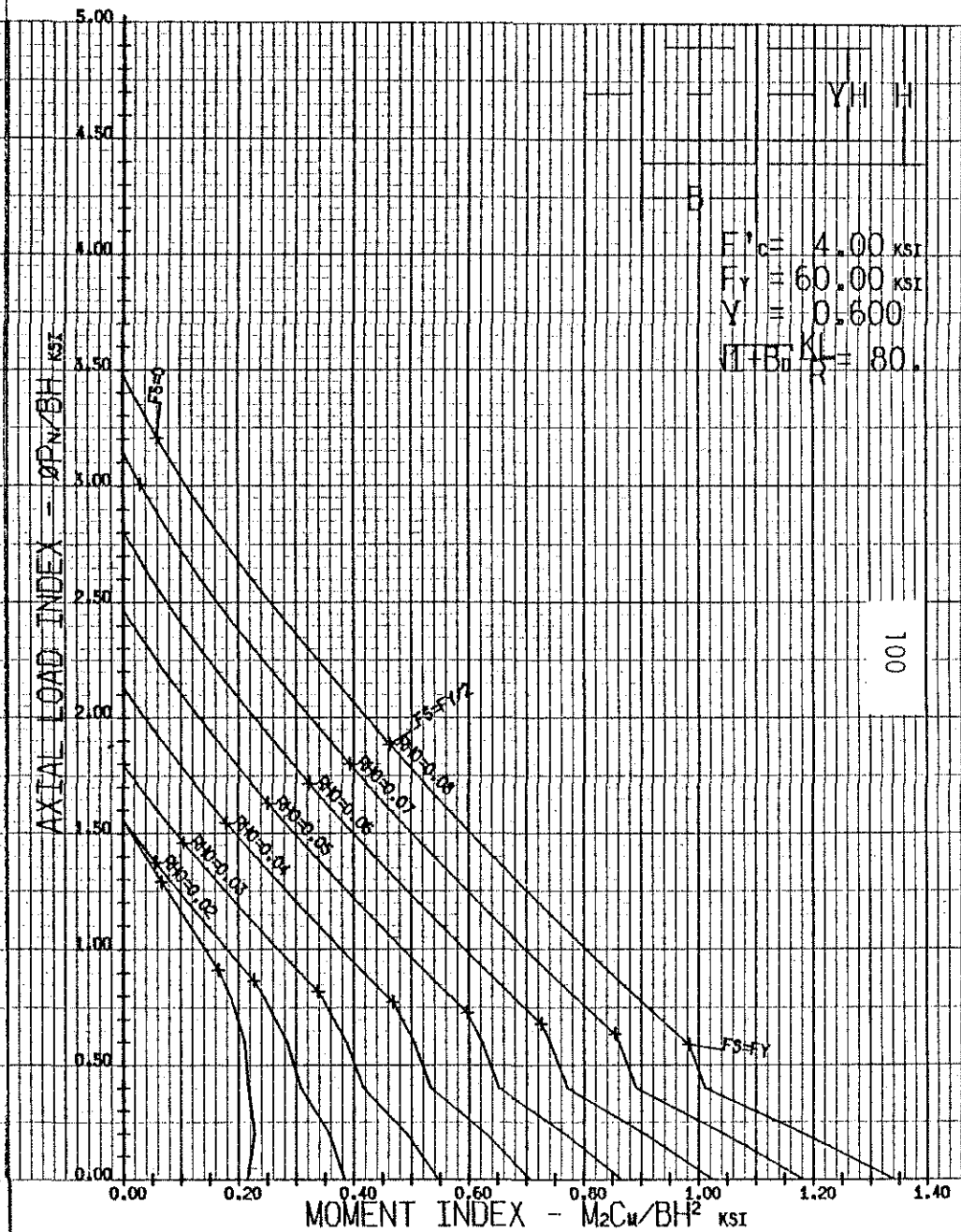


Fig. E4-60.60-80 - Interaction Diagram

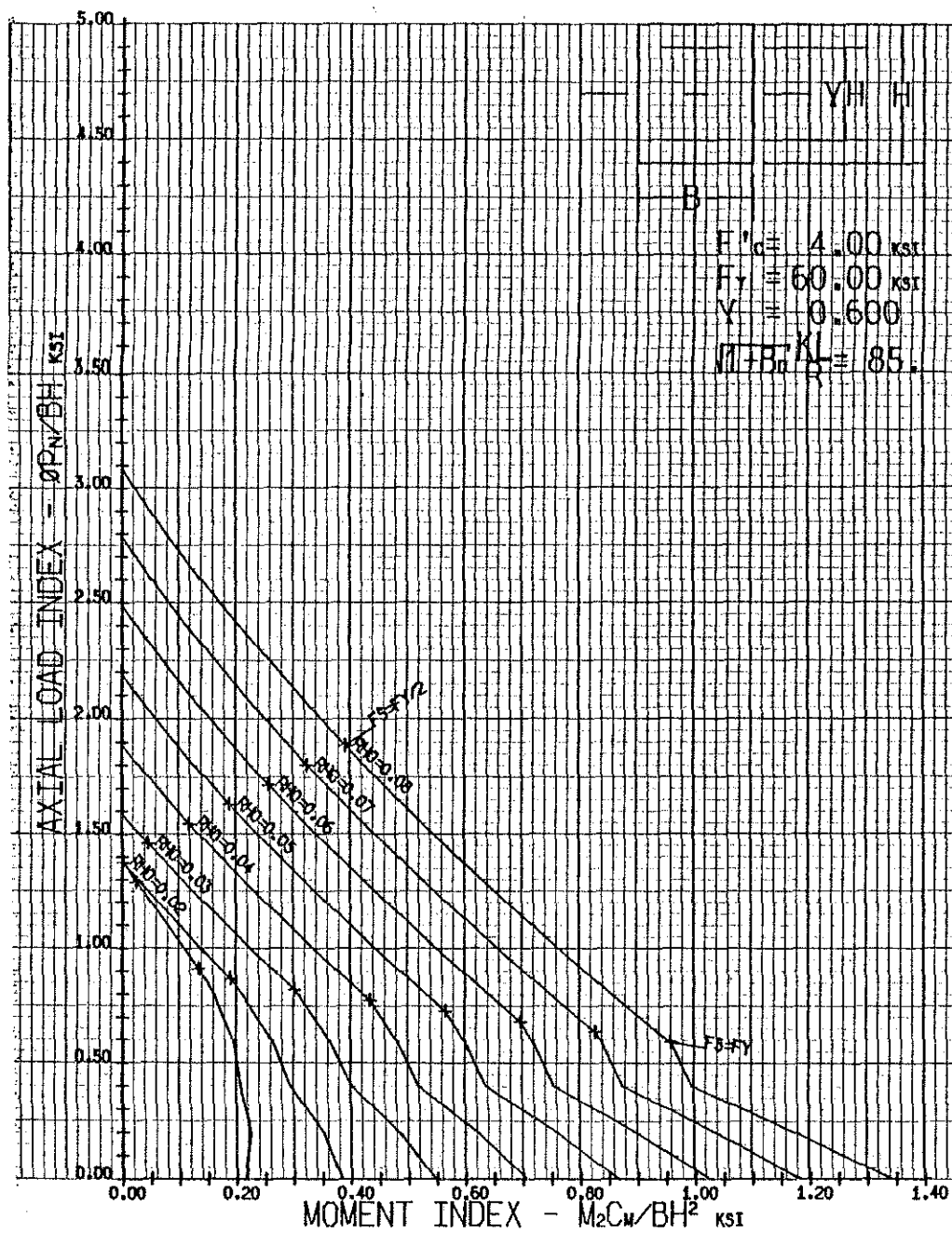


Fig. E4-60.60-85 - Interaction Diagram

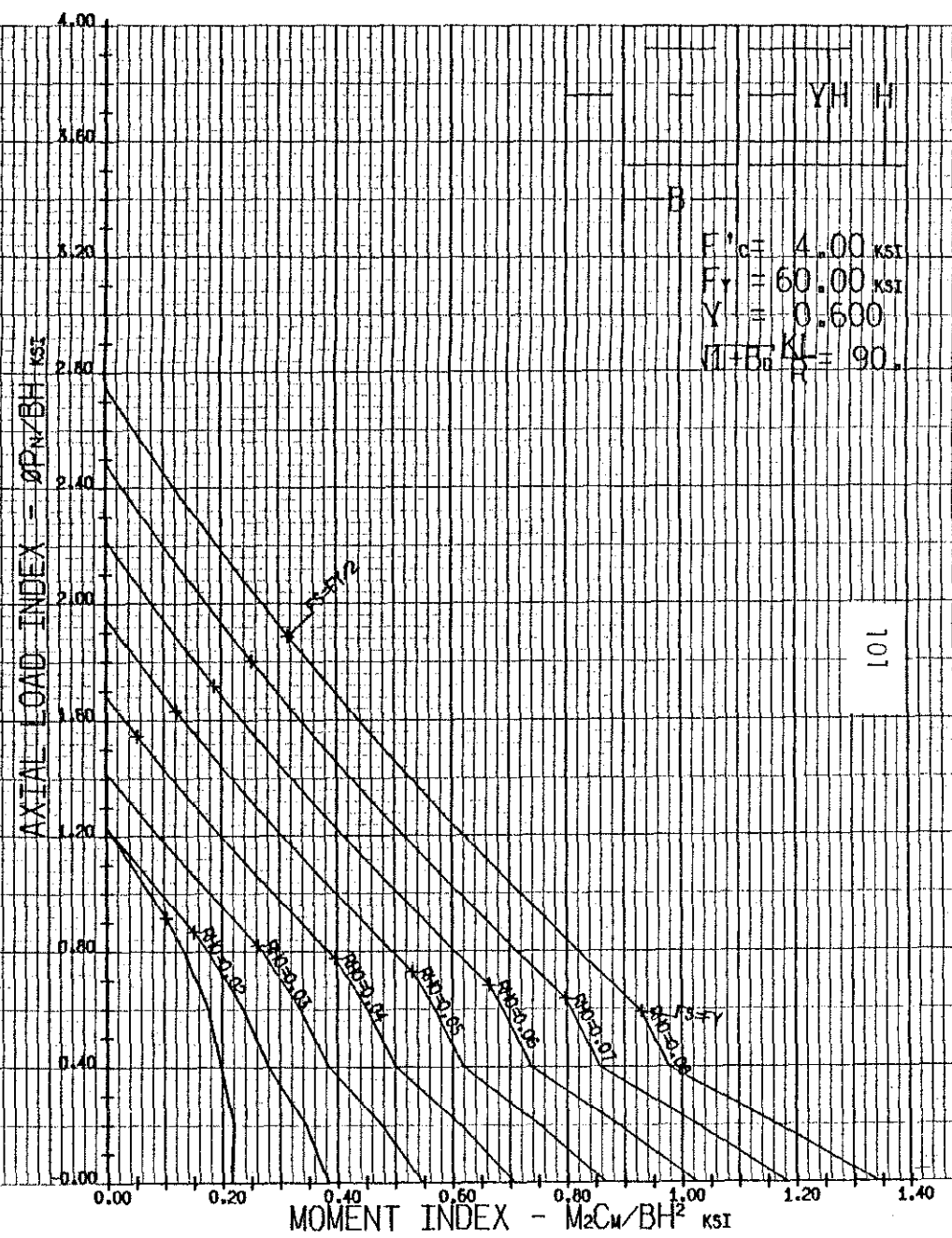


Fig. E4-60.60-90 - Interaction Diagram



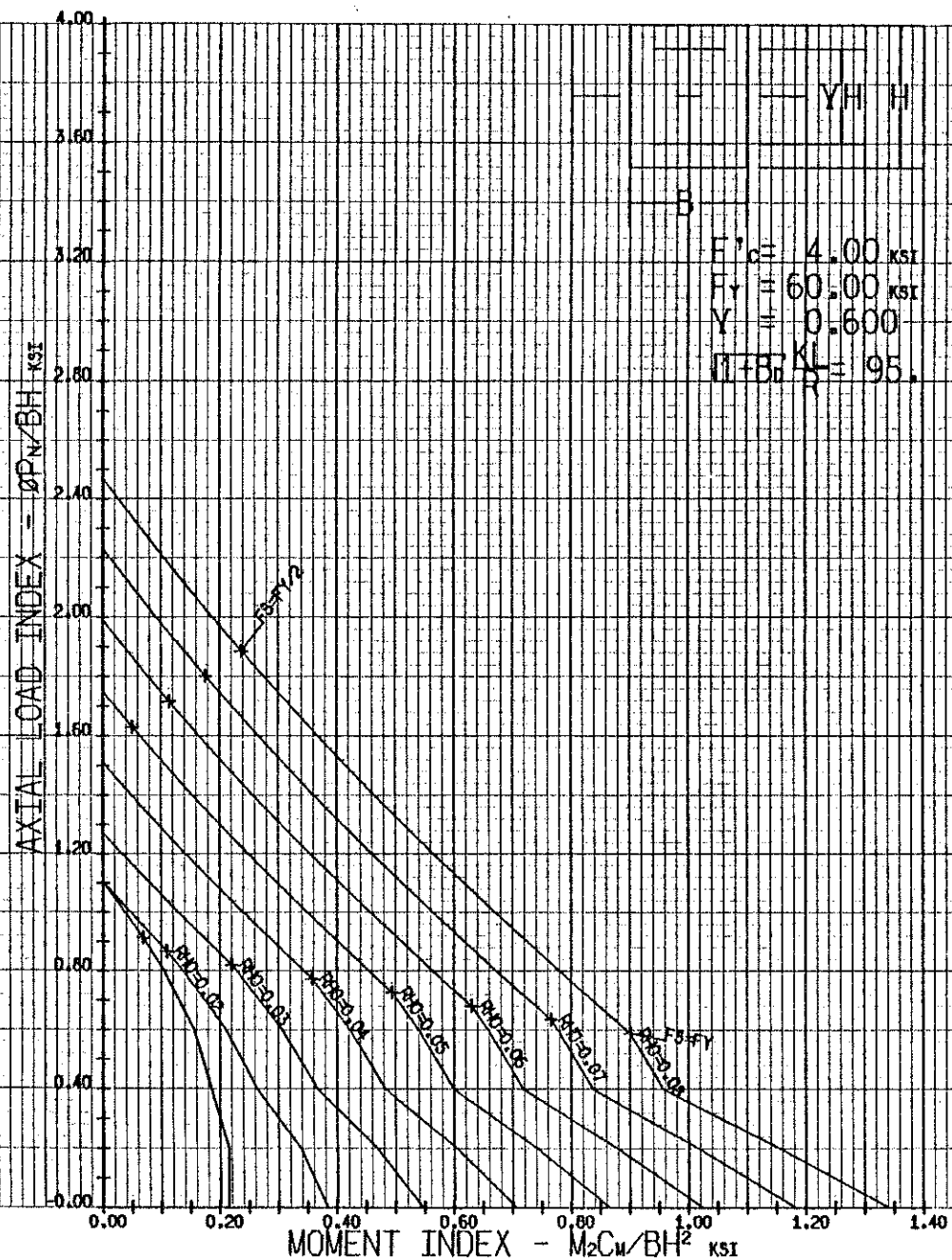


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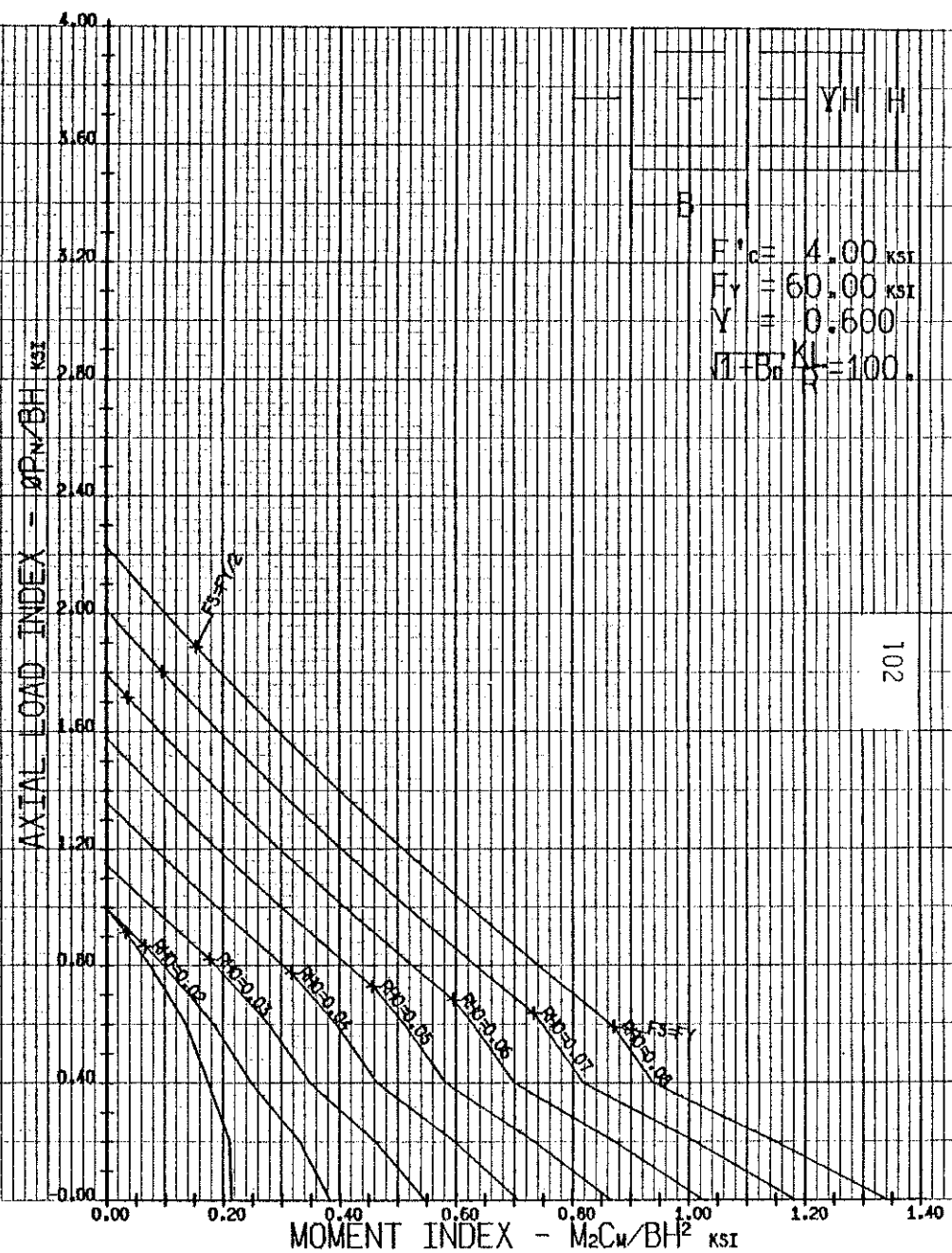


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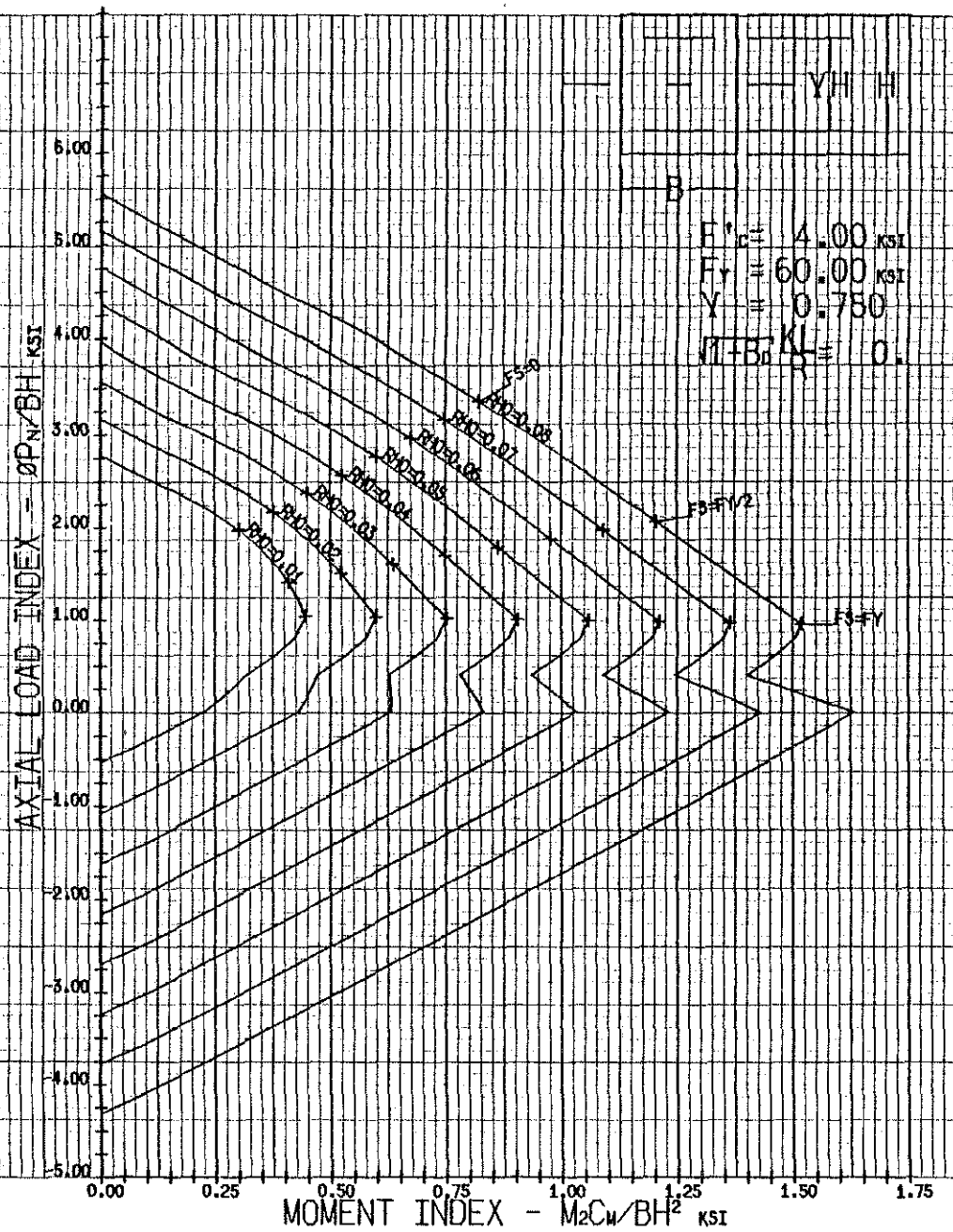


Fig. E4-60.75-0 - Interaction Diagram

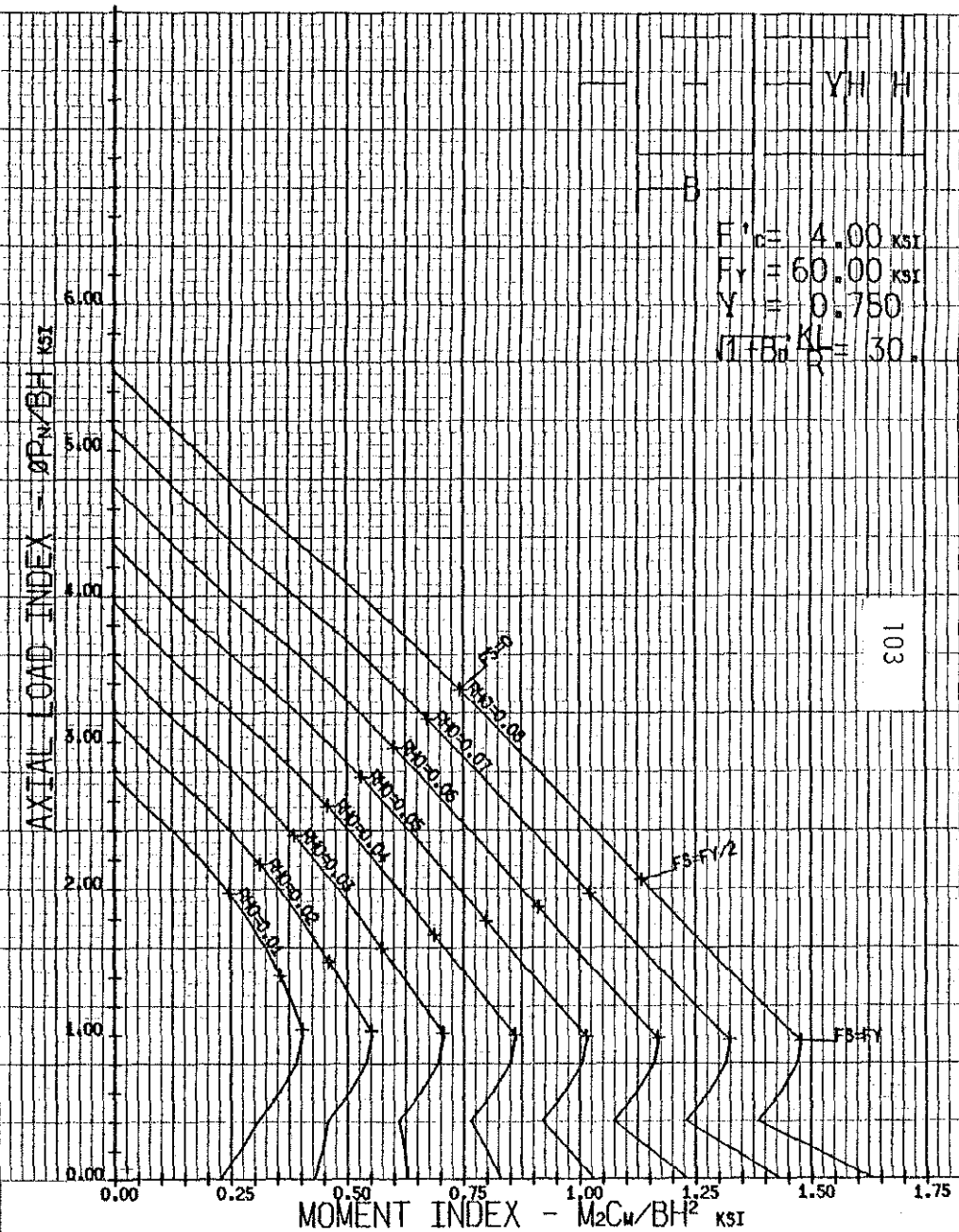


Fig. E4-60.75-30 - Interaction Diagram

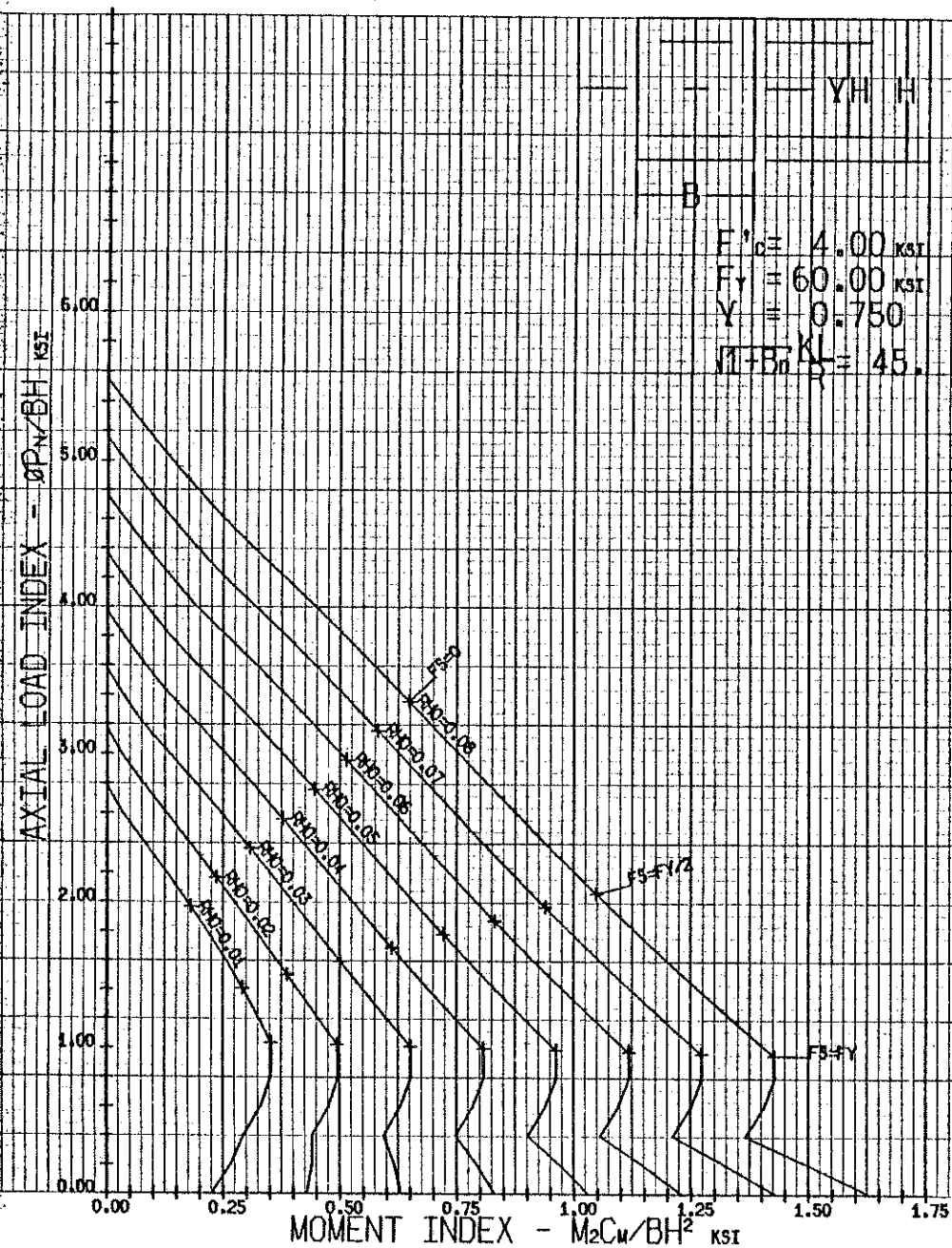


Fig. E4-60.75-45 - Interaction Diagram

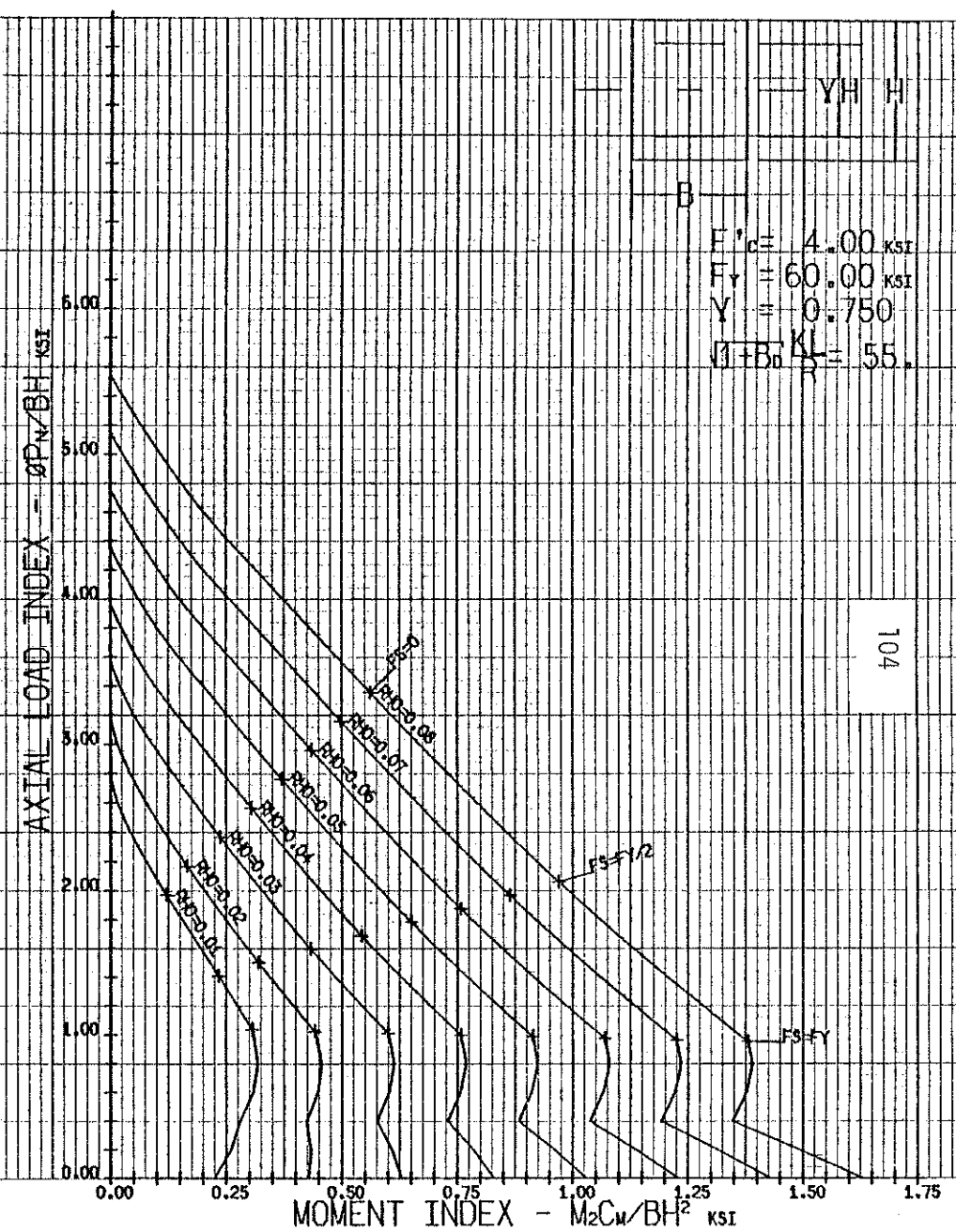


Fig. E4-60.75-55 - Interaction Diagram

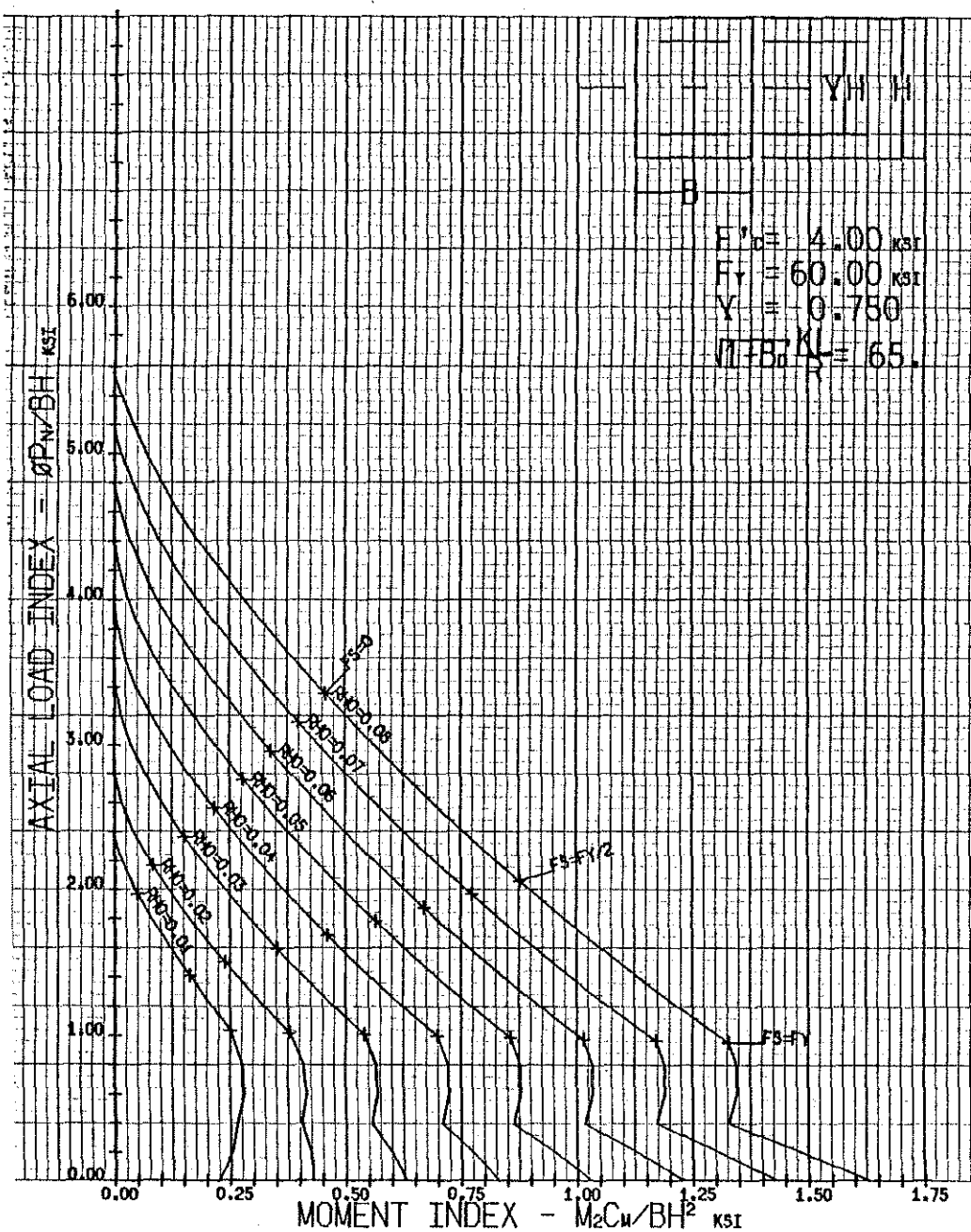


Fig. E4-60.75-65 - Interaction Diagram

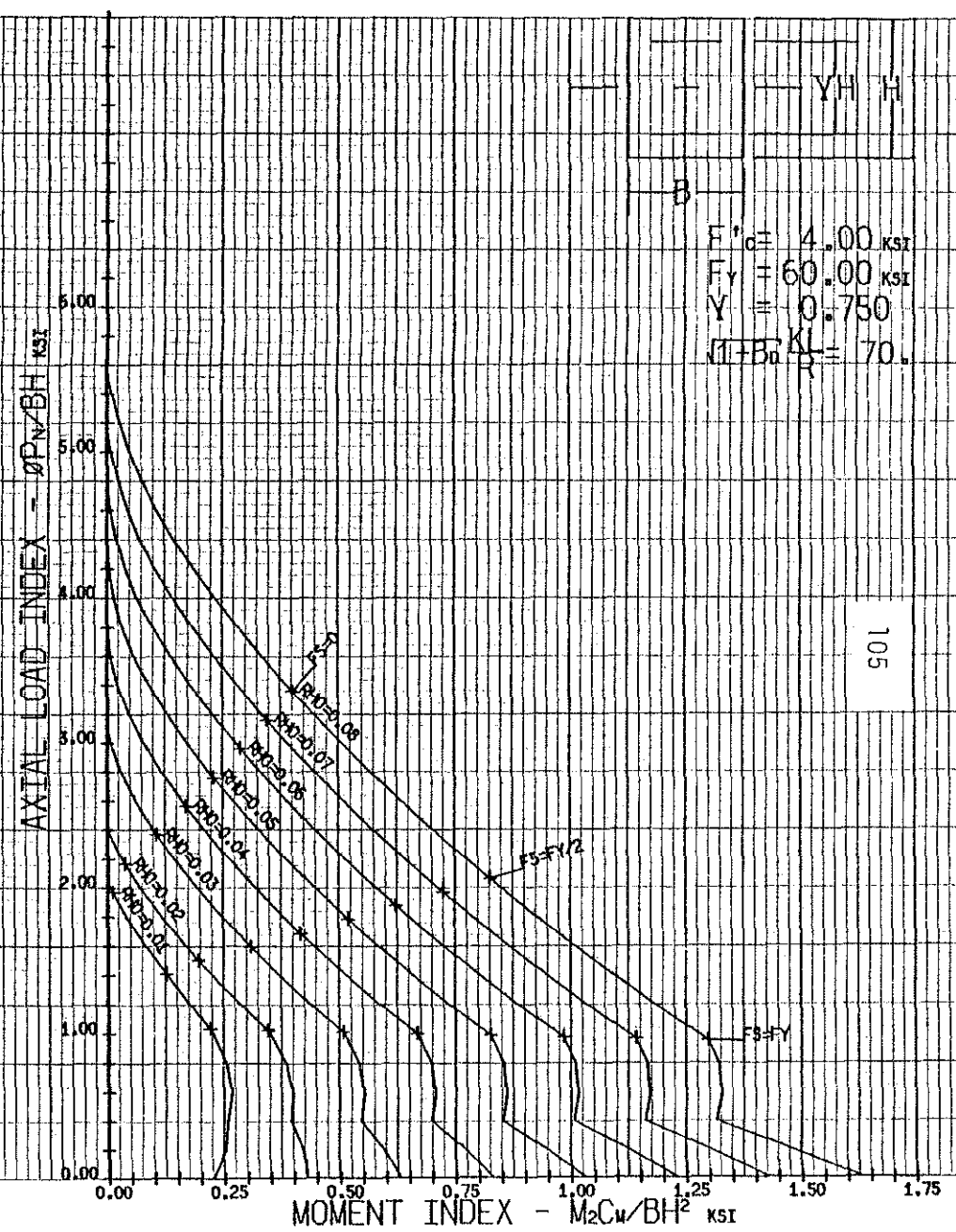


Fig. E4-60.75-70 - Interaction Diagram

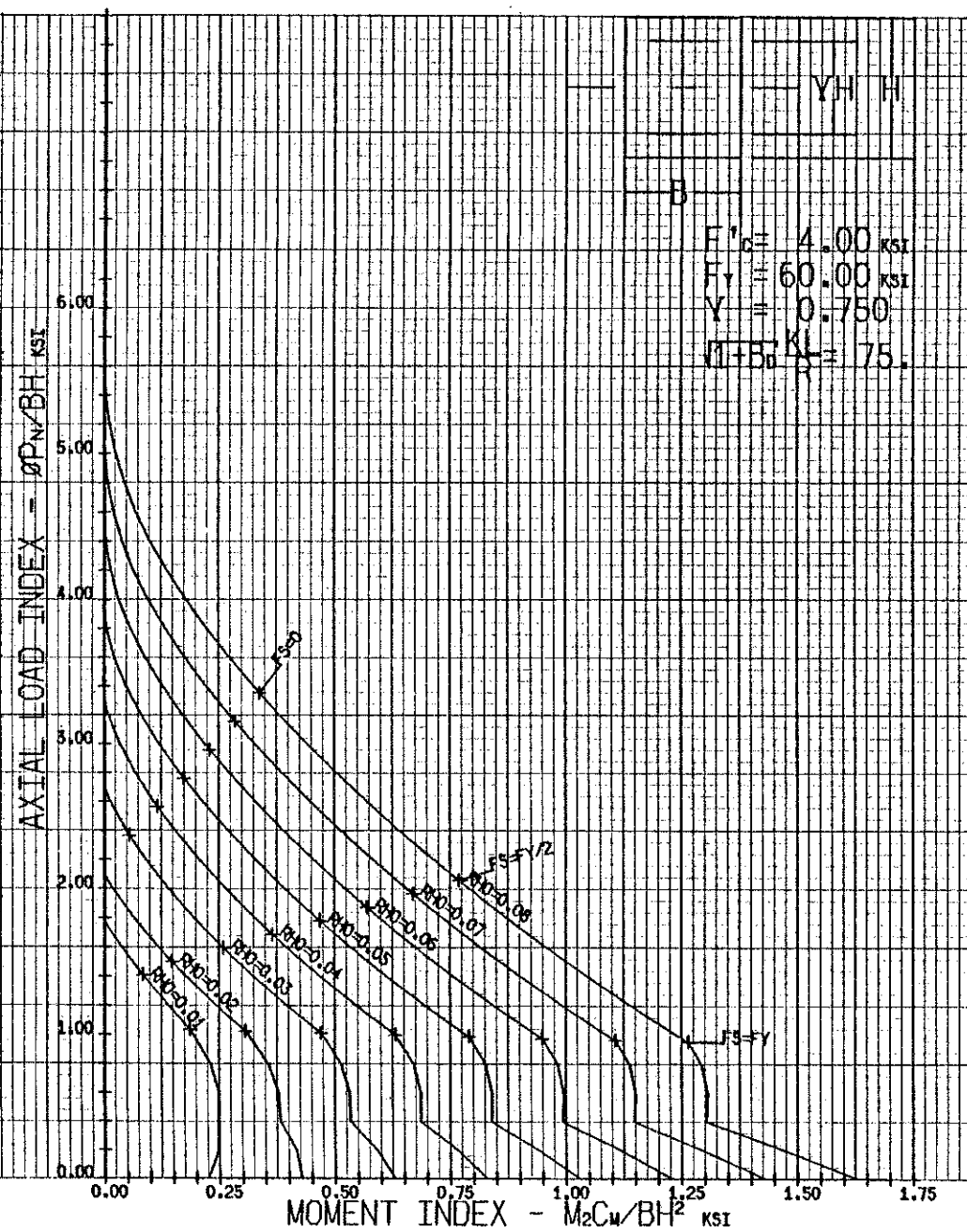


Fig. E4-60.75-75 - Interaction Diagram

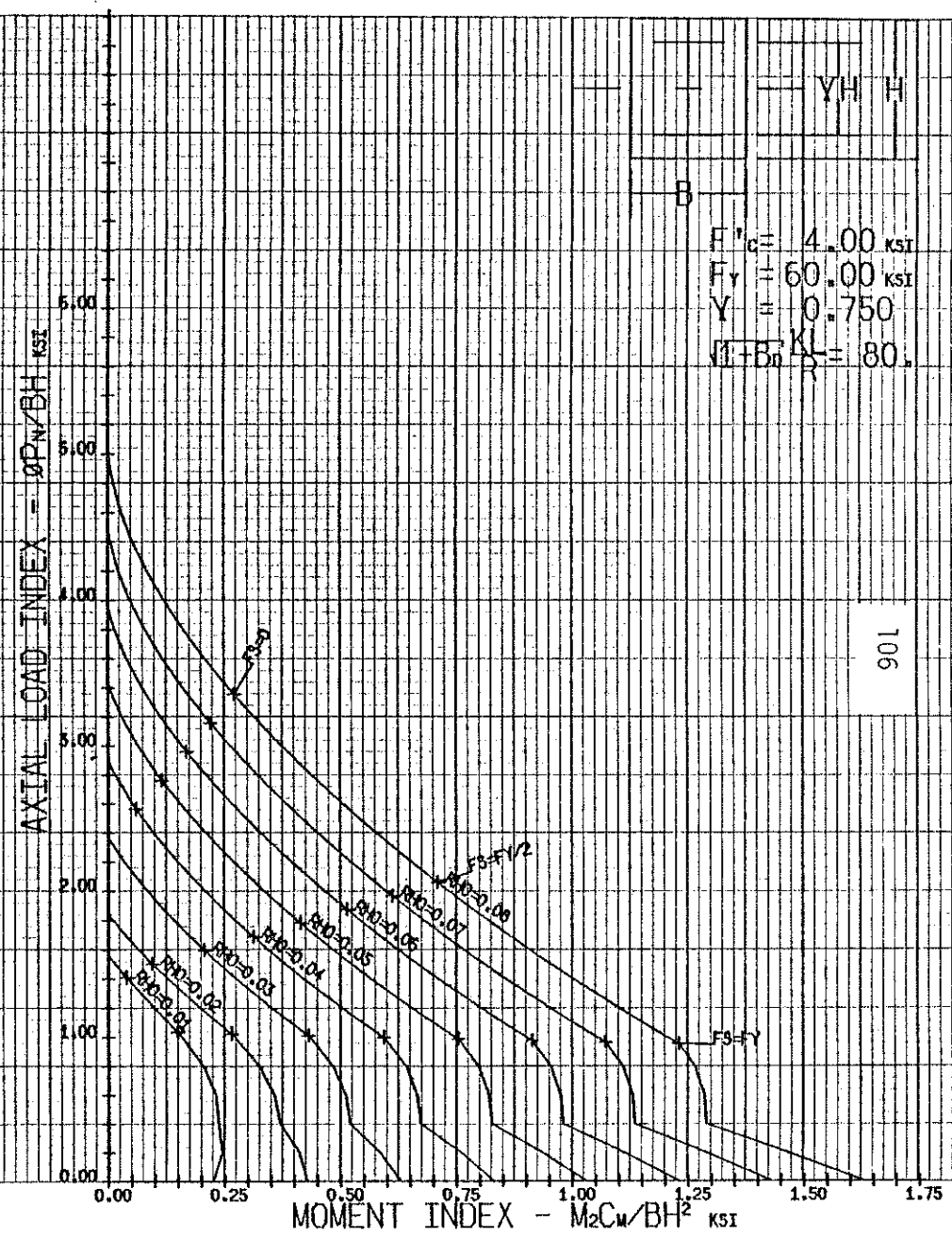


Fig. E4-60.75-80 - Interaction Diagram



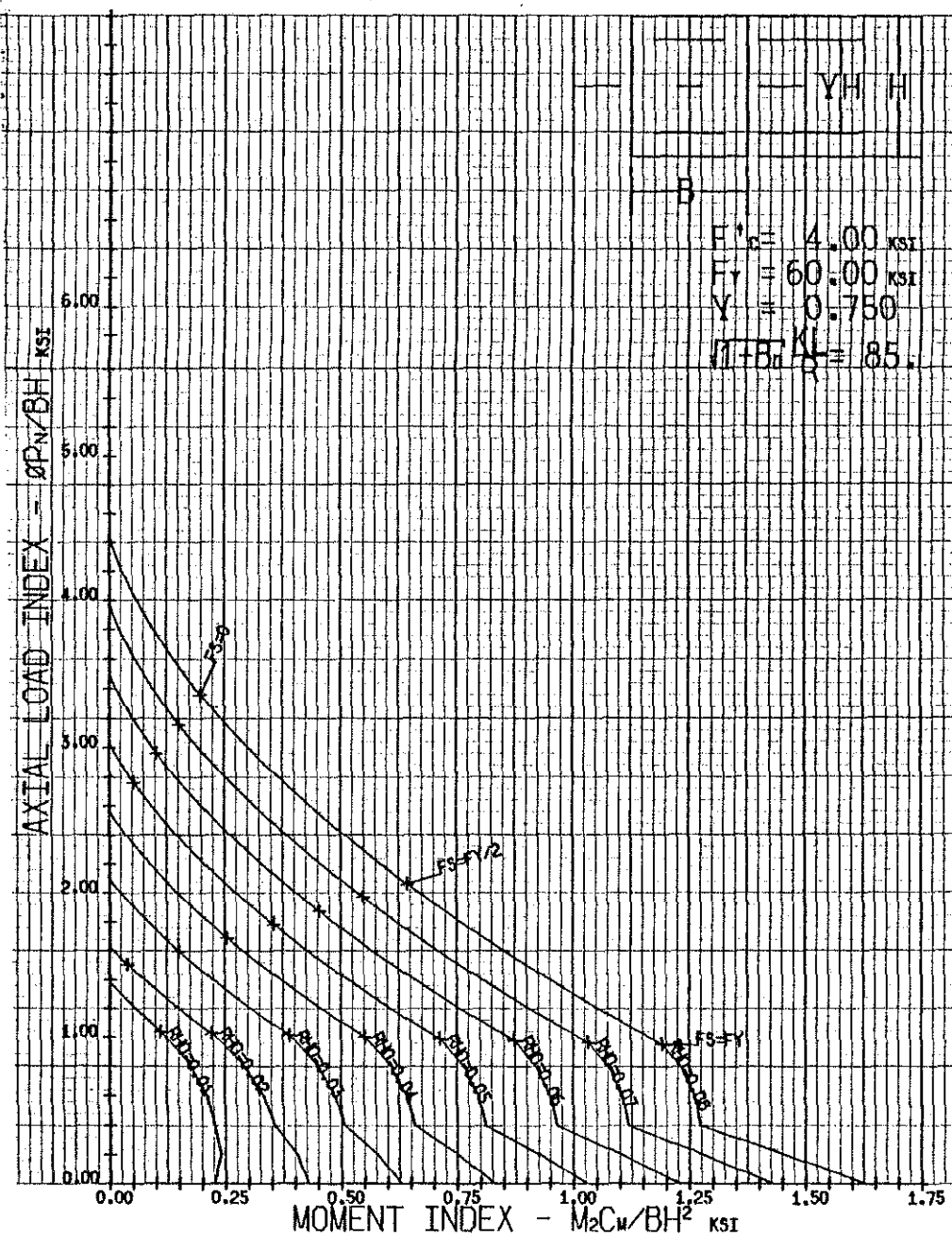


Fig. E4-60.75-85 - Interaction Diagram

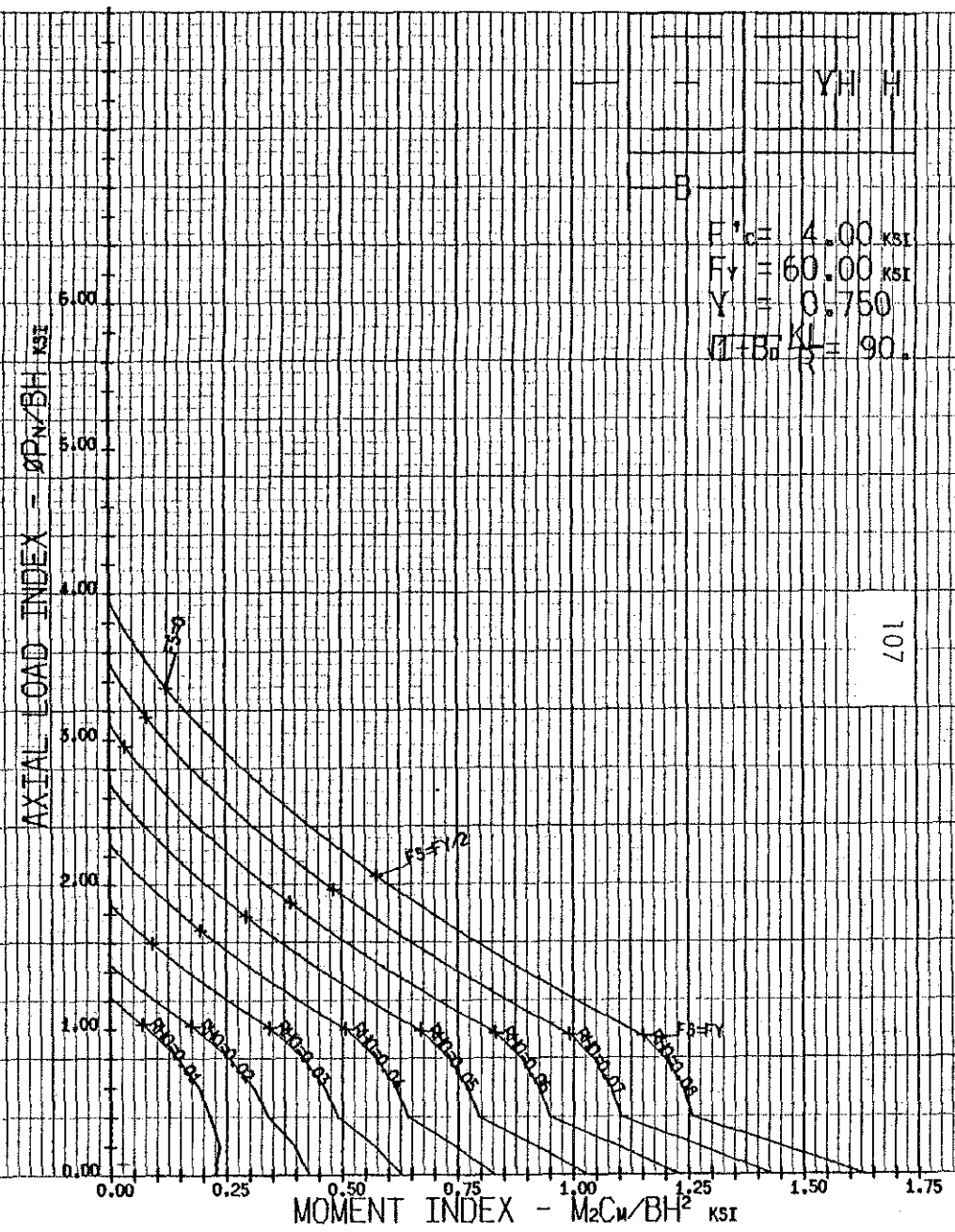


Fig. E4-60.75-90 - Interaction Diagram

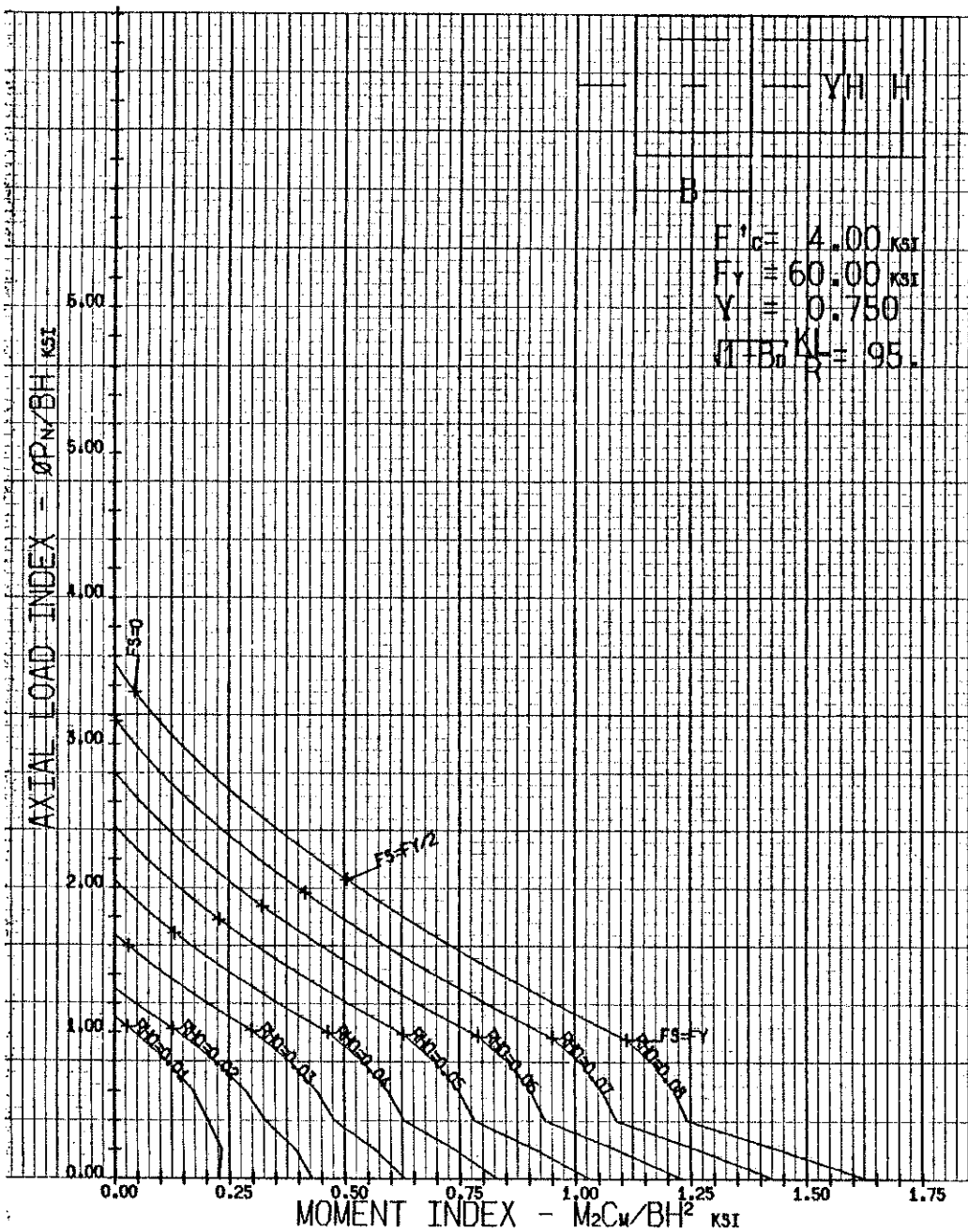


Fig. E4-60.75-95 - Interaction Diagram

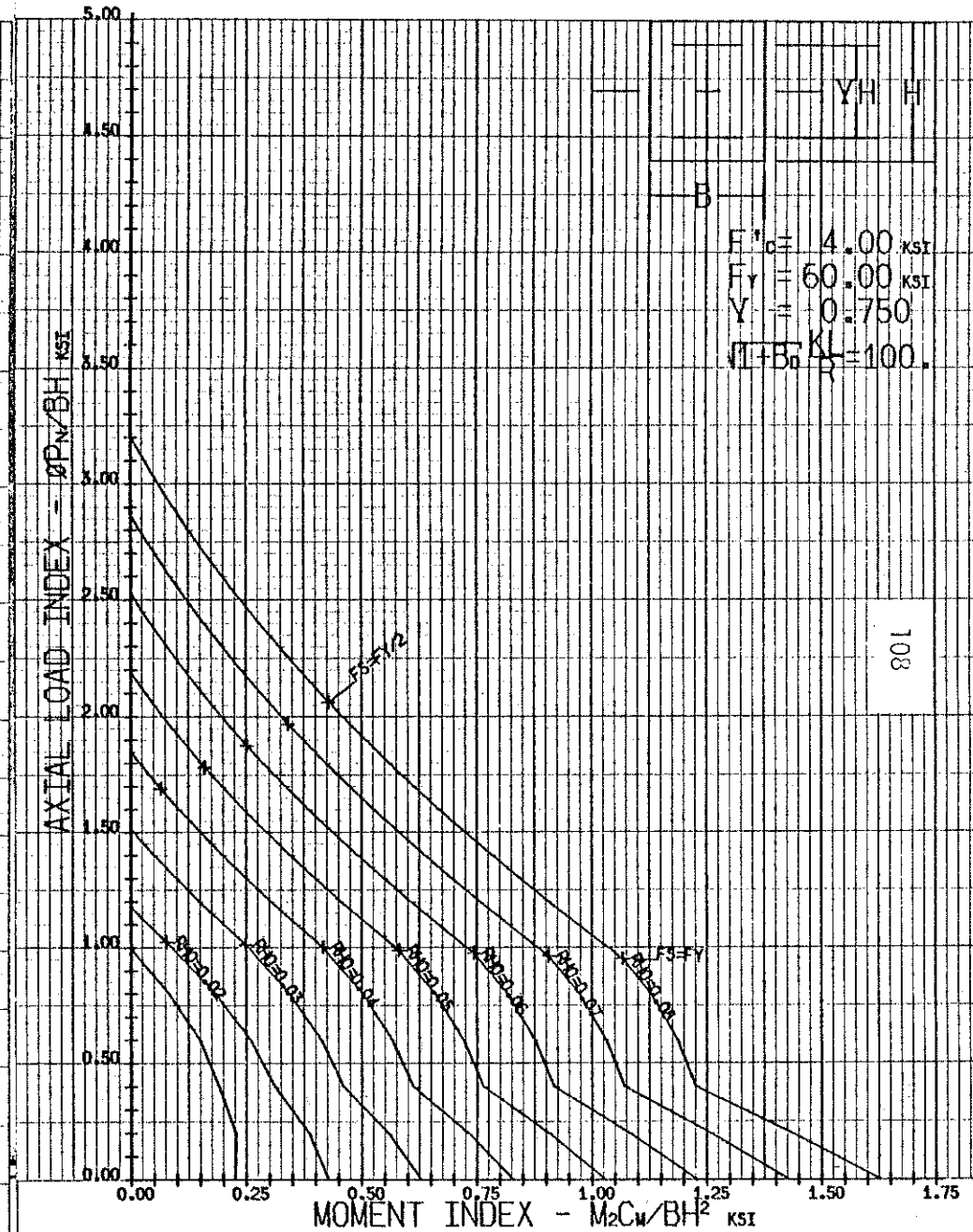


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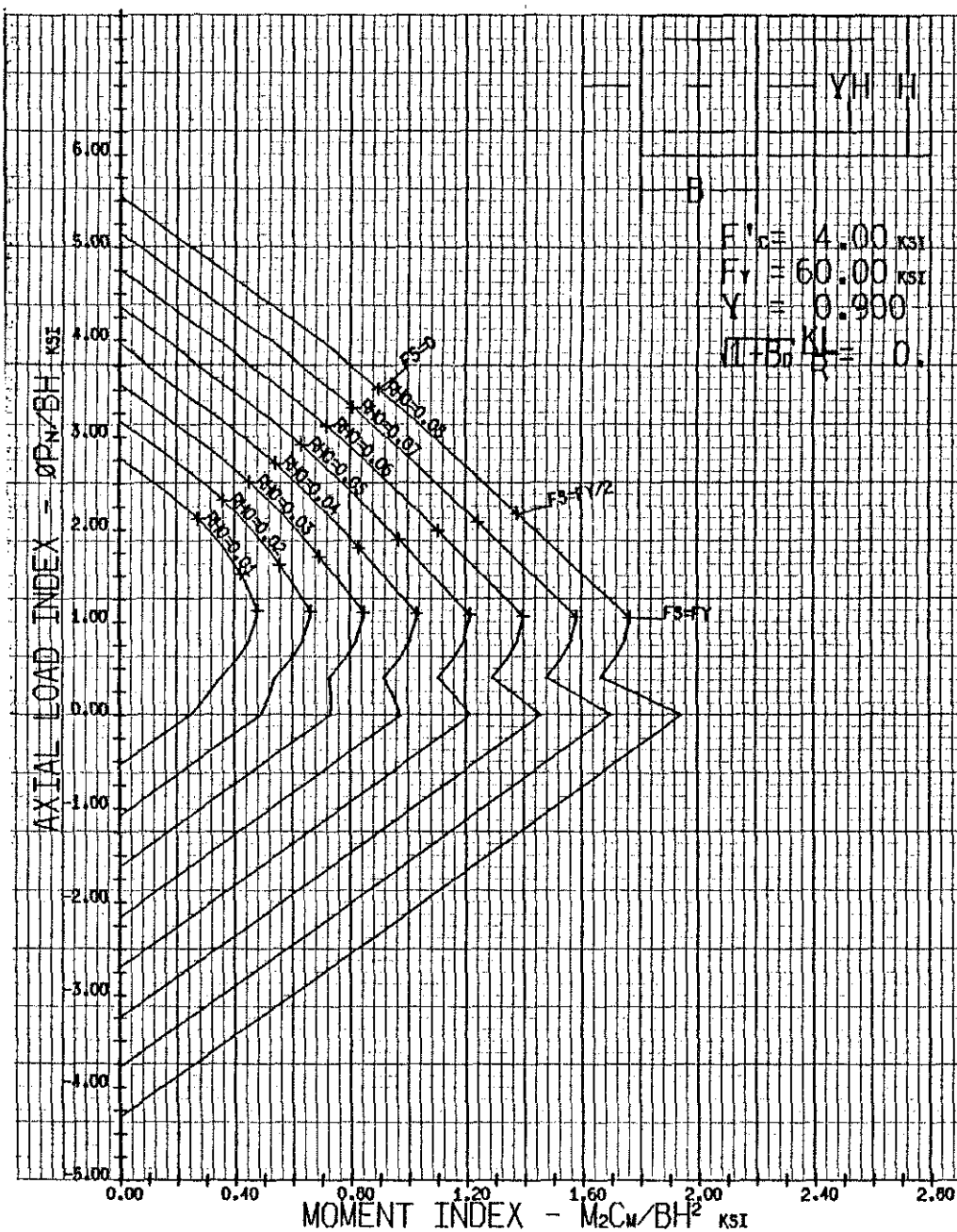


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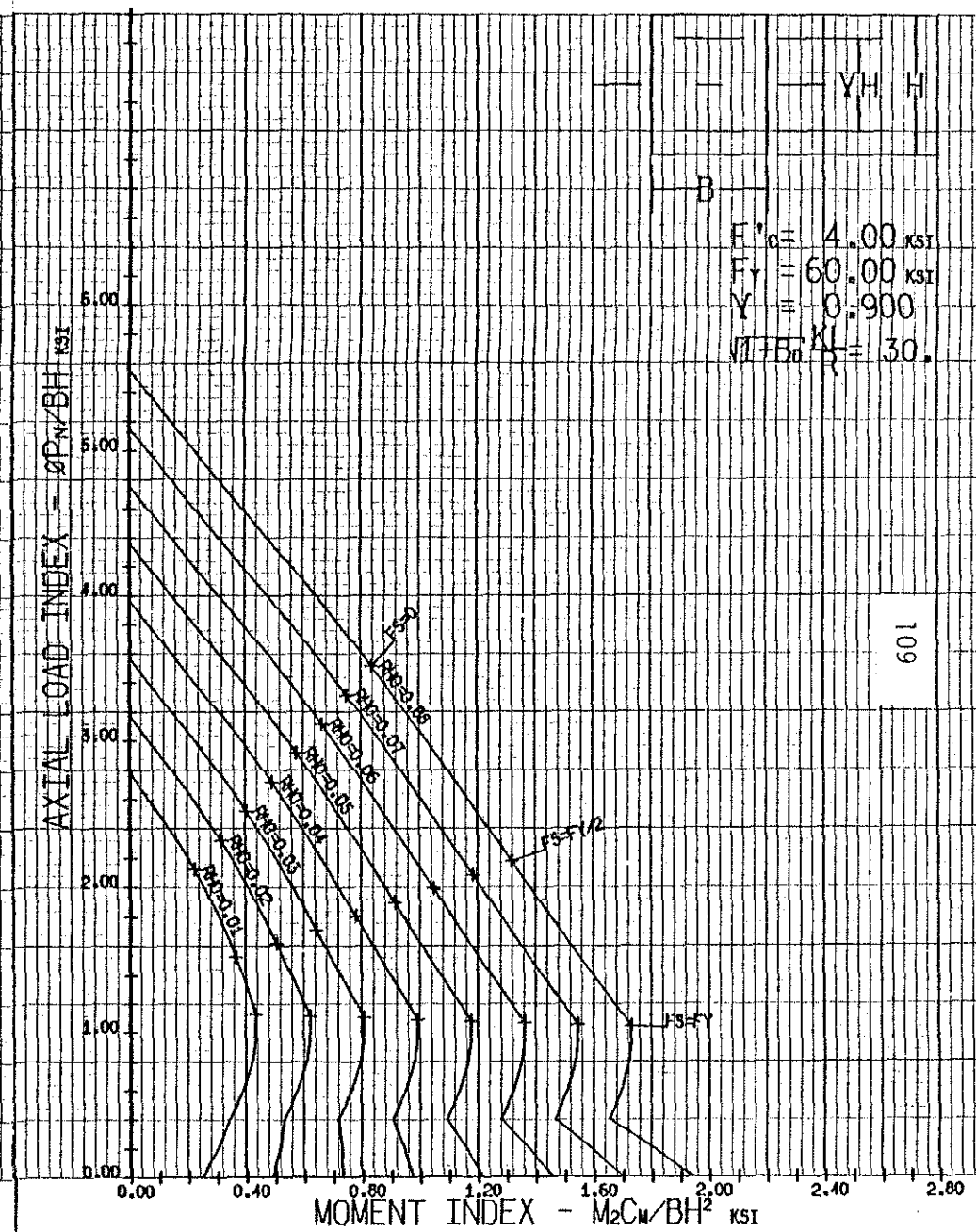


Fig. E4-60.90-30 - Interaction Diagram



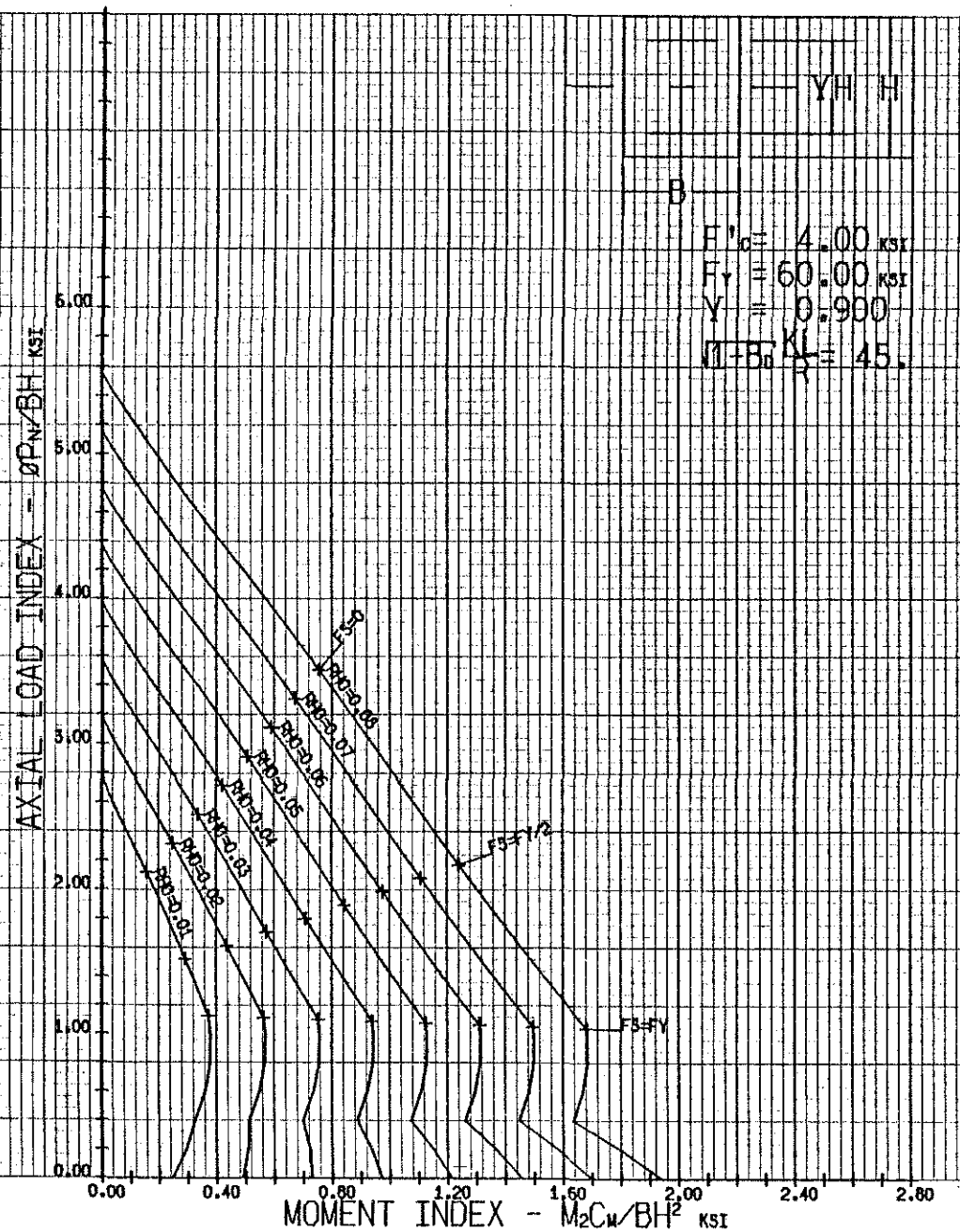


Fig. E4-60.90-45 - Interaction Diagram

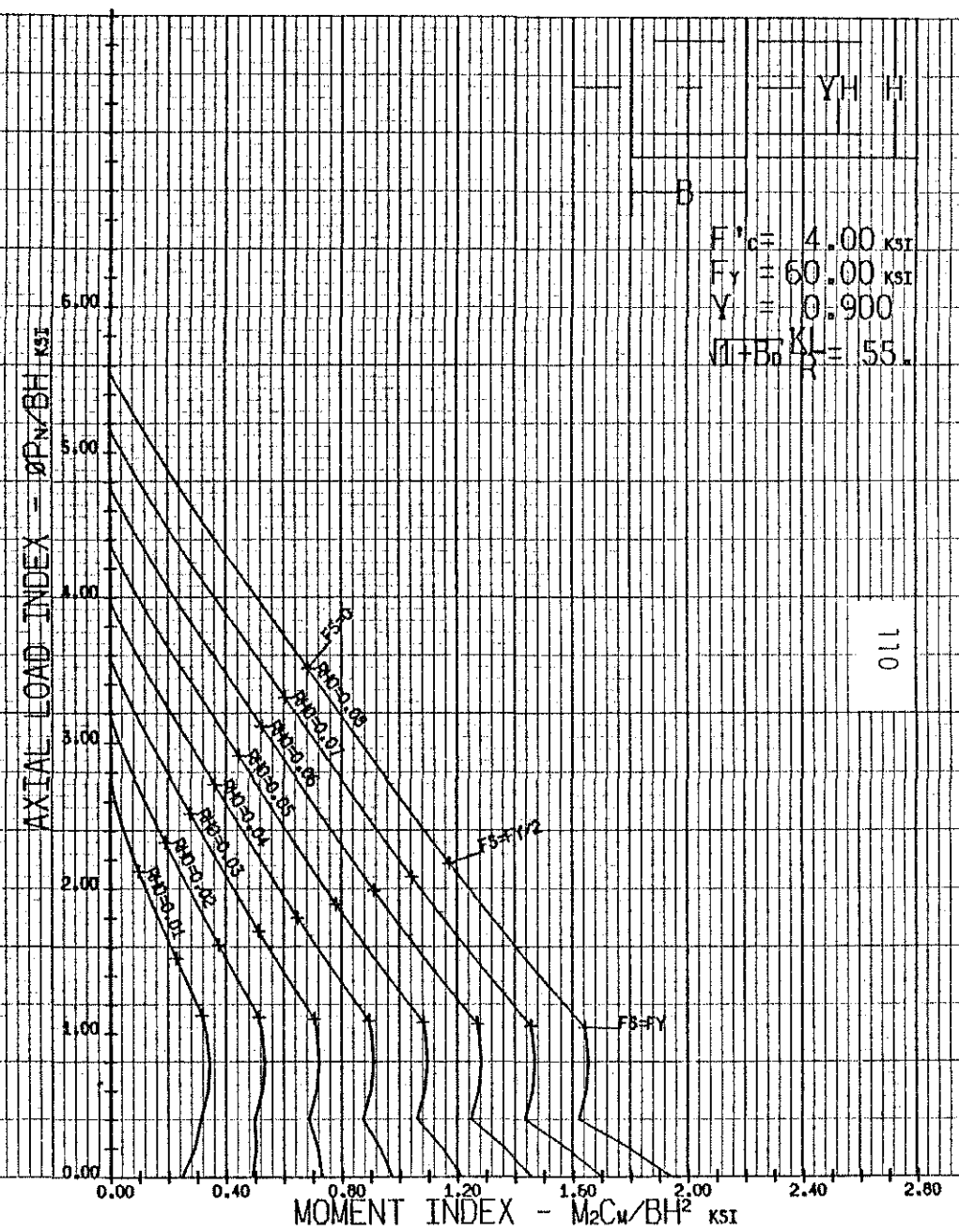


Fig. E4-60.90-55 - Interaction Diagram

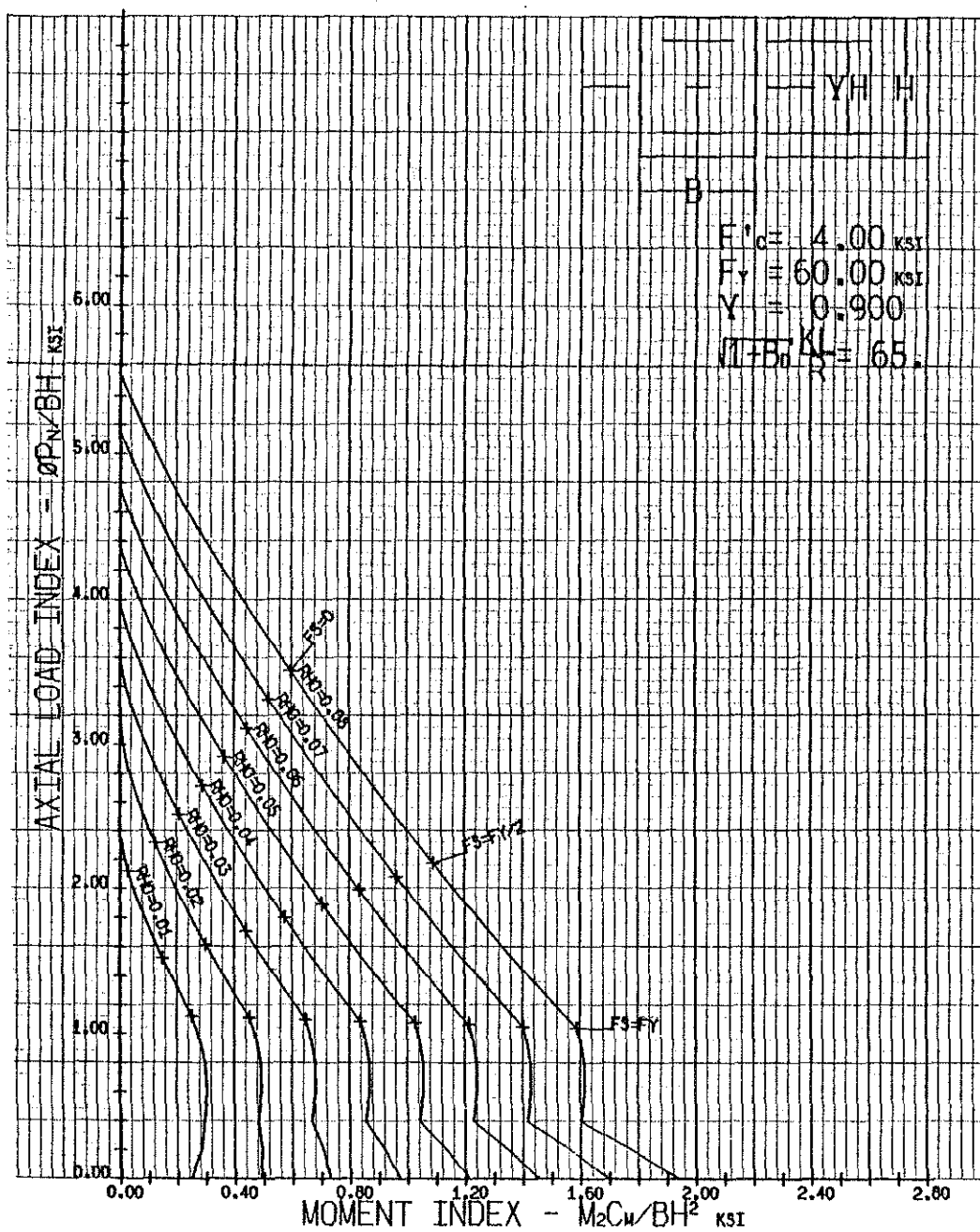


Fig. E4-60.90-65 - Interaction Diagram

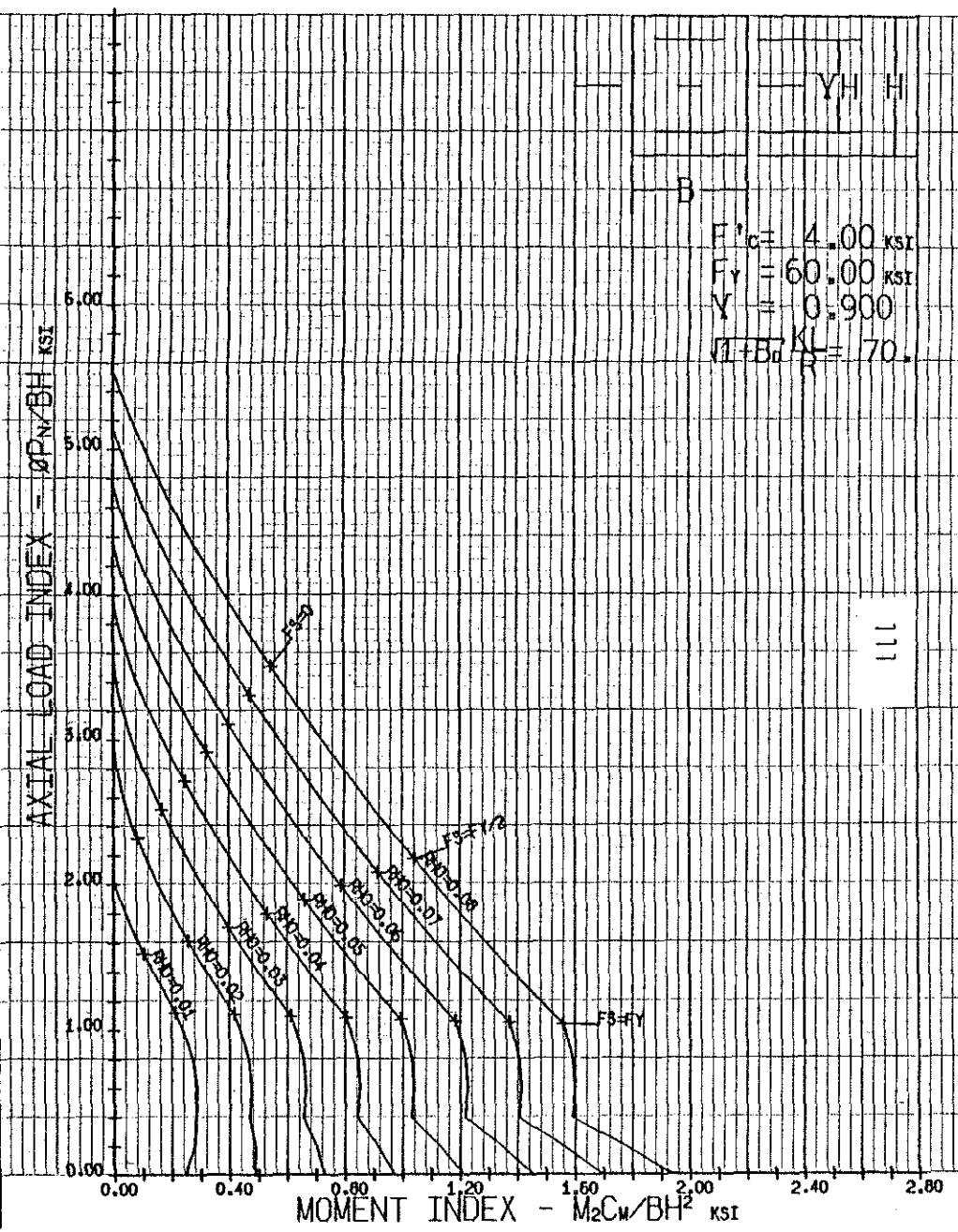


Fig. E4-60.90-70 - Interaction Diagram

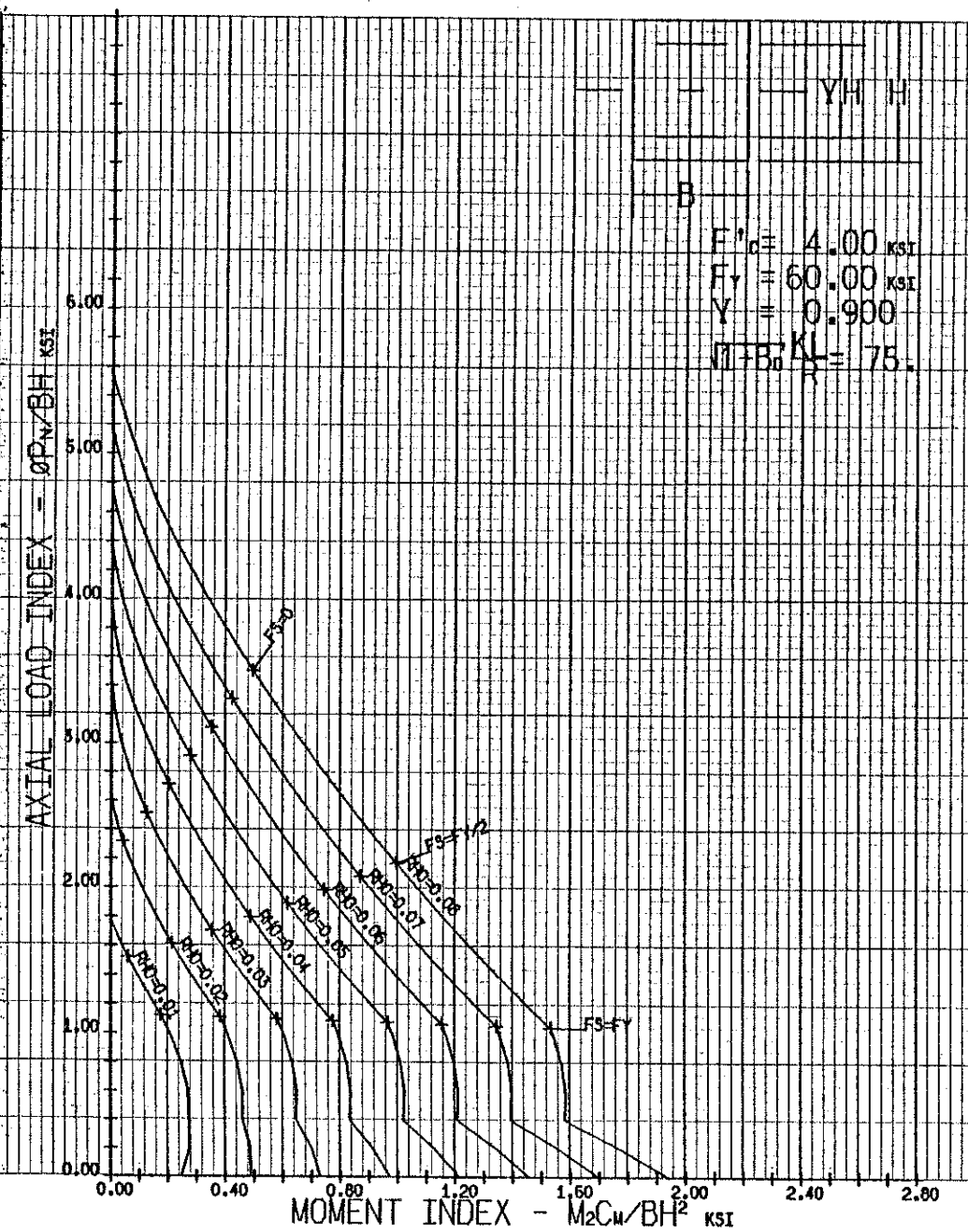


Fig. E4-60.90-75 - Interaction Diagram

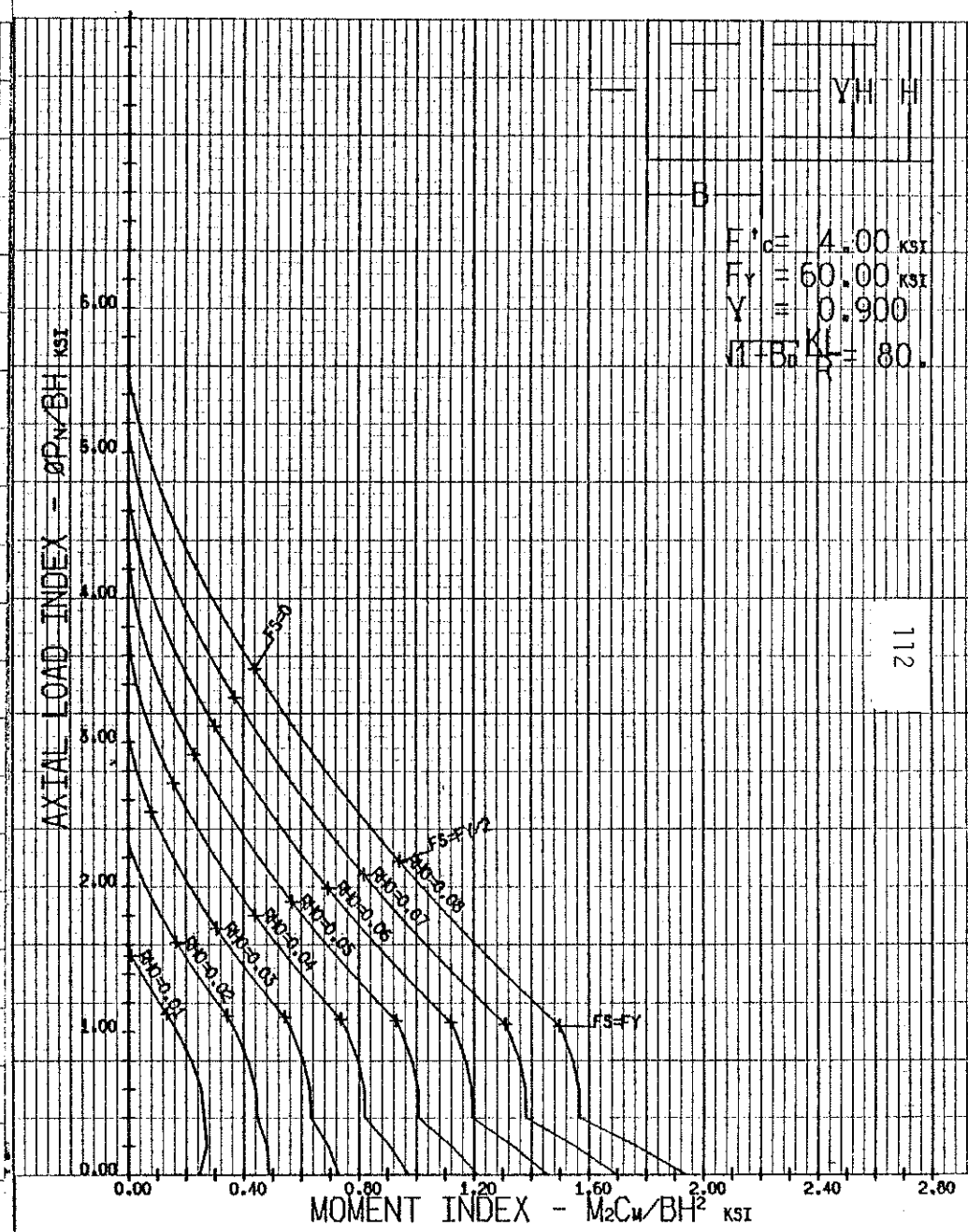
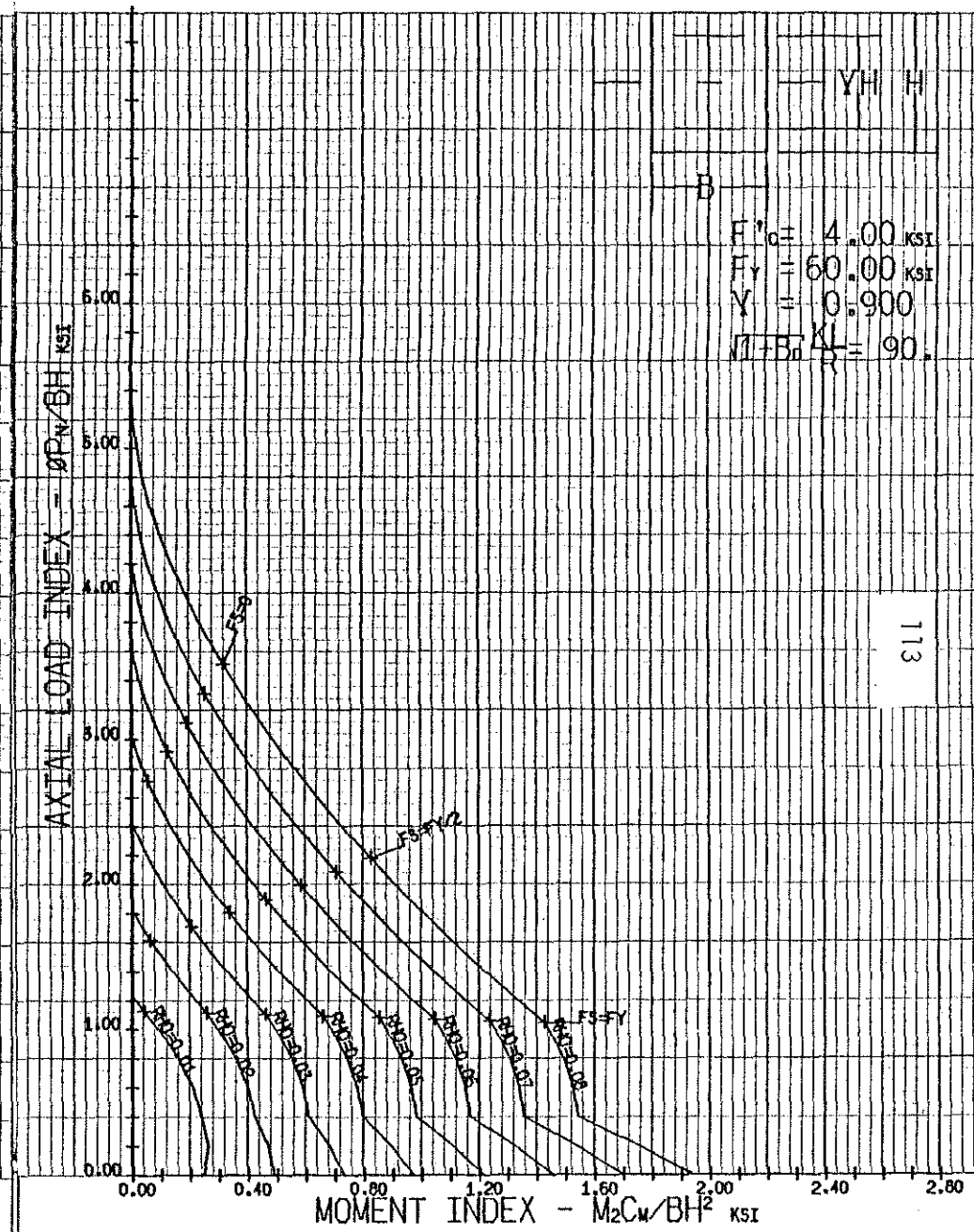
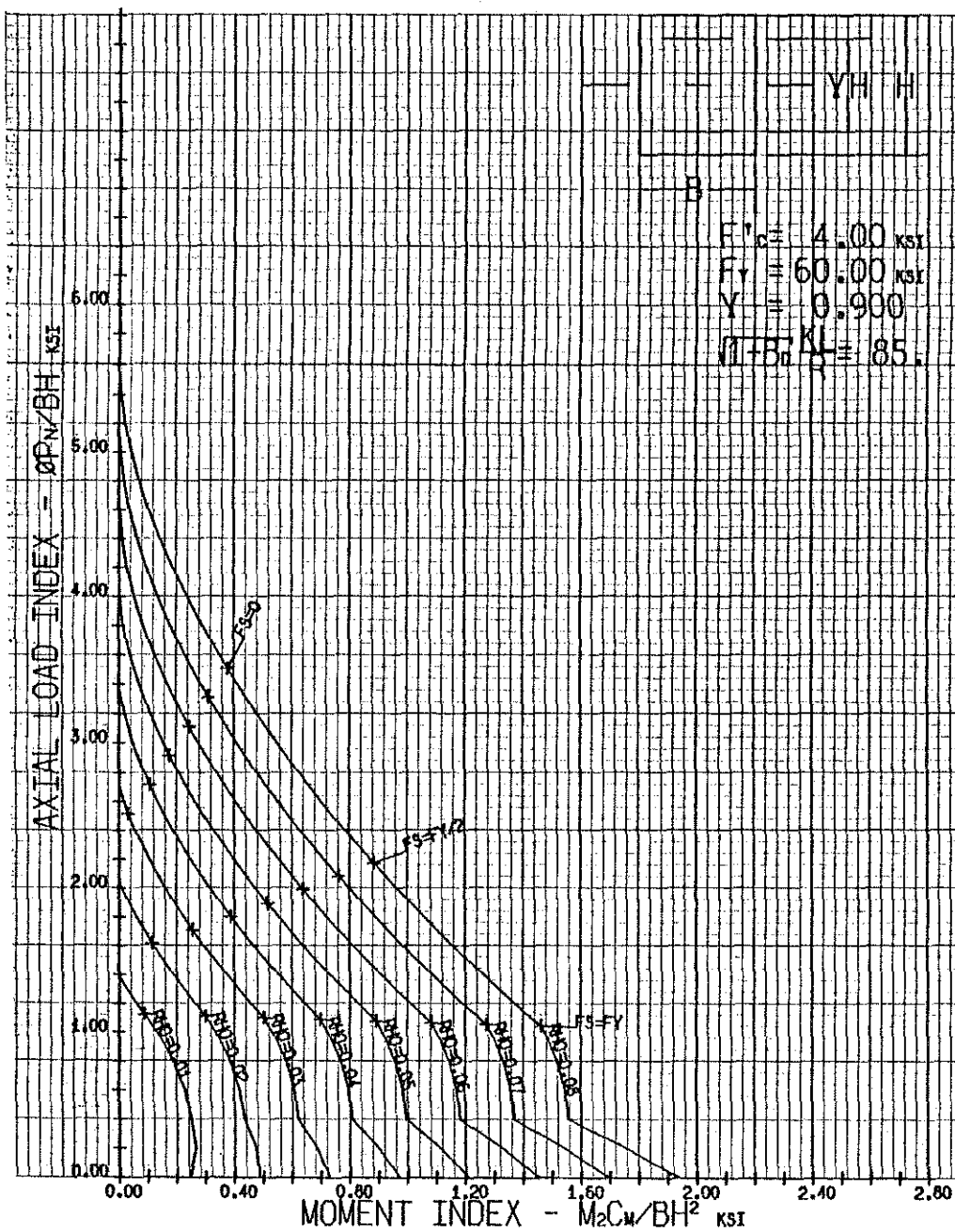


Fig. E4-60.90-80 - Interaction Diagram



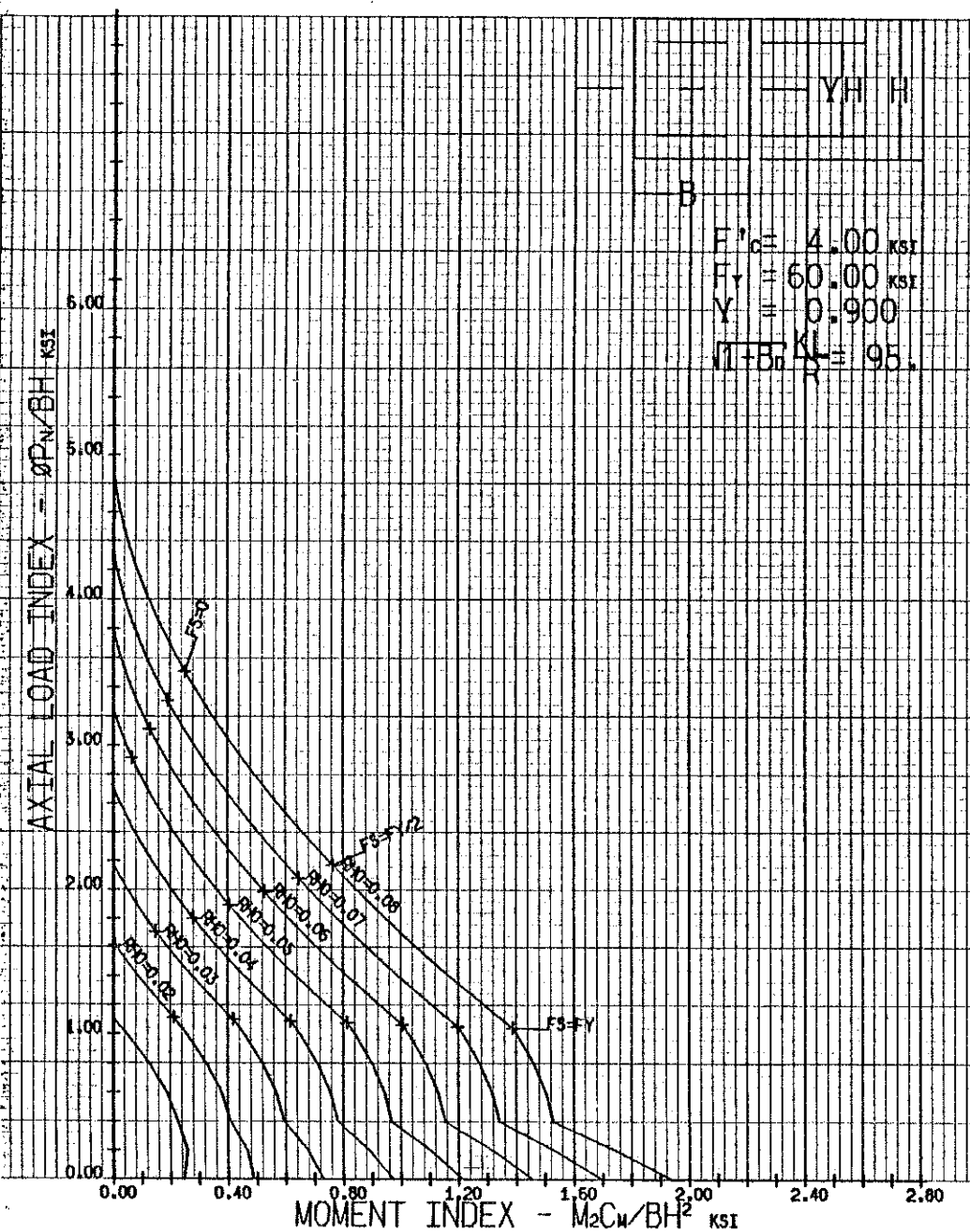


Fig. E4-60.90-95 - Interaction Diagram

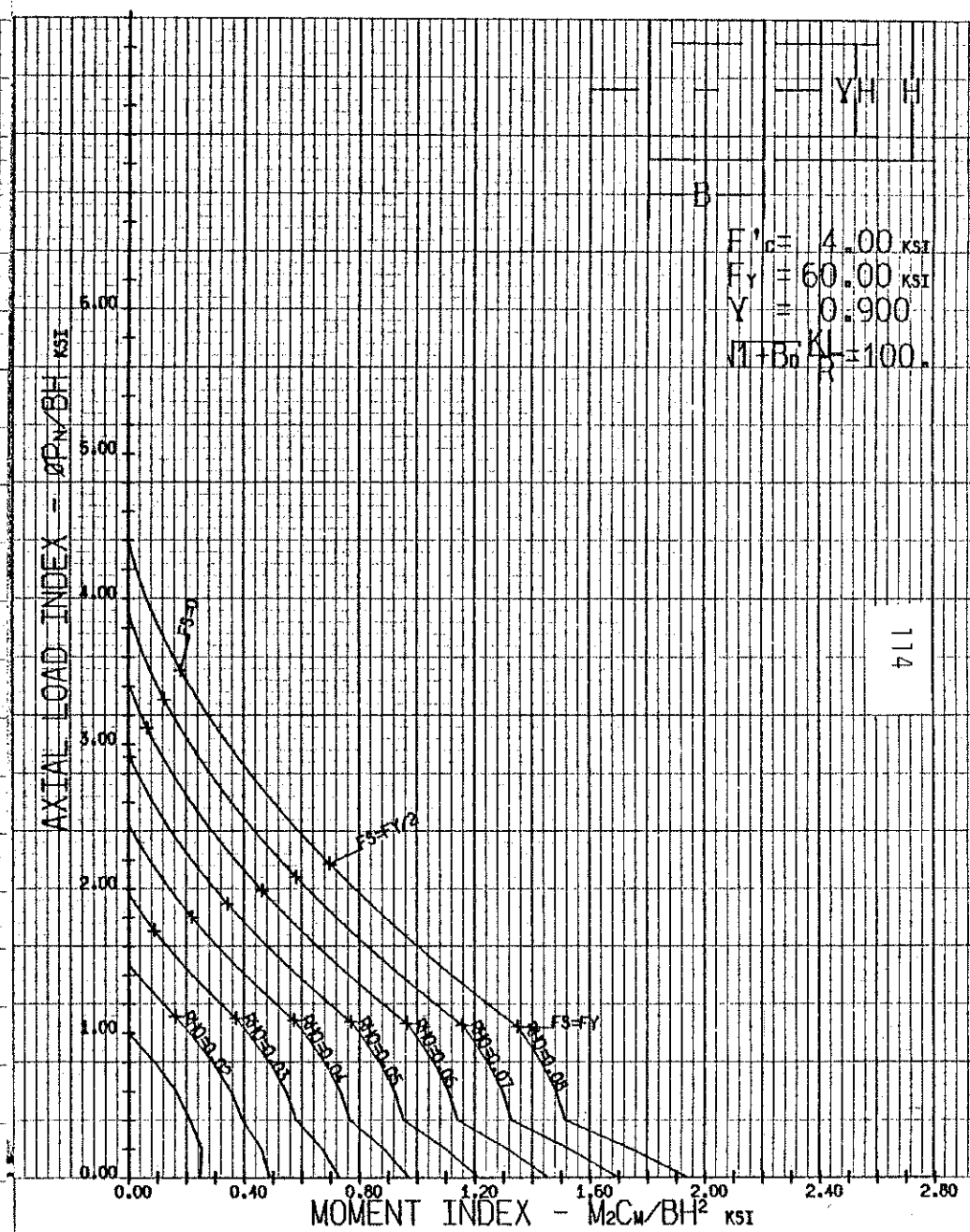


Fig. E4-60.90-100 - Interaction Diagram



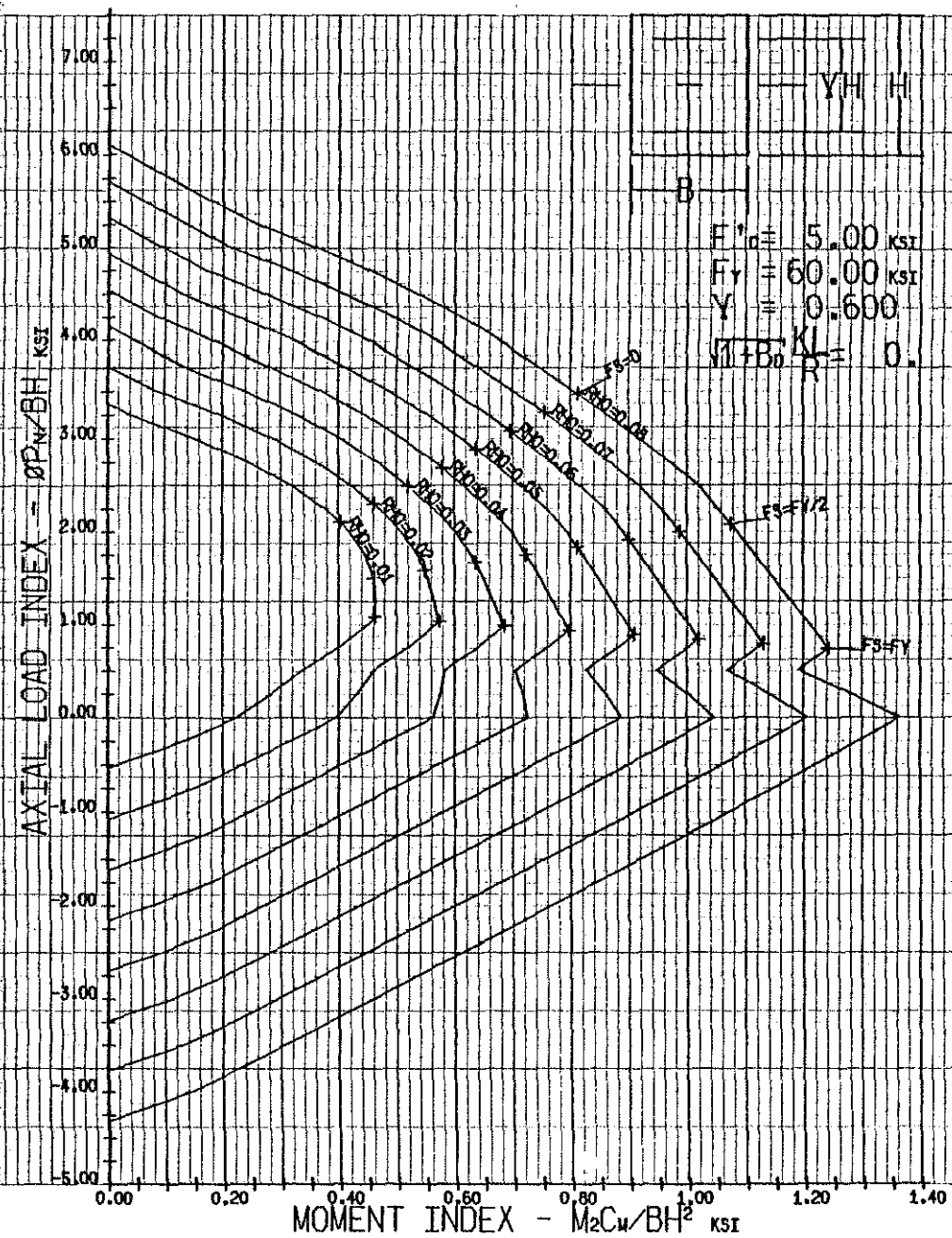


Fig. E5-60.60-0 - Interaction Diagram

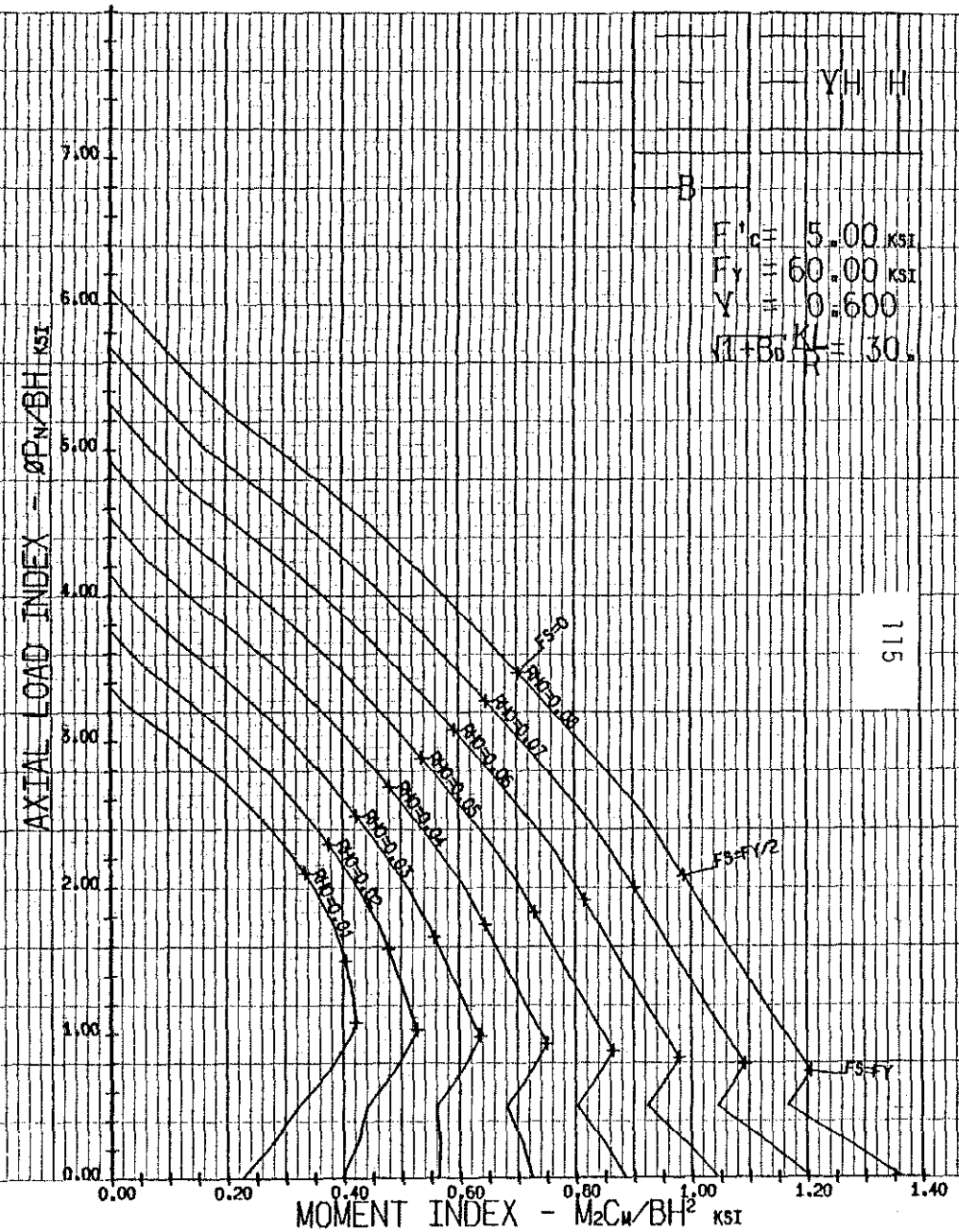


Fig. E5-60.60-30 - Interaction Diagram

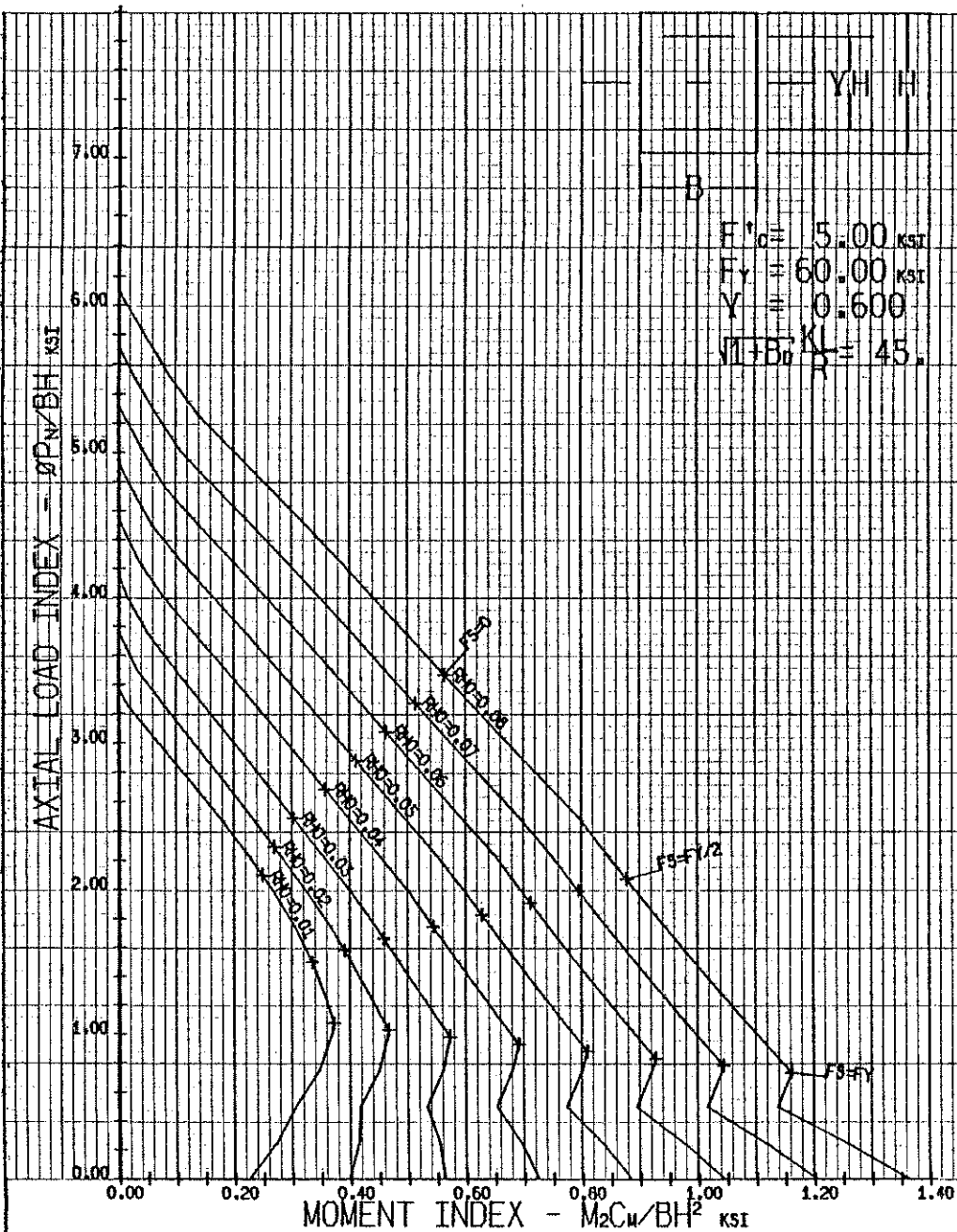


Fig. E5-60.60-45 - Interaction Diagram

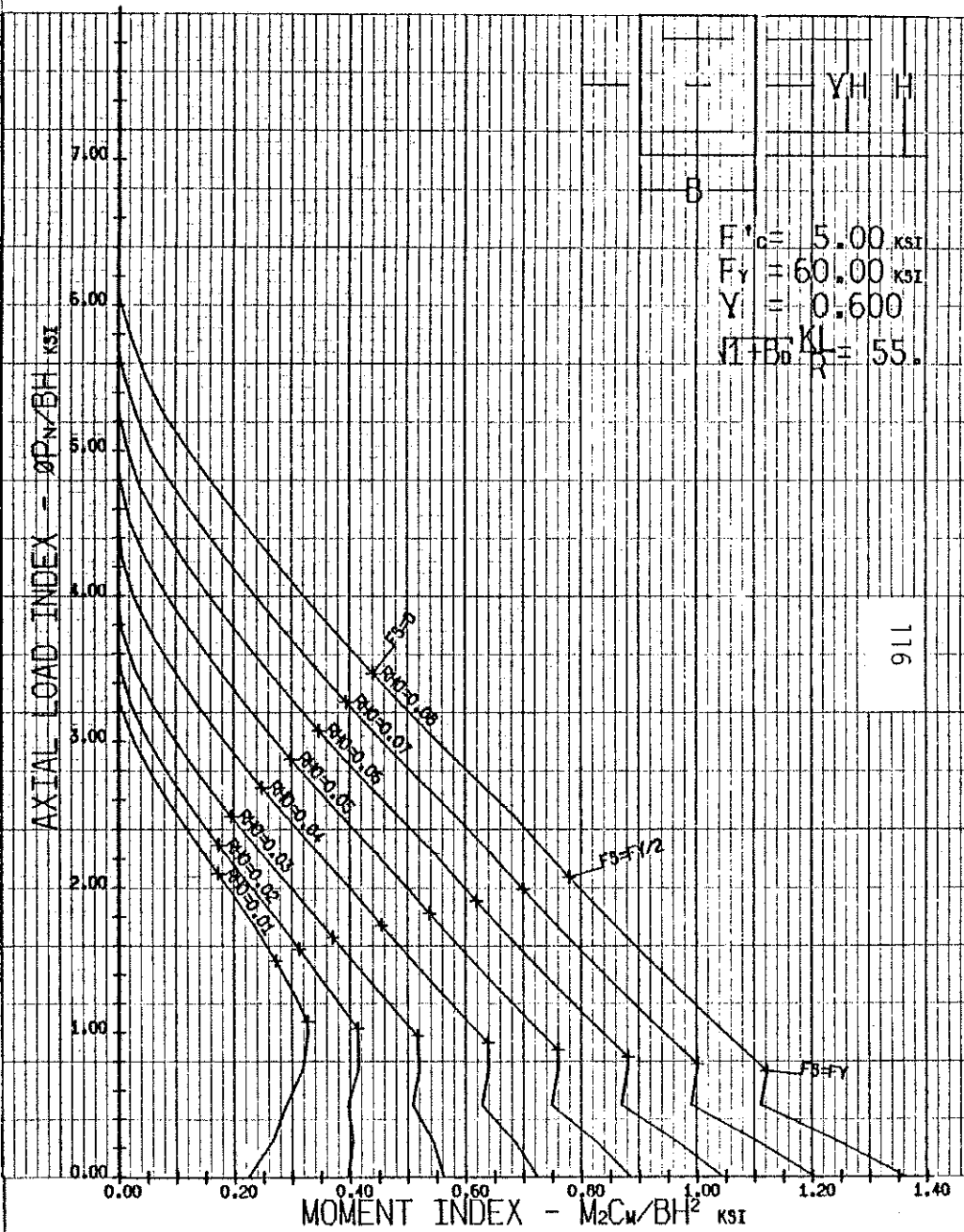


Fig. E5-60.60-55 - Interaction Diagram

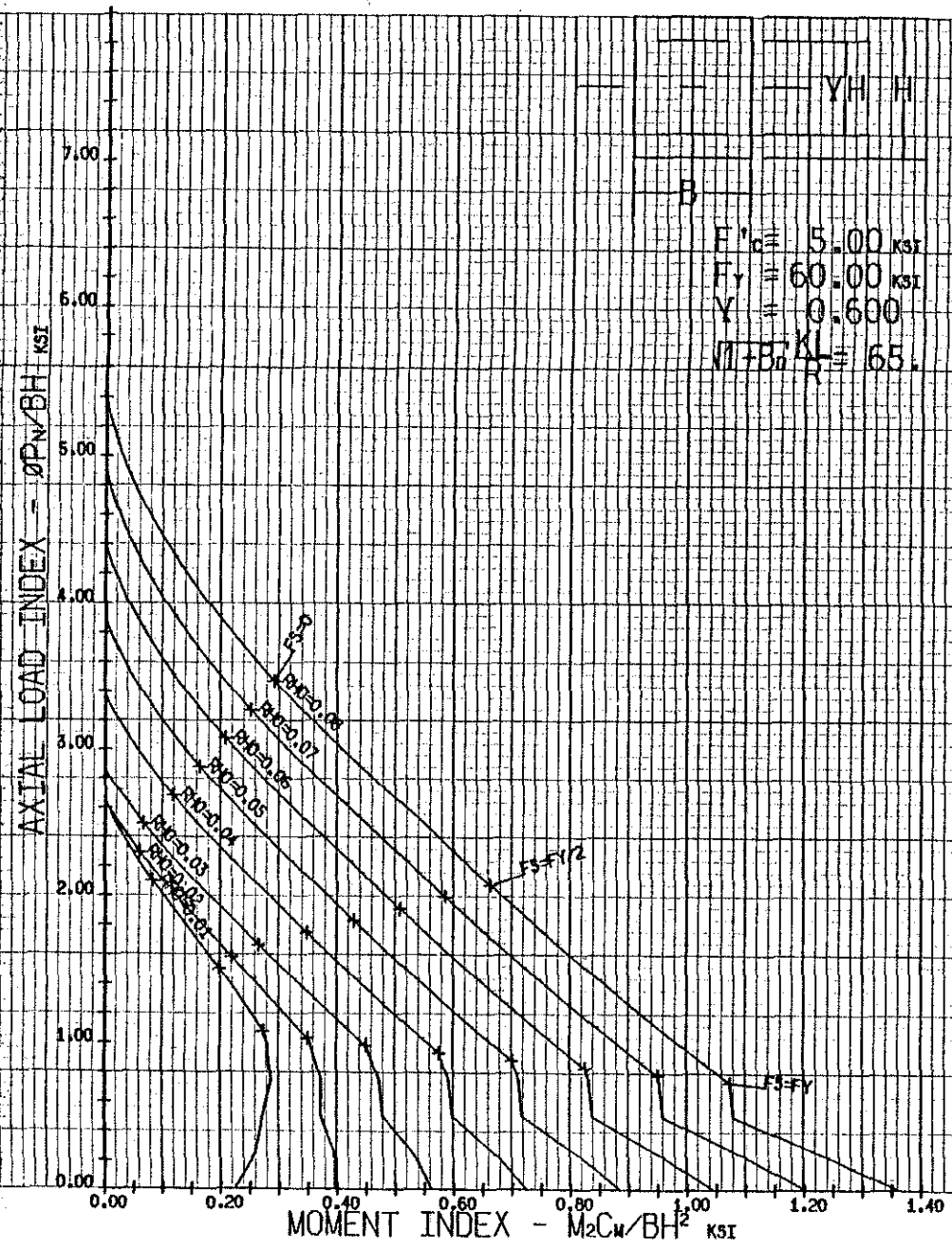


Fig. E5-60.60-65 - Interaction Diagram

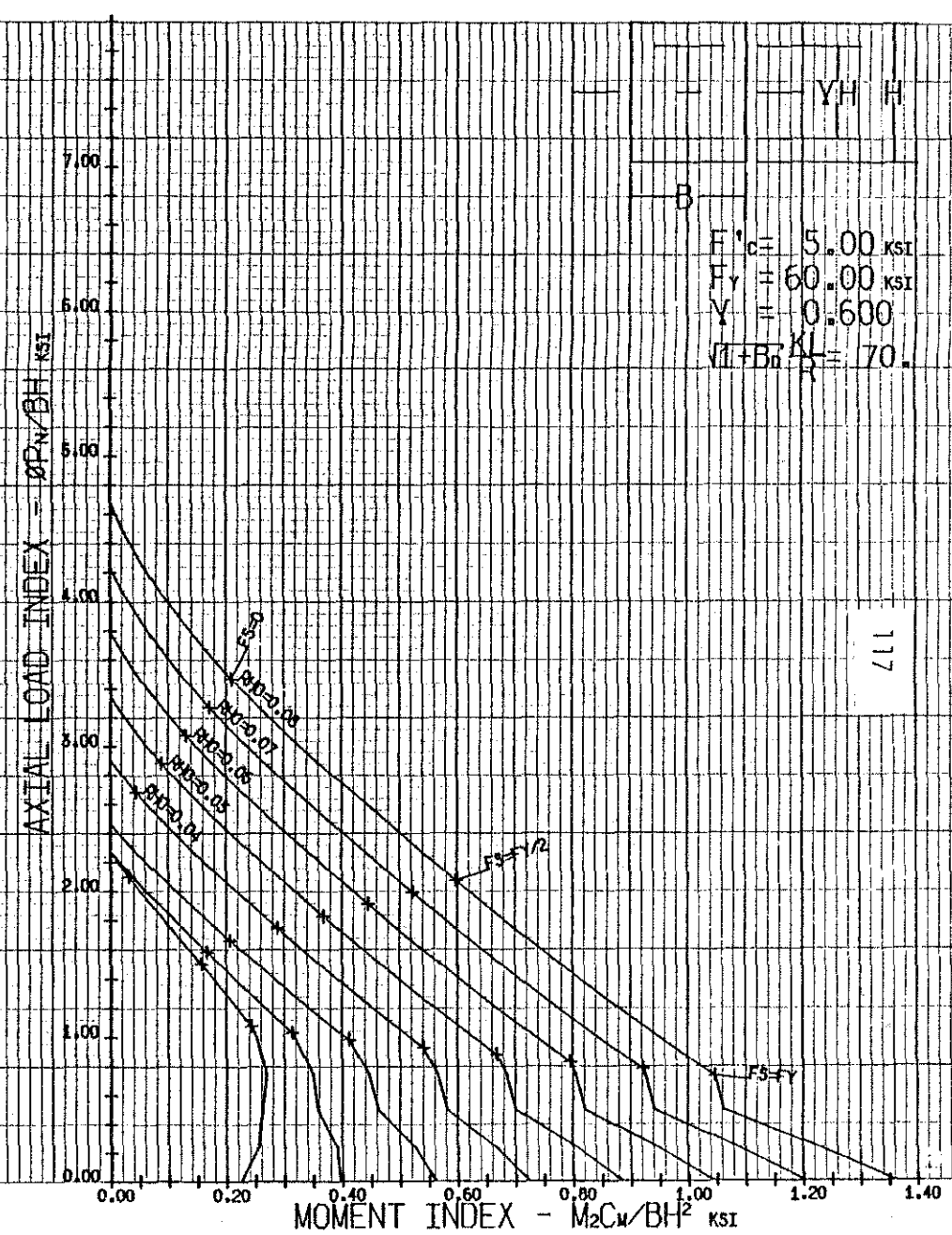


Fig. E5-60.60-70 - Interaction Diagram



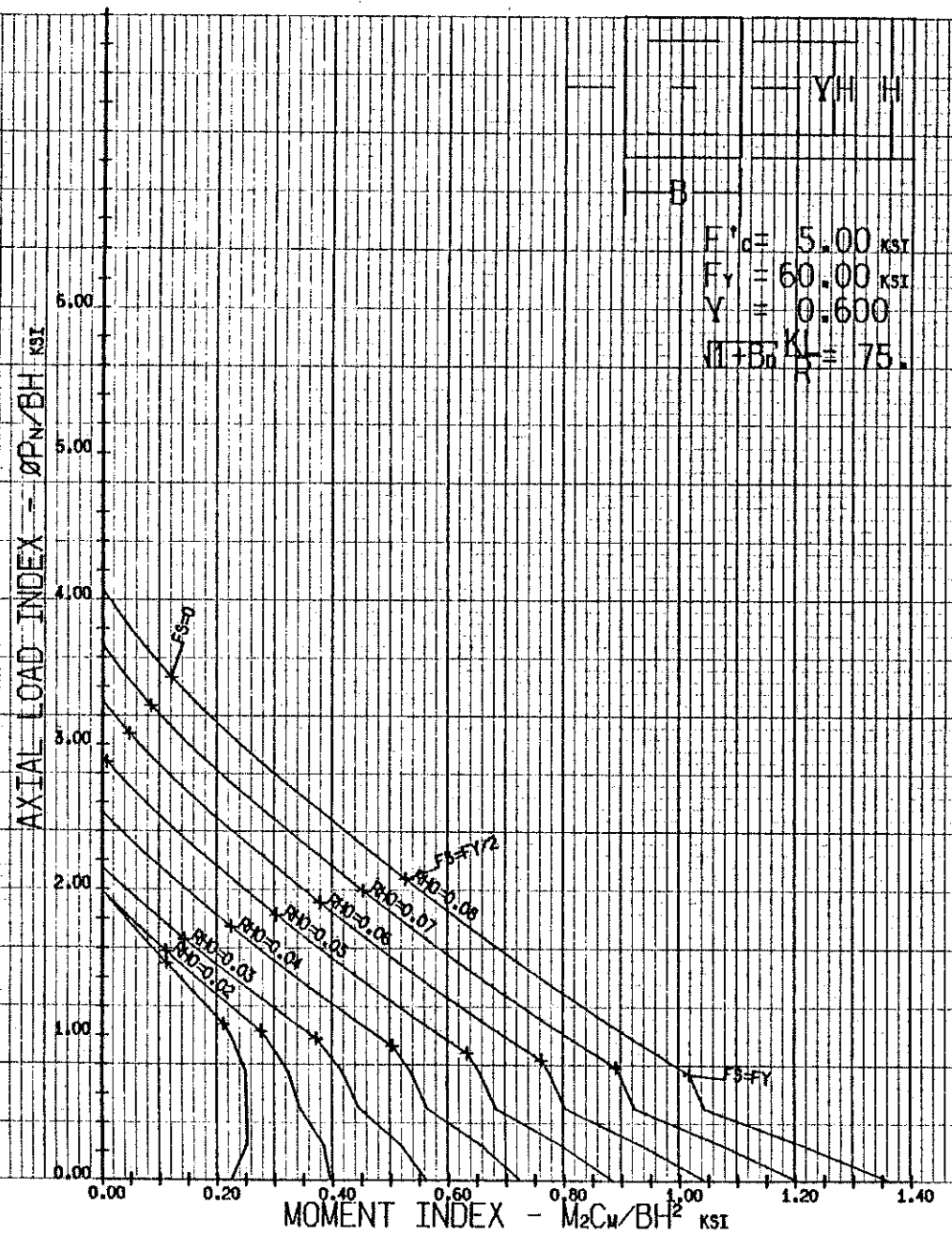


Fig. E5-60.60-75 - Interaction Diagram

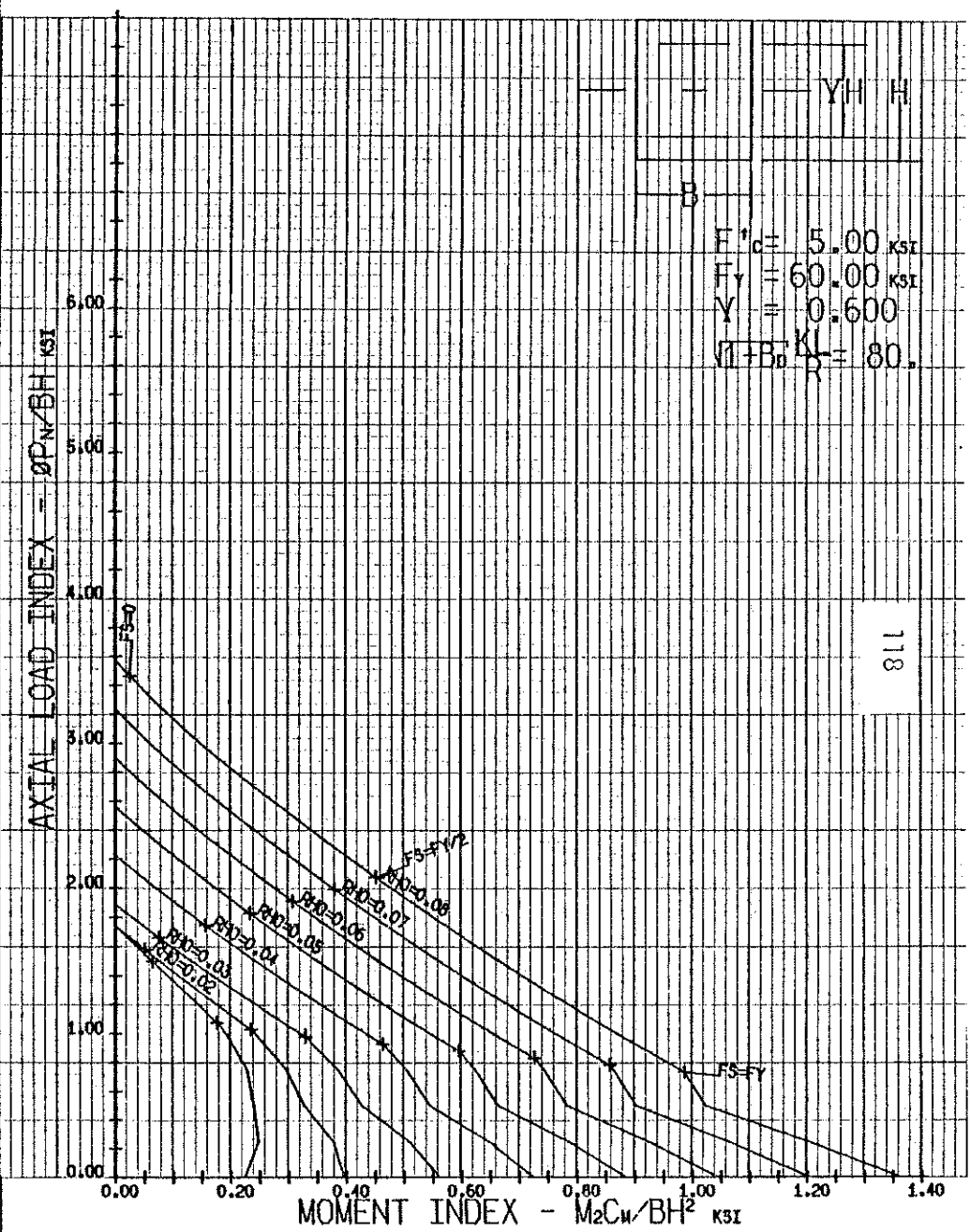


Fig. E5-60.60-80 - Interaction Diagram

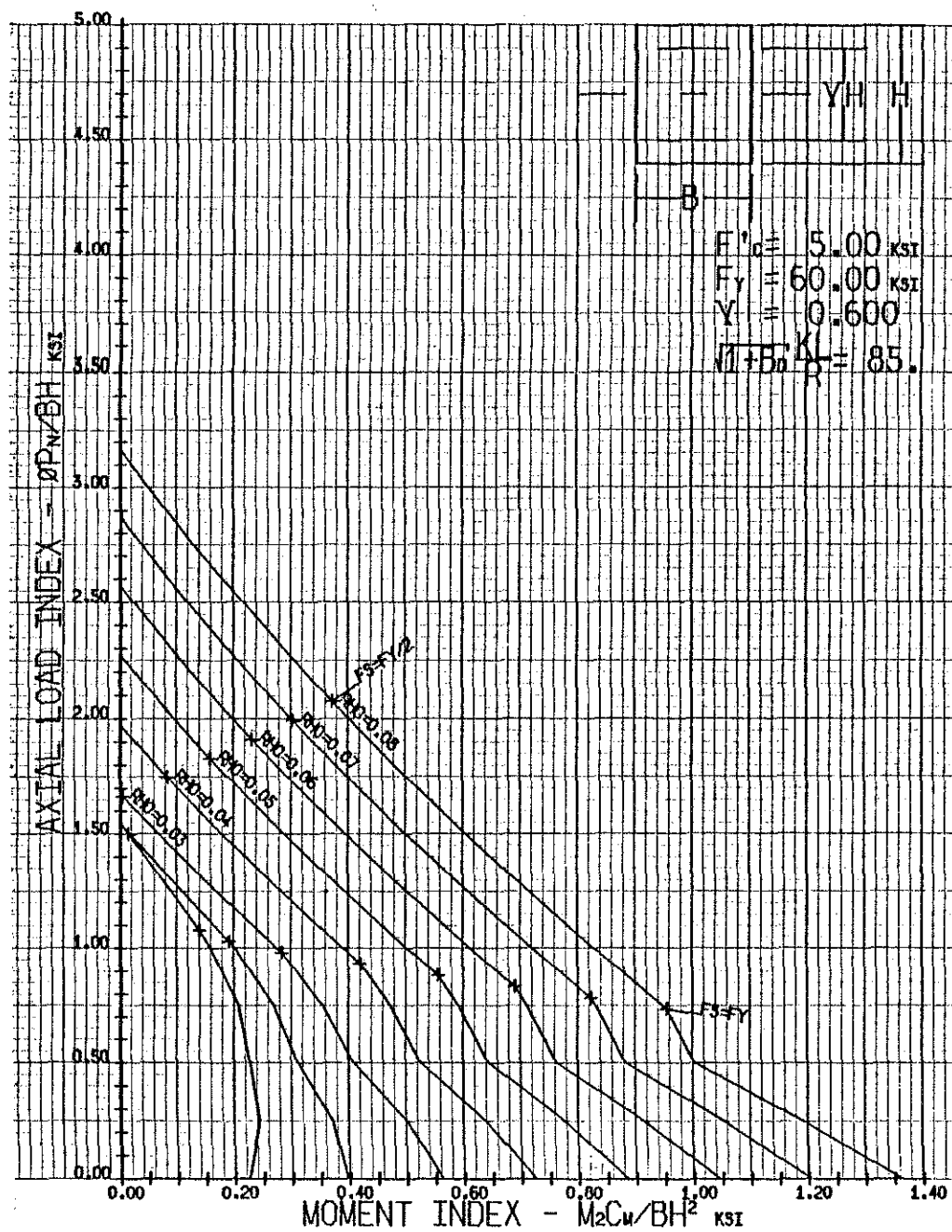


Fig. E5-60.60-85 - Interaction Diagram

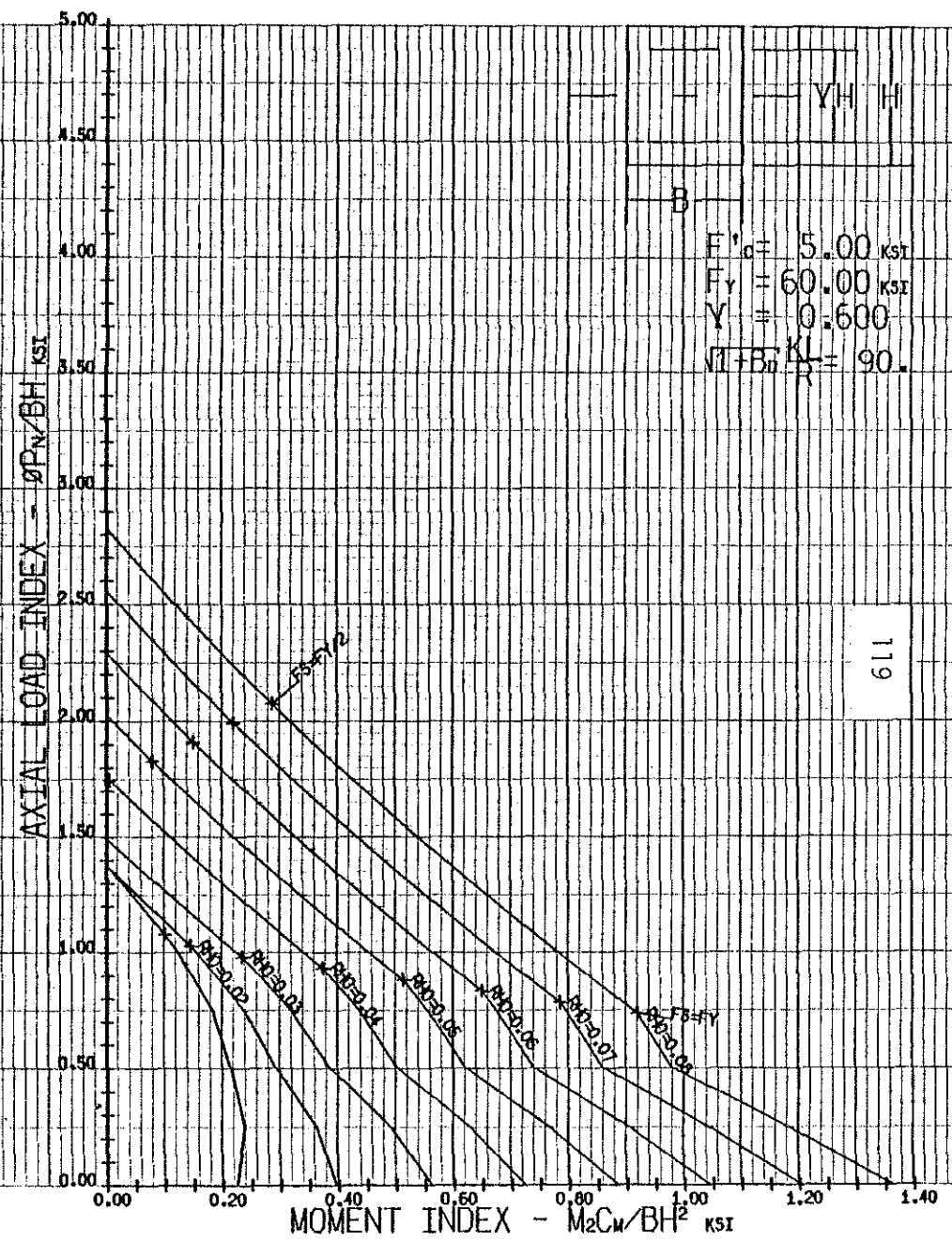


Fig. E5-60.60-90 - Interaction Diagram

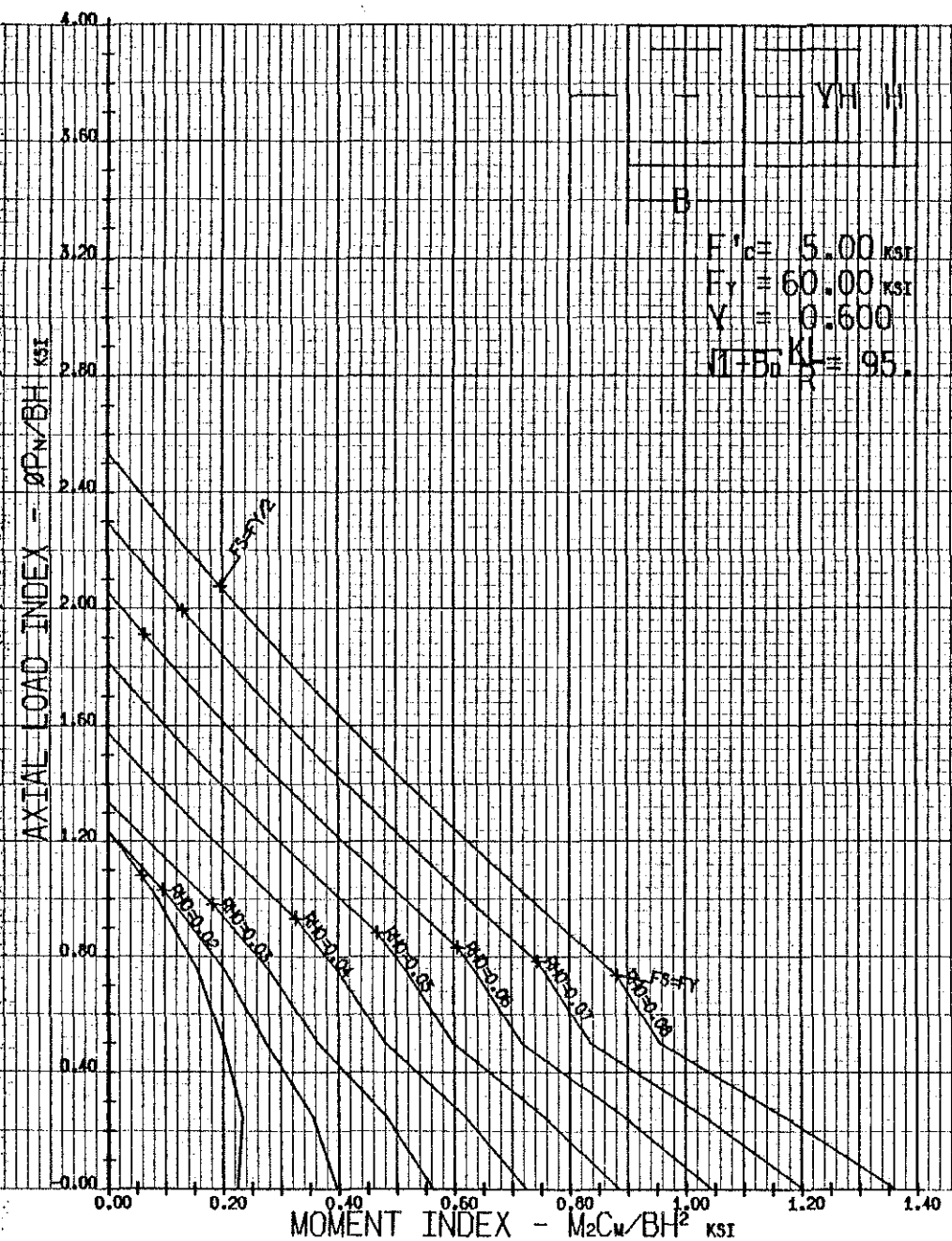


Fig. E5-60.60-95 - Interaction Diagram

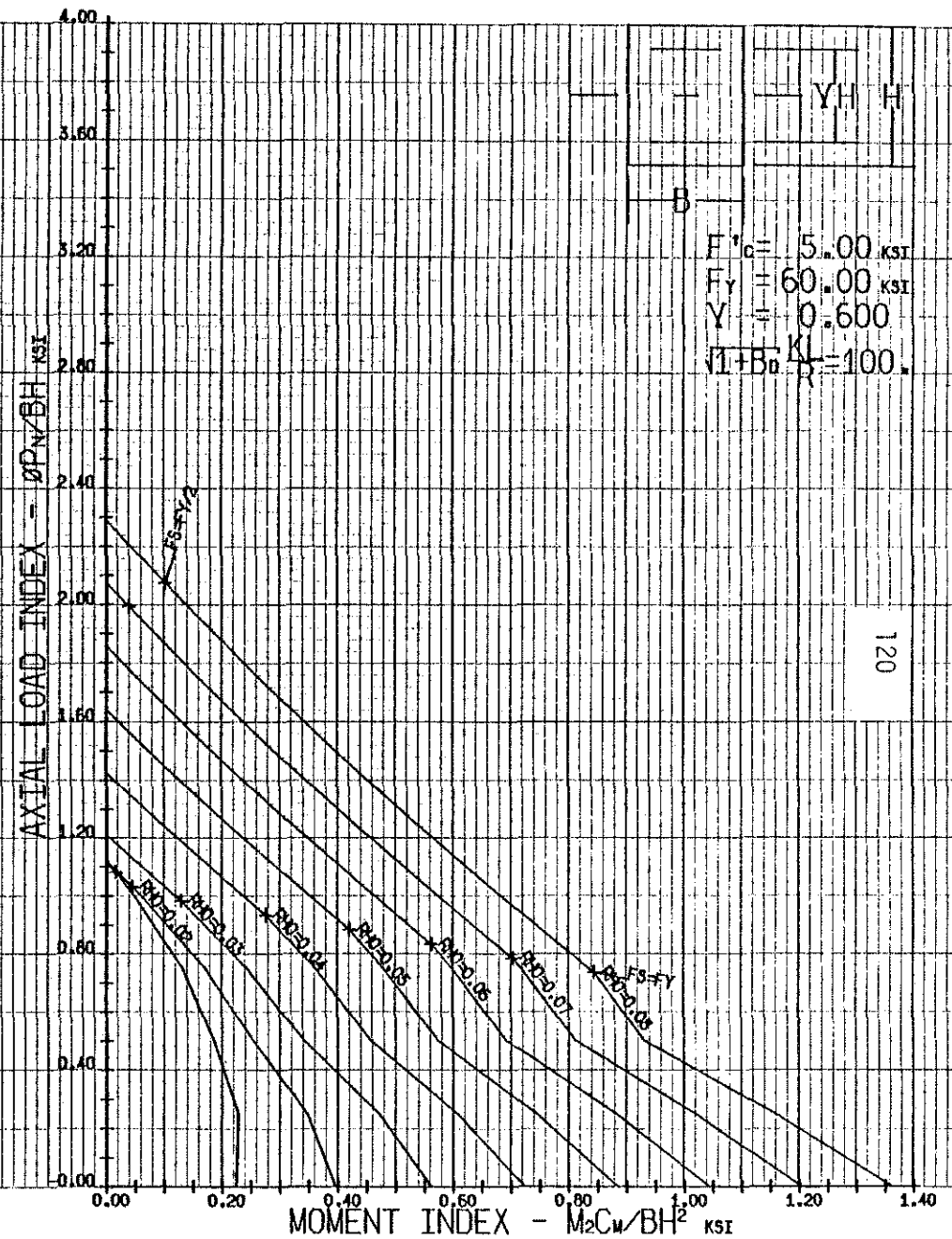


Fig. E5-60.60-100 - Interaction Diagram

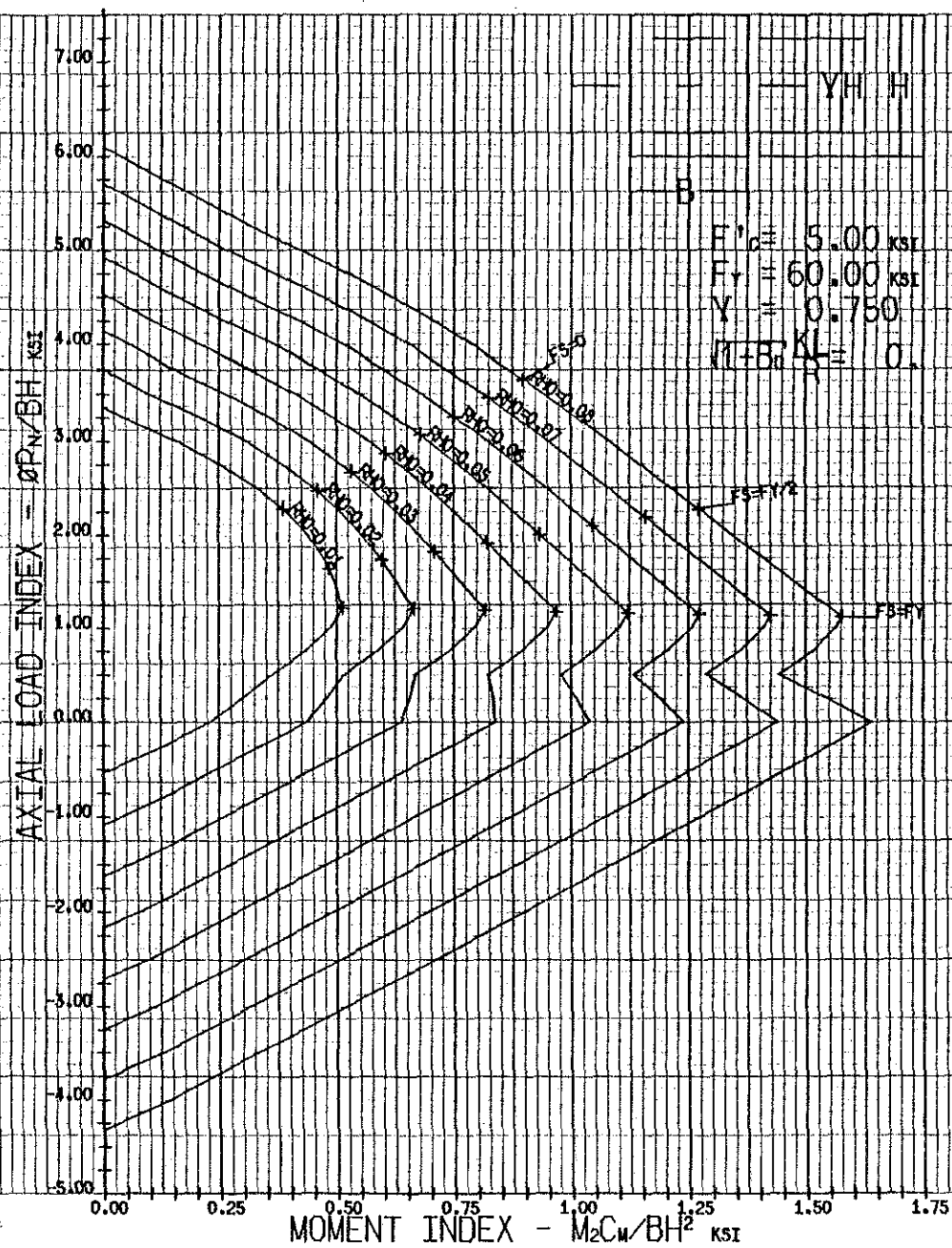


Fig. E5-60.75-0 - Interaction Diagram

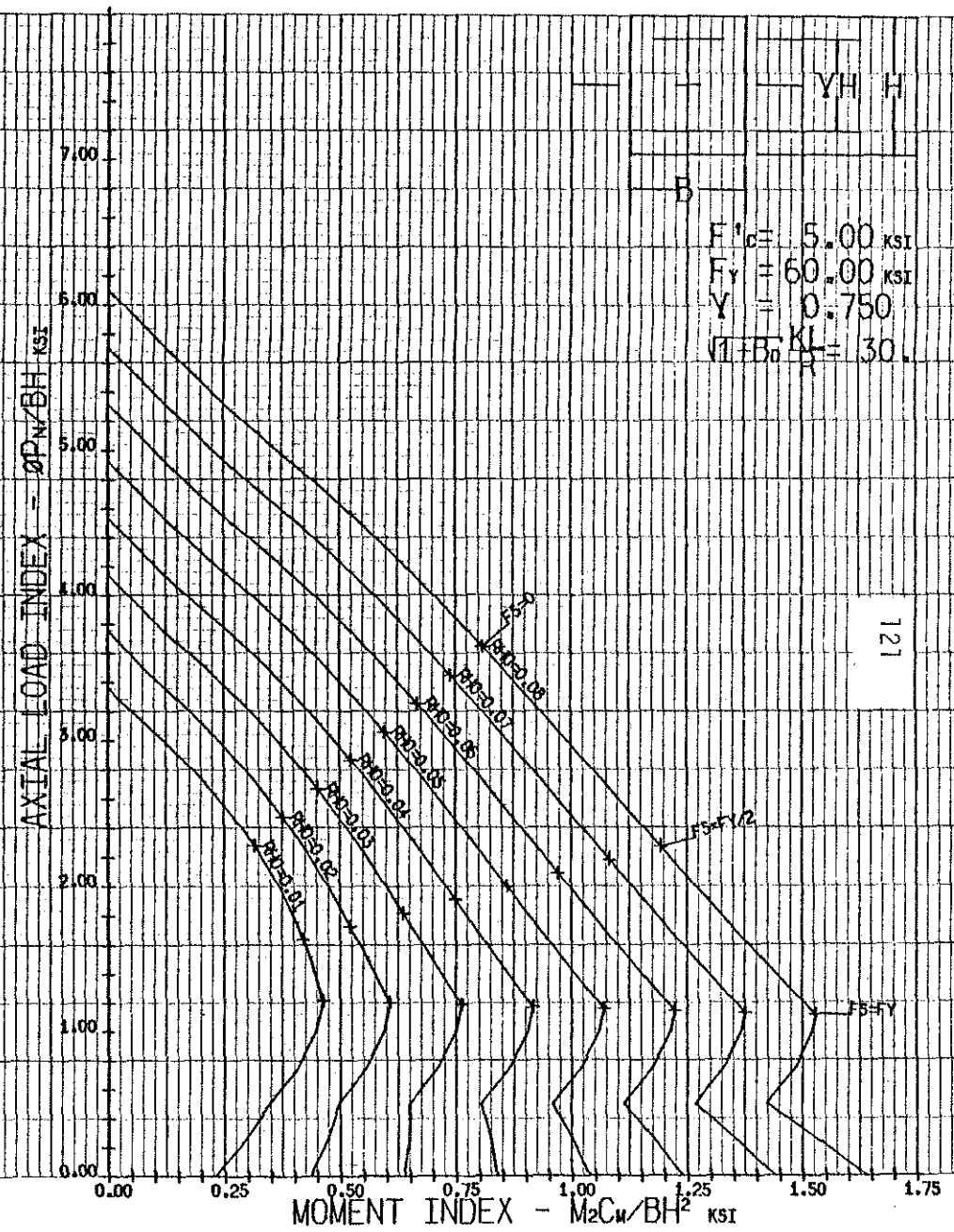


Fig. E5-60.75-30 - Interaction Diagram

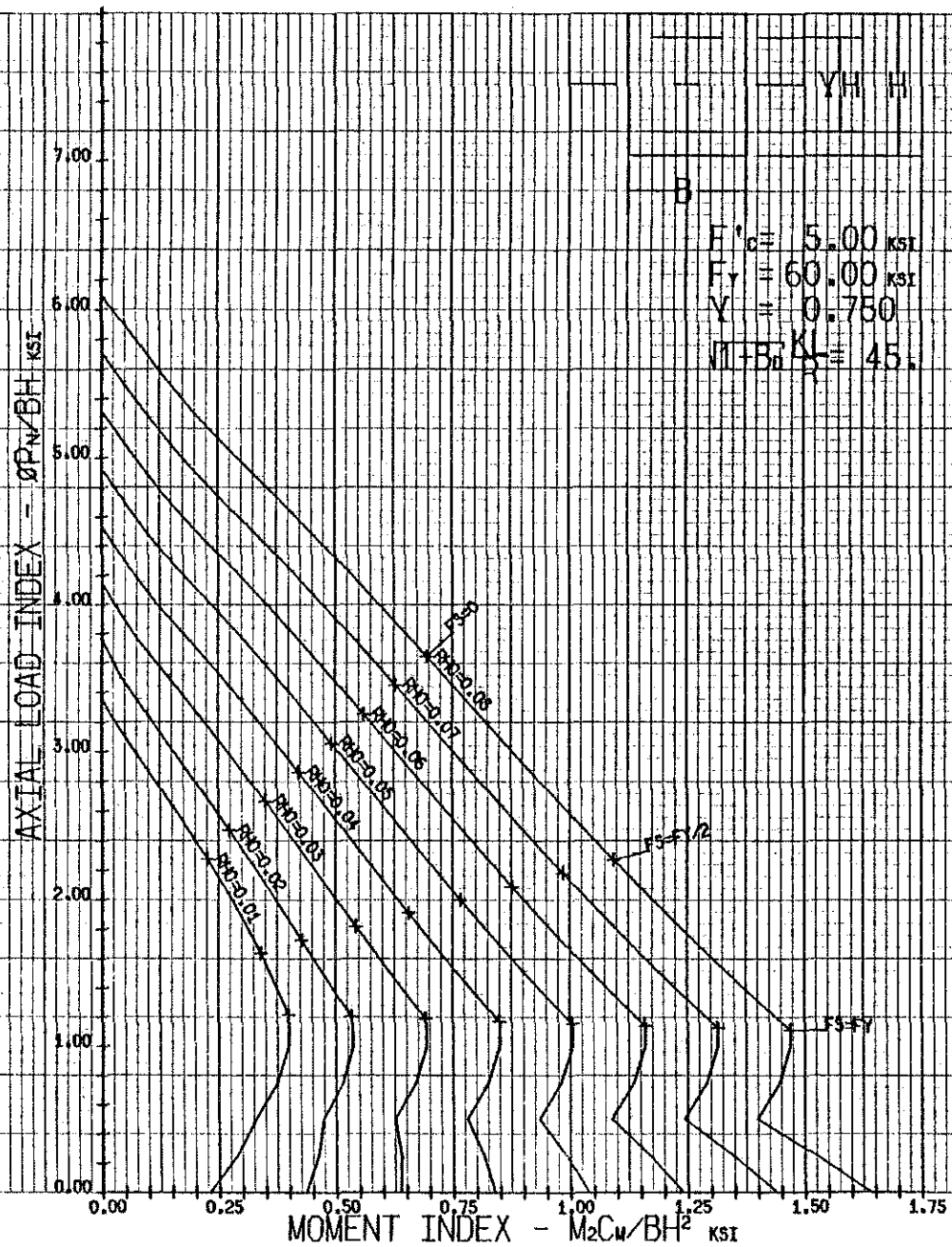


Fig. E5-60.75-45 - Interaction Diagram

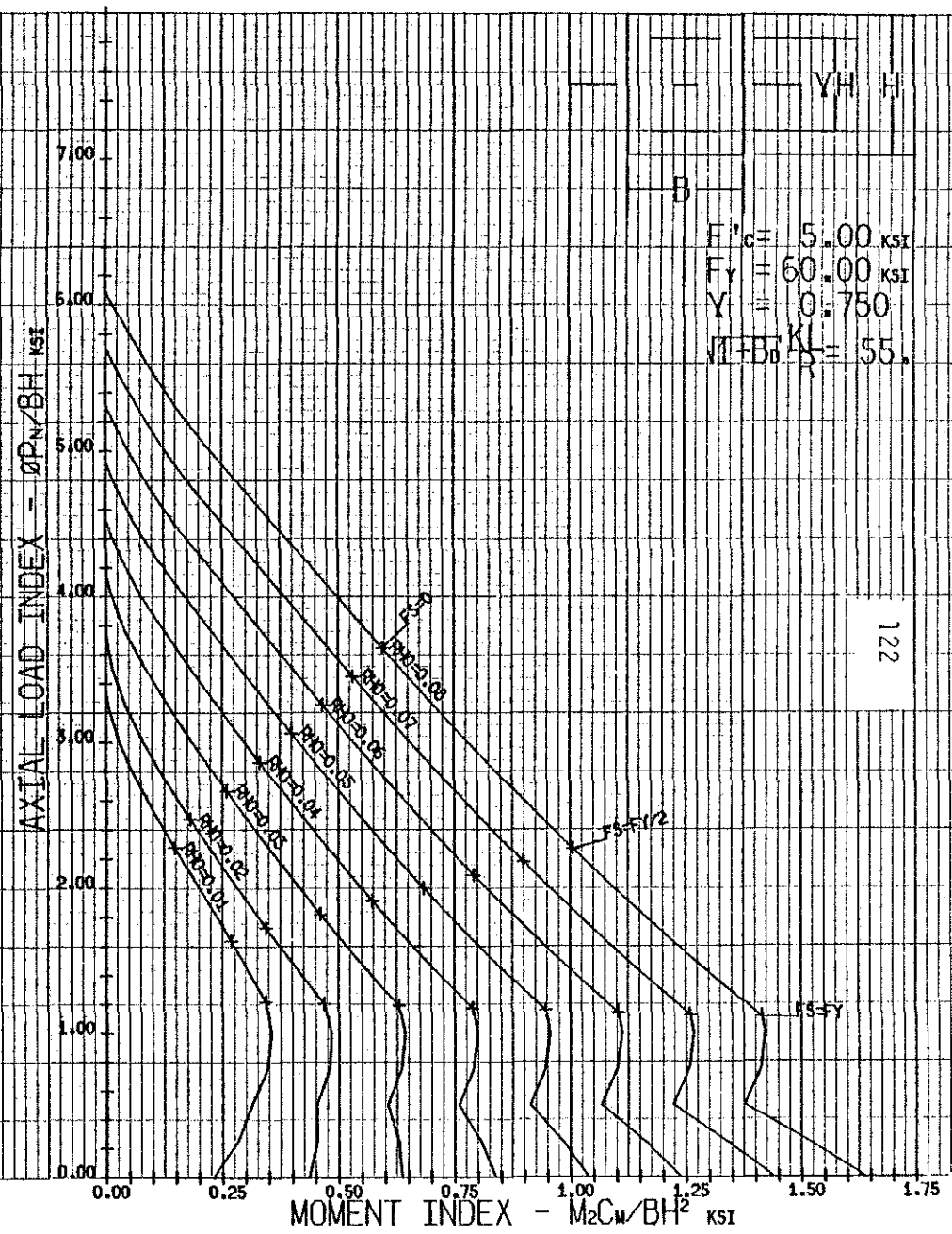


Fig. E5-60.75-55 - Interaction Diagram



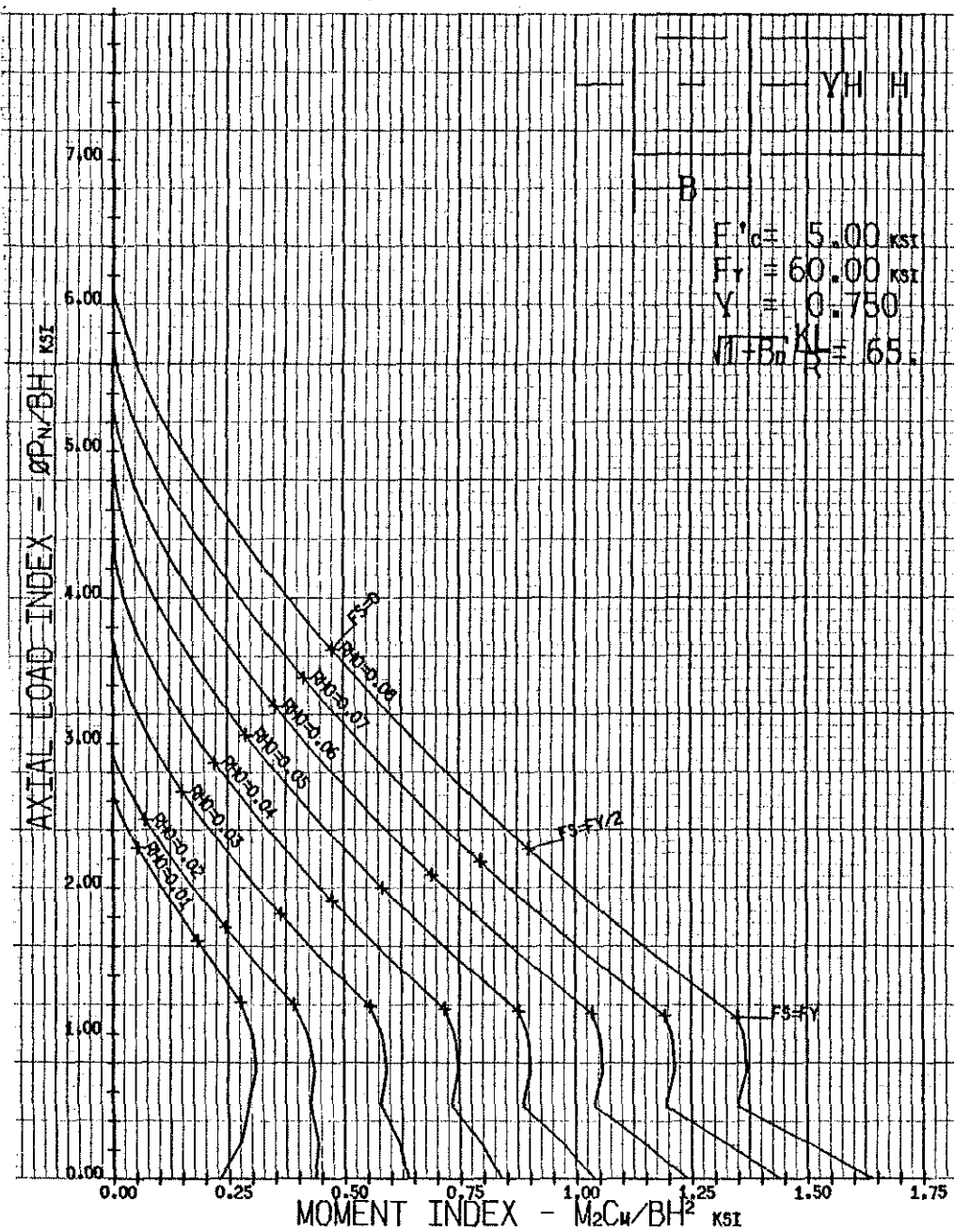


Fig. E5-60.75-65 - Interaction Diagram

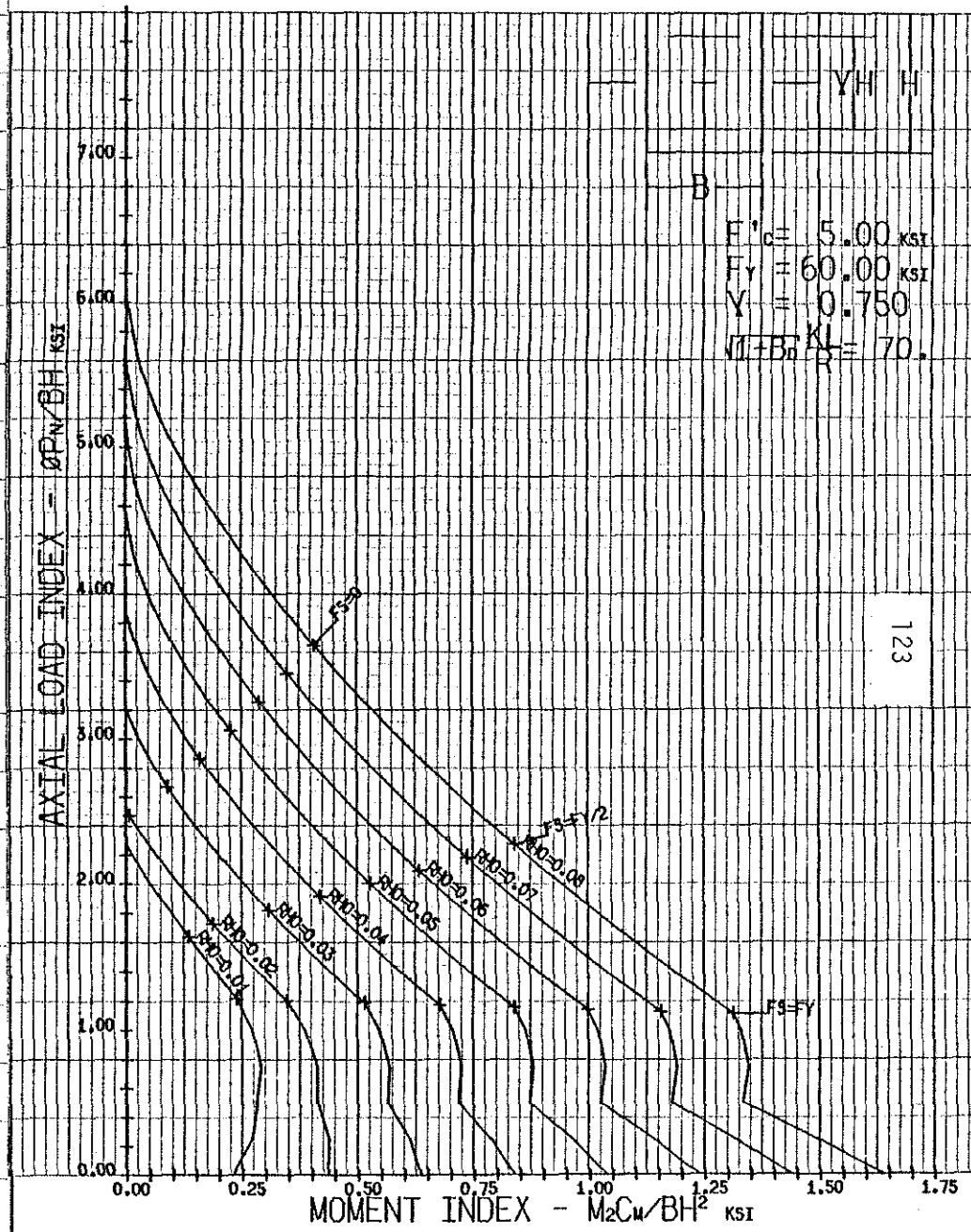


Fig. E5-60.75-70 - Interaction Diagram

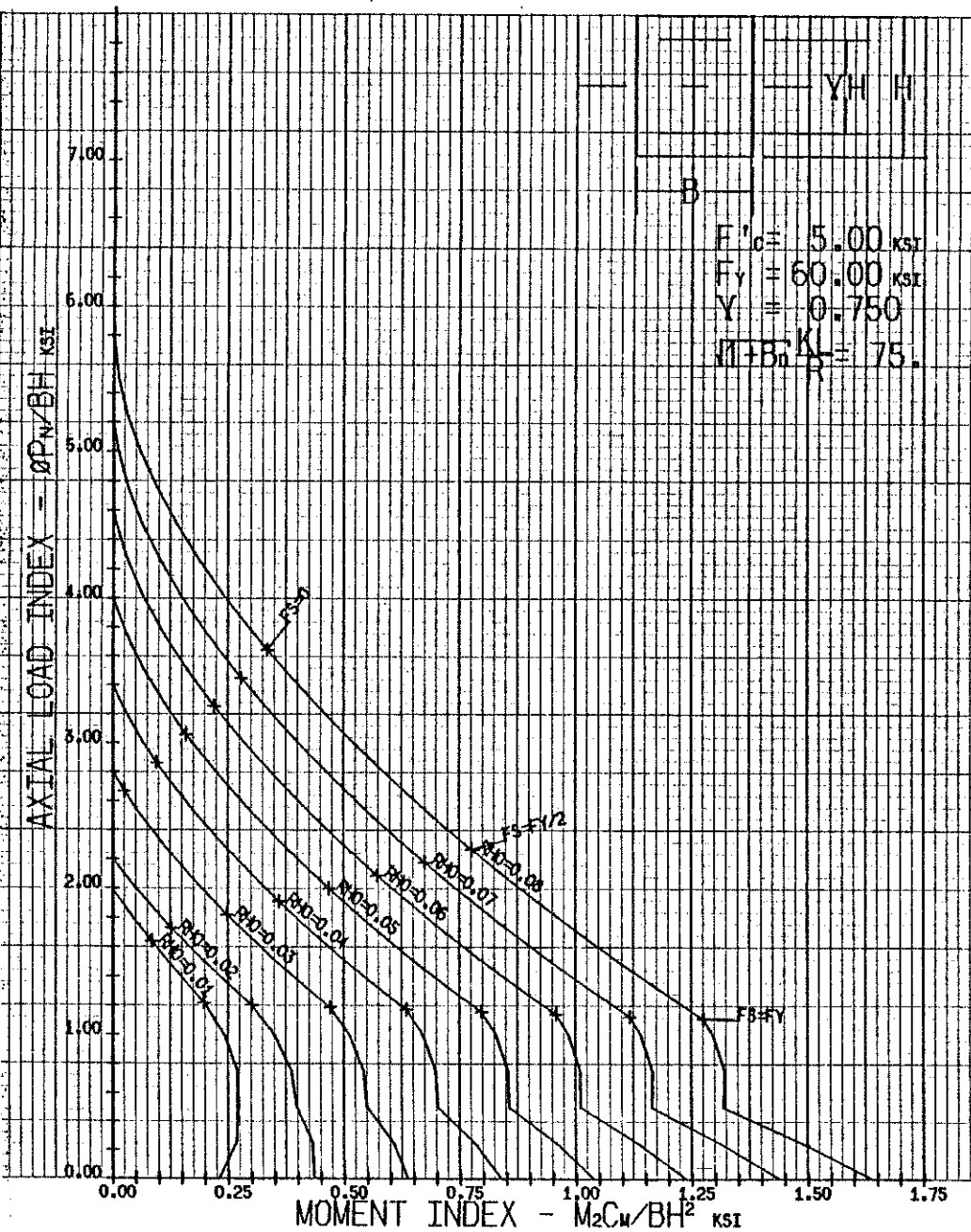


Fig. E5-60.75-75 - Interaction Diagram

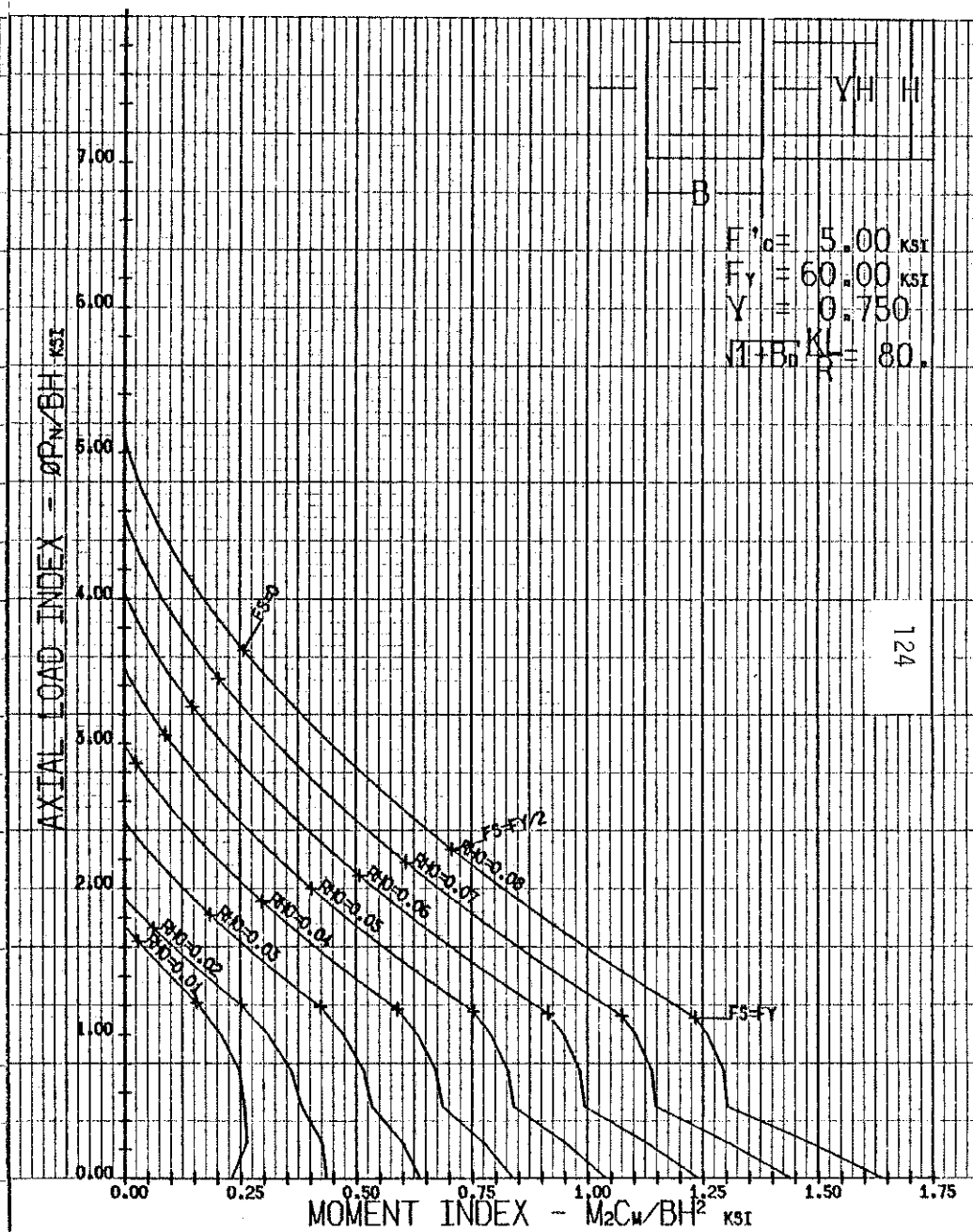


Fig. E5-60.75-80 - Interaction Diagram

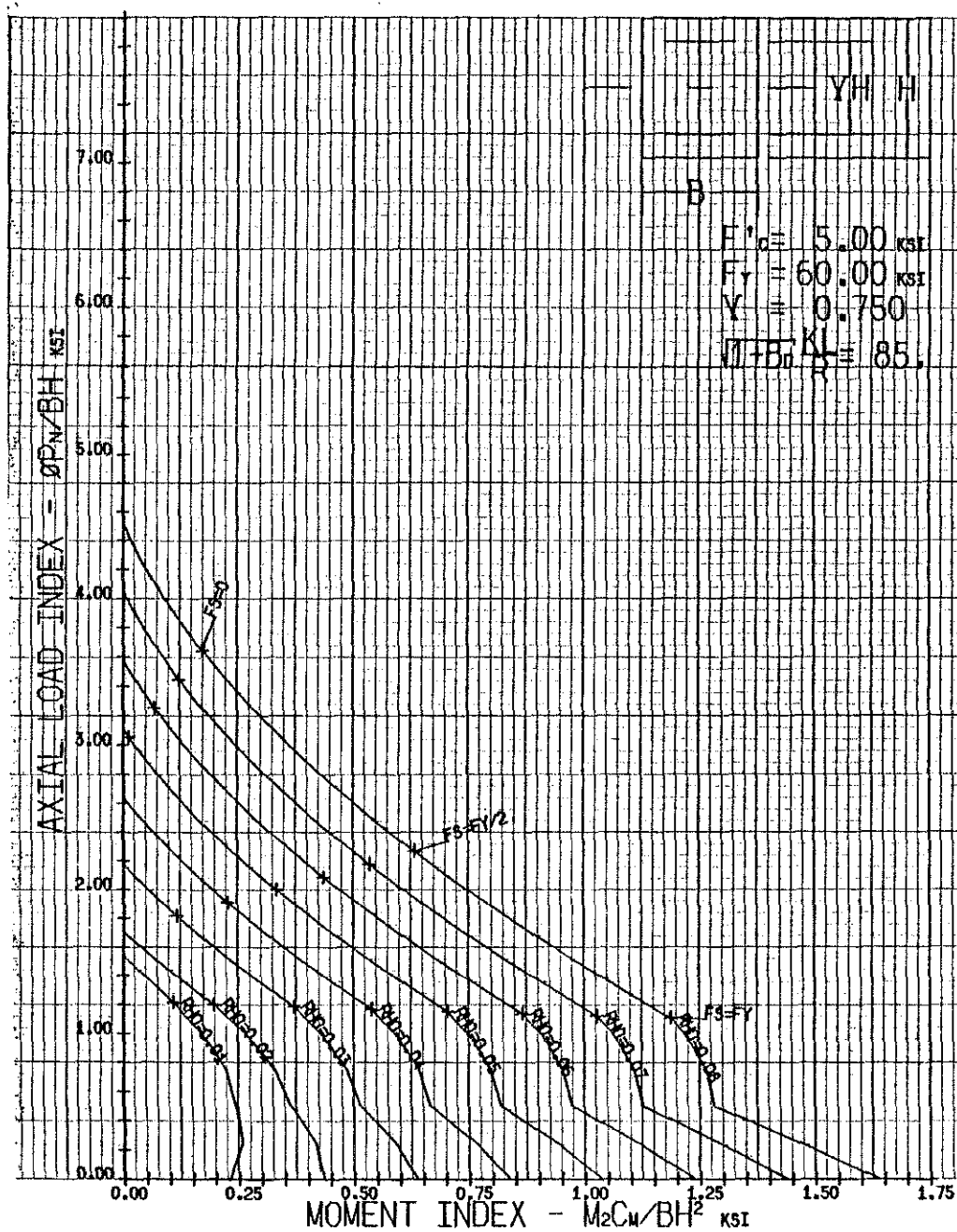


Fig. E5-60.75-85 - Interaction Diagram

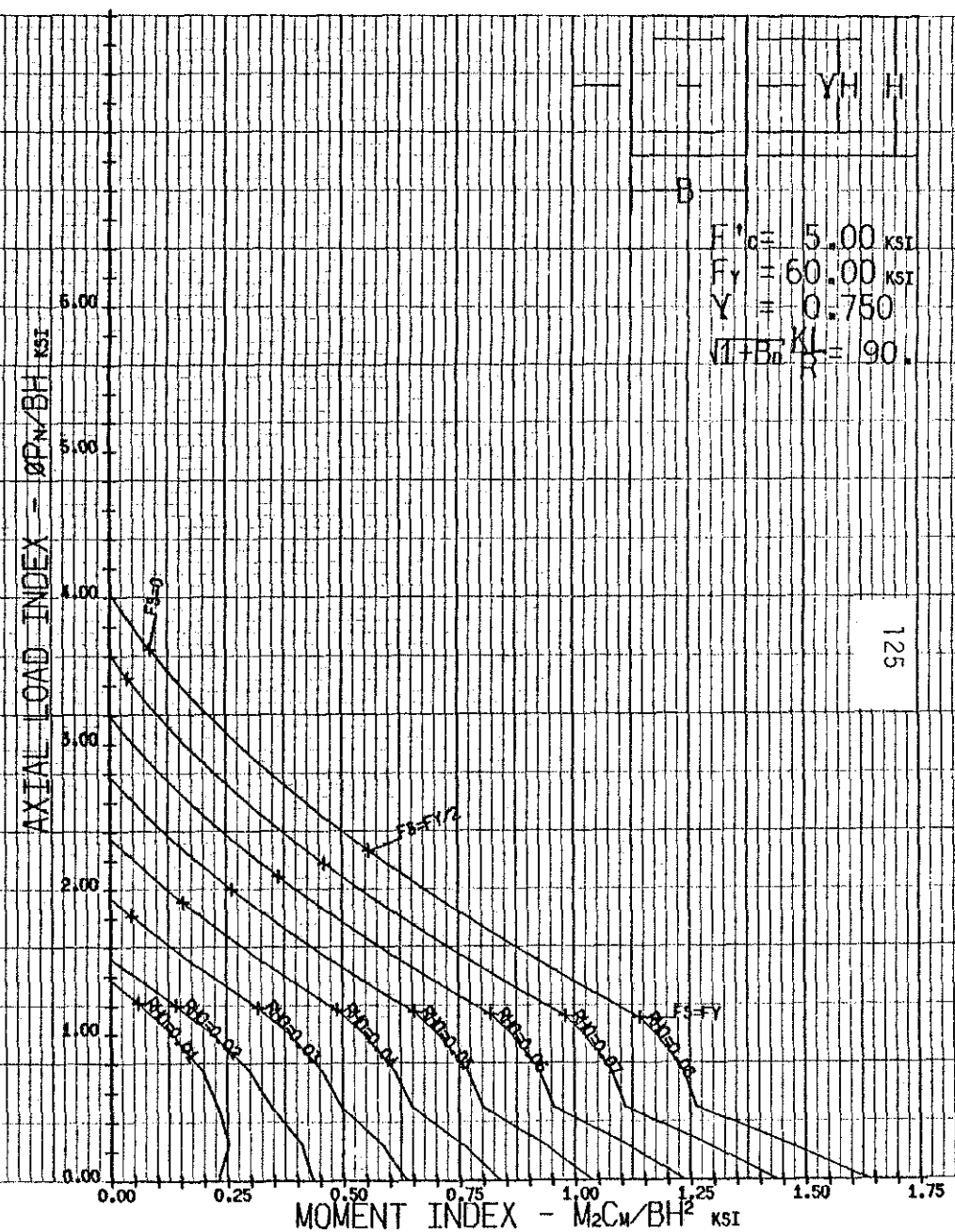


Fig. E5-60.75-90 - Interaction Diagram



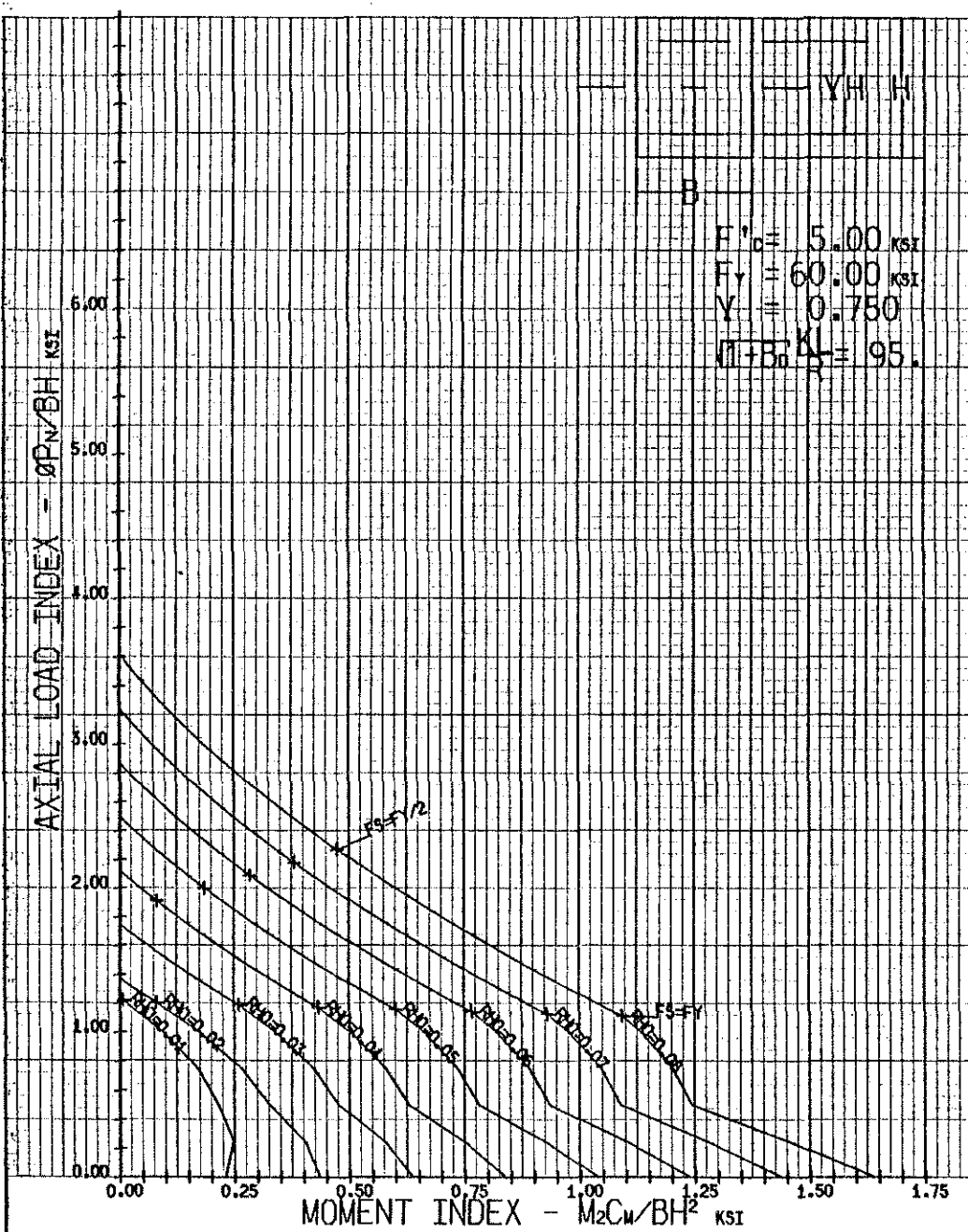


Fig. E5-60.75-95 - Interaction Diagram

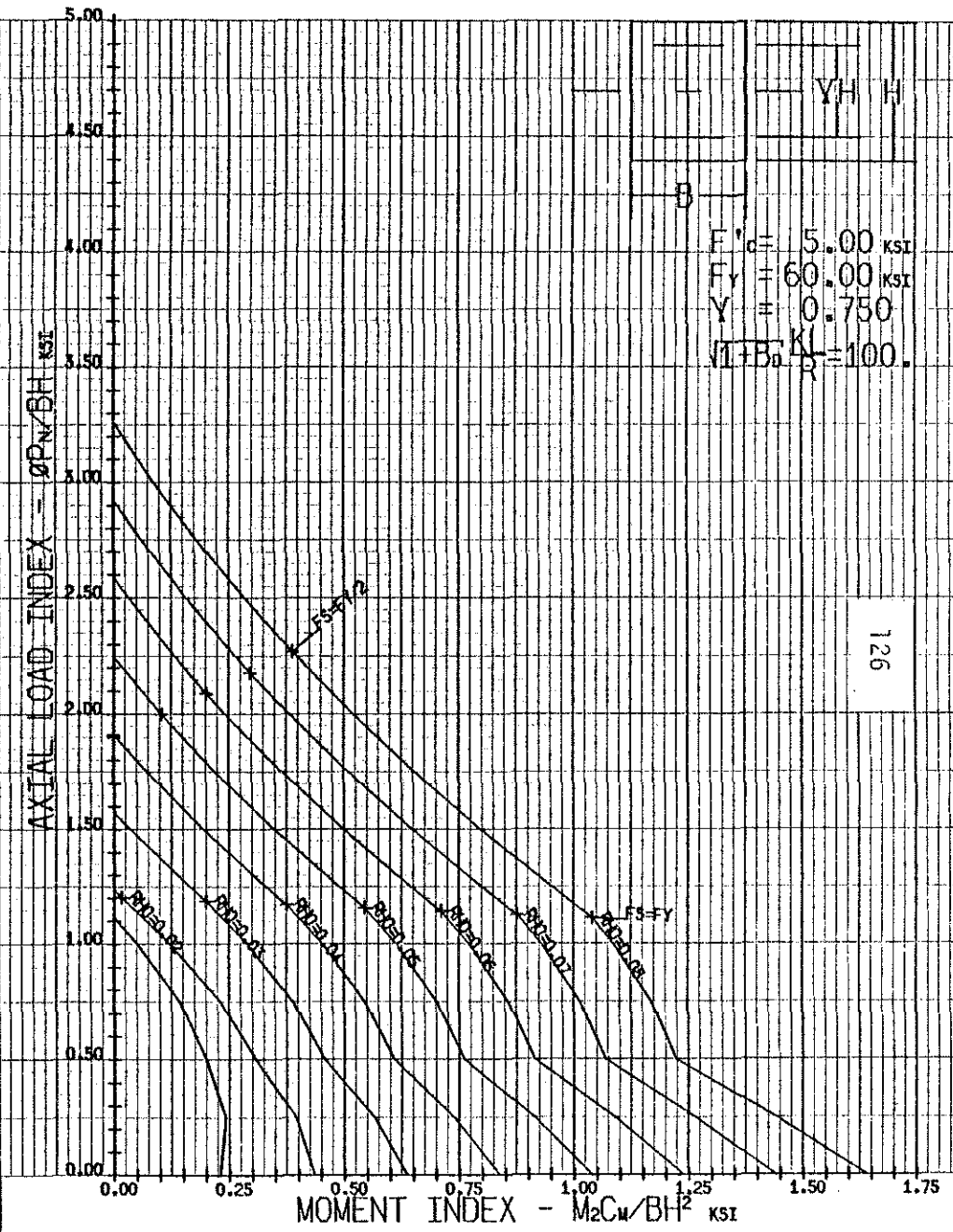


Fig. E5-60.75-100 - Interaction Diagram

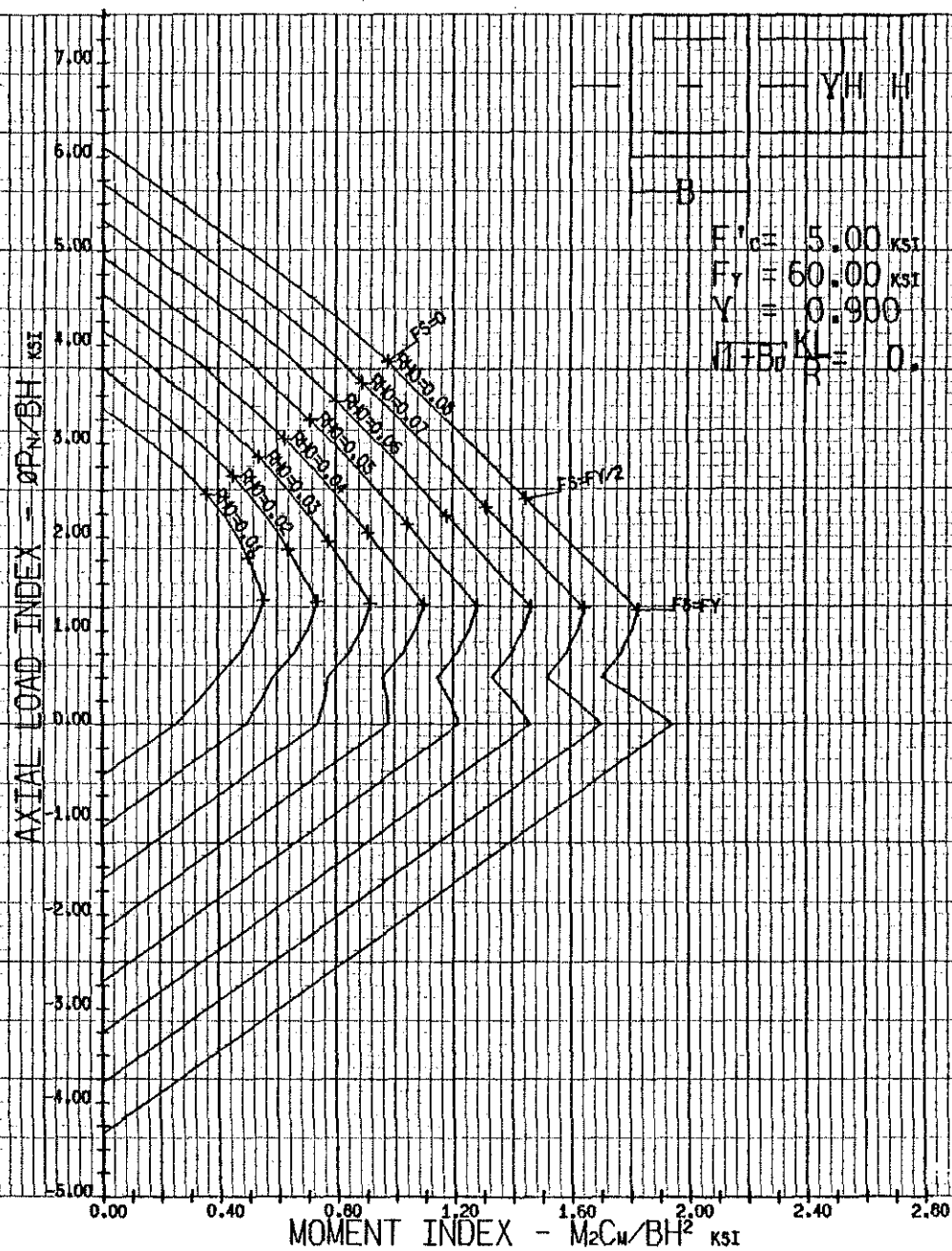


Fig. E5-60.90-0 - Interaction Diagram

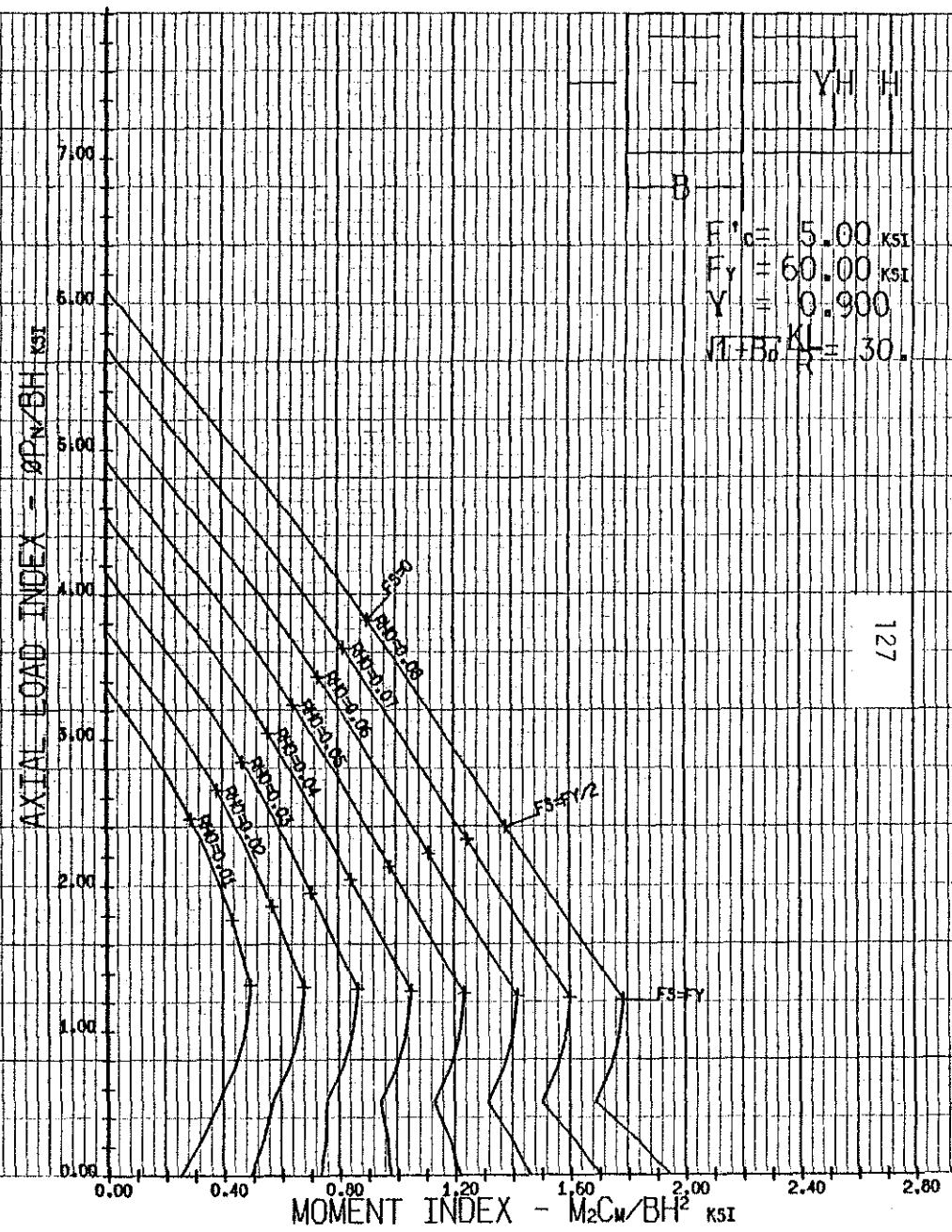


Fig. E5-60.90-30 - Interaction Diagram

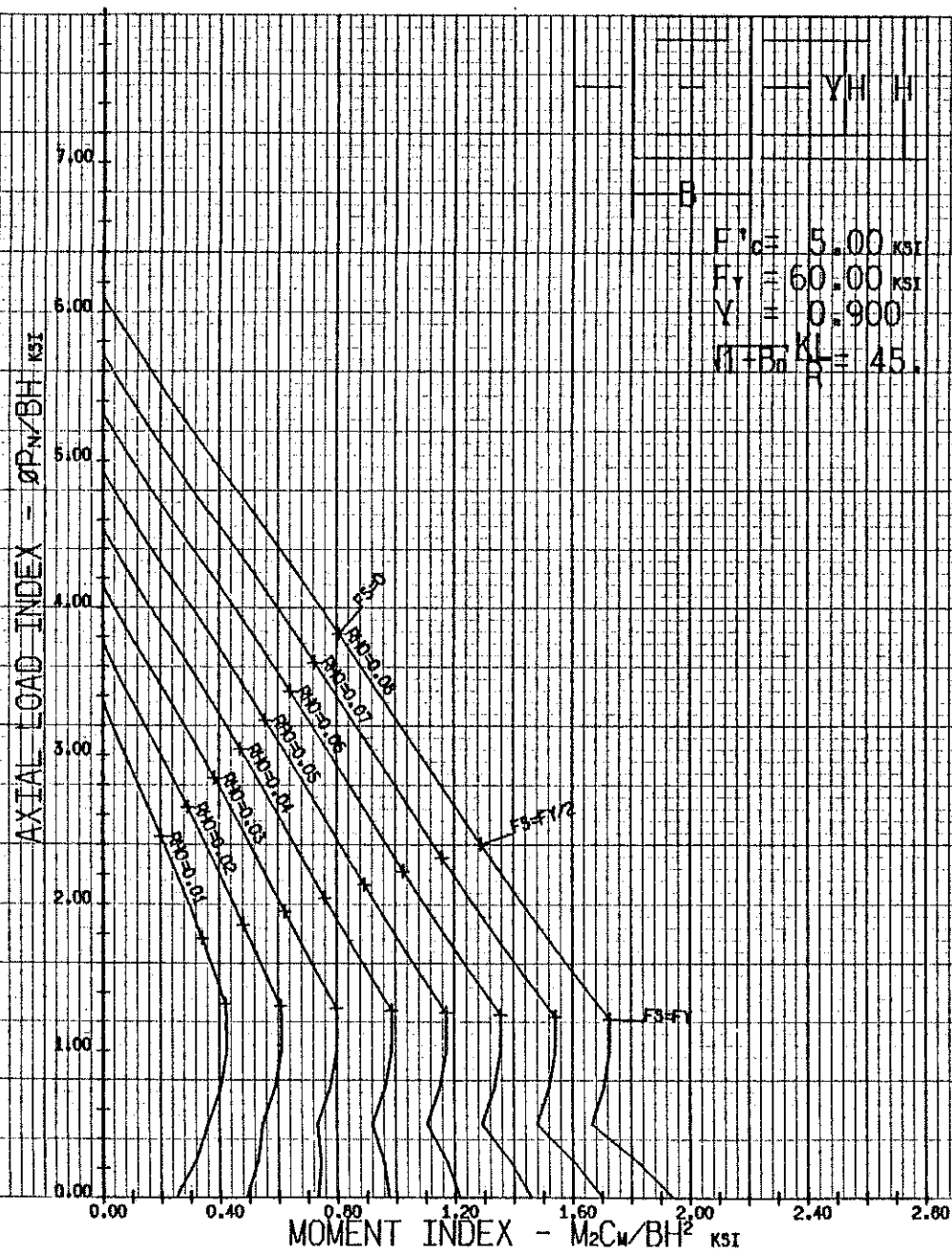


Fig. E5-60.90-45 - Interaction Diagram

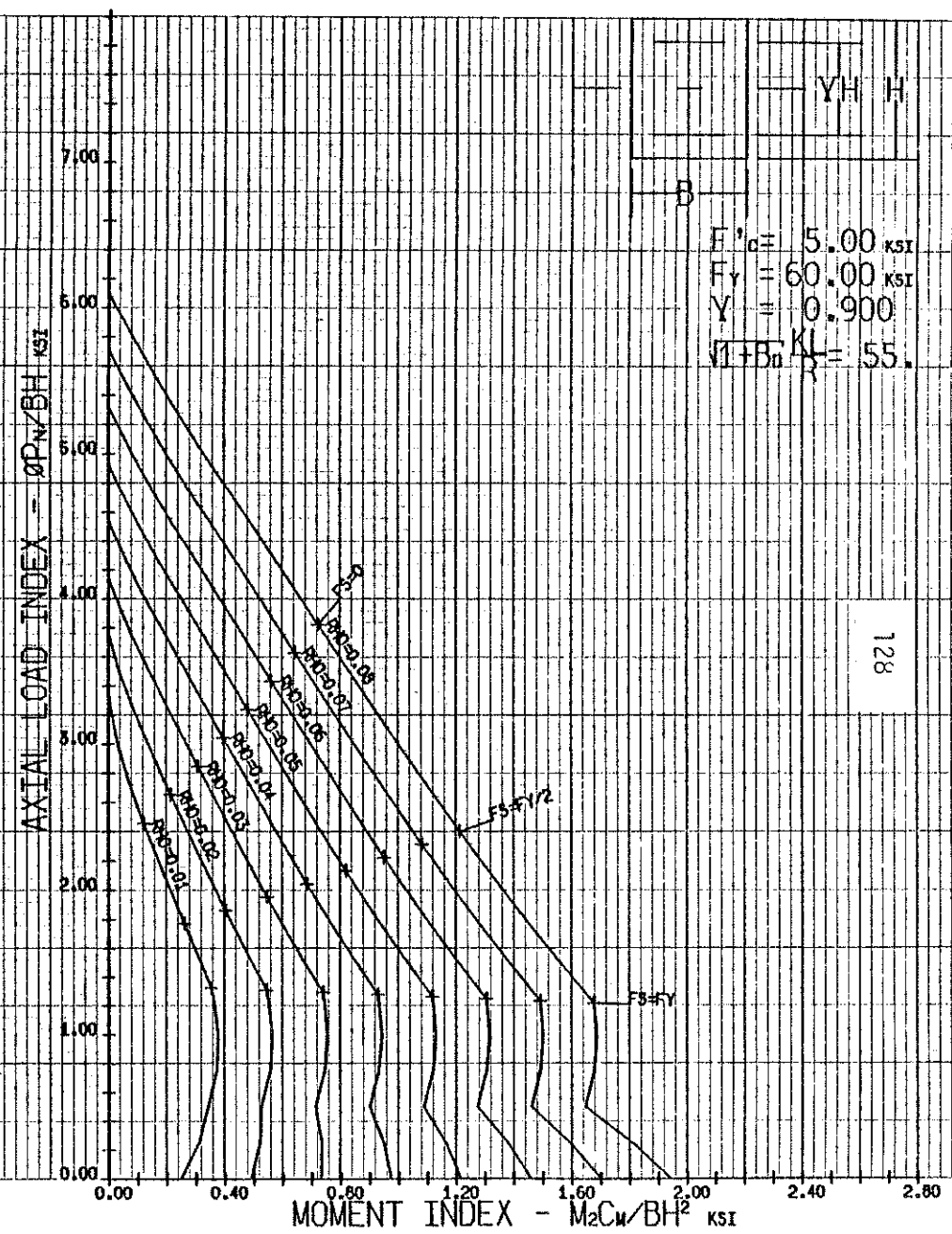


Fig. E5-60.90-55 - Interaction Diagram

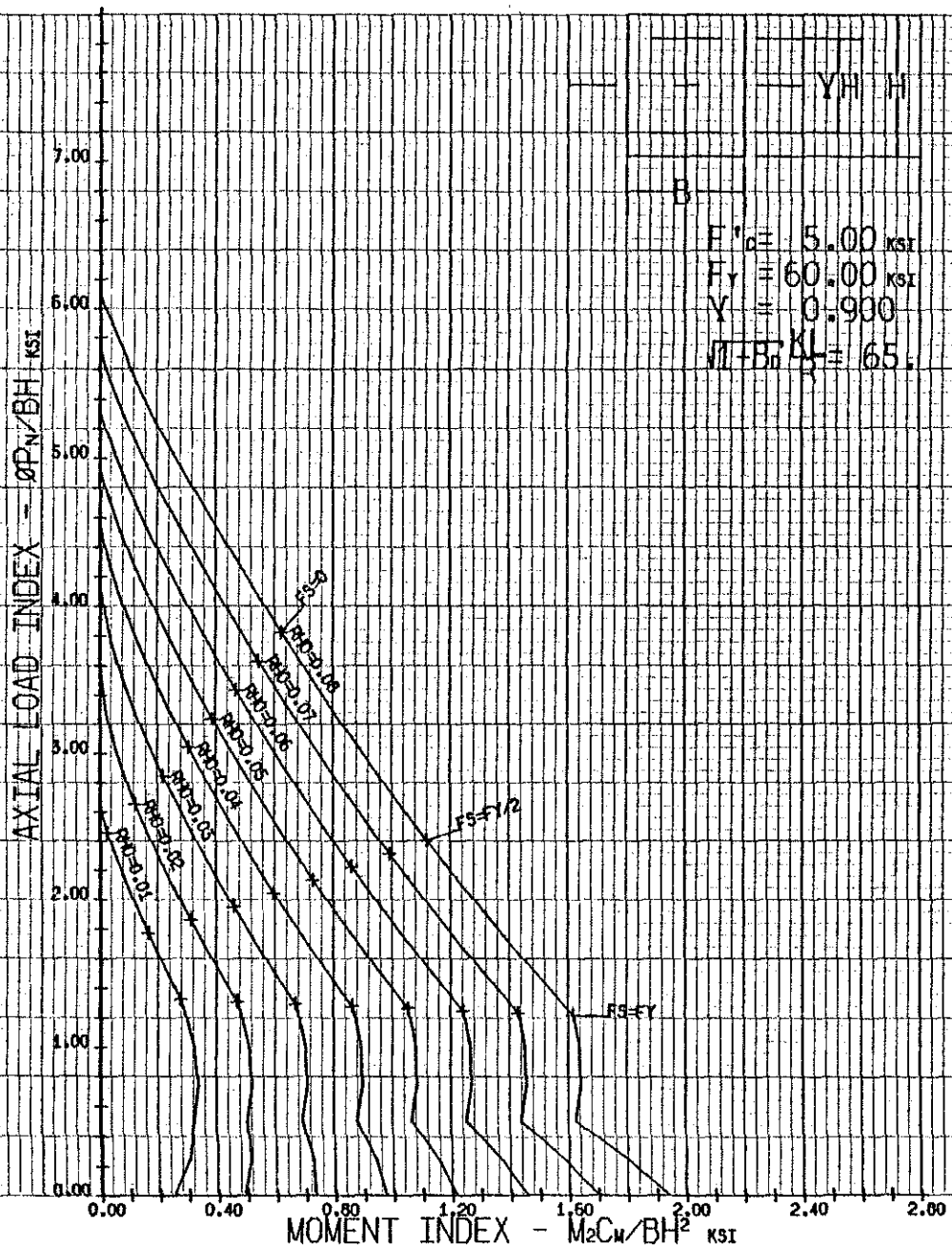


Fig. E5-60.90-65 - Interaction Diagram

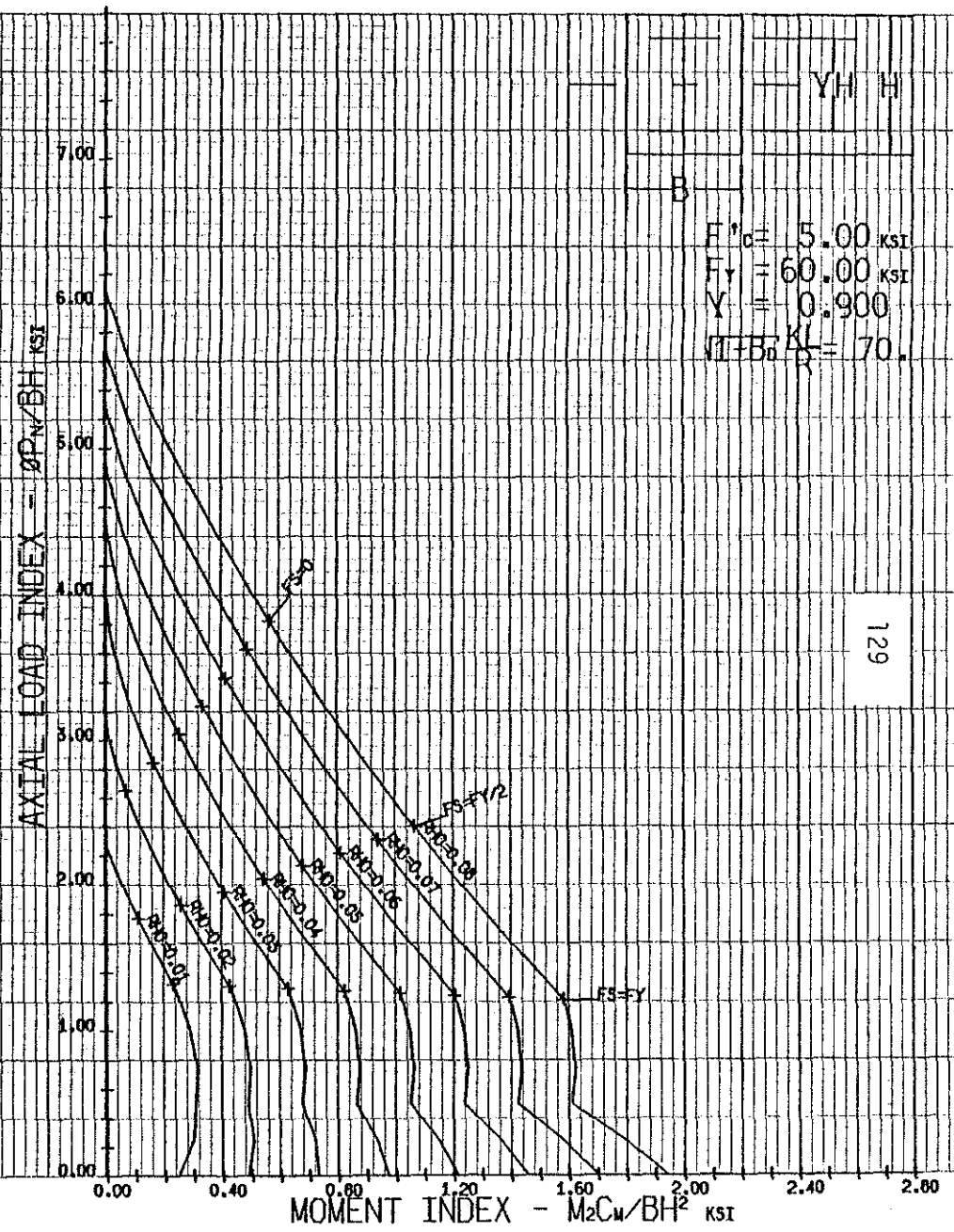


Fig. E5-60.90-70 - Interaction Diagram

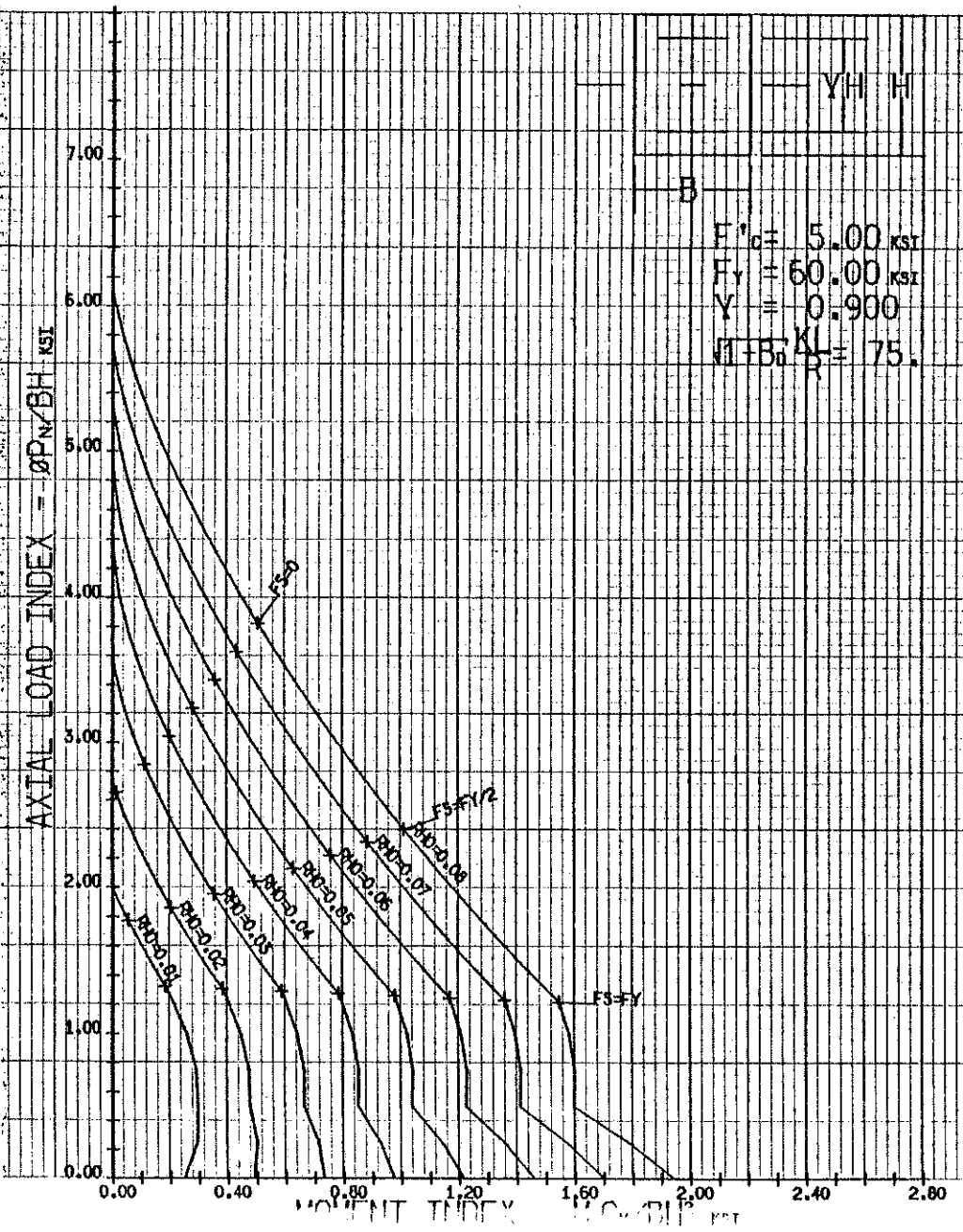


Fig. E5-60.90-75 - Interaction Diagram

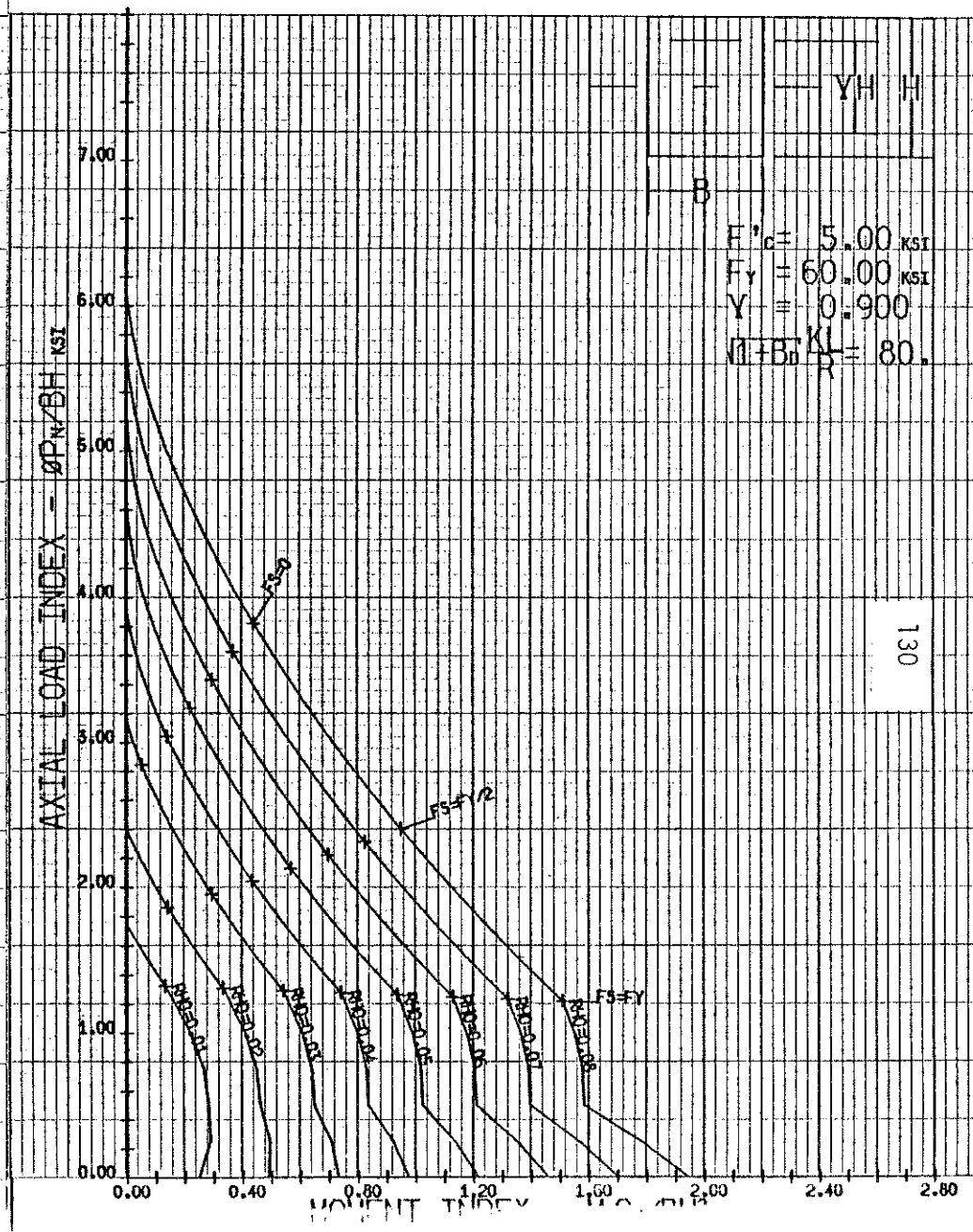


Fig. E5-60.90-80 - Interaction Diagram



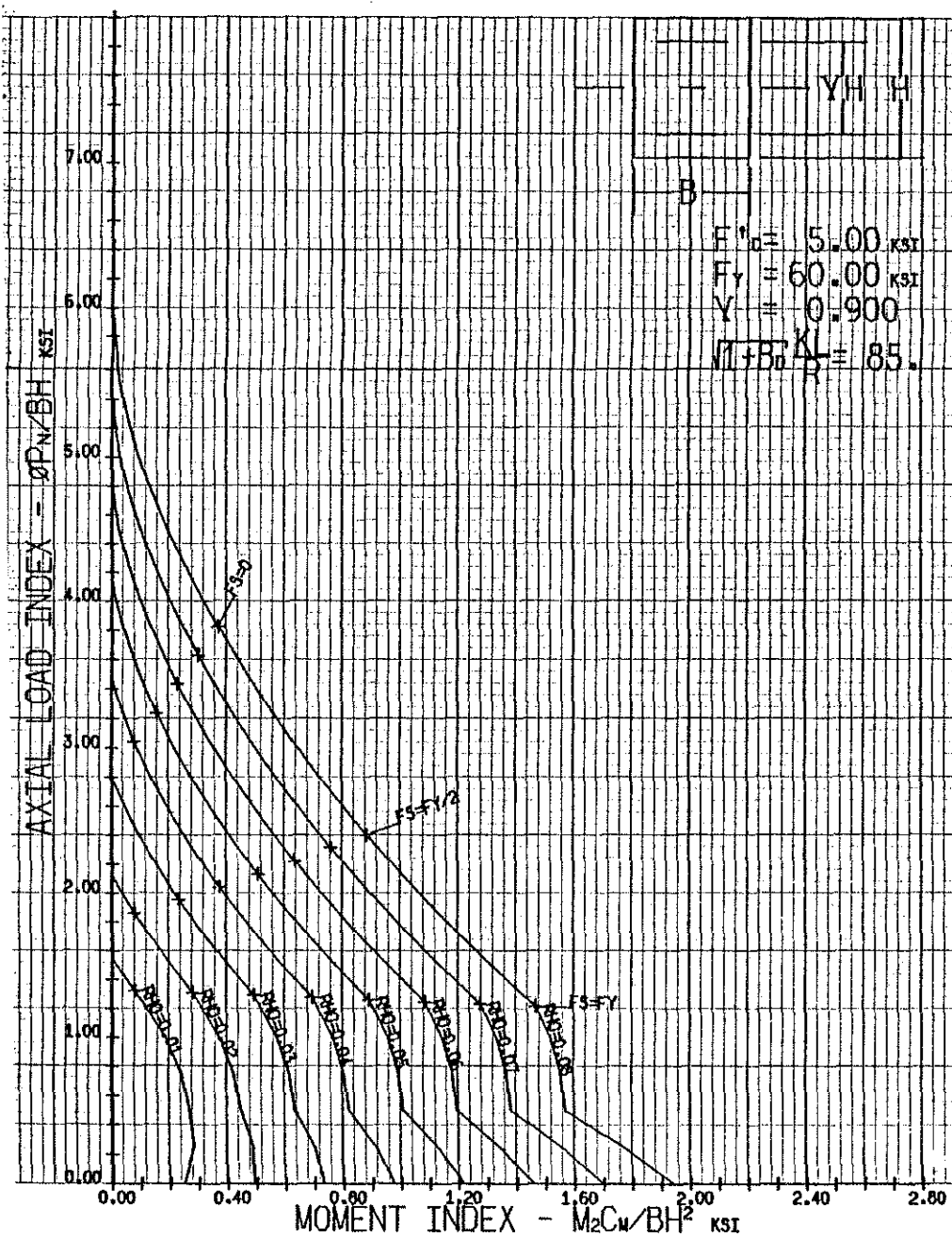


Fig. E5-60.90-85 - Interaction Diagram

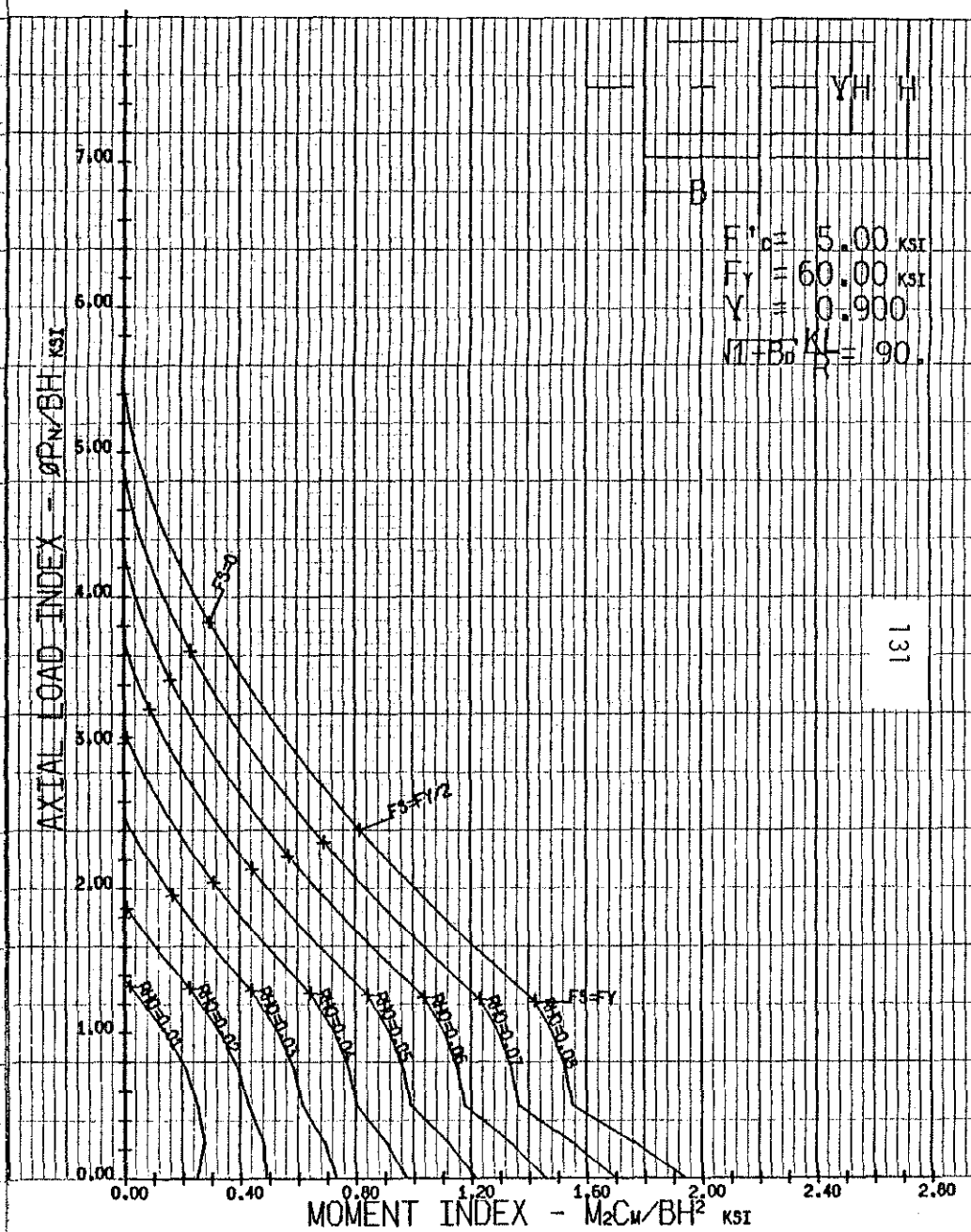


Fig. E5-60.90-90 - Interaction Diagram

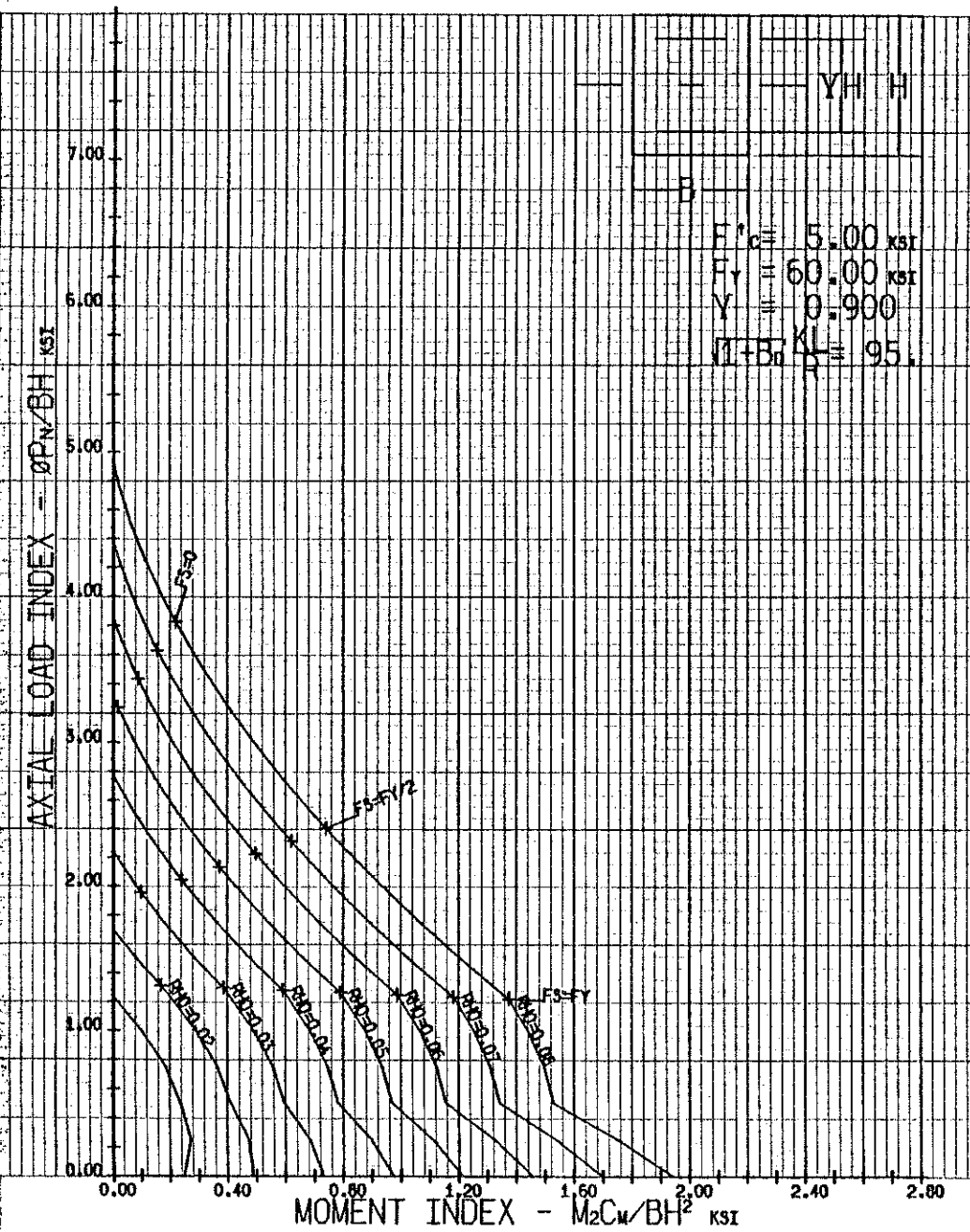


Fig. E5-60.90-95 - Interaction Diagram

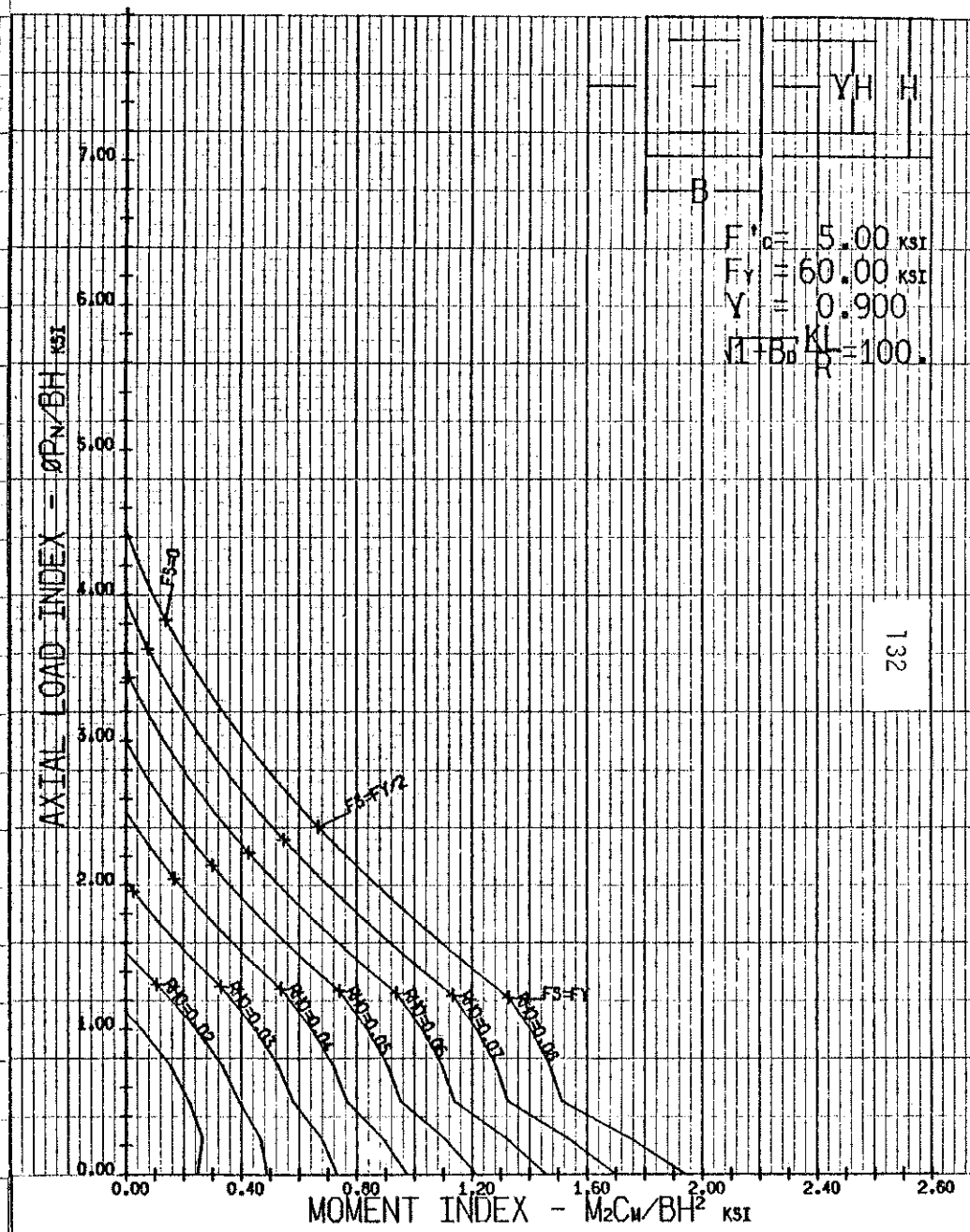


Fig. E5-60.90-100 - Interaction Diagram

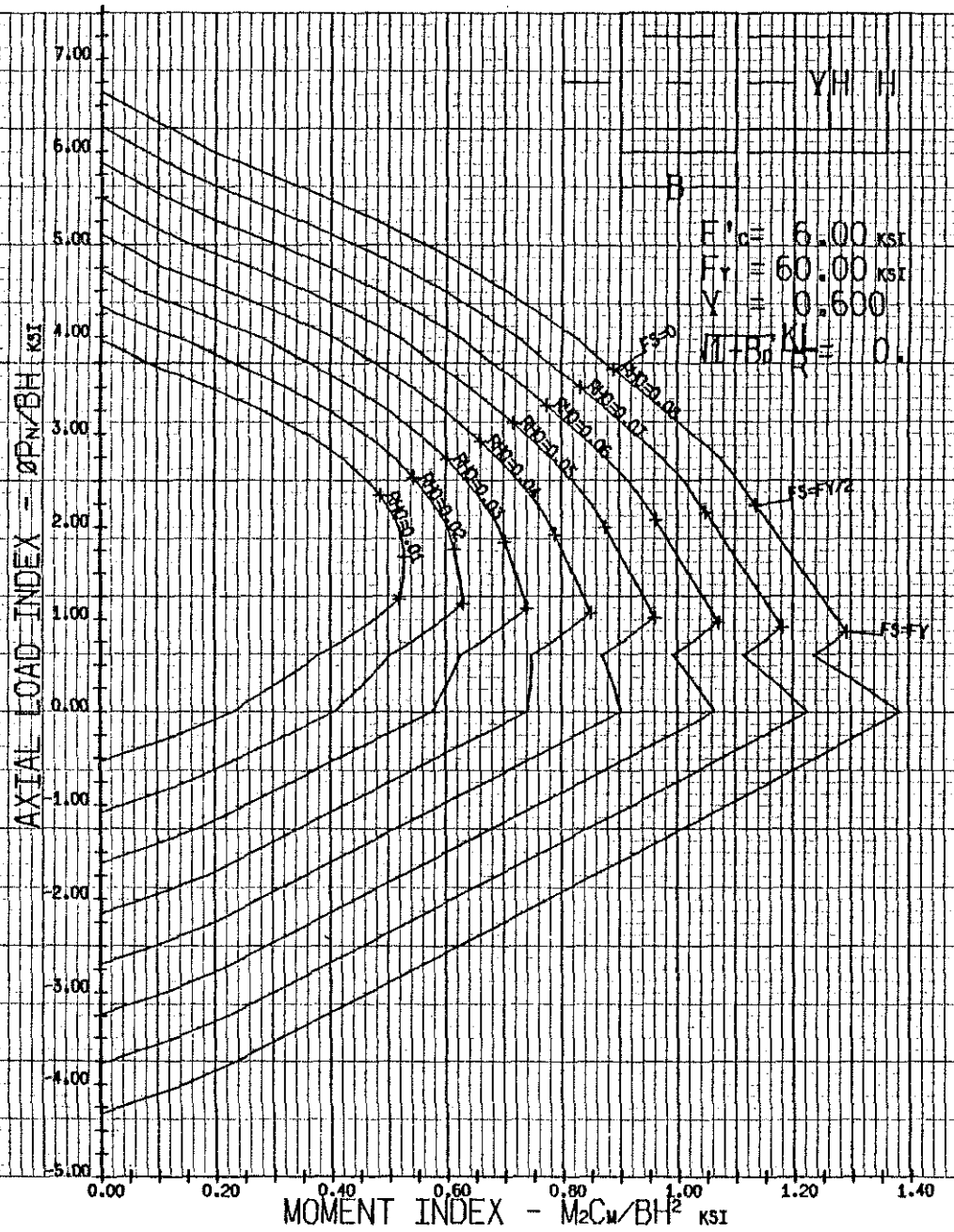


Fig. E6-60.60-0 - Interaction Diagram

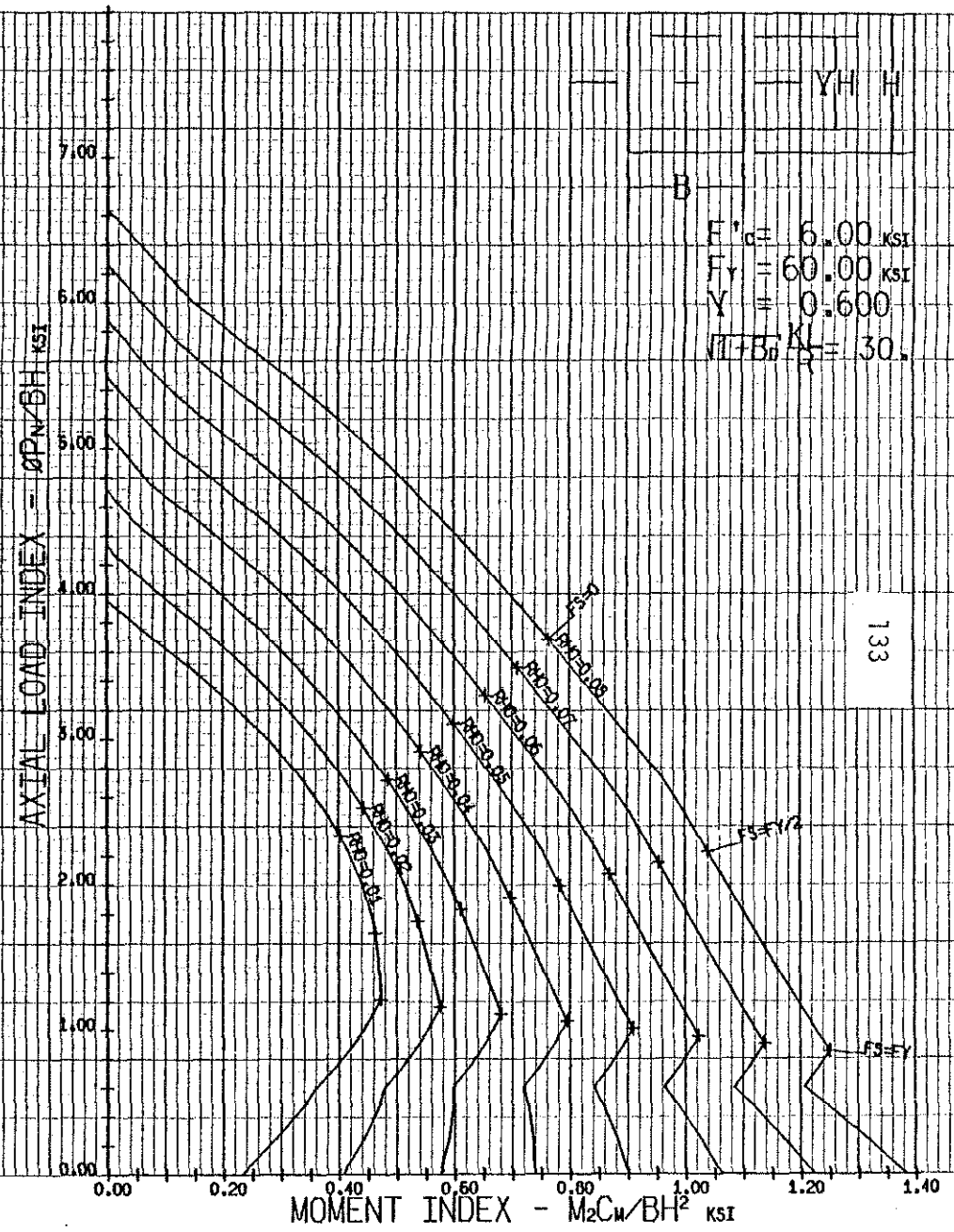


Fig. E6-60.60-30 - Interaction Diagram



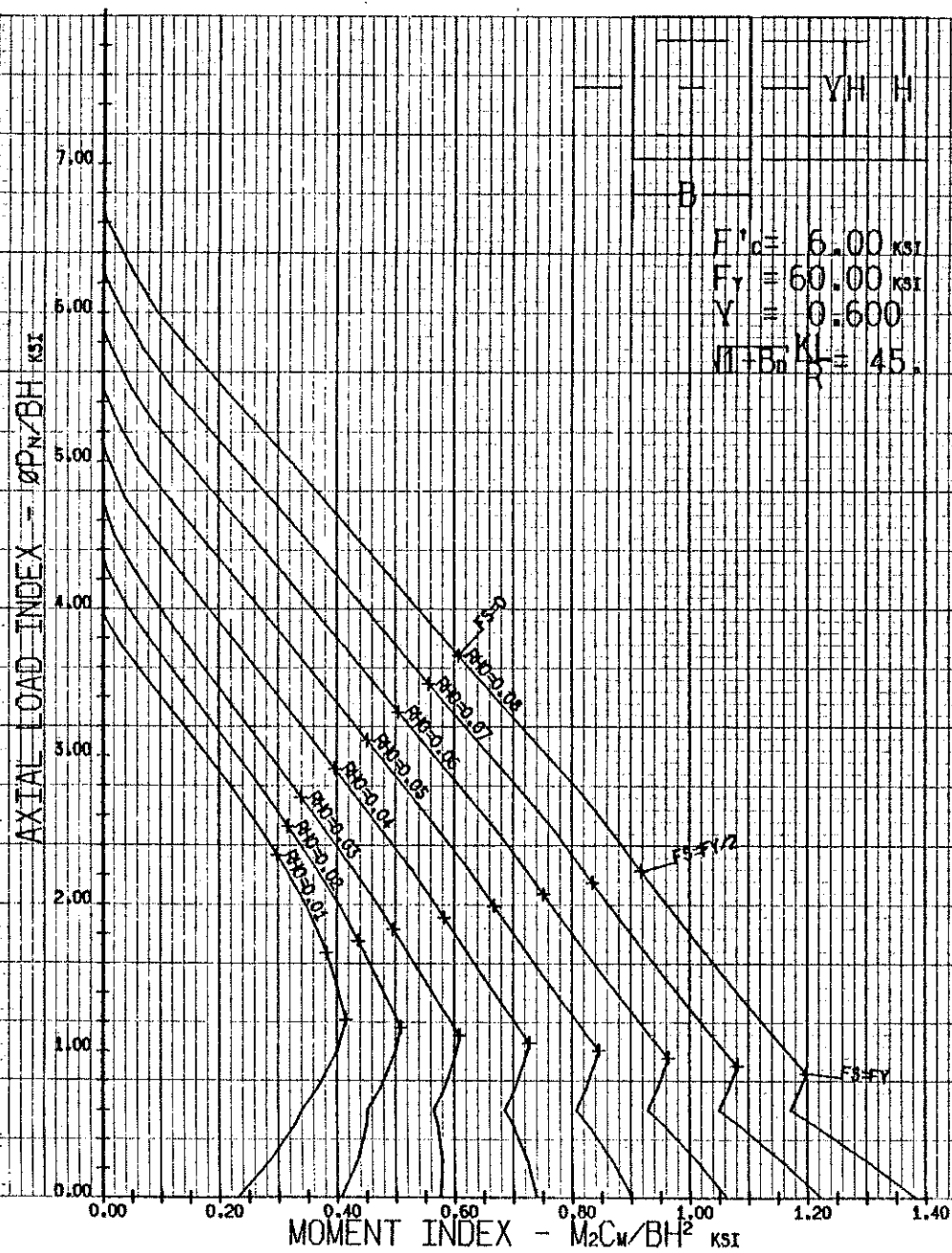


Fig. E6-60.60-45 - Interaction Diagram

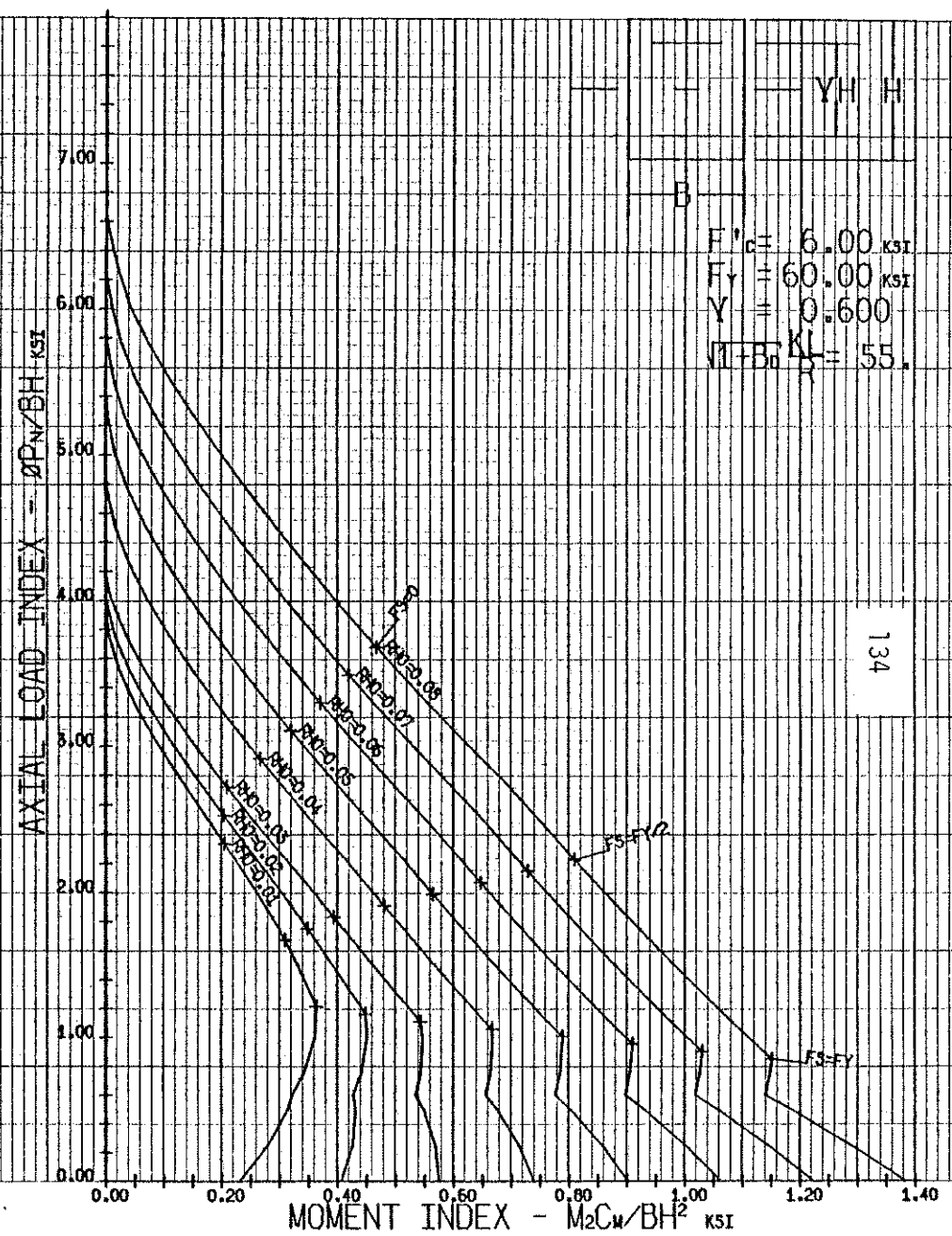


Fig. E6-60.60-55 - Interaction Diagram

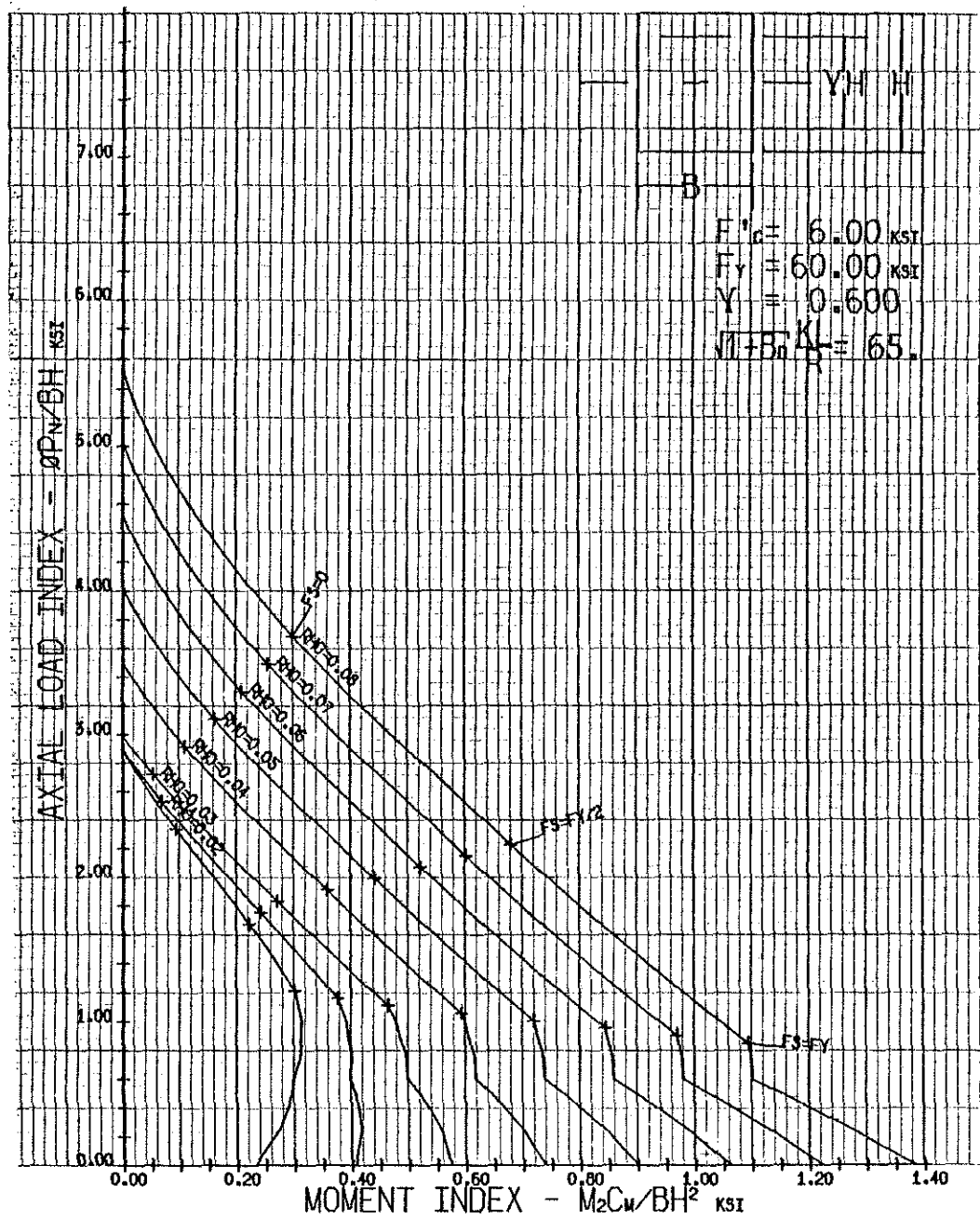


Fig. E6-60.60-65 - Interaction Diagram

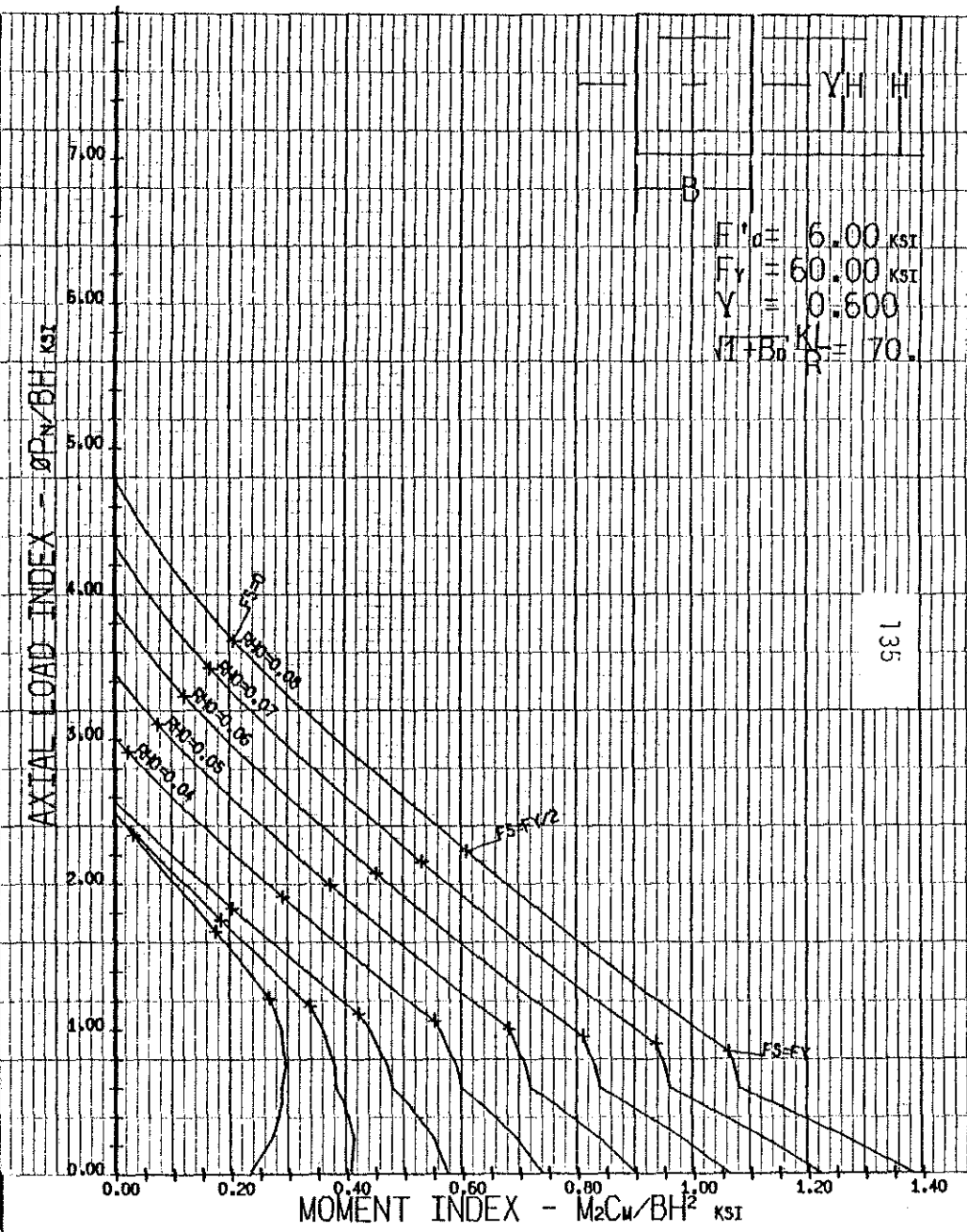


Fig. E6-60.60-70 - Interaction Diagram

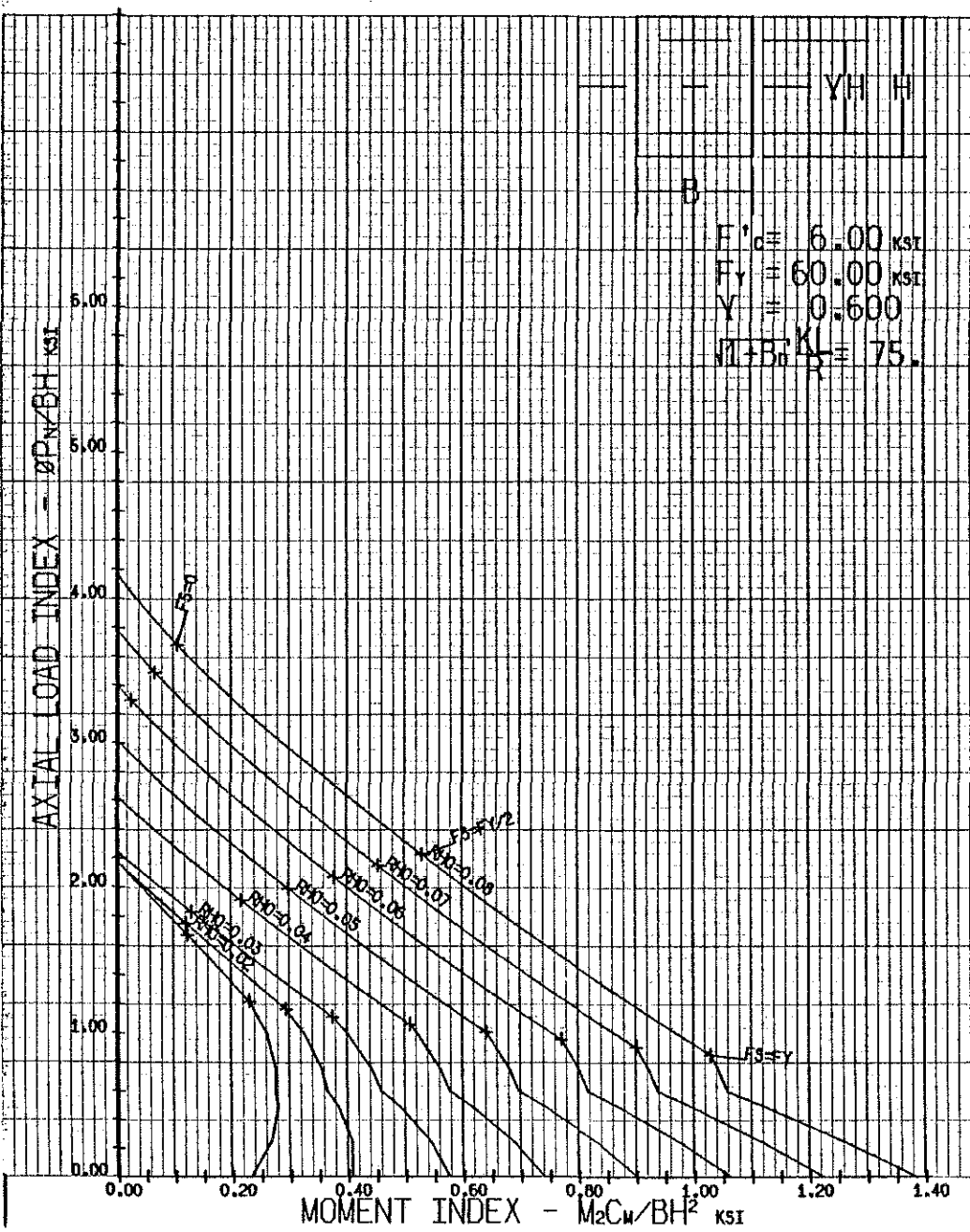


Fig. E6-60.60-75 - Interaction Diagram

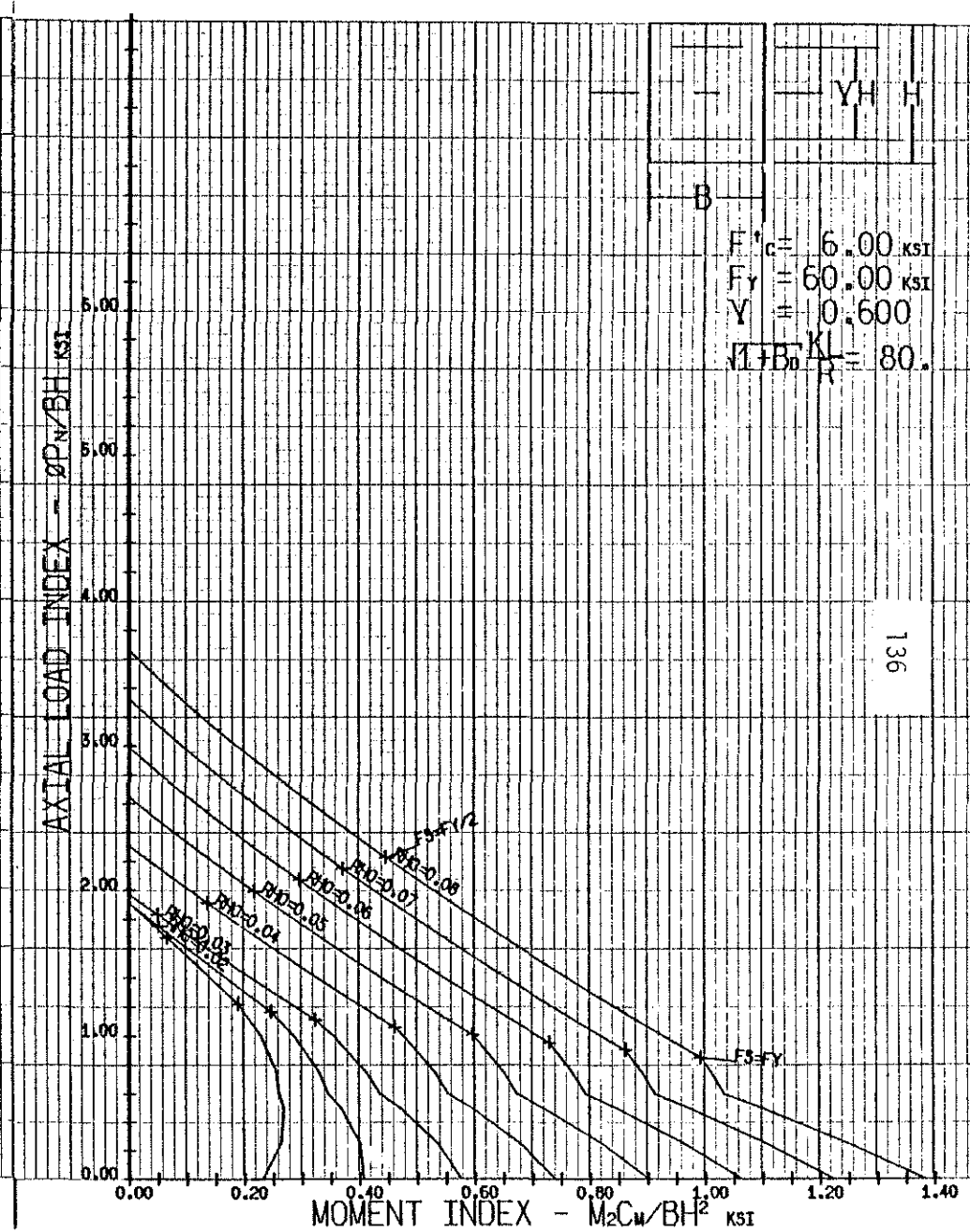


Fig. E6-60.60-80 - Interaction Diagram

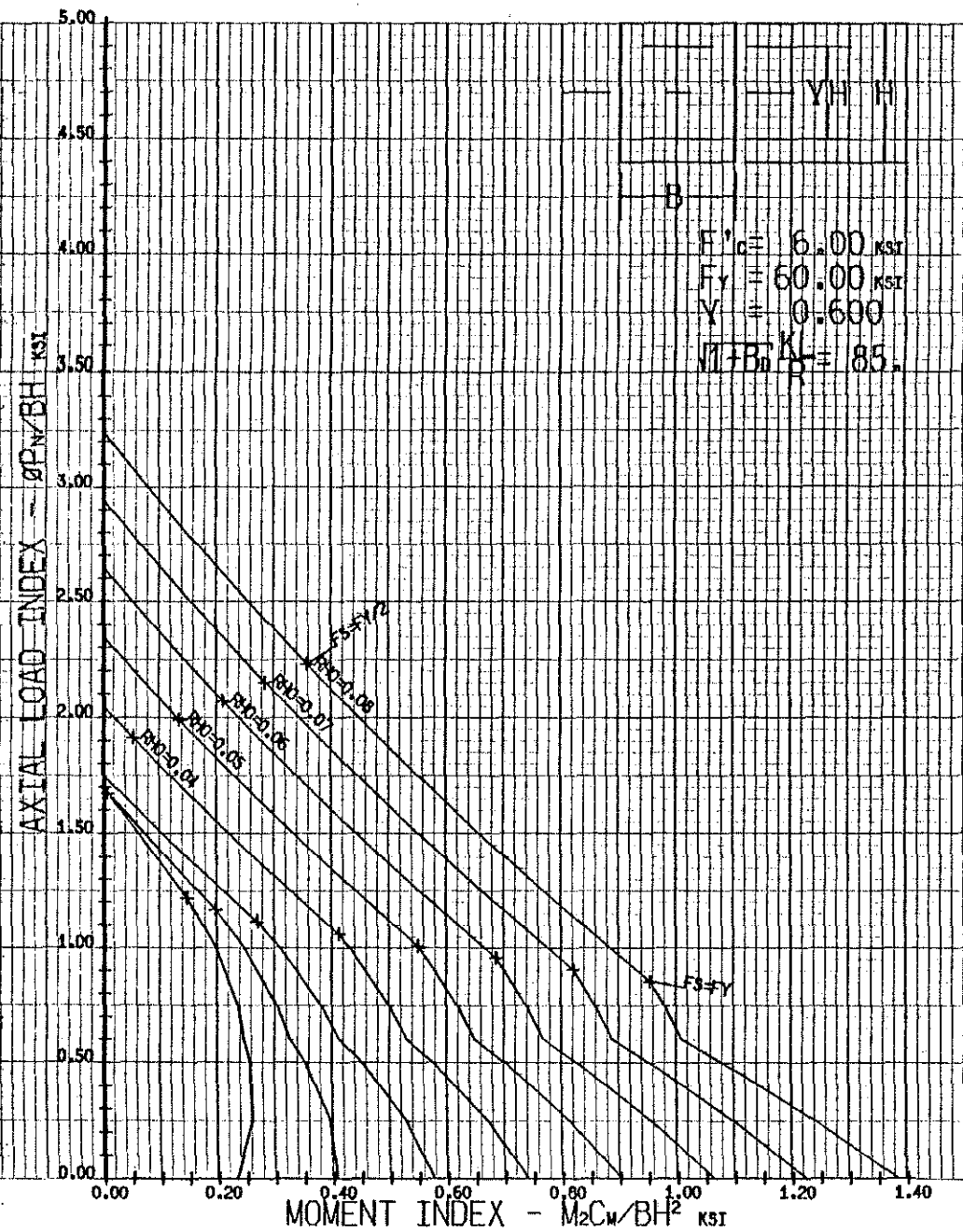


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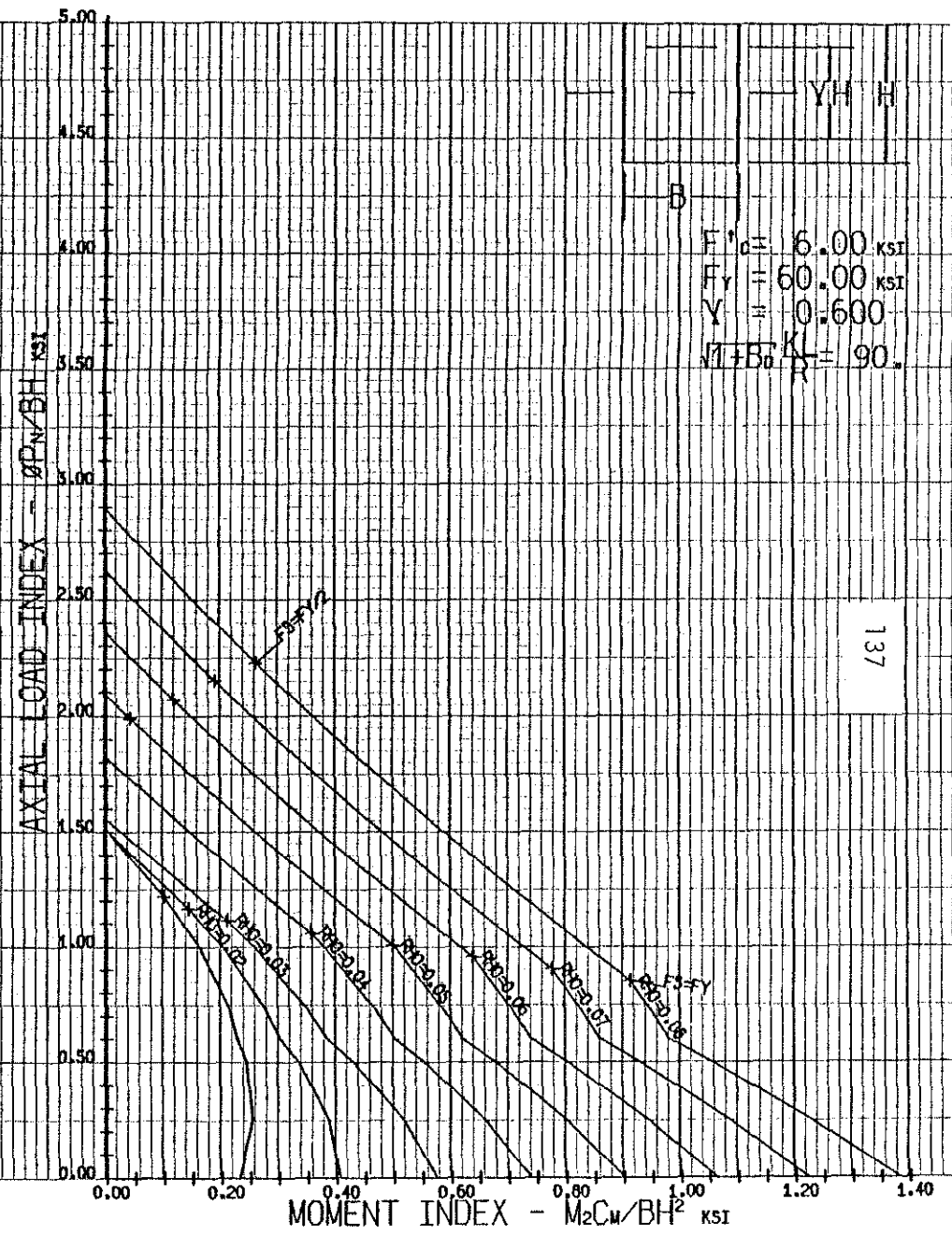


Fig. E6-60.60-90 - Interaction Diagram





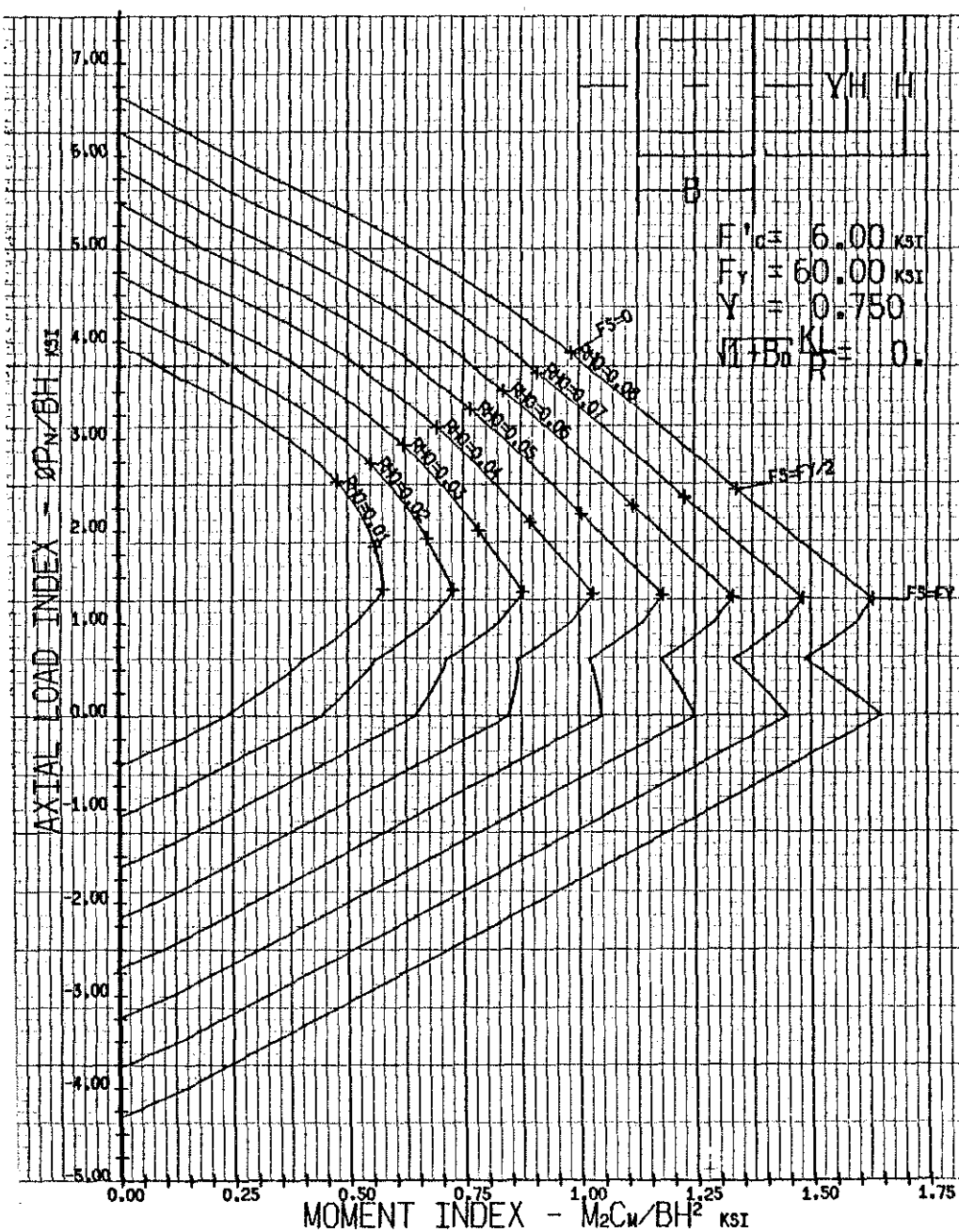


Fig. E6-60.75-0 - Interaction Diagram

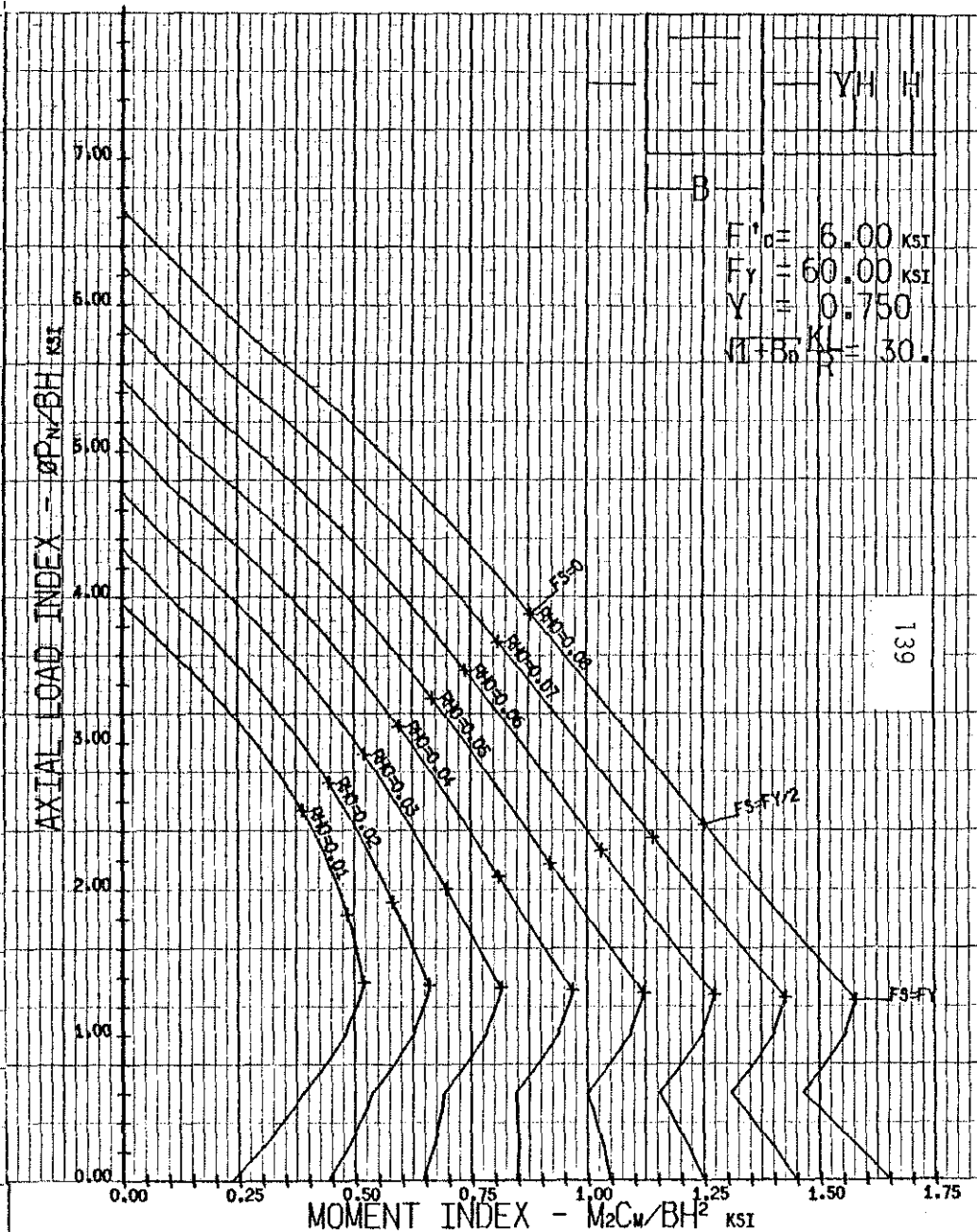


Fig. E6-60.75-30 - Interaction Diagram

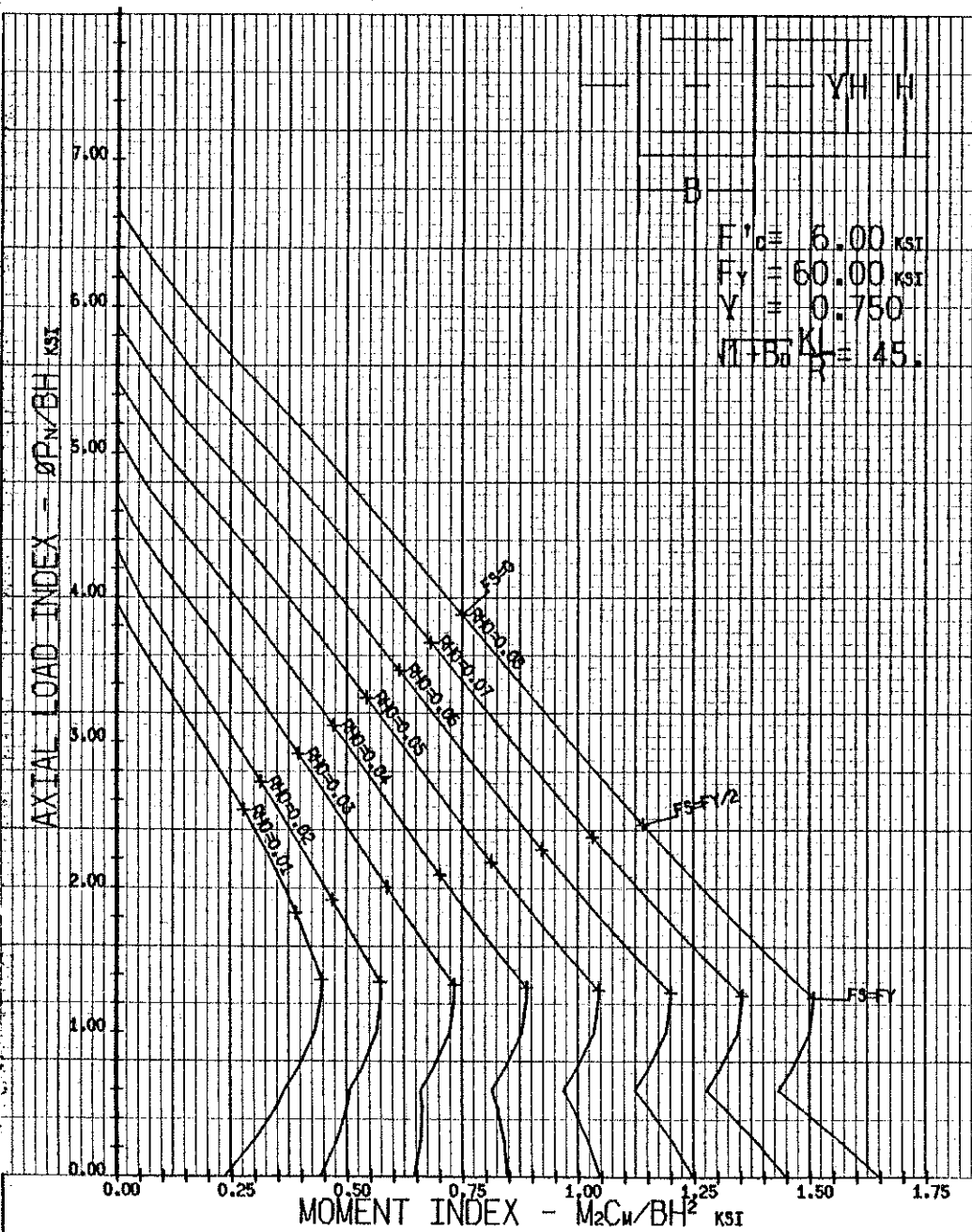


Fig. E6-60.75-45 - Interaction Diagram

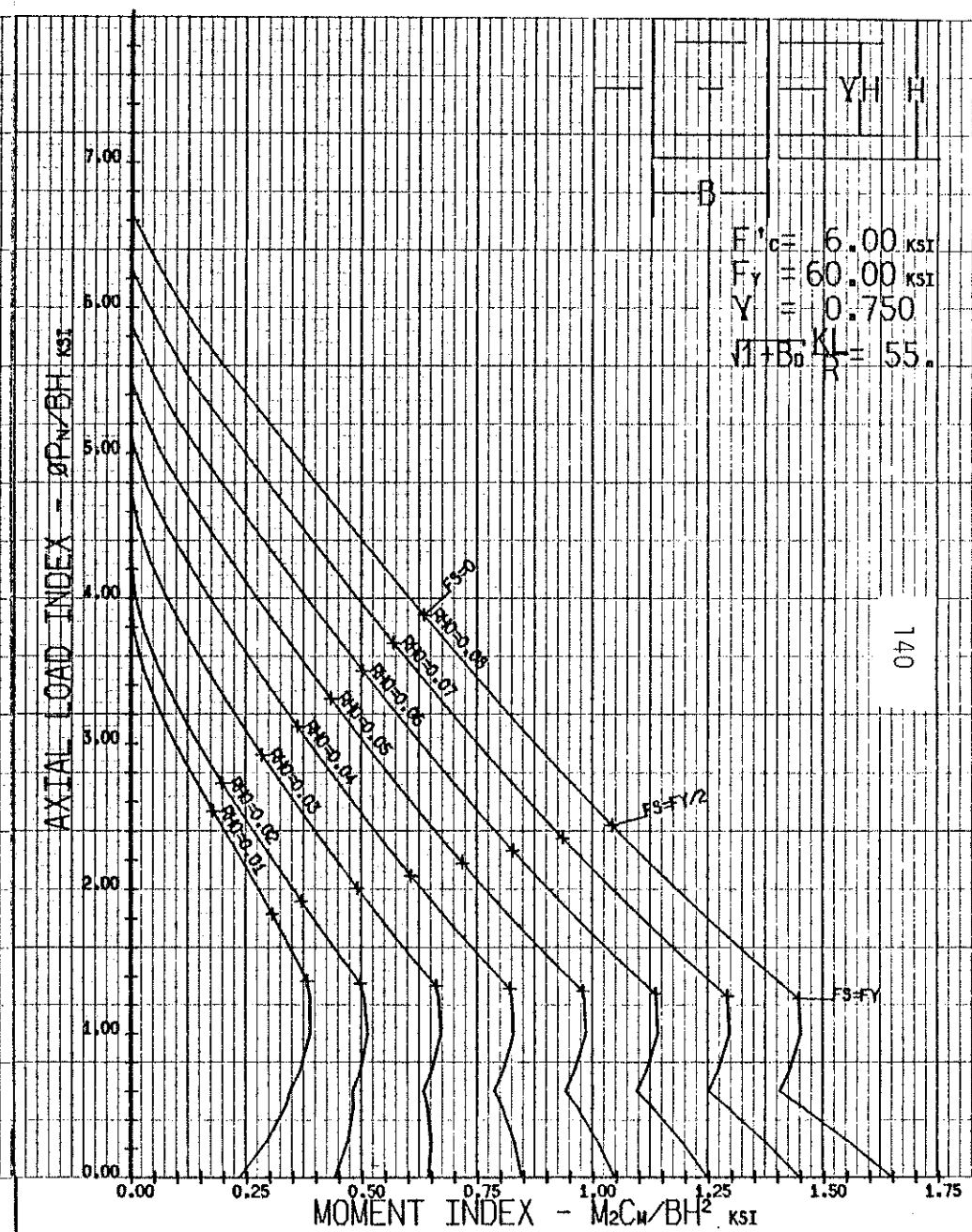


Fig. E6-60.75-55 - Interaction Diagram

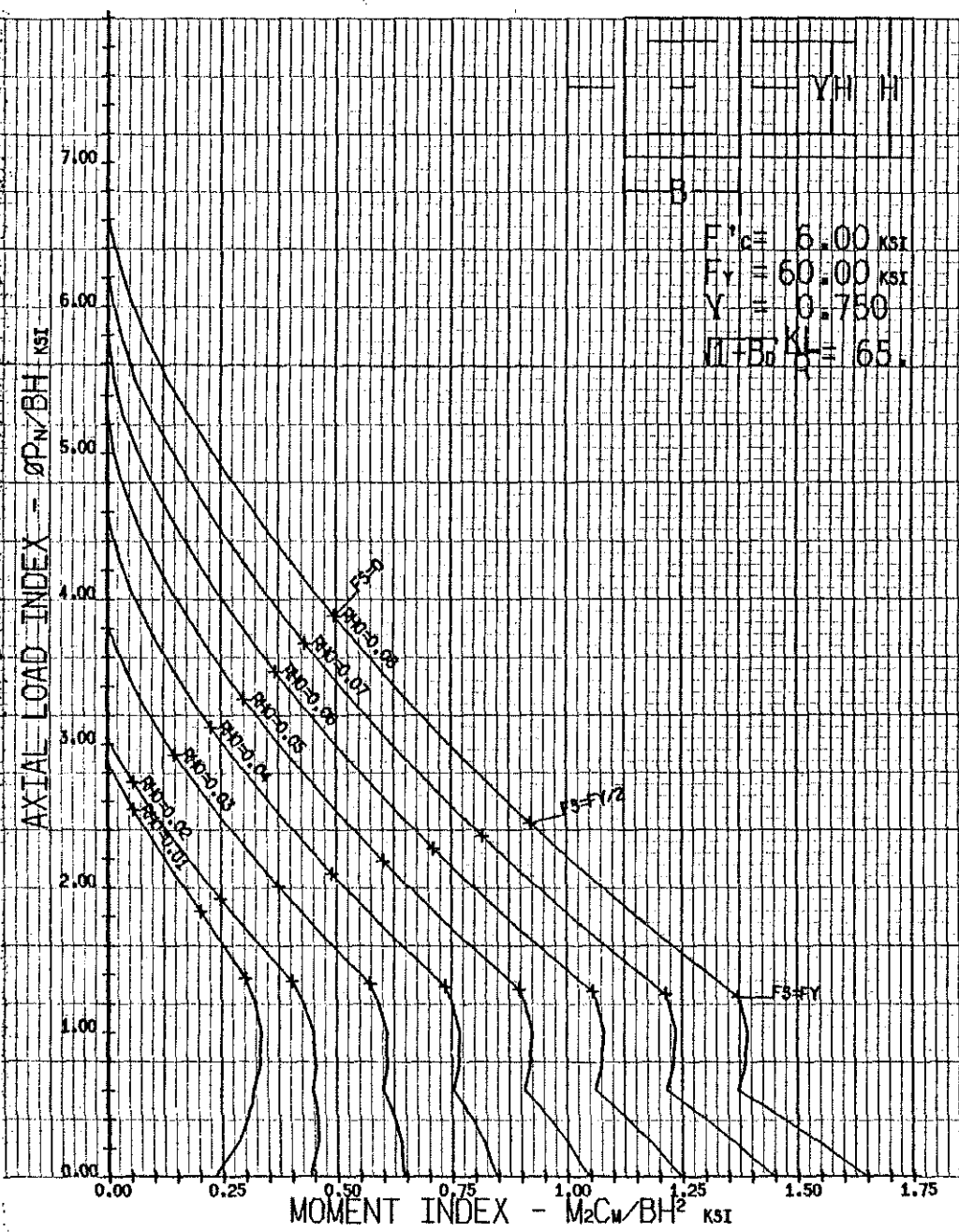


Fig. E6-60.75-65 - Interaction Diagram

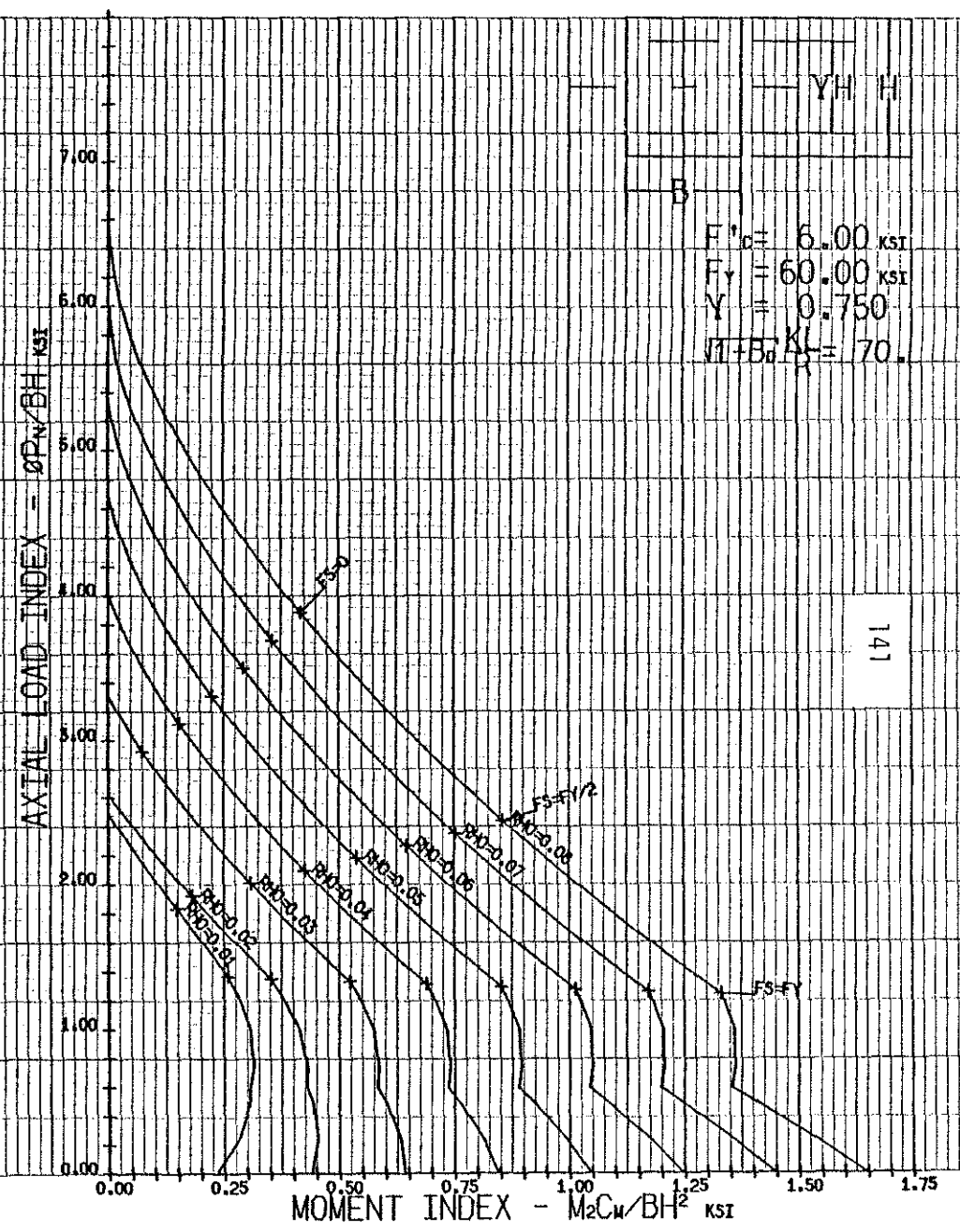


Fig. E6-60.75-70 - Interaction Diagram



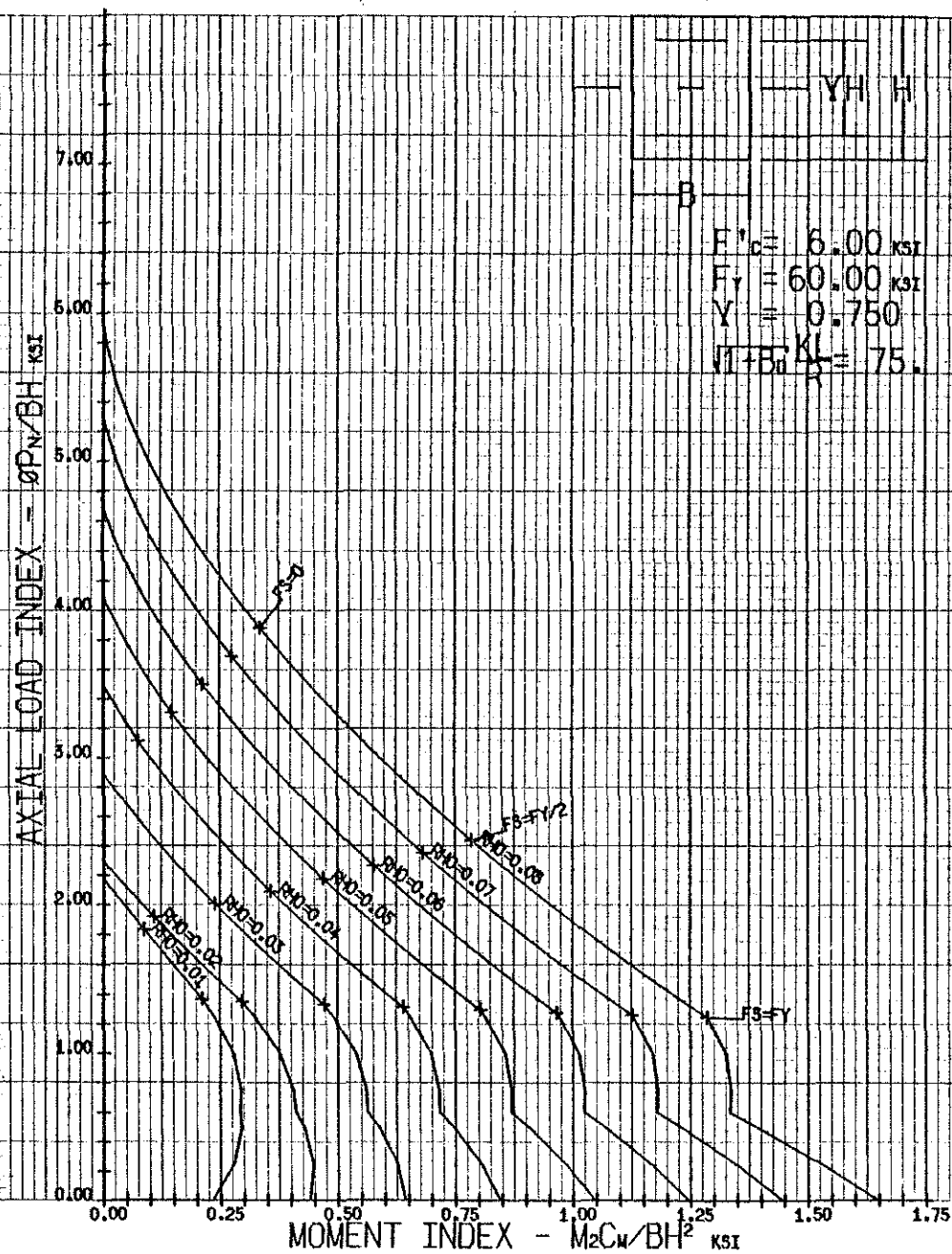


Fig. E6-60.75-75 - Interaction Diagram

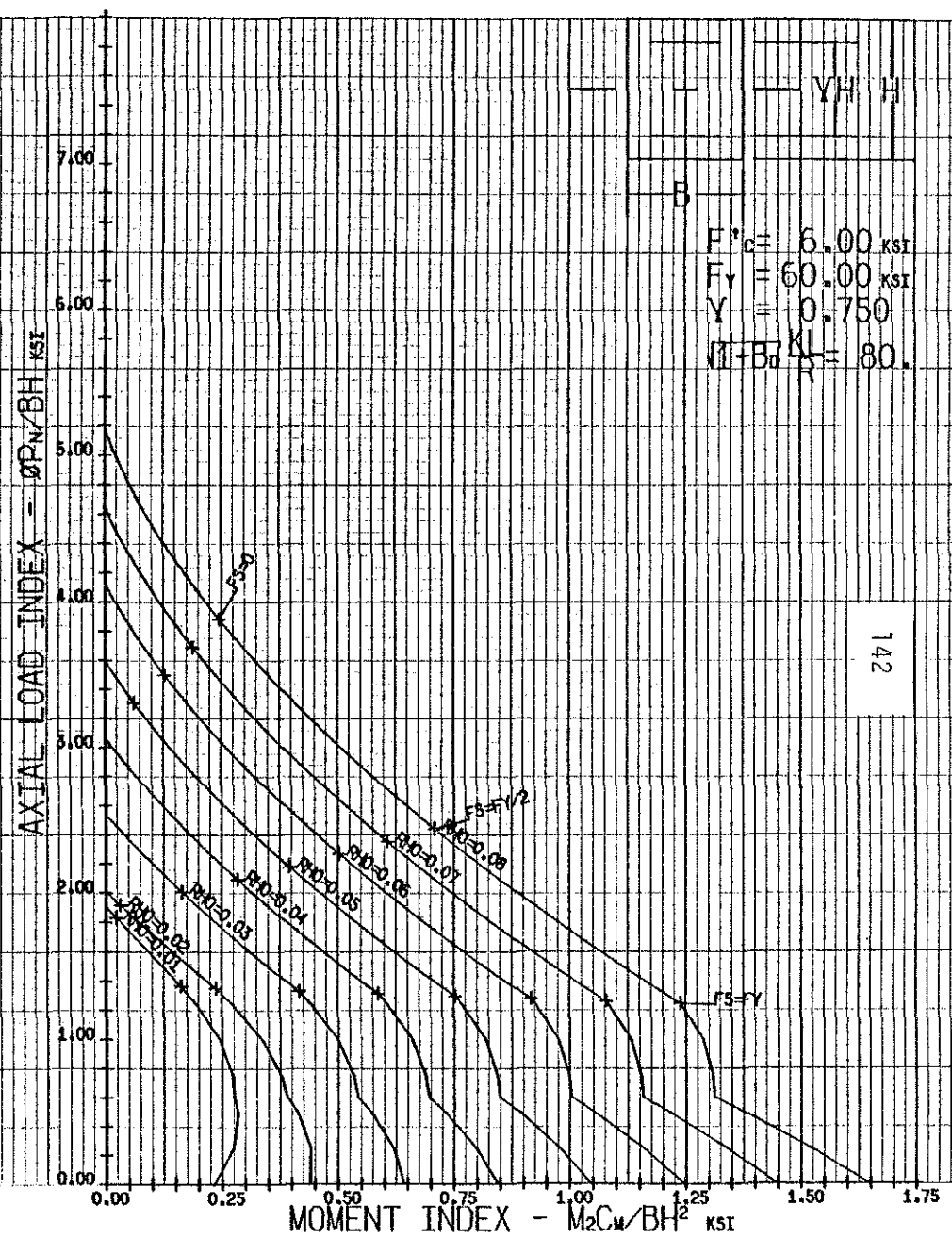


Fig. E6-60.75-80 - Interaction Diagram

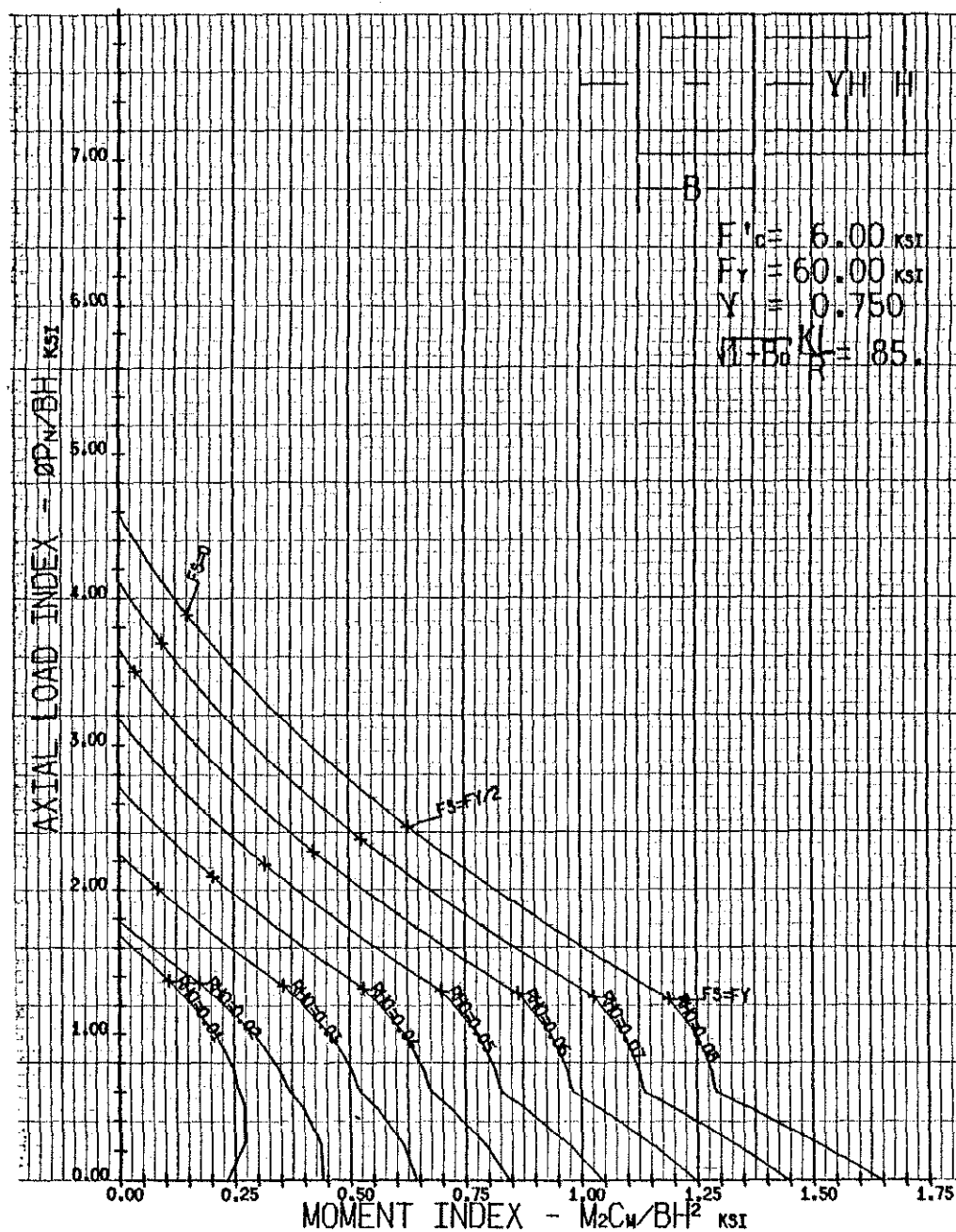


Fig. E6-60.75-85 - Interaction Diagram

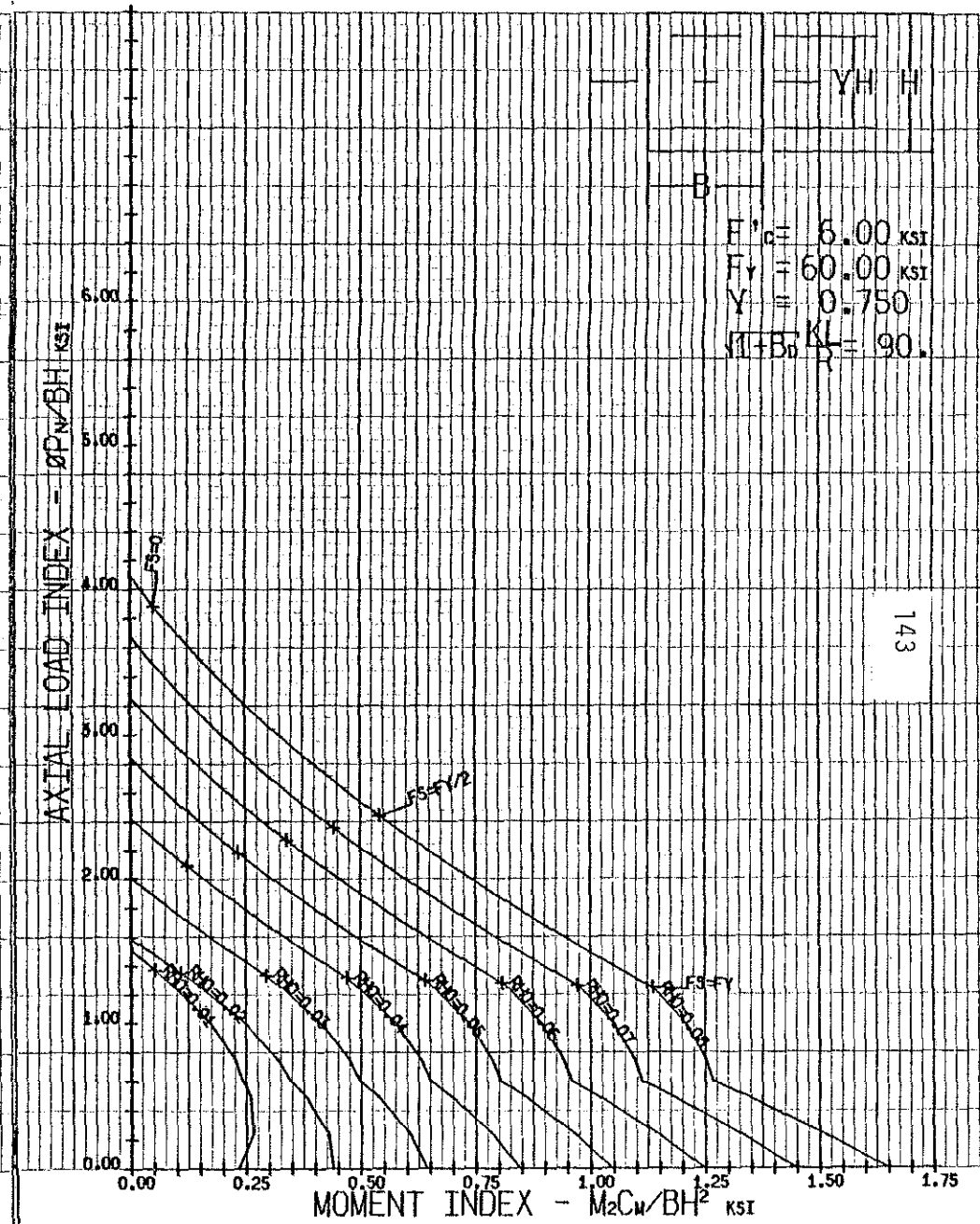


Fig. E6-60.75-90 - Interaction Diagram

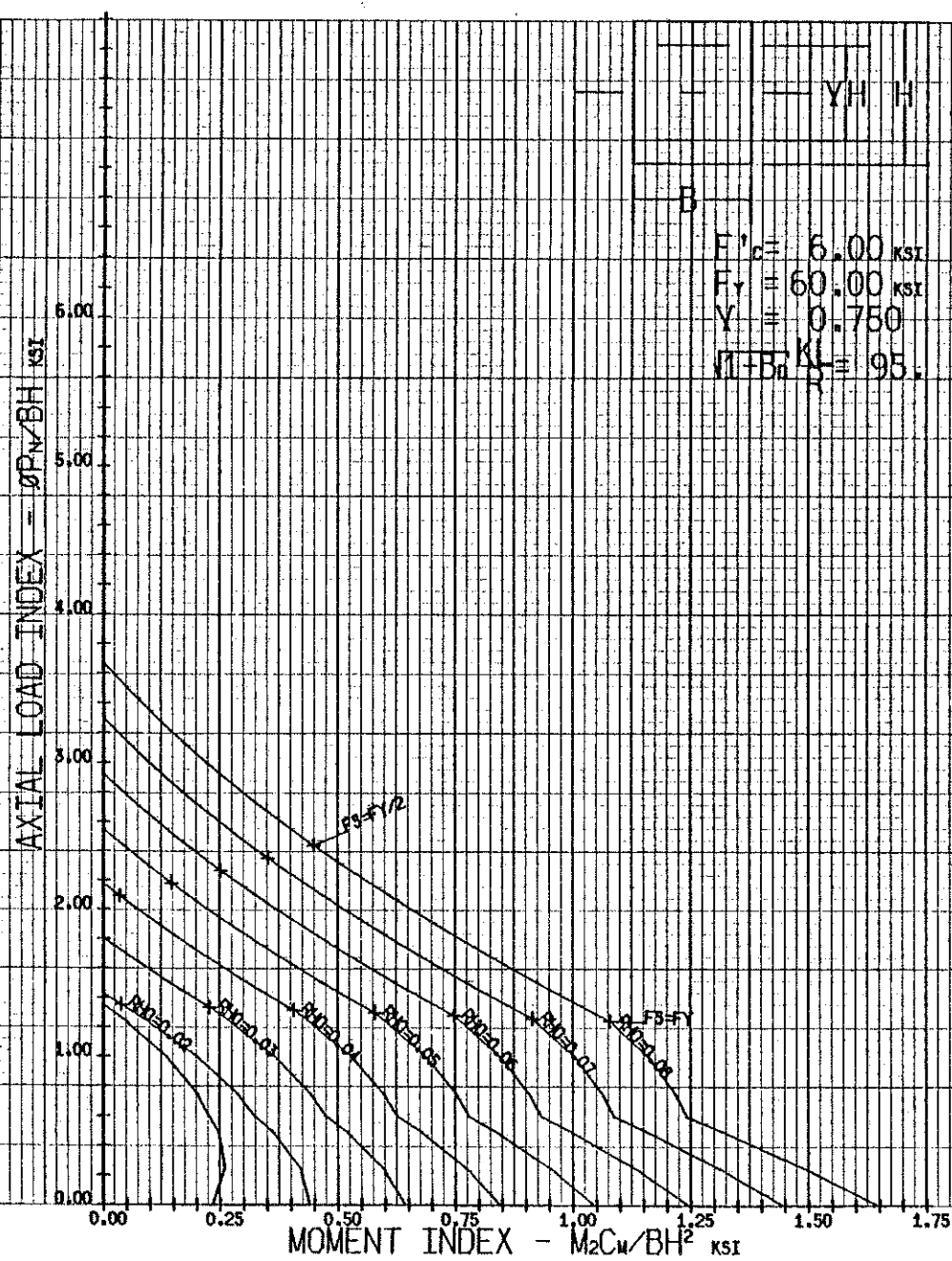


Fig. E6-60.75-95 - Interaction Diagram

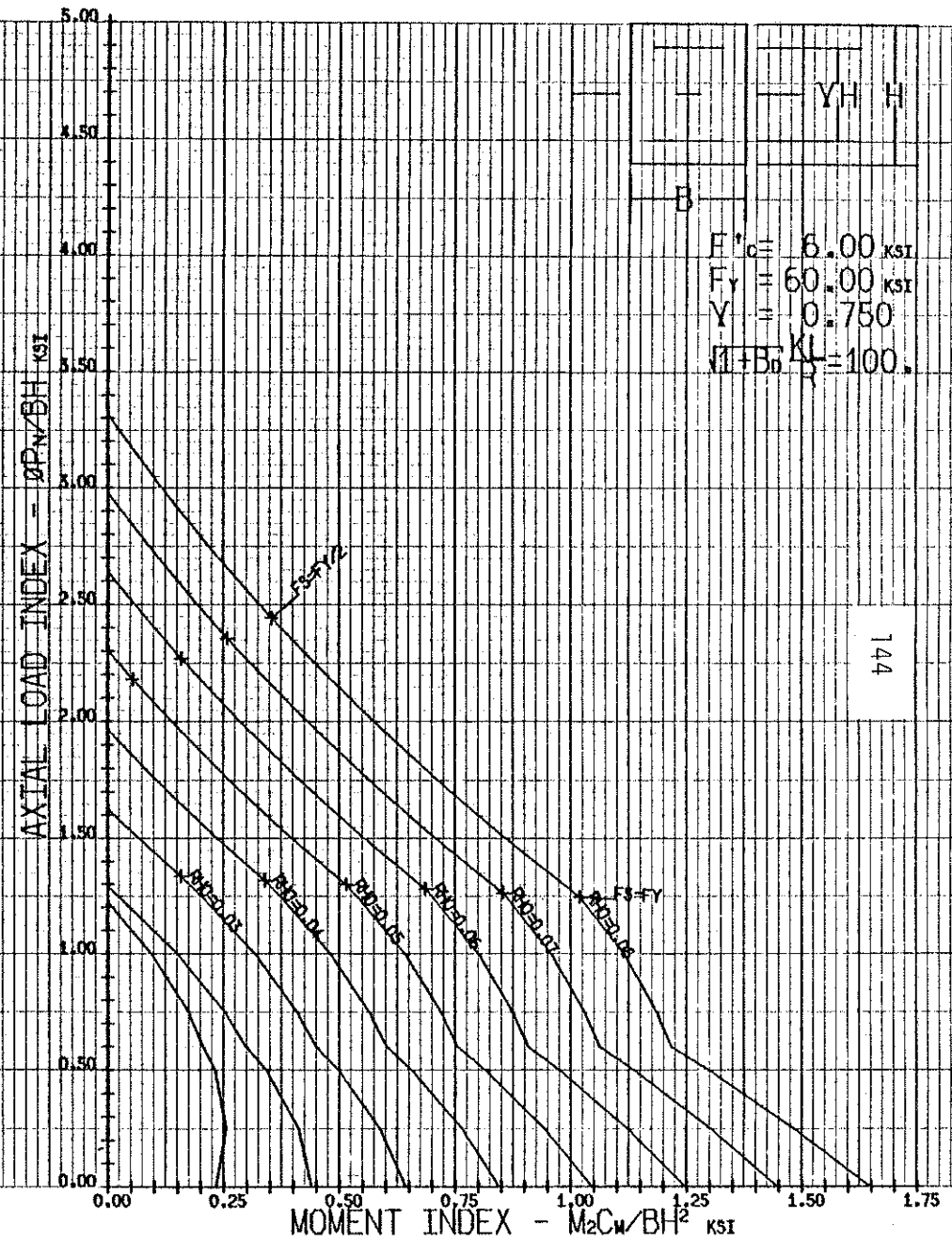


Fig. E6-60.75-100 - Interaction Diagram

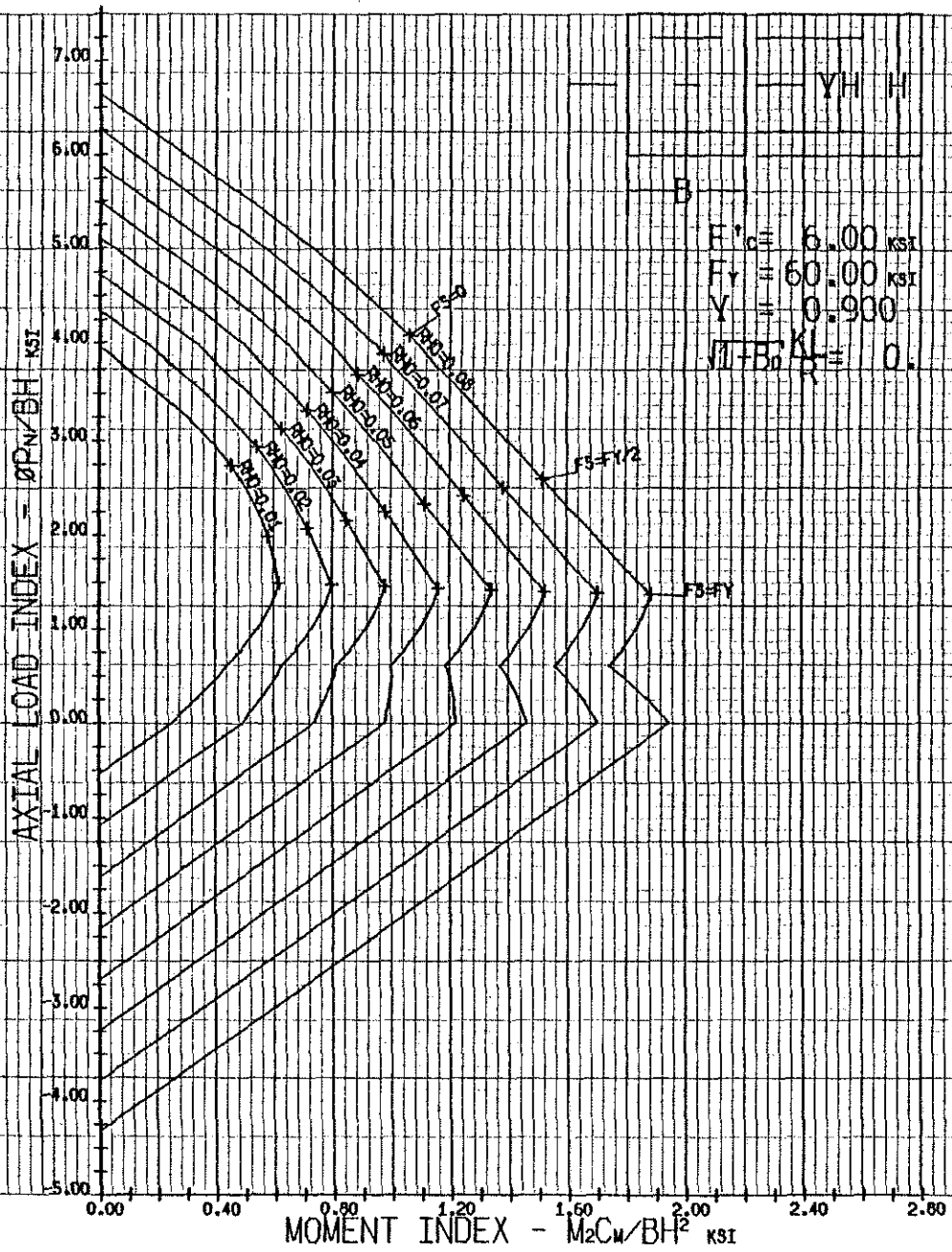


Fig. E6-60.90-0 - Interaction Diagram

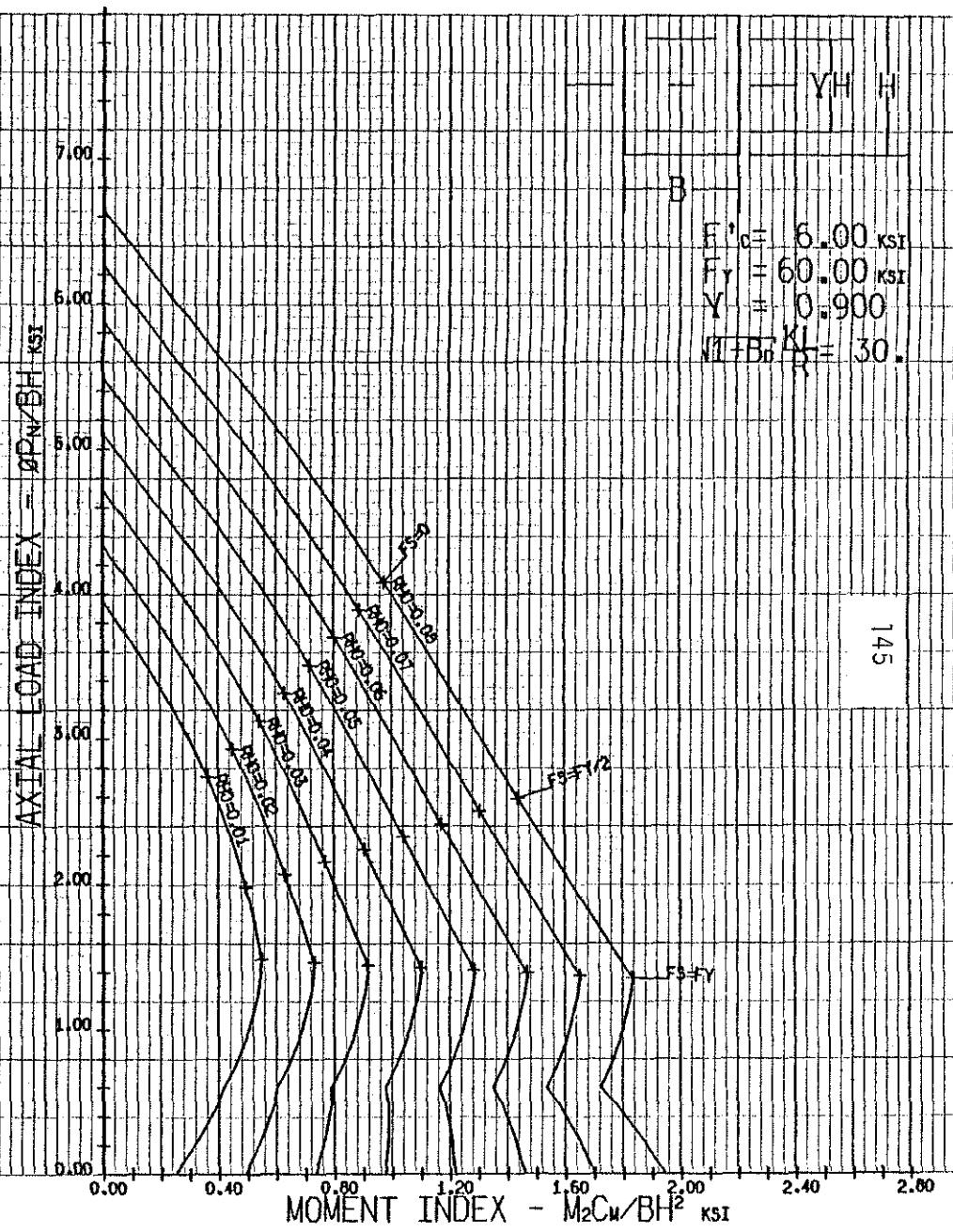


Fig. E6-60.90-30 - Interaction Diagram

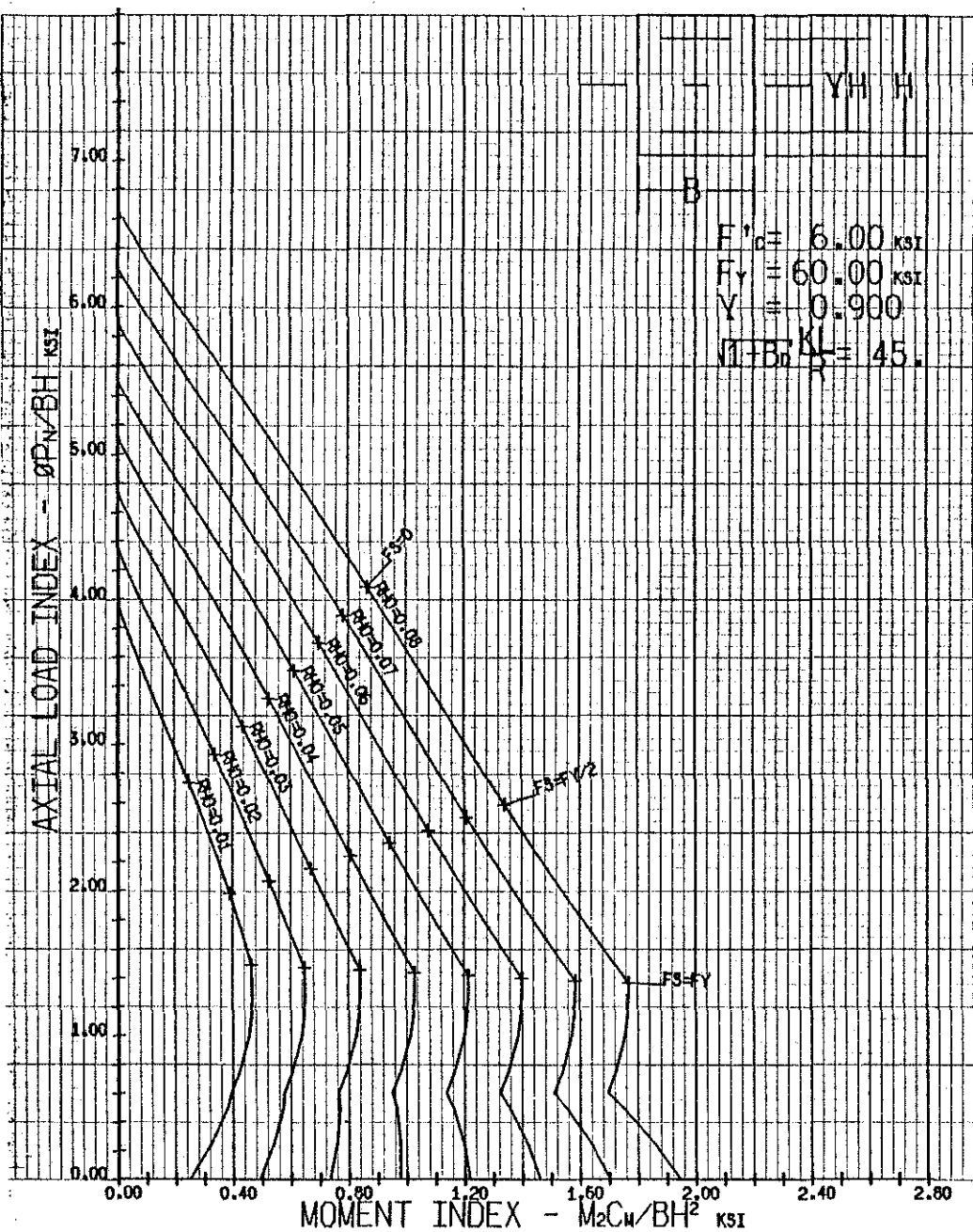


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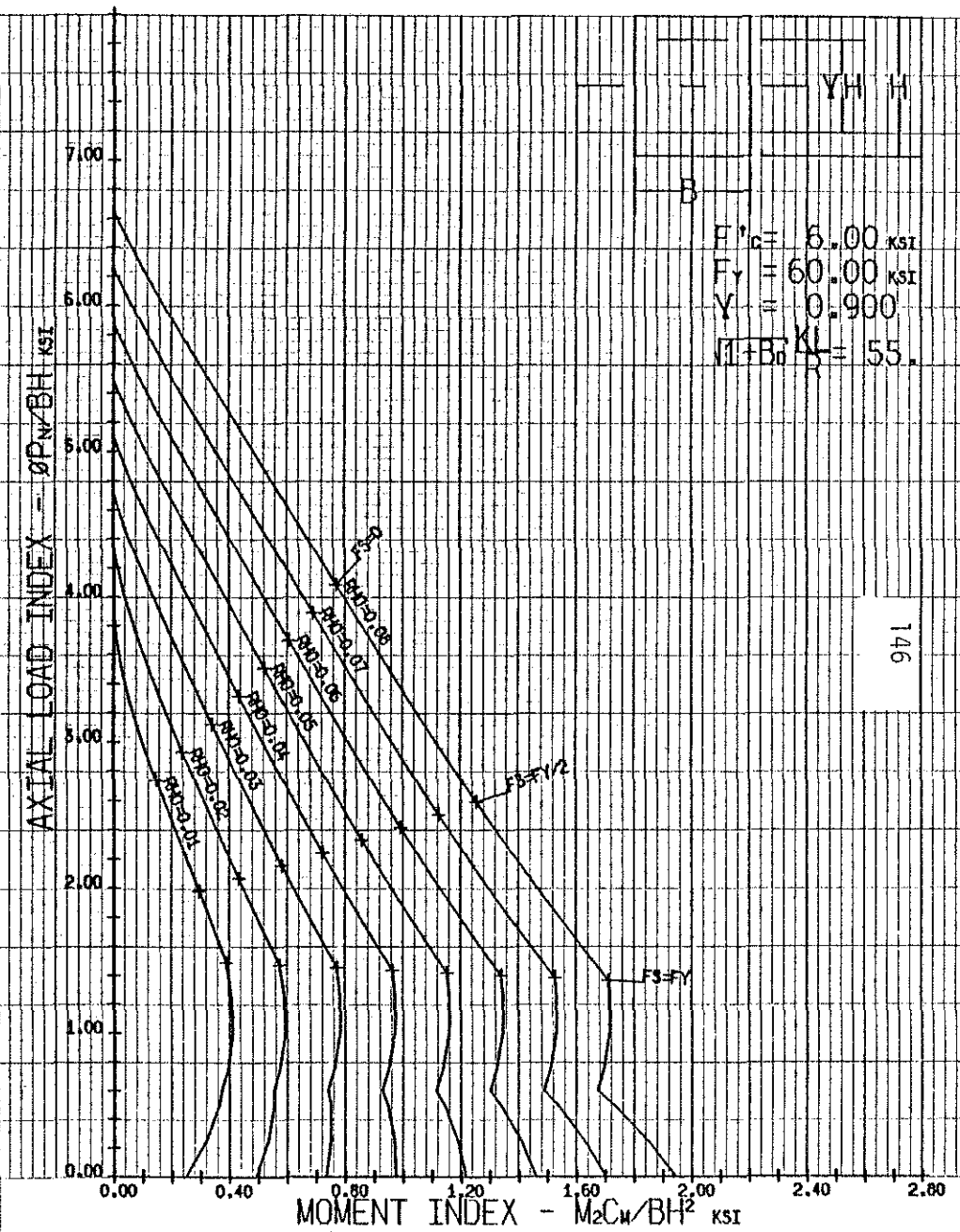


Fig. E6-60.90-55 - Interaction Diagram



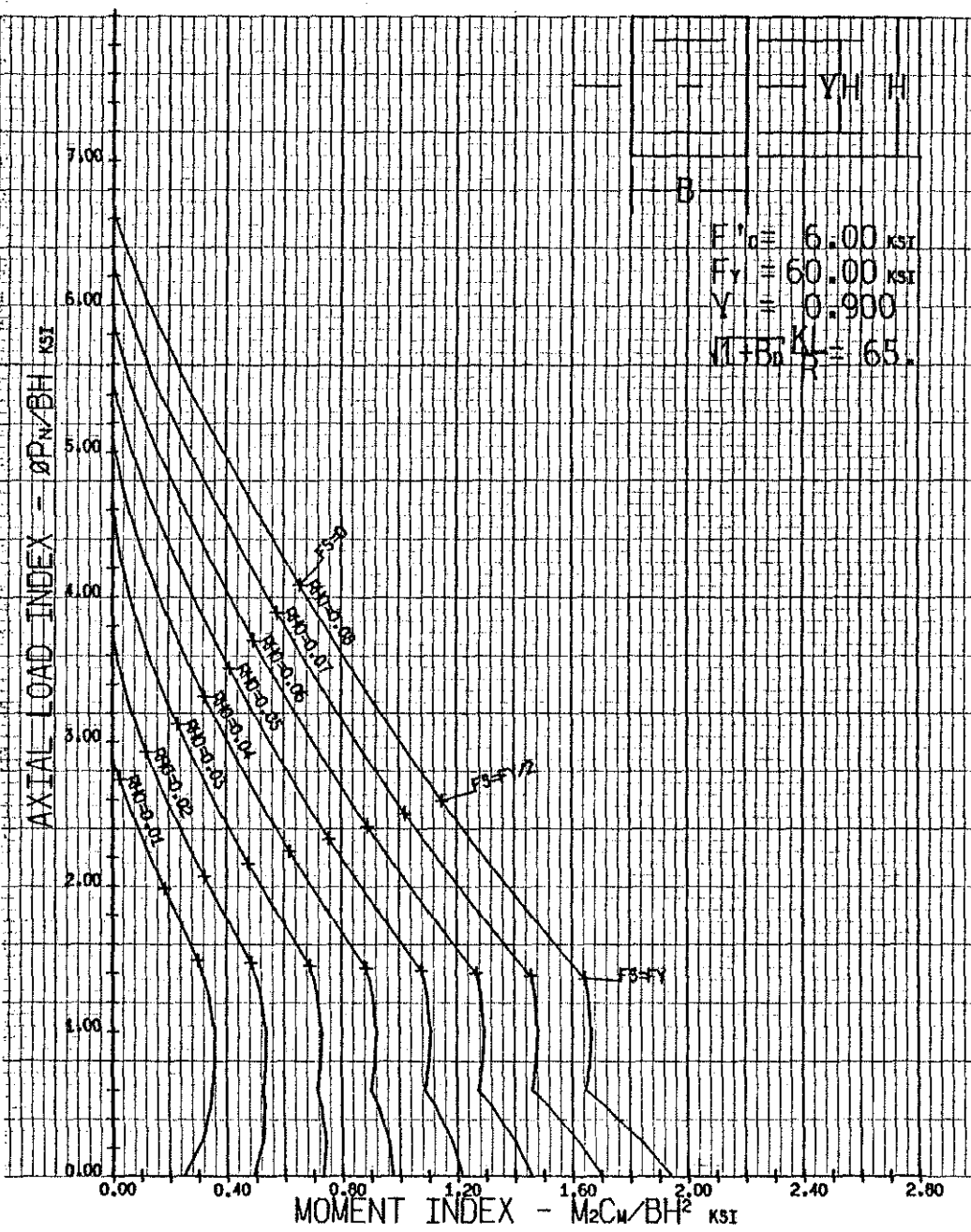


Fig. E6-60.90-65 - Interaction Diagram

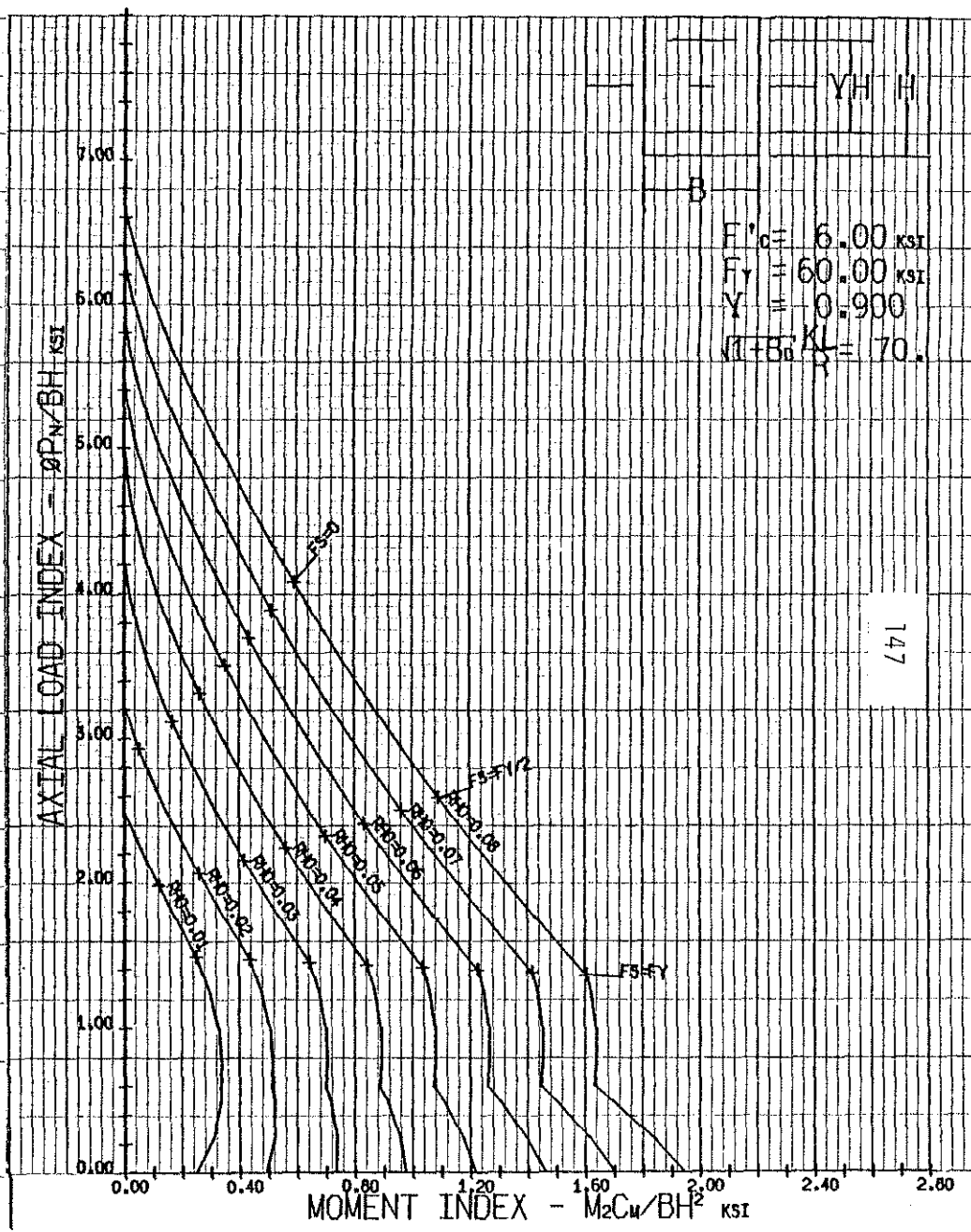


Fig. E6-60.90-70 - Interaction Diagram

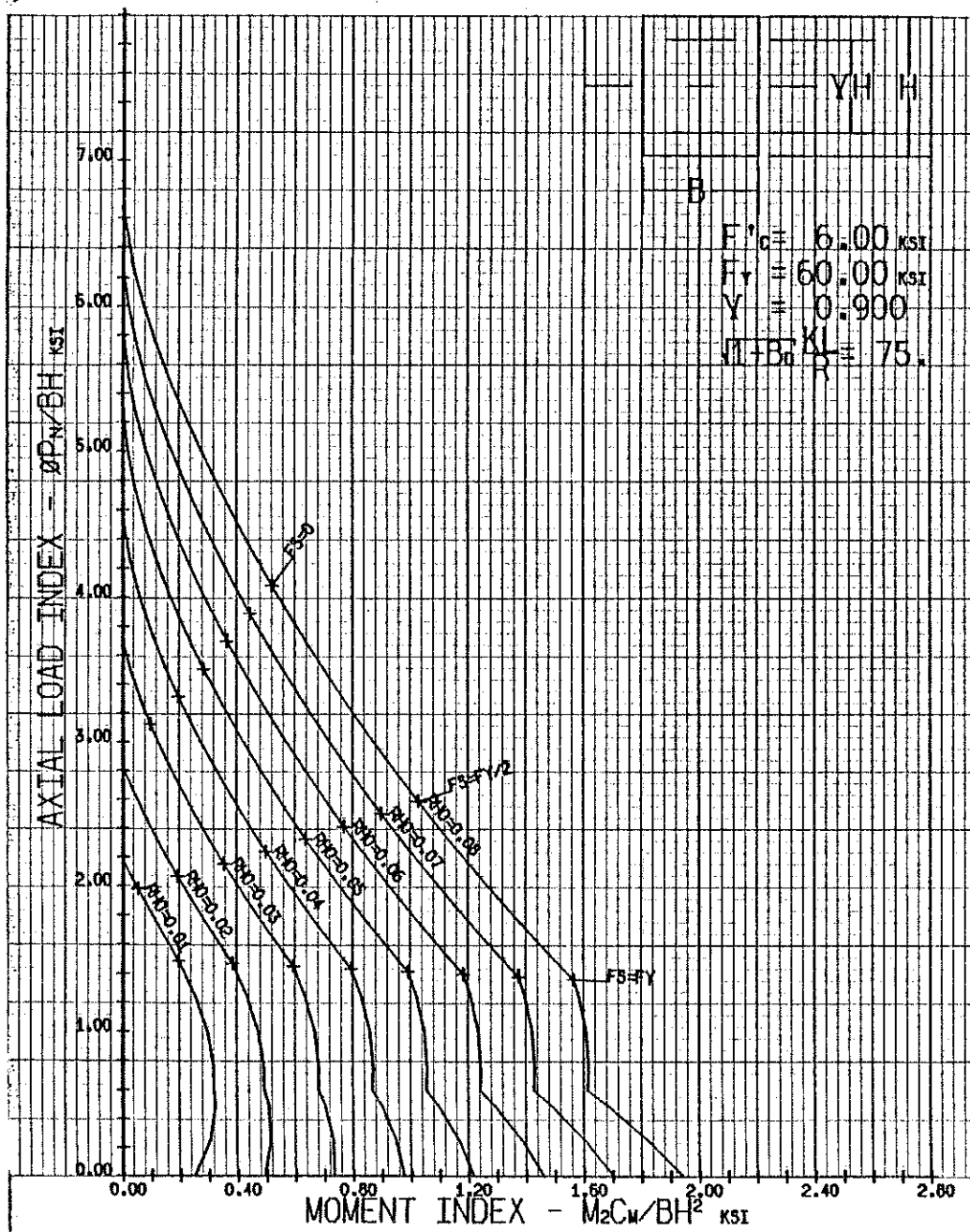


Fig. E6-60.90-75 - Interaction Diagram

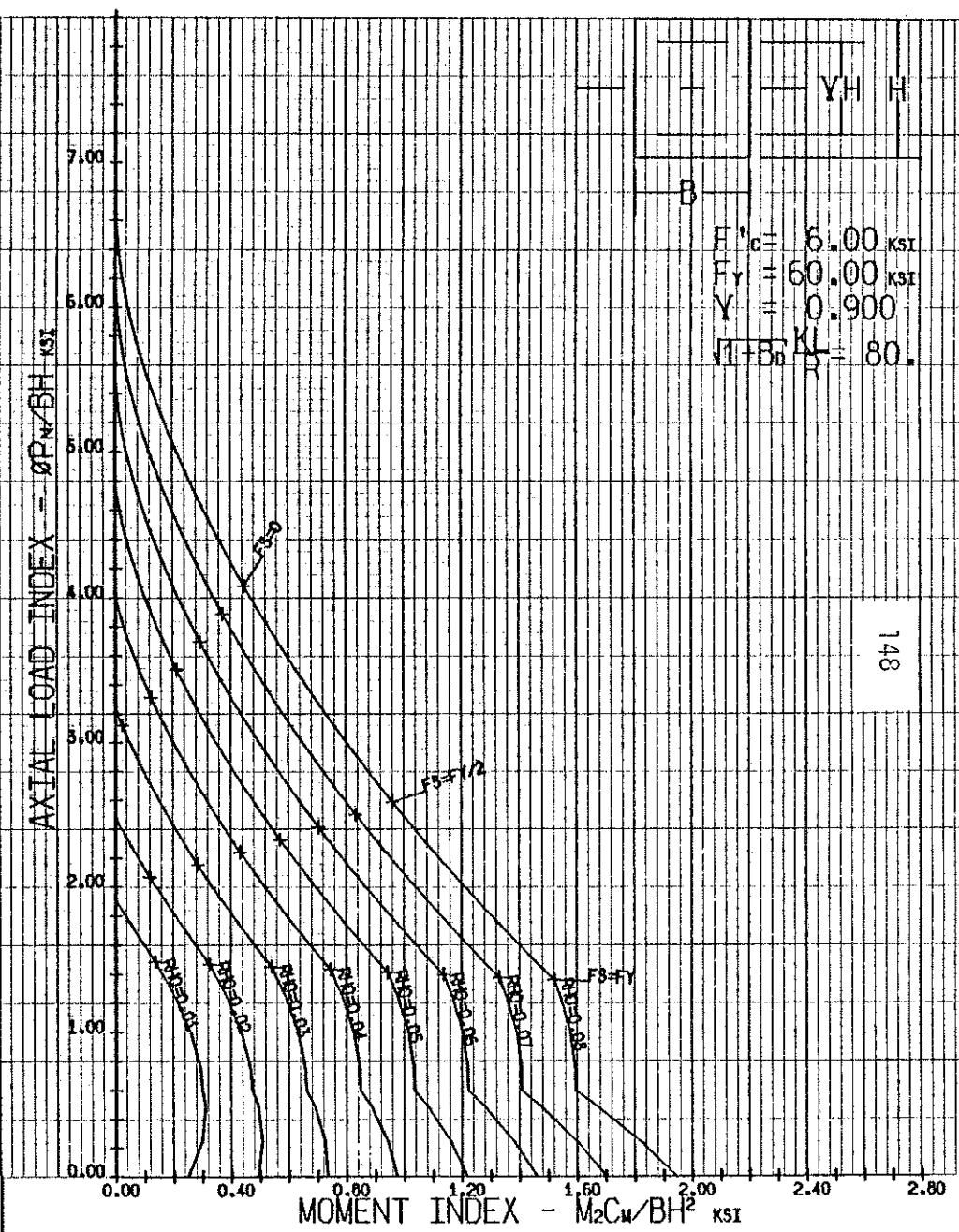


Fig. E6-60.90-80 - Interaction Diagram



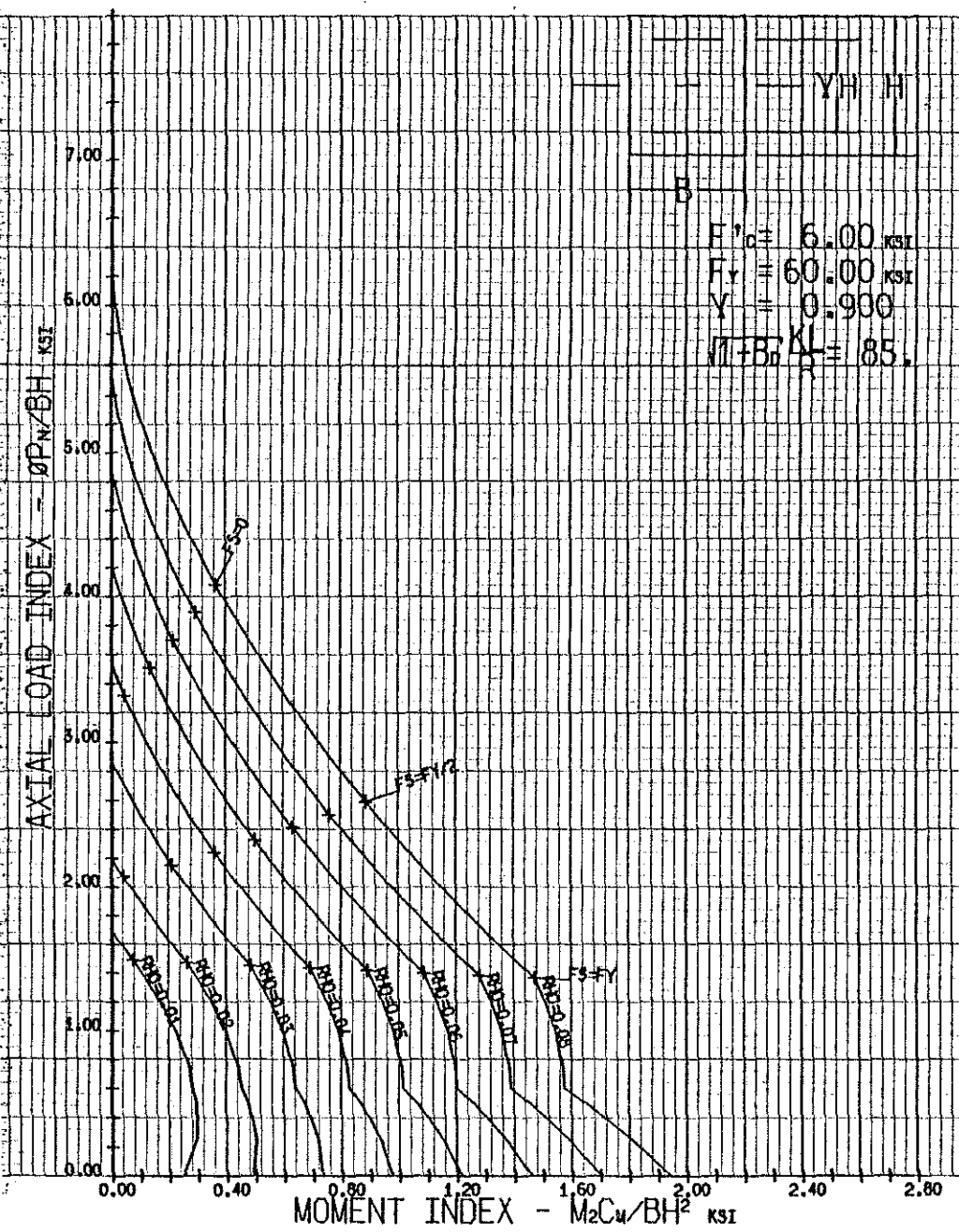


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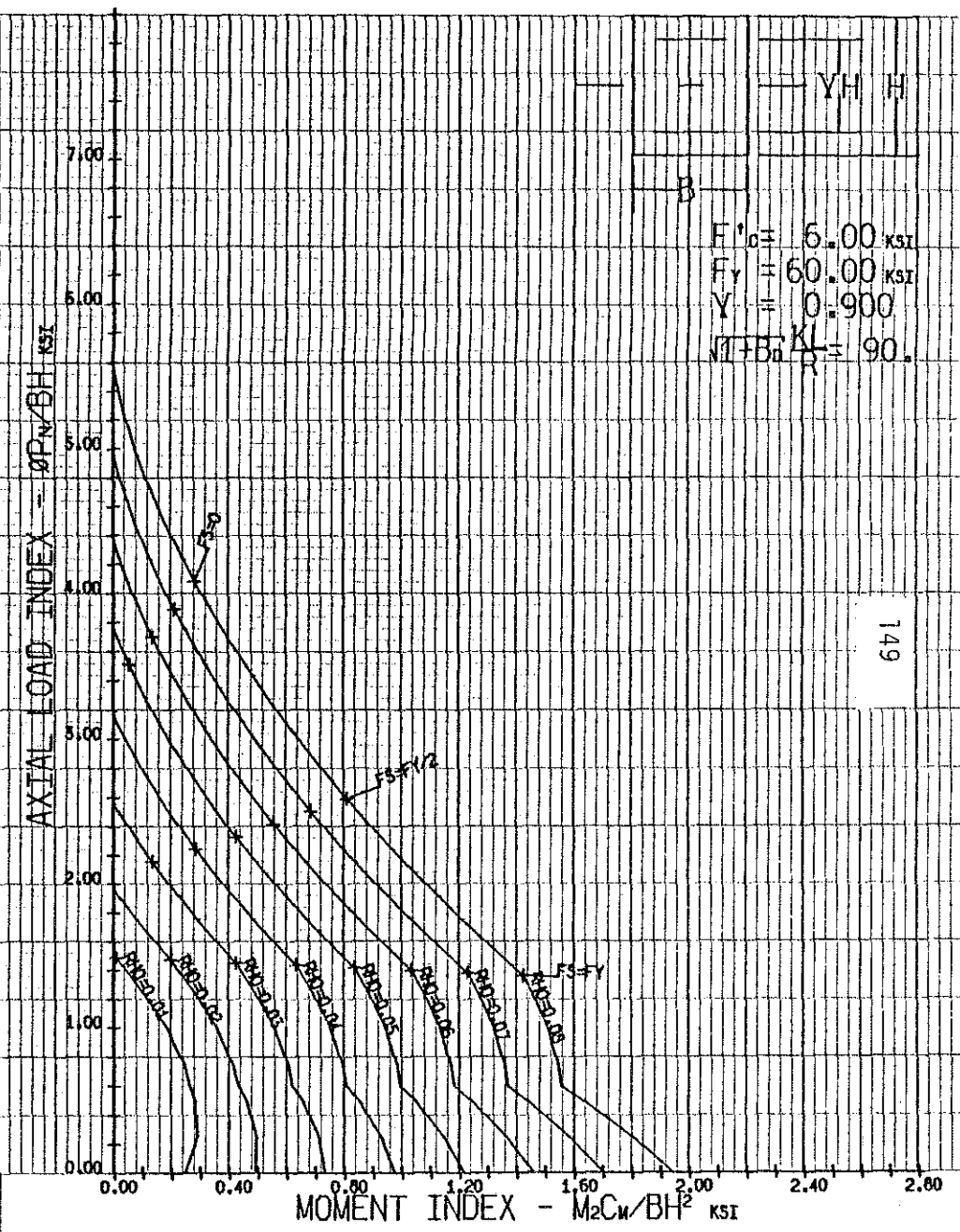


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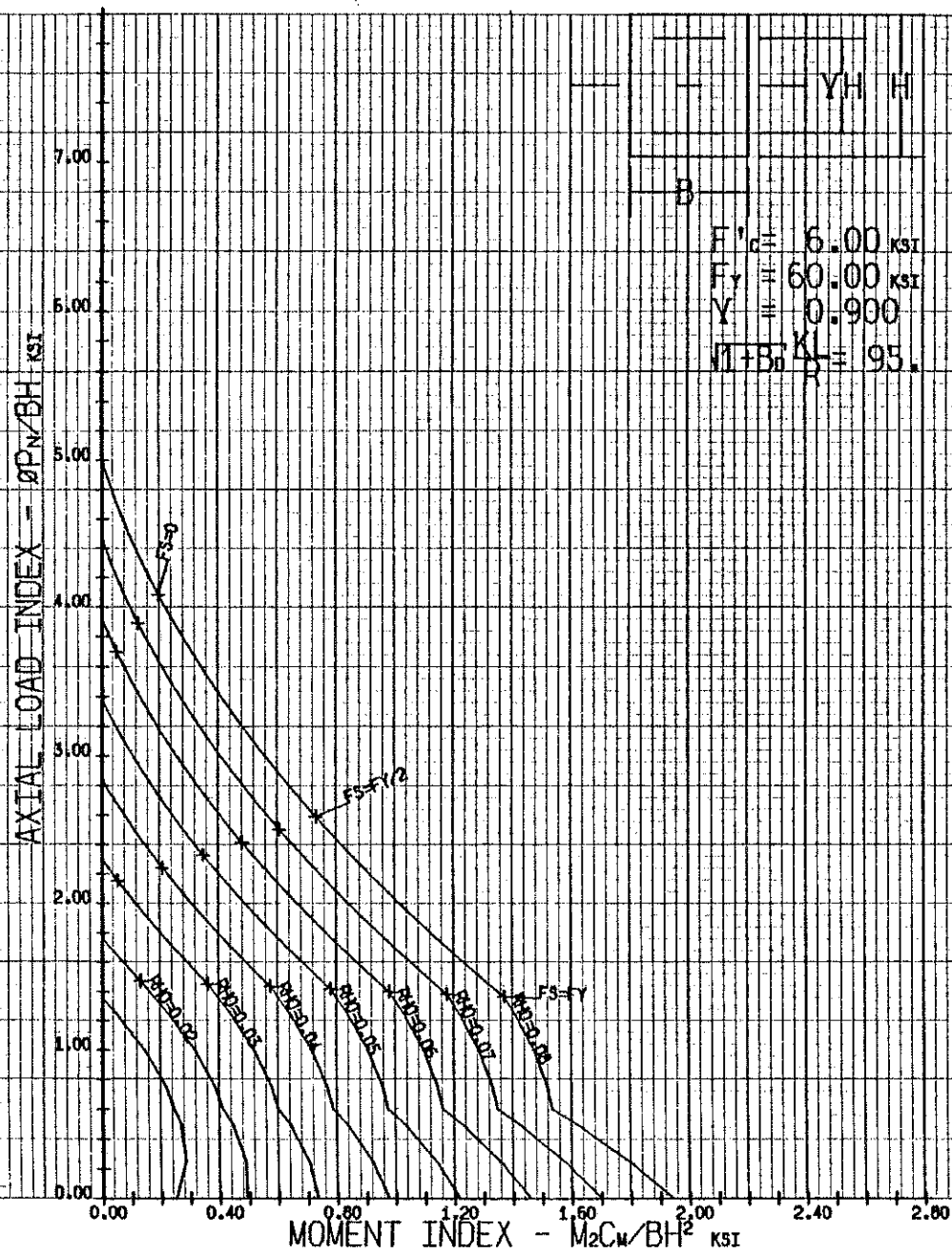


Fig. E6-60.90-95 - Interaction Diagram

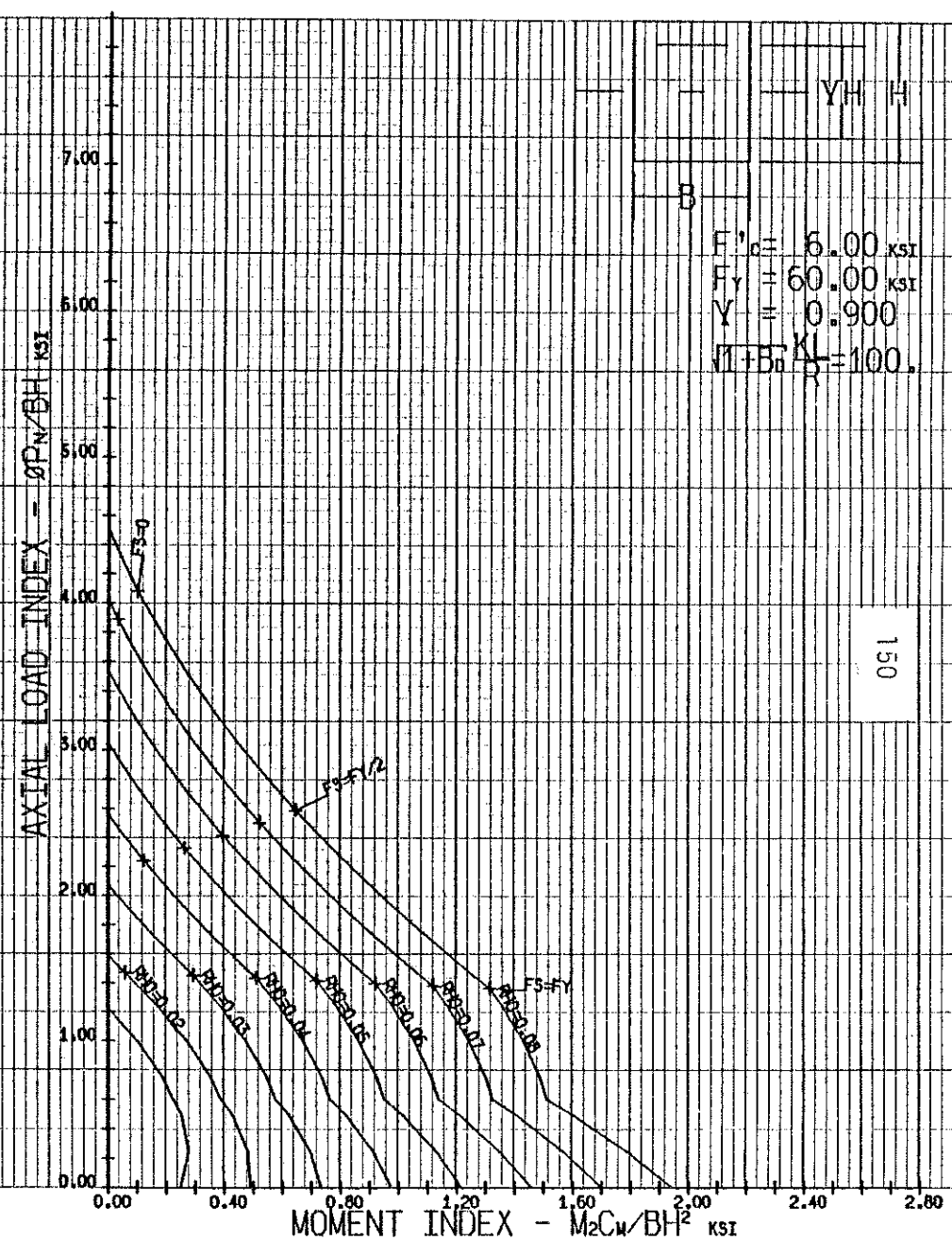


Fig. E6-60.90-100 - Interaction Diagram

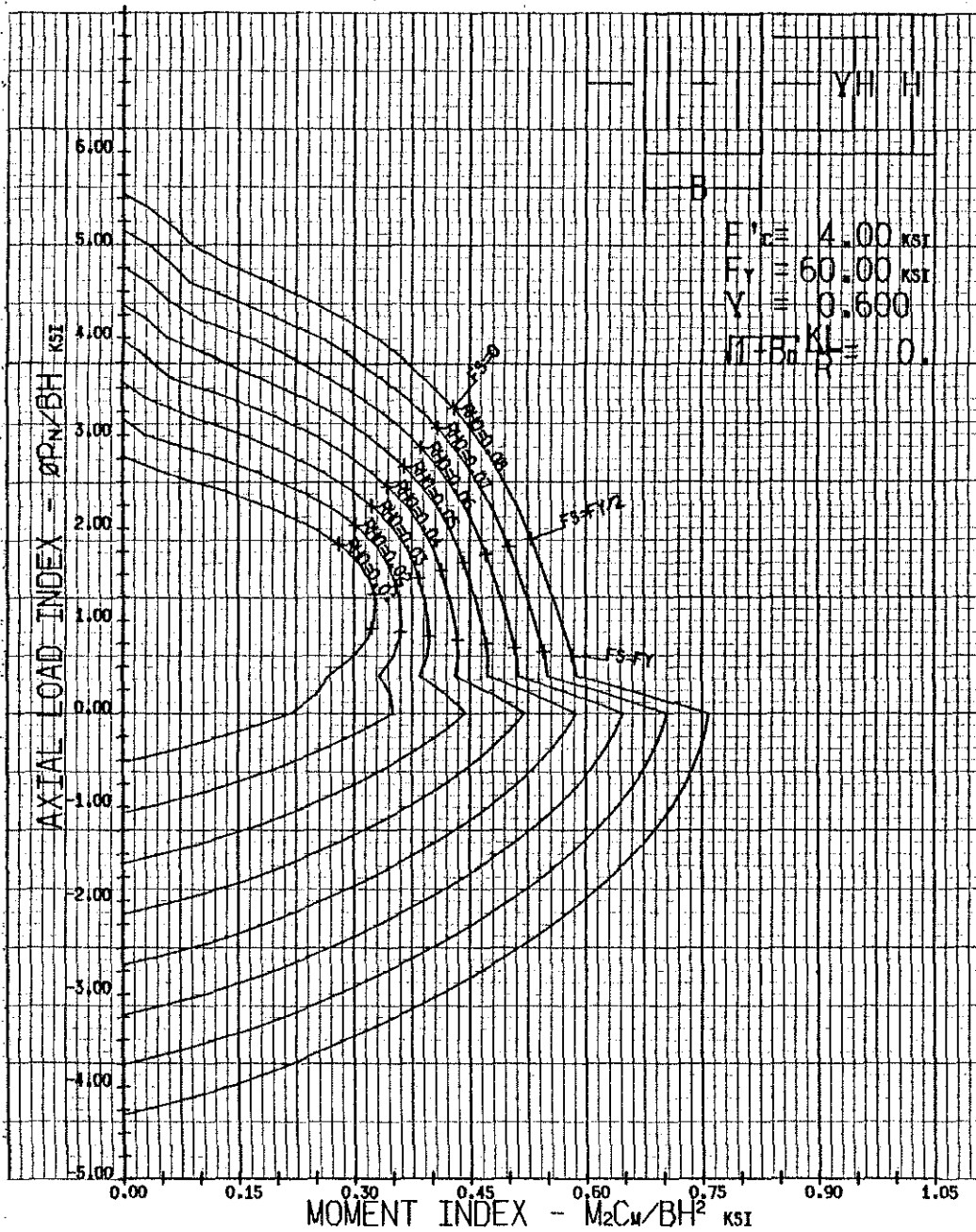


Fig. L4-60.60-0 - Interaction Diagram

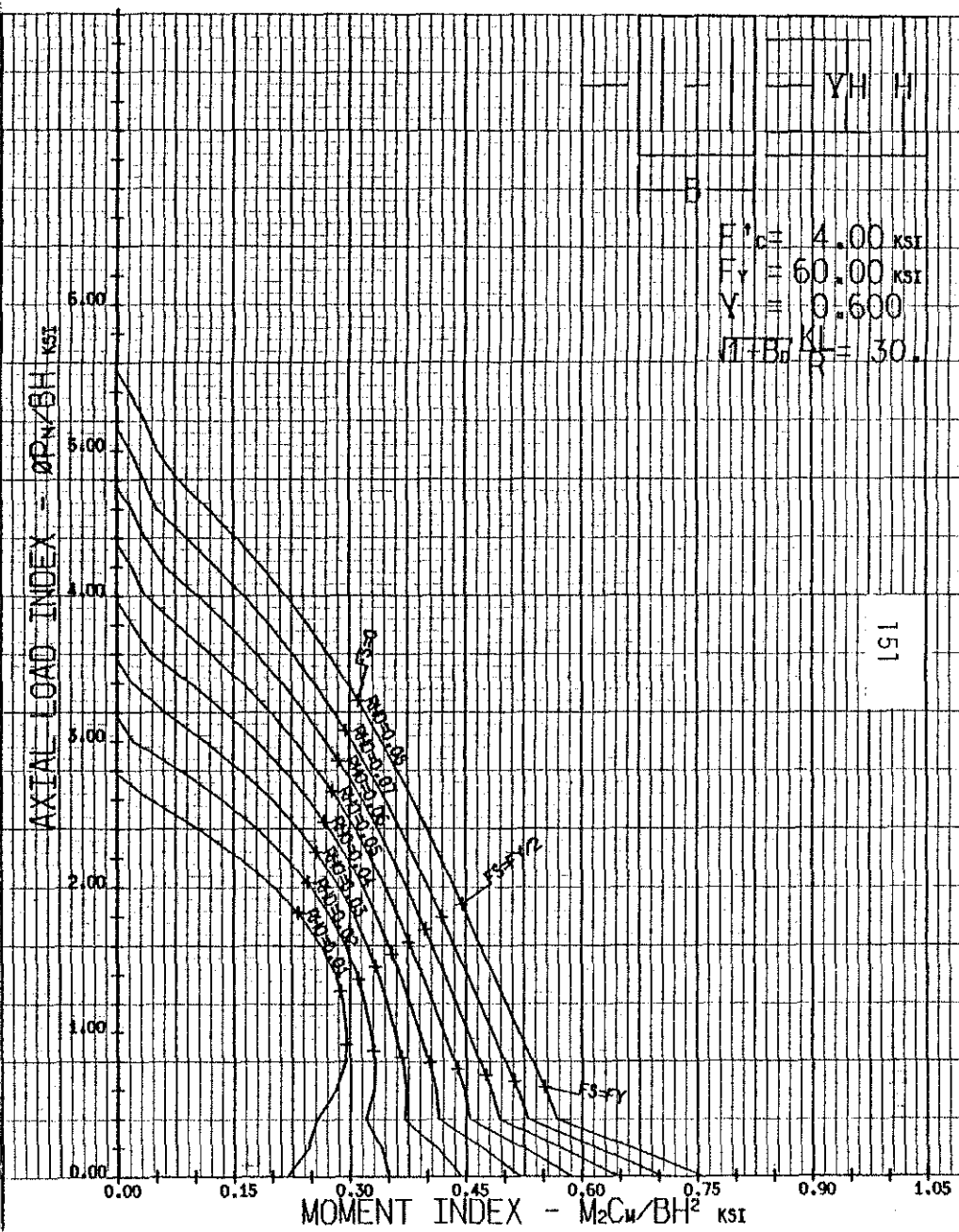


Fig. L4-60.60-30 - Interaction Diagram

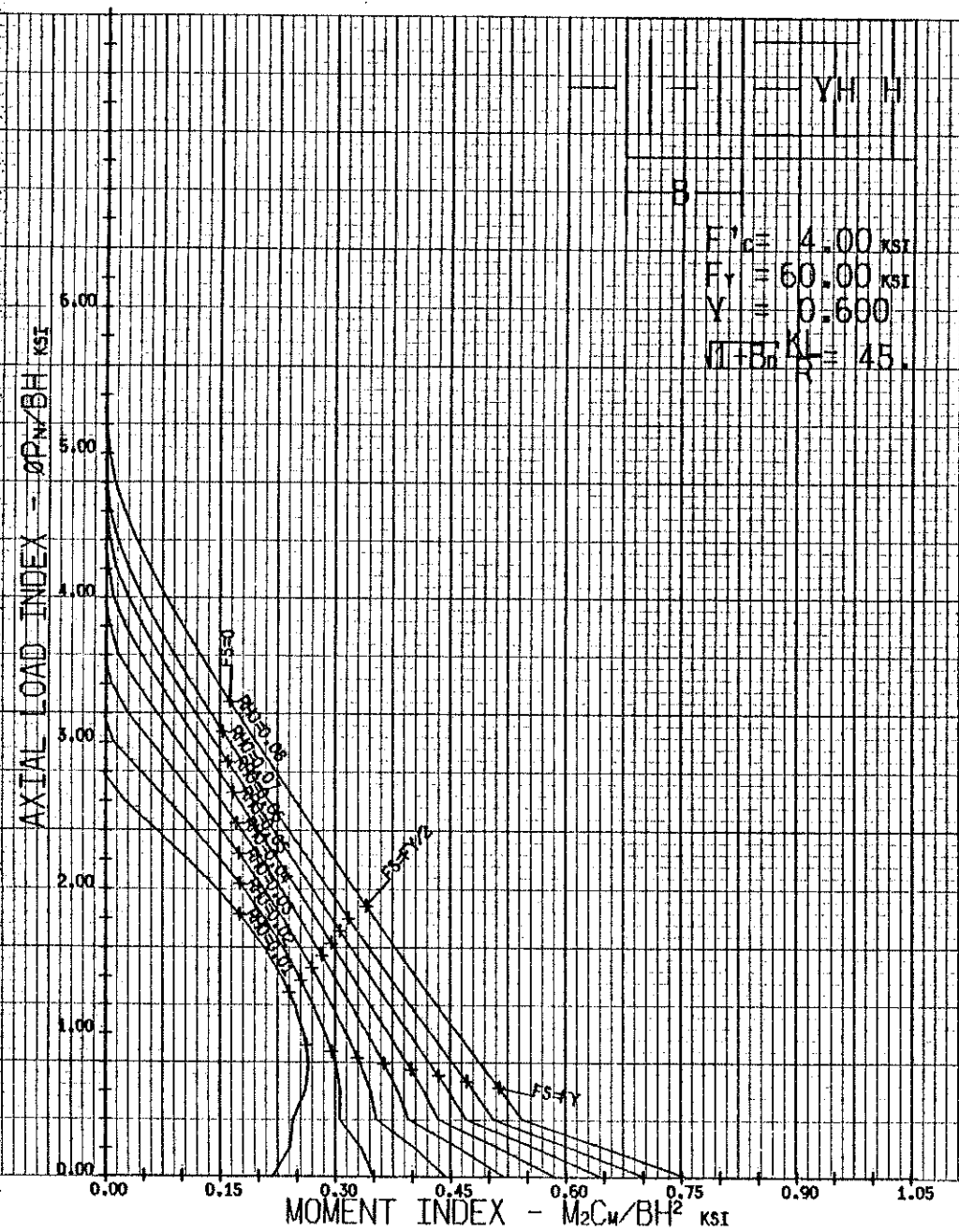


Fig. L4-60.60-45 - Interaction Diagram

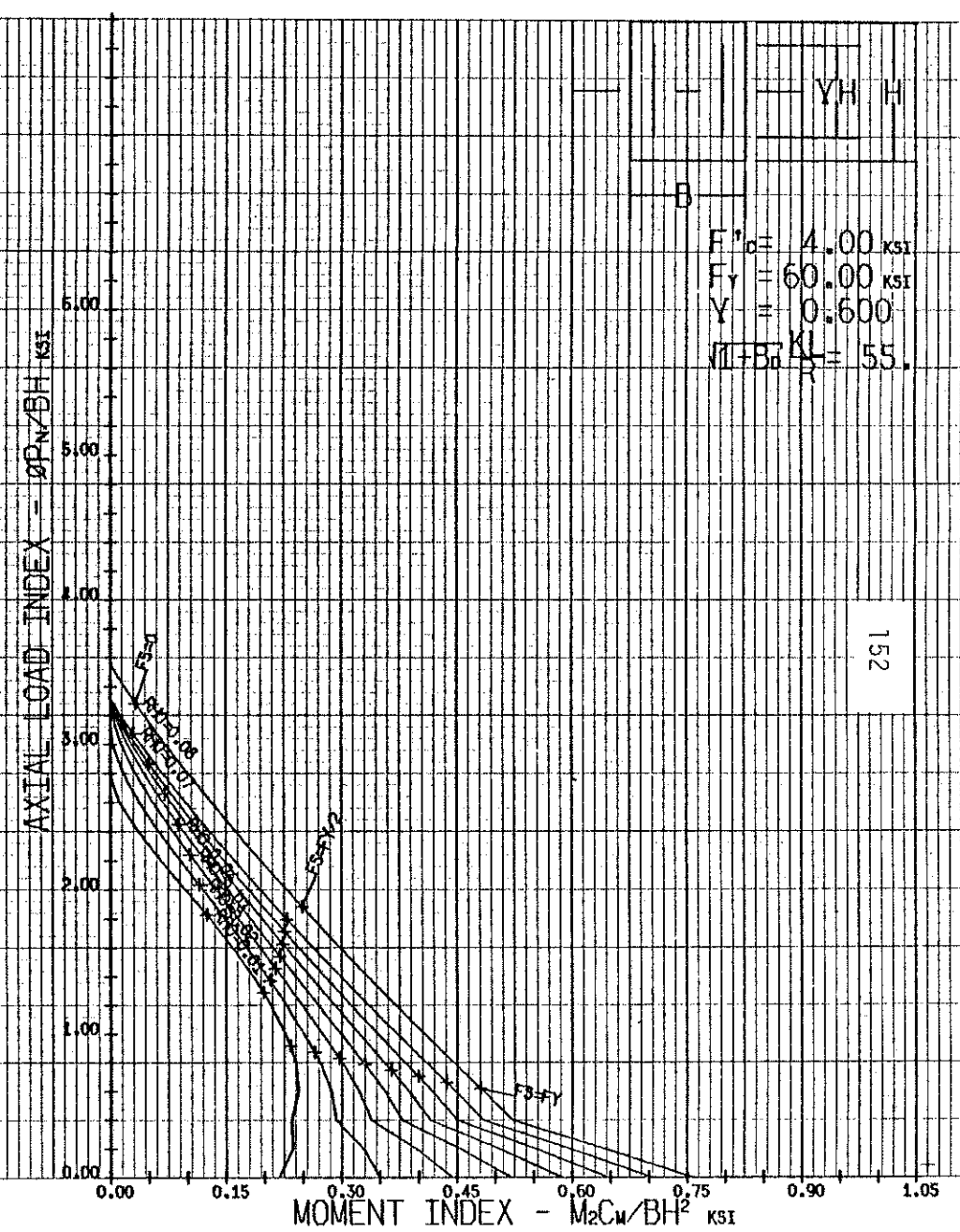


Fig. L4-60.60-55 - Interaction Diagram

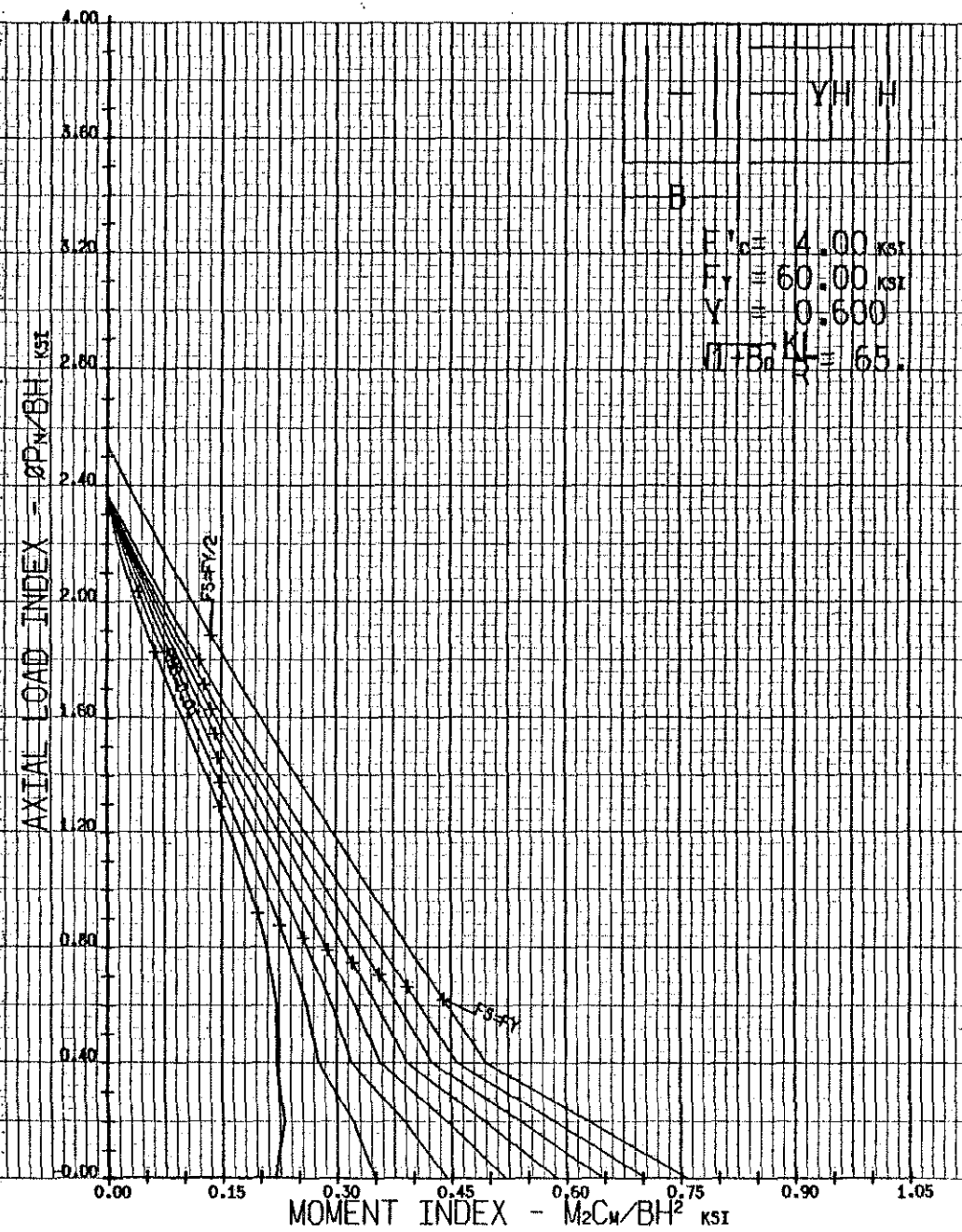


Fig. L4-60.60-65 - Interaction Diagram

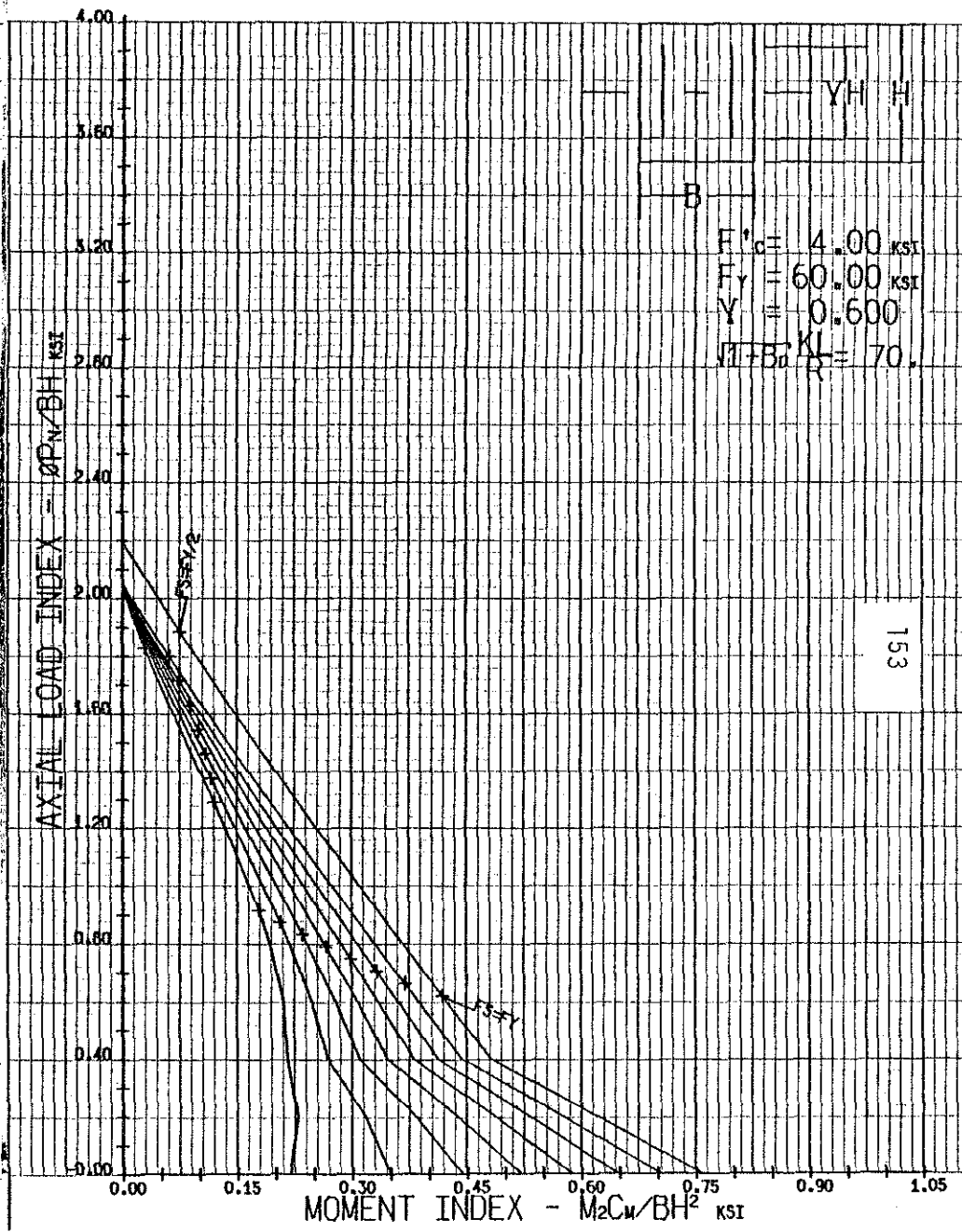


Fig. L4-60.60-70 - Interaction Diagram



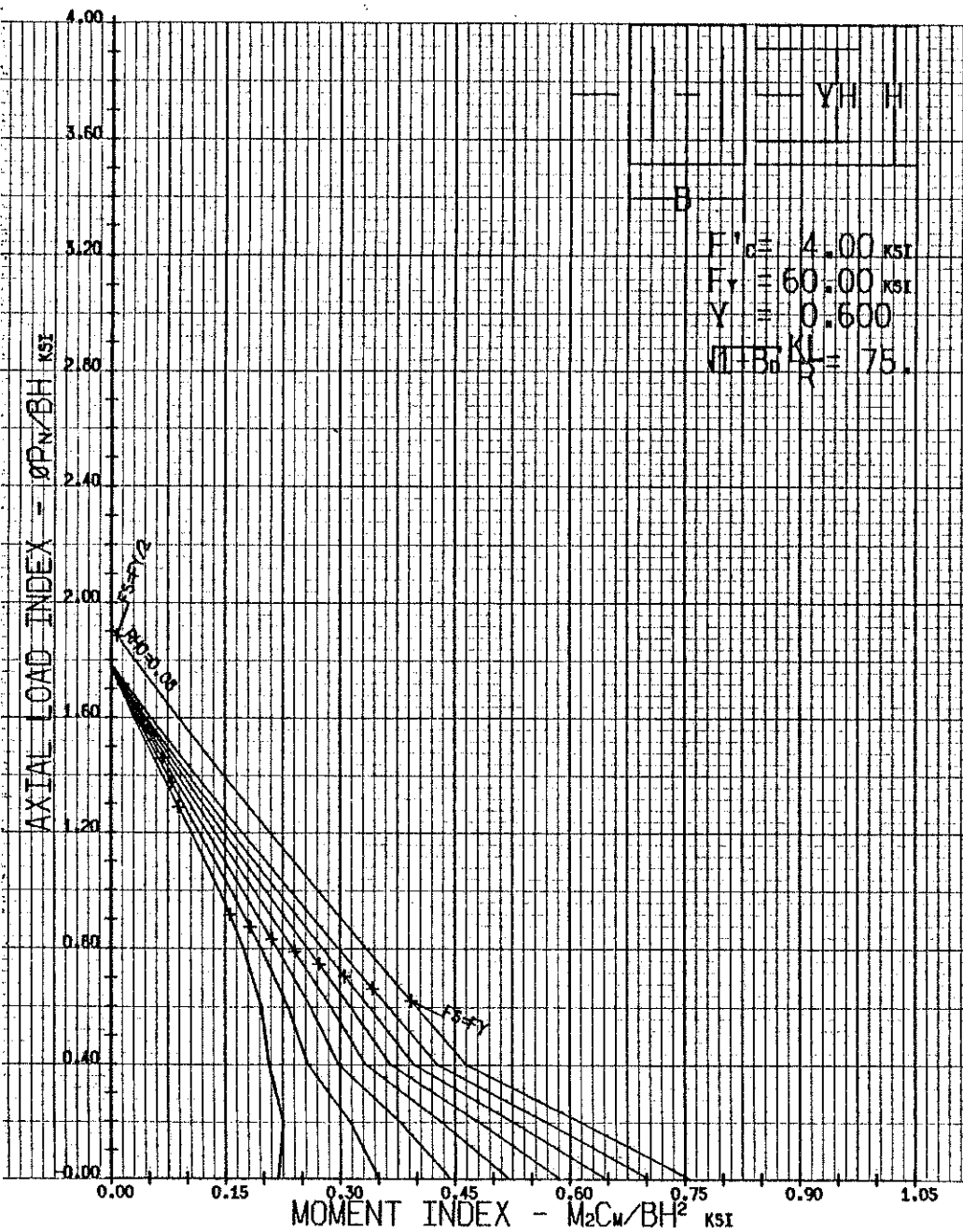


Fig. L4-60.60-75 - Interaction Diagram

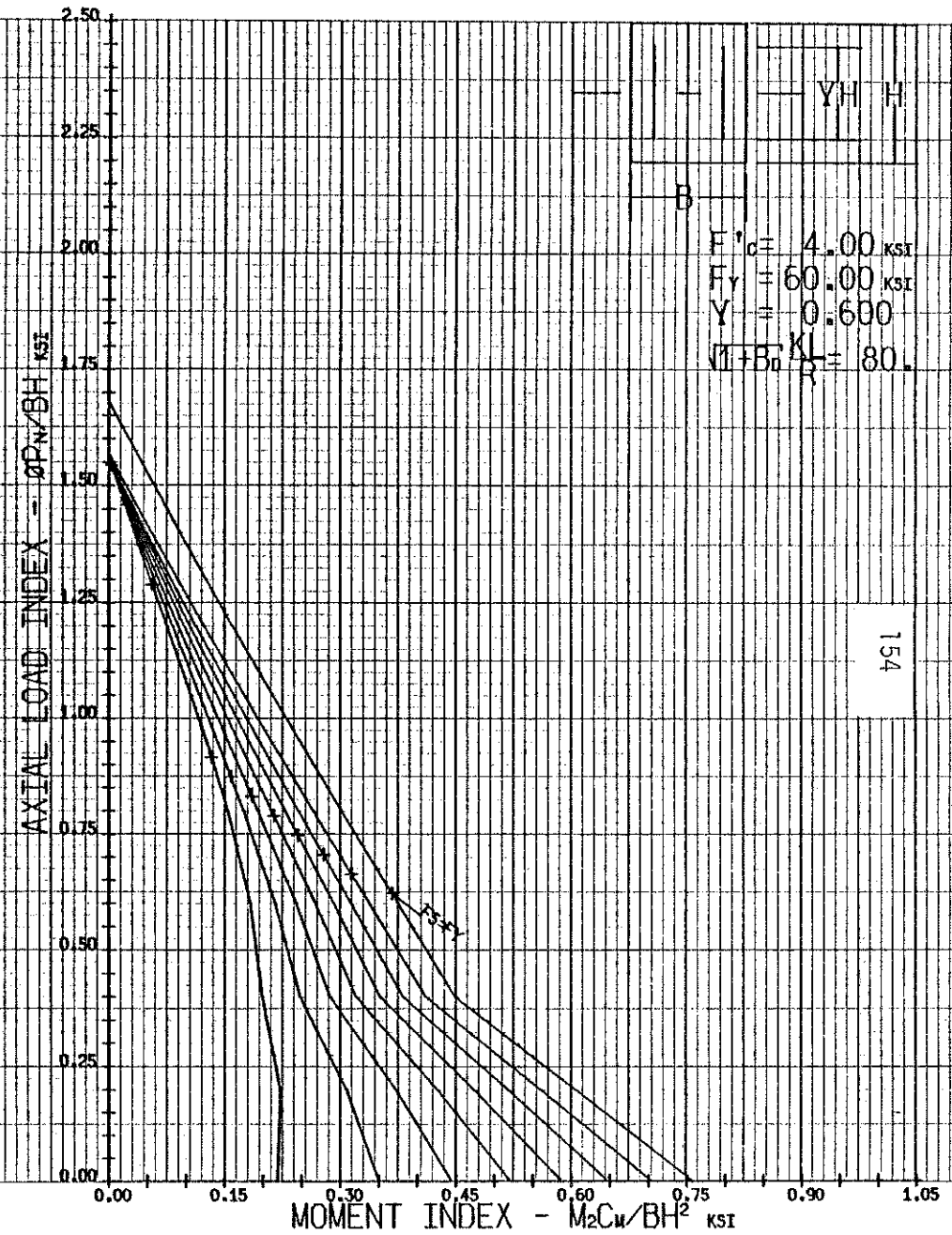


Fig. L4-60.60-80 - Interaction Diagram

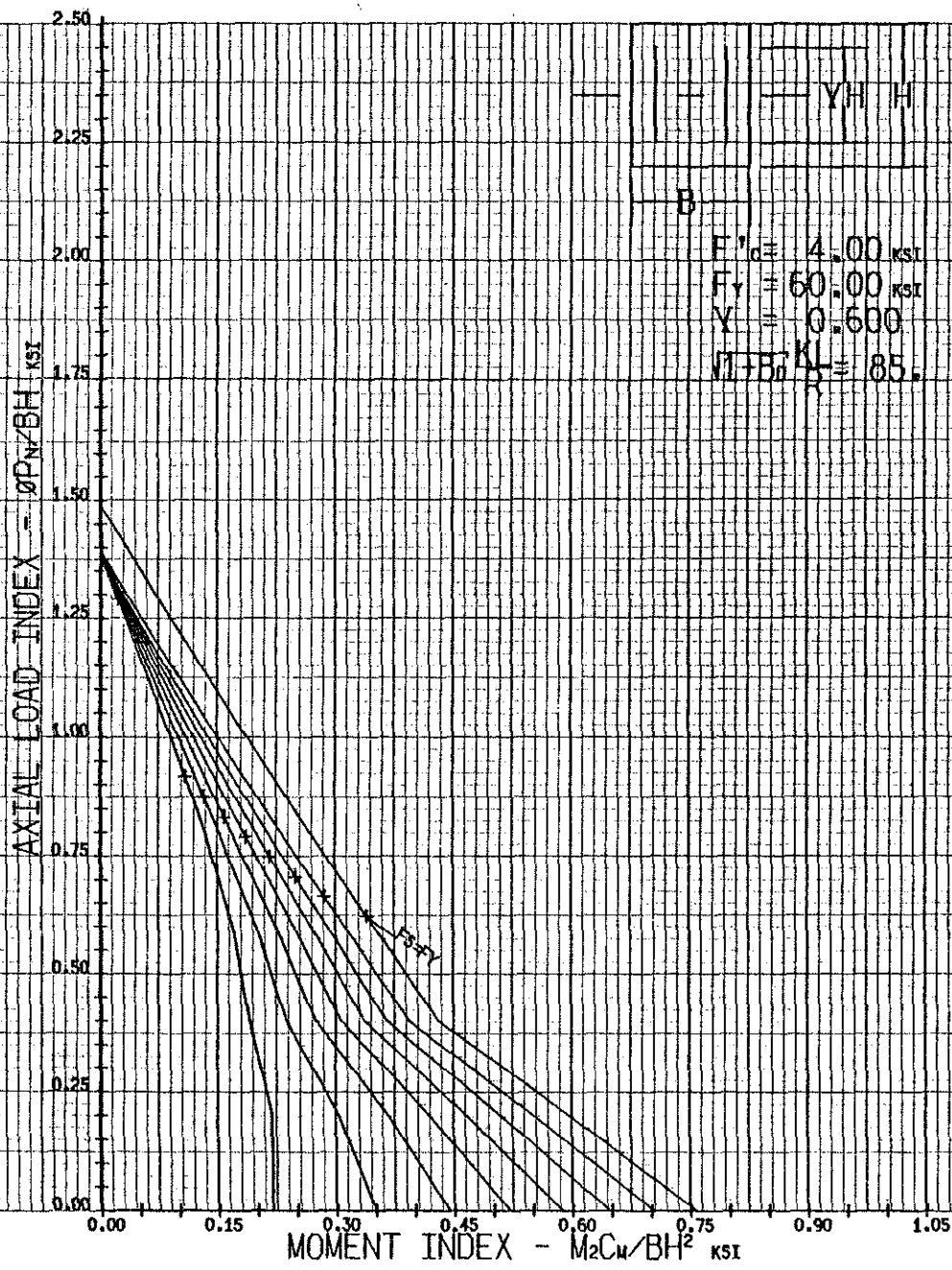


Fig. L4-60.60-85 - Interaction Diagram

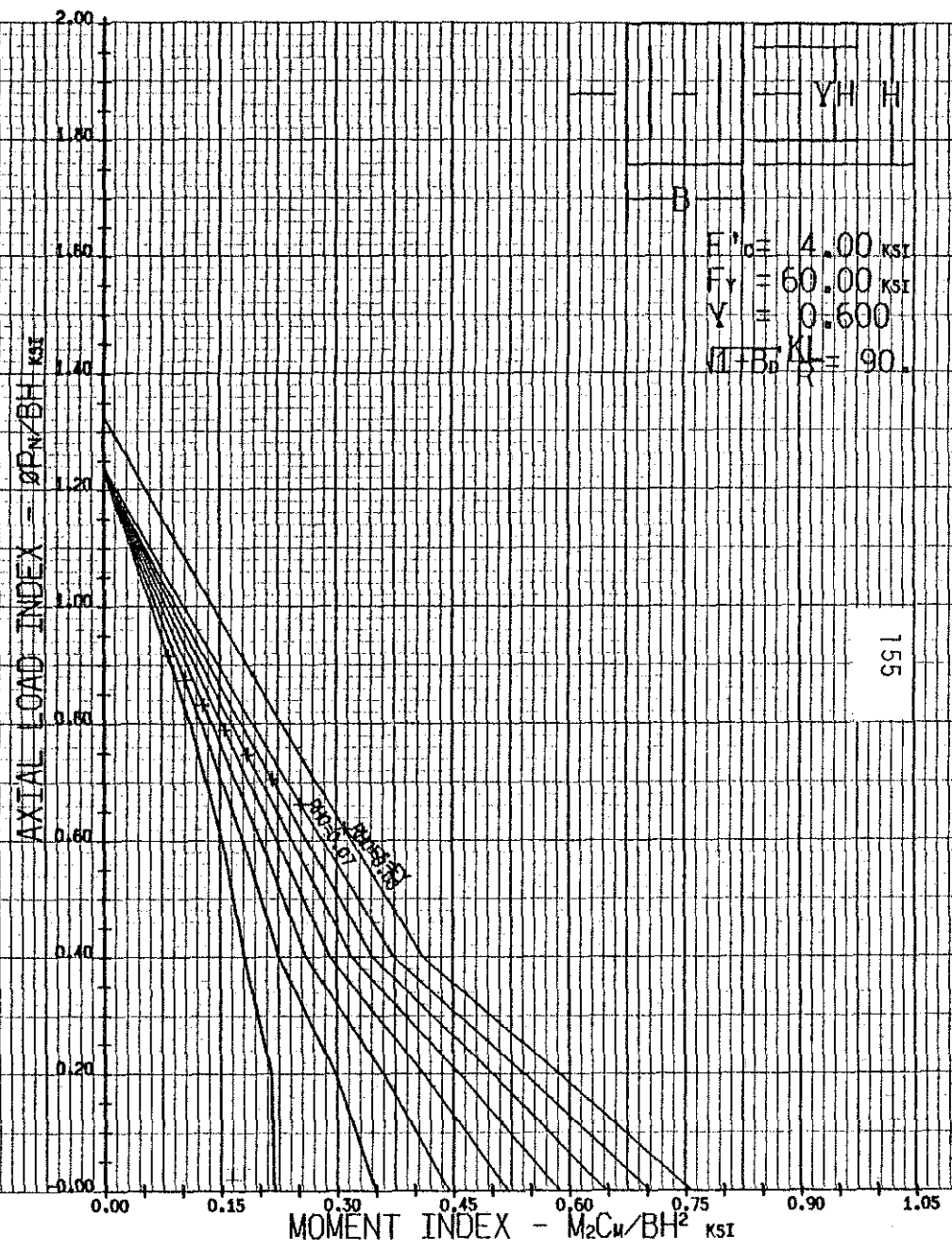


Fig. L4-60.60-90 - Interaction Diagram



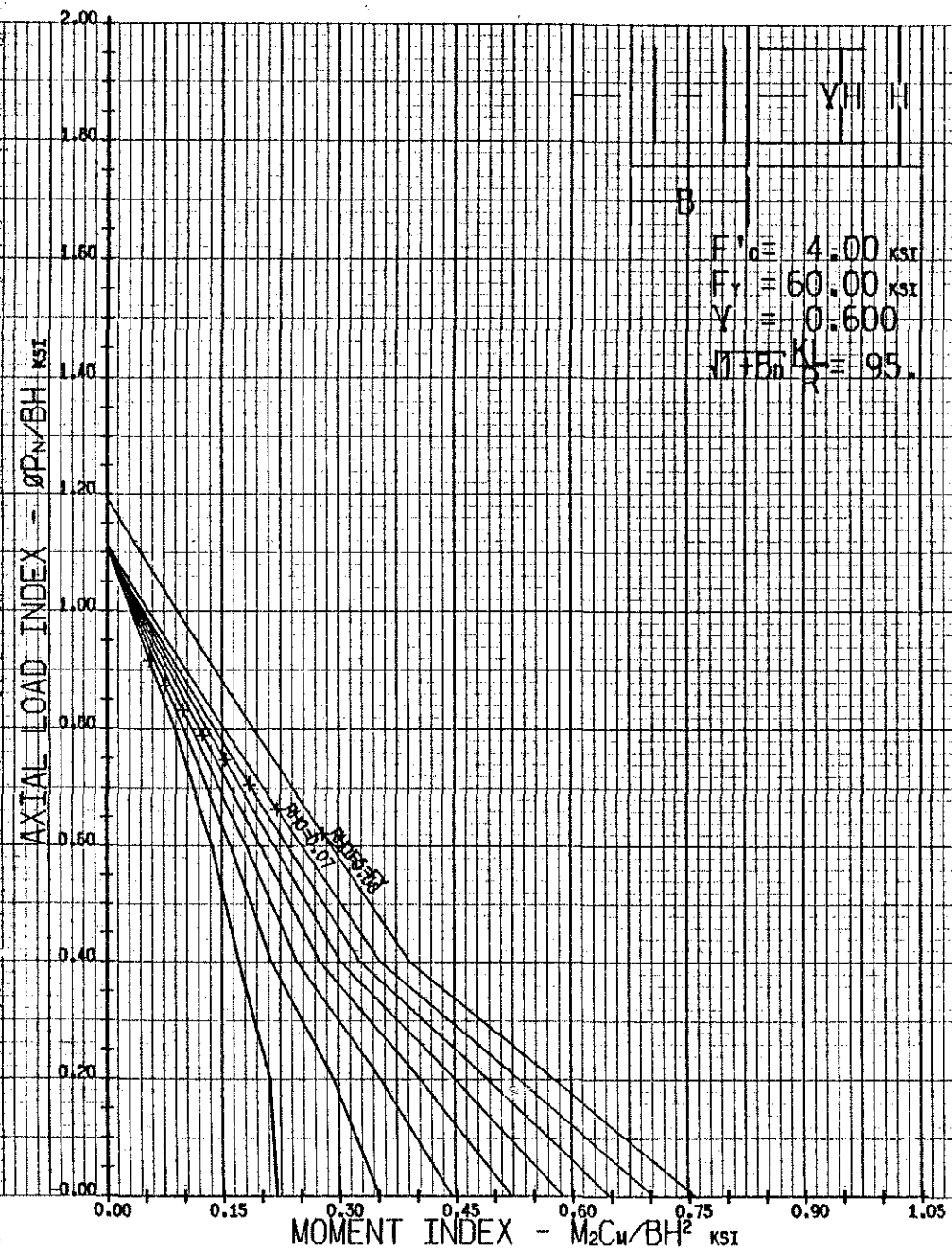


Fig. L4-60.60-95 - Interaction Diagram

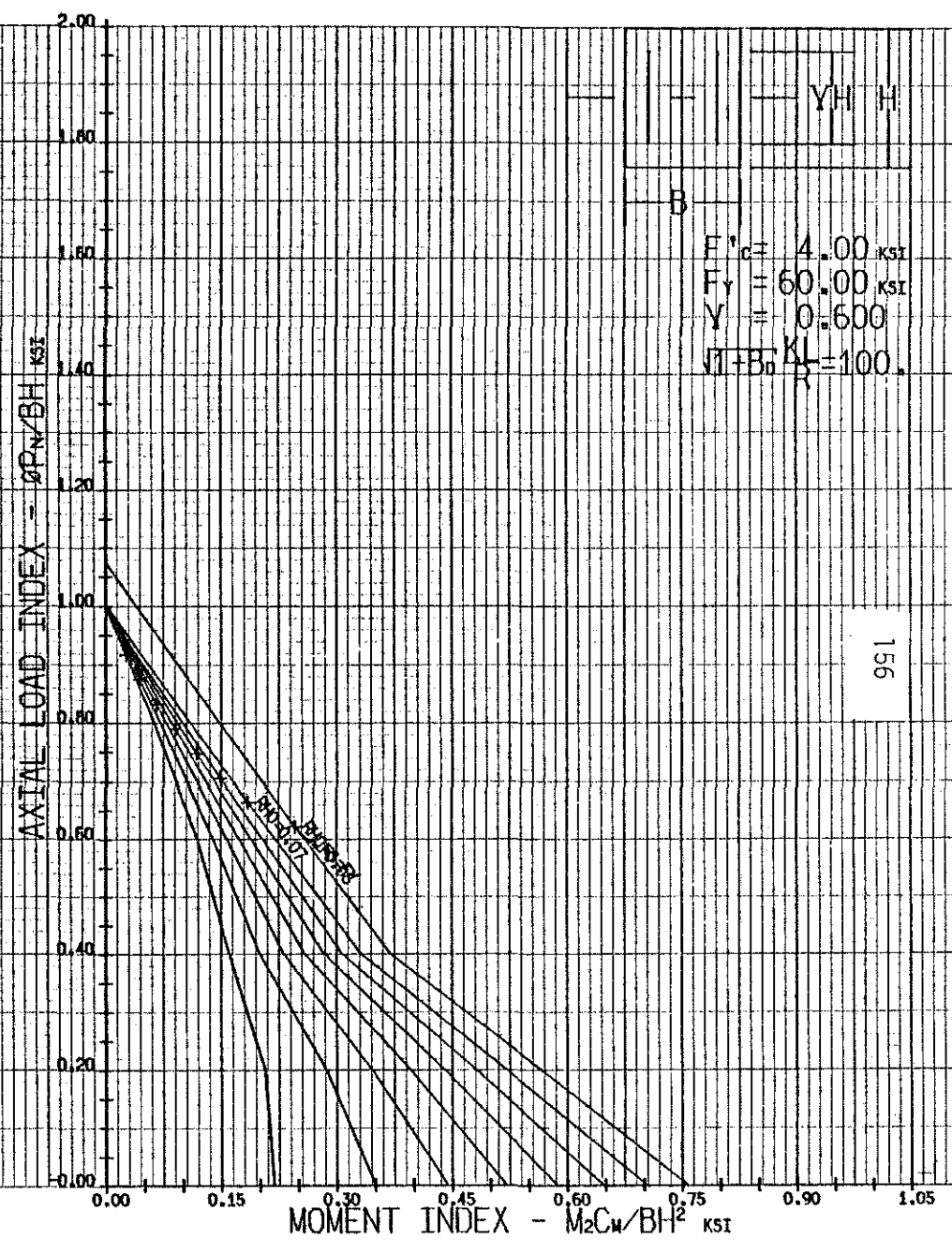


Fig. L4-60.60-100 - Interaction Diagram

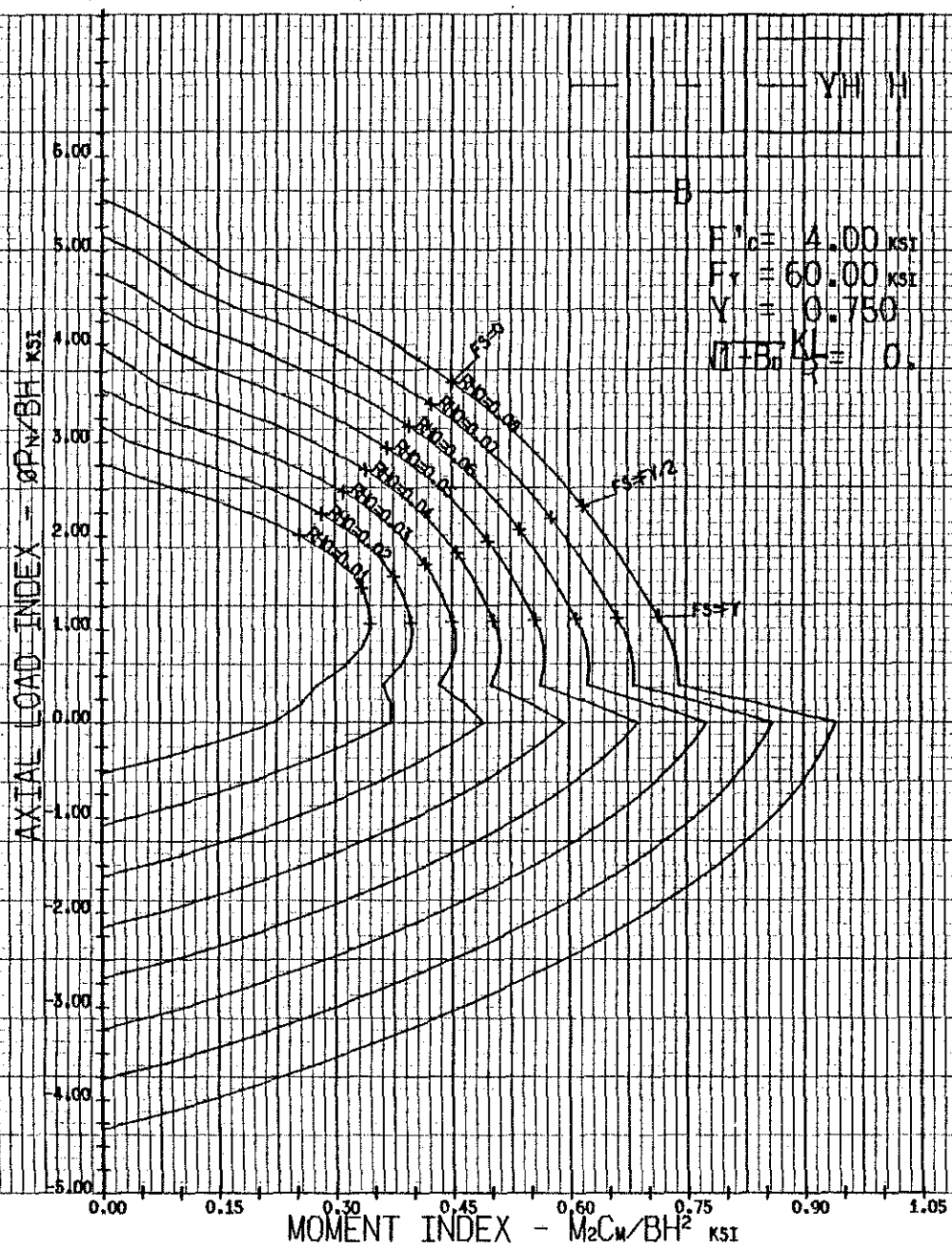


Fig. L4-60.75-0 - Interaction Diagram

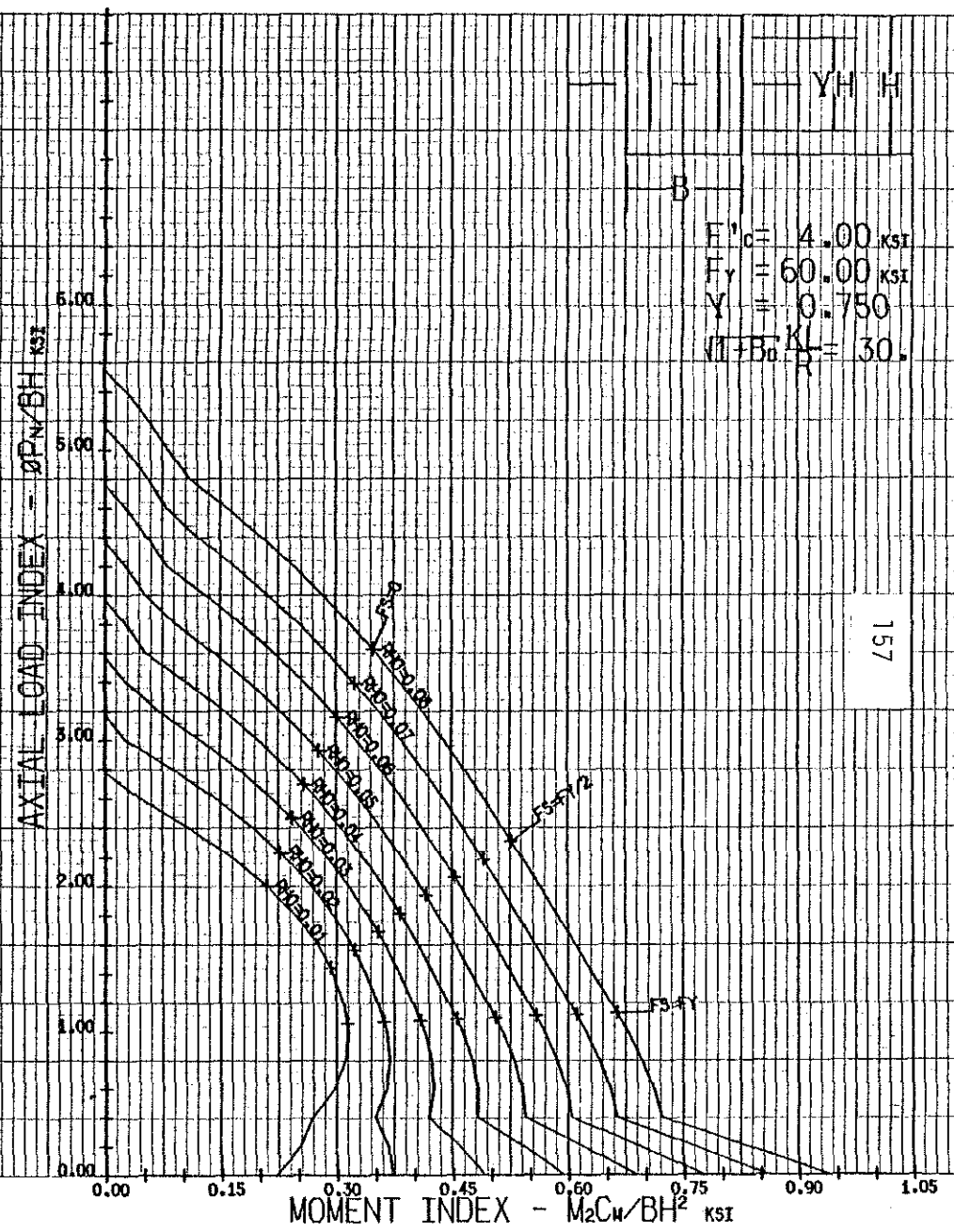


Fig. L4-60.75-30 - Interaction Diagram

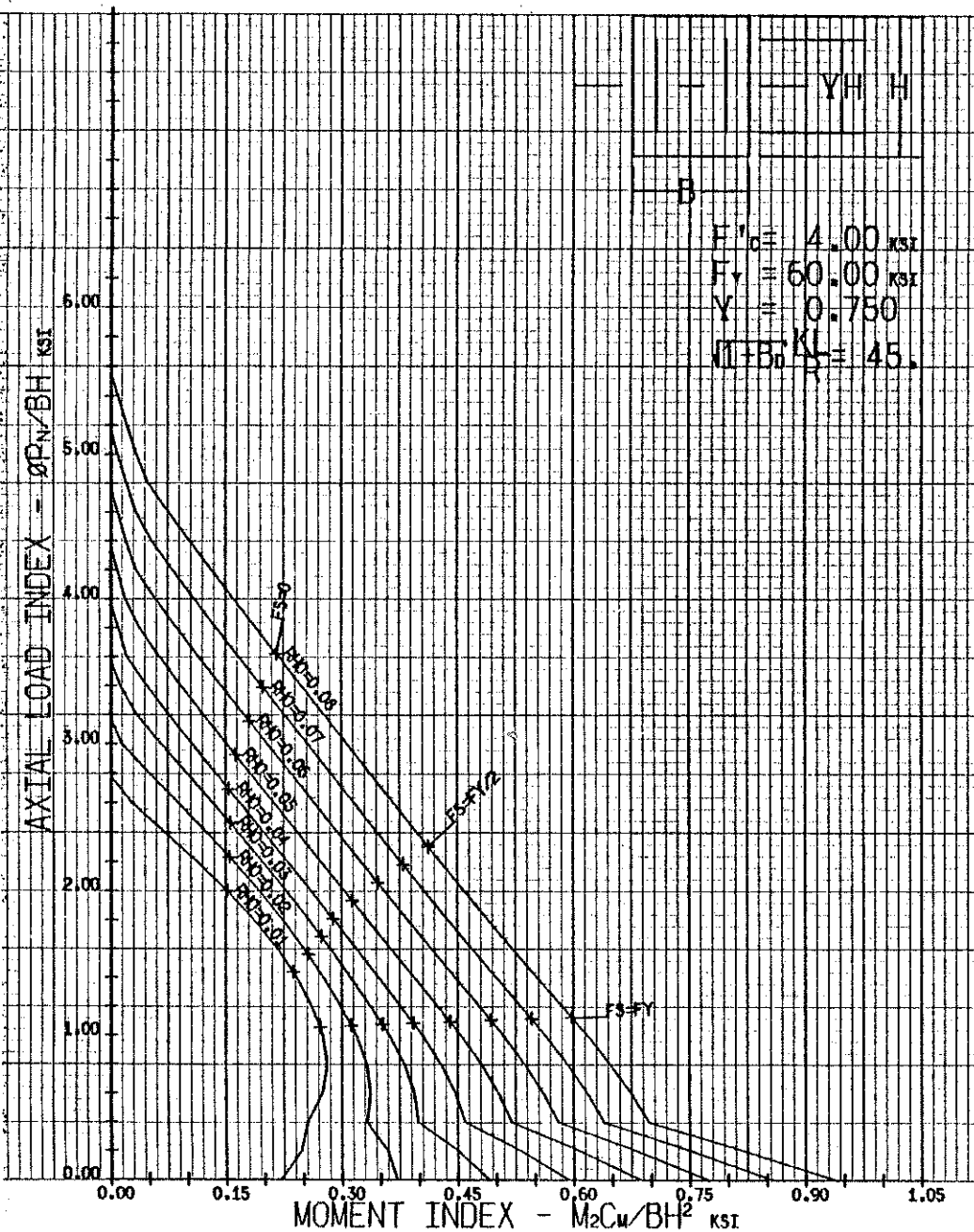


Fig. L4-60.75-45 - Interaction Diagram

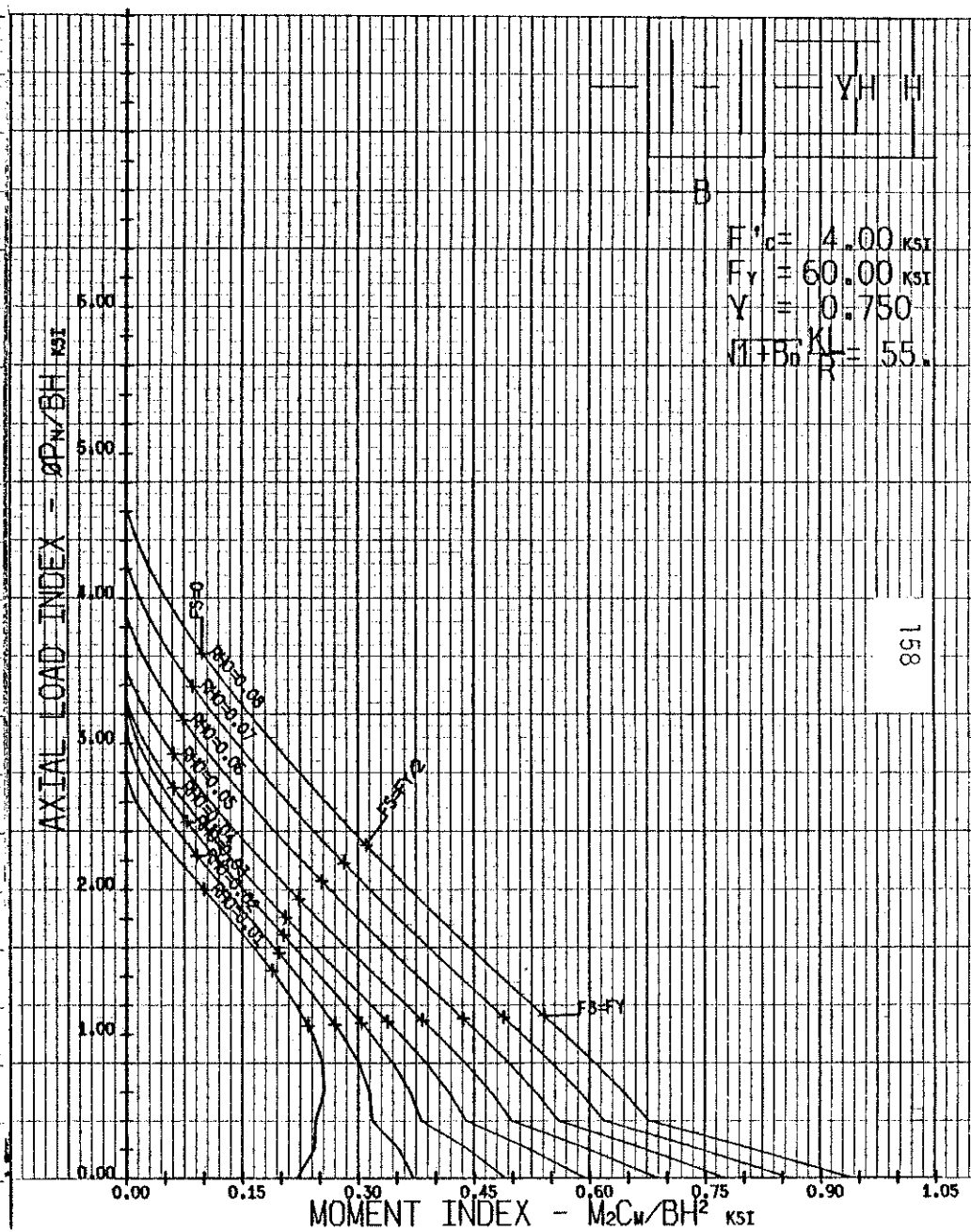


Fig. L4-60.75-55 - Interaction Diagram

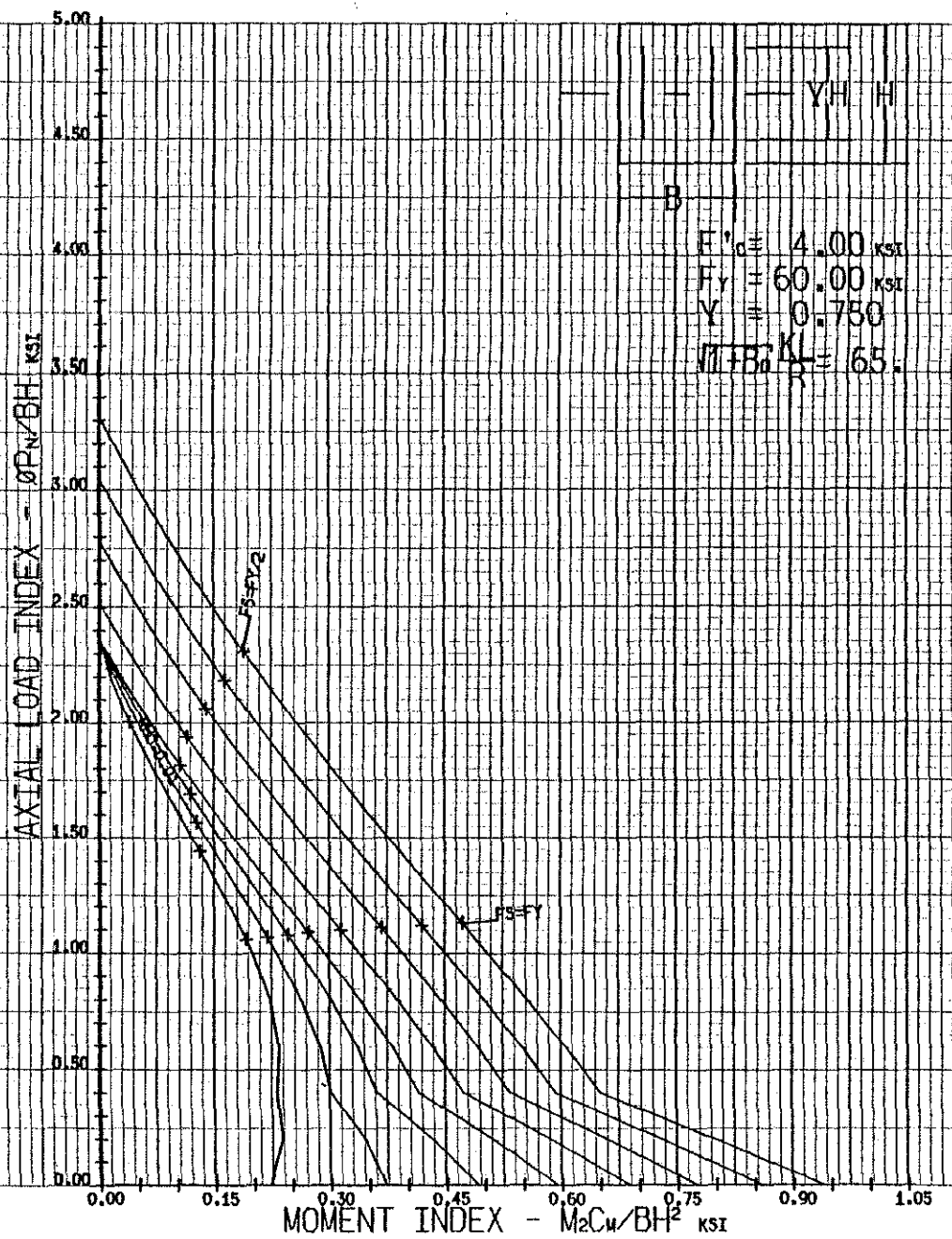


Fig. L4-60.75-65 - Interaction Diagram

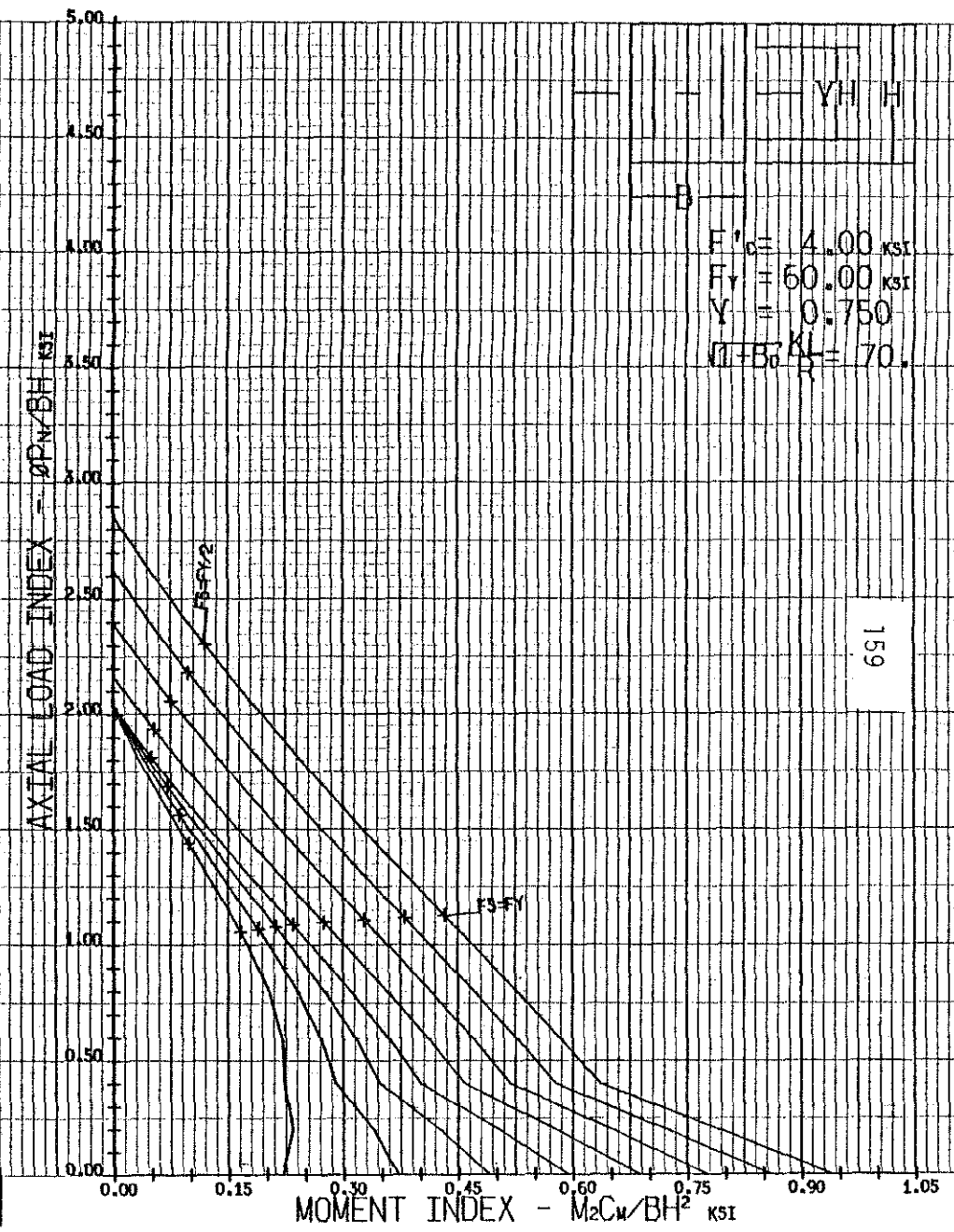


Fig. L4-60.75-70 - Interaction Diagram

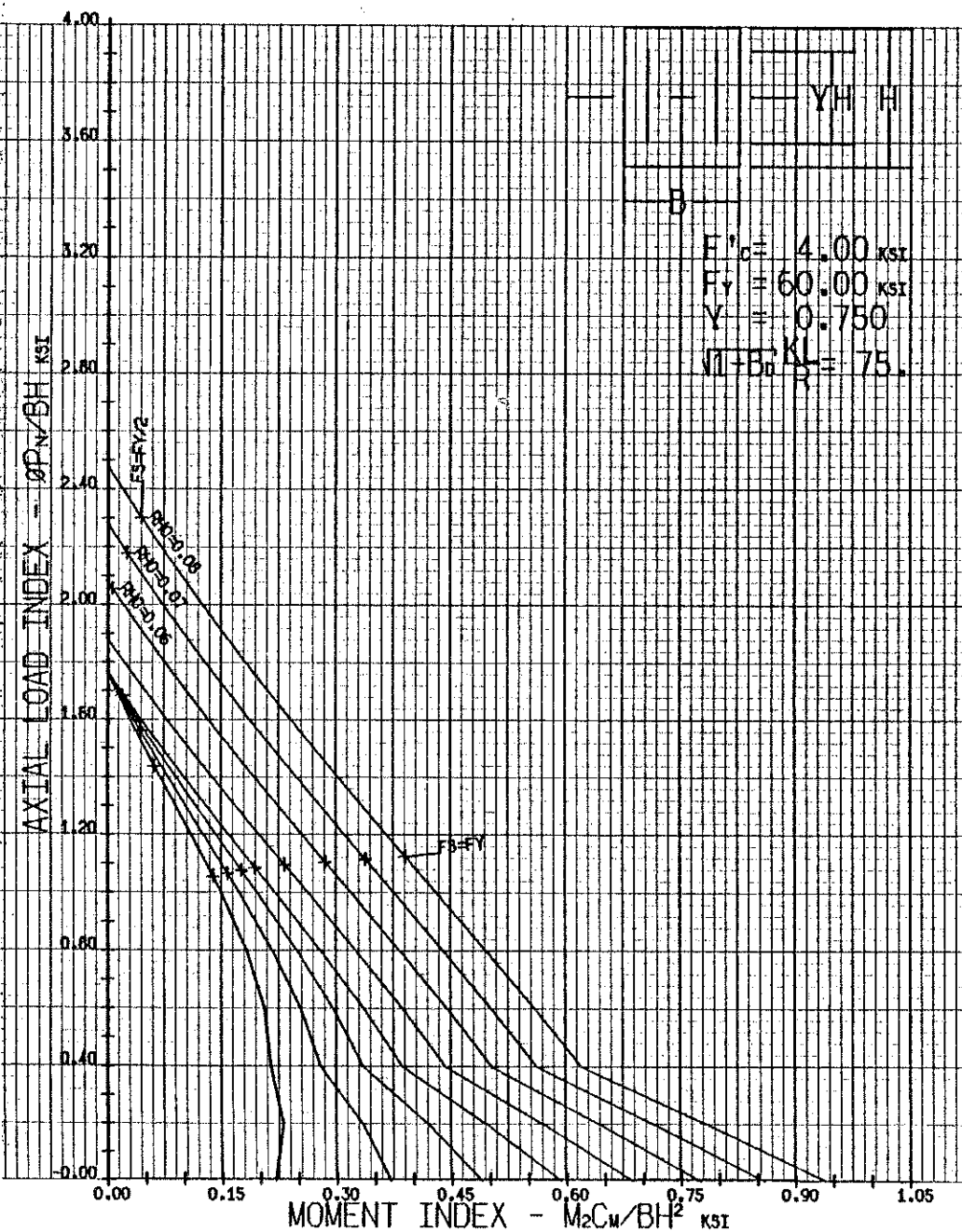


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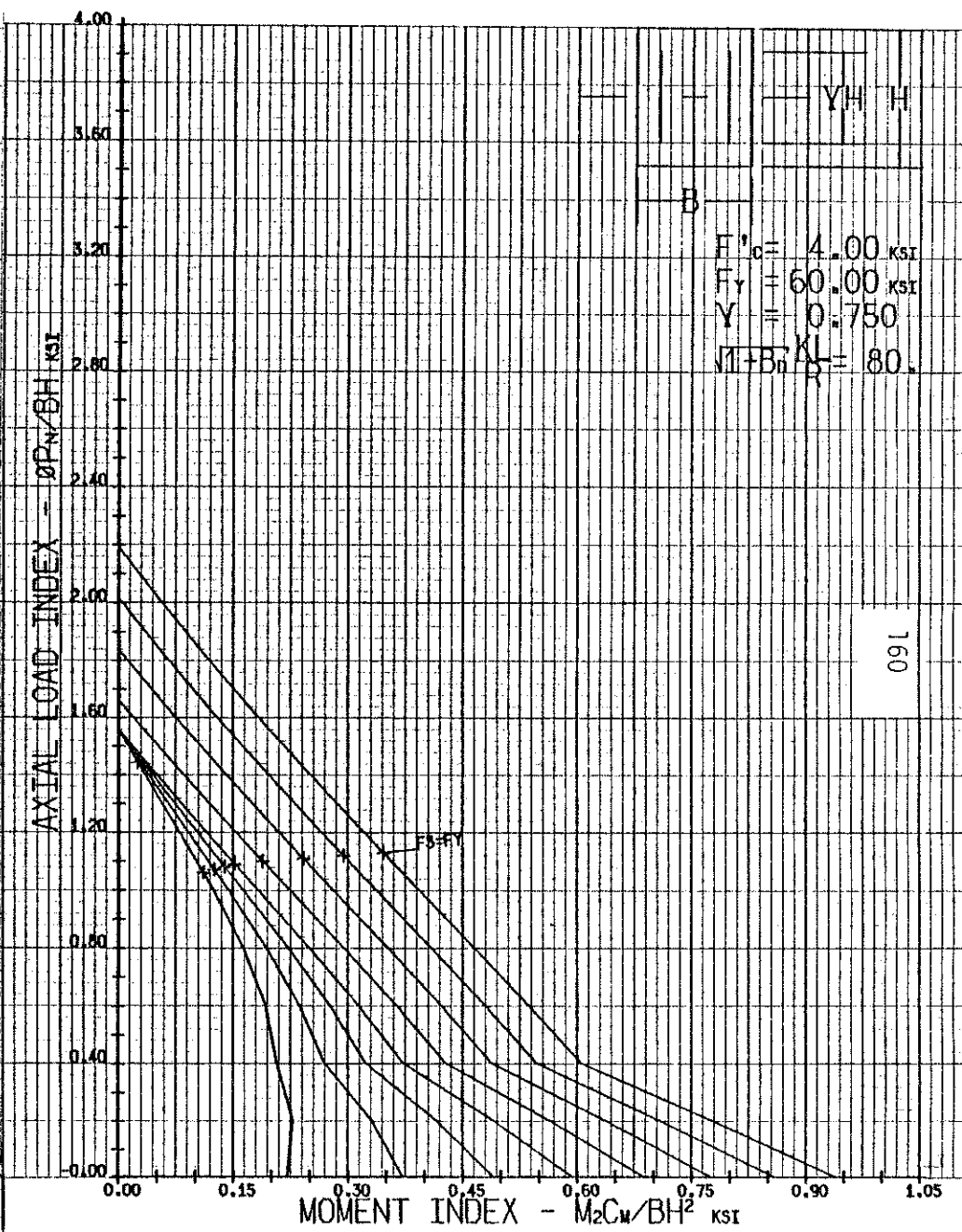


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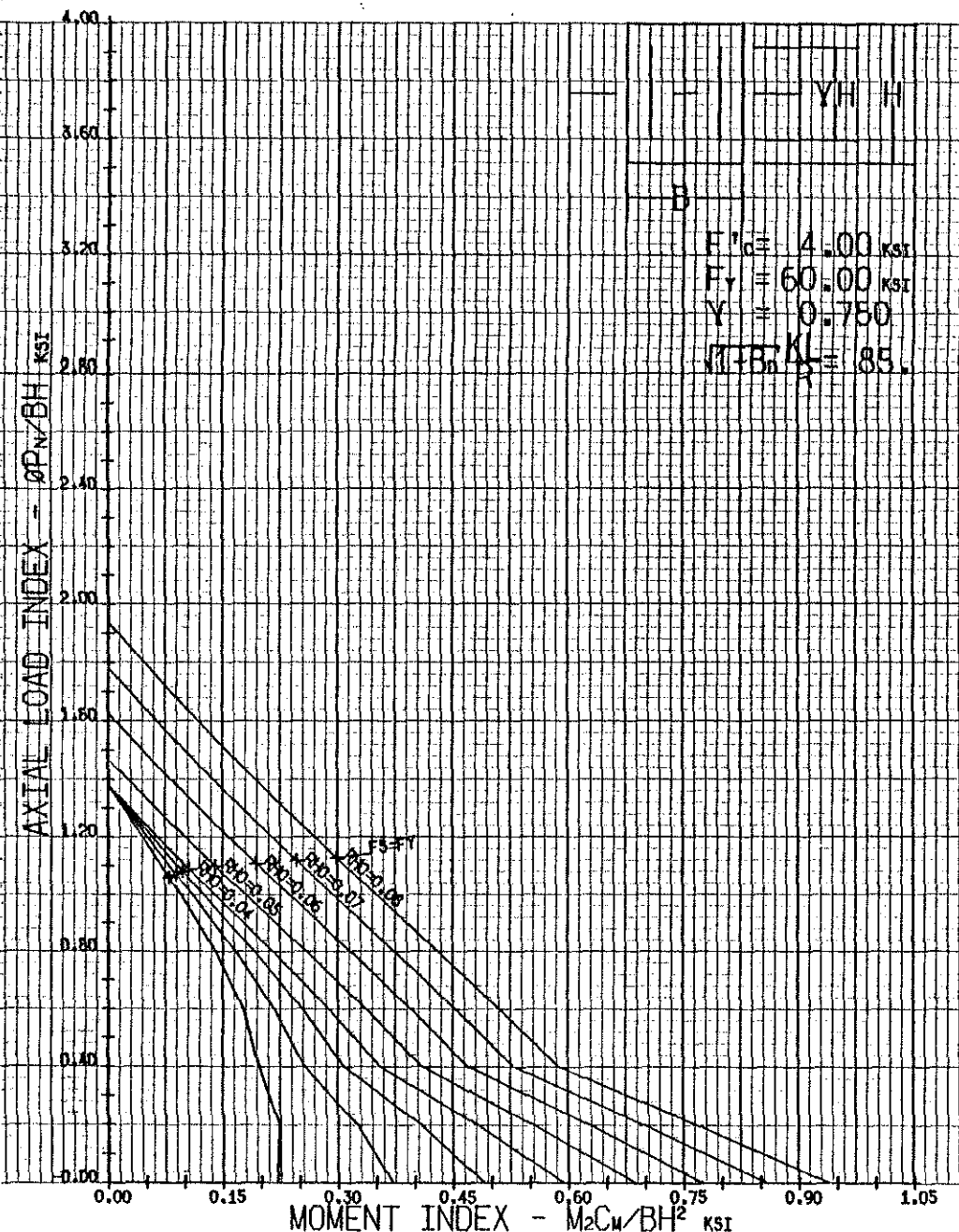


Fig. L4-60.75-85 - Interaction Diagram

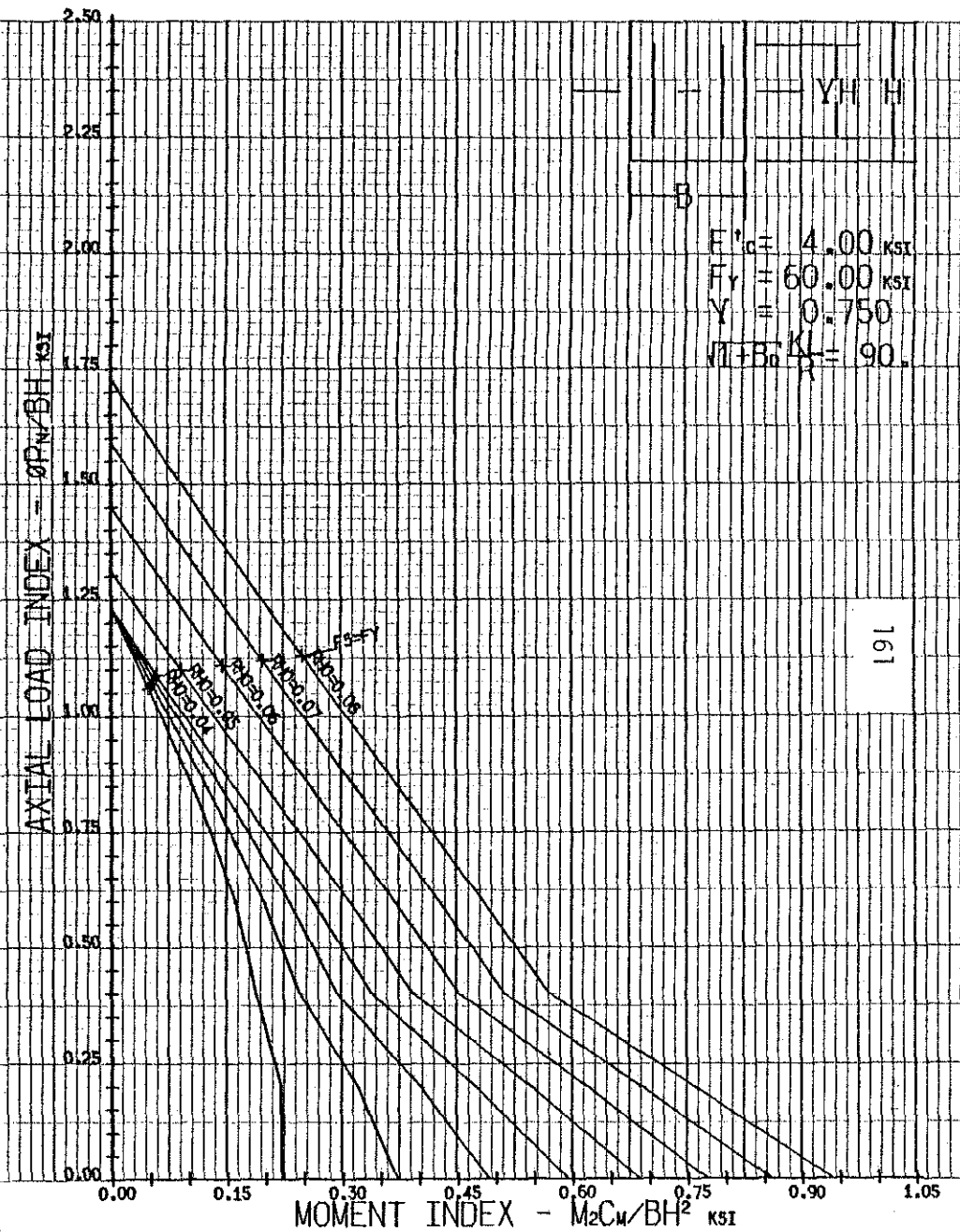


Fig. L4-60.75-90 - Interaction Diagram

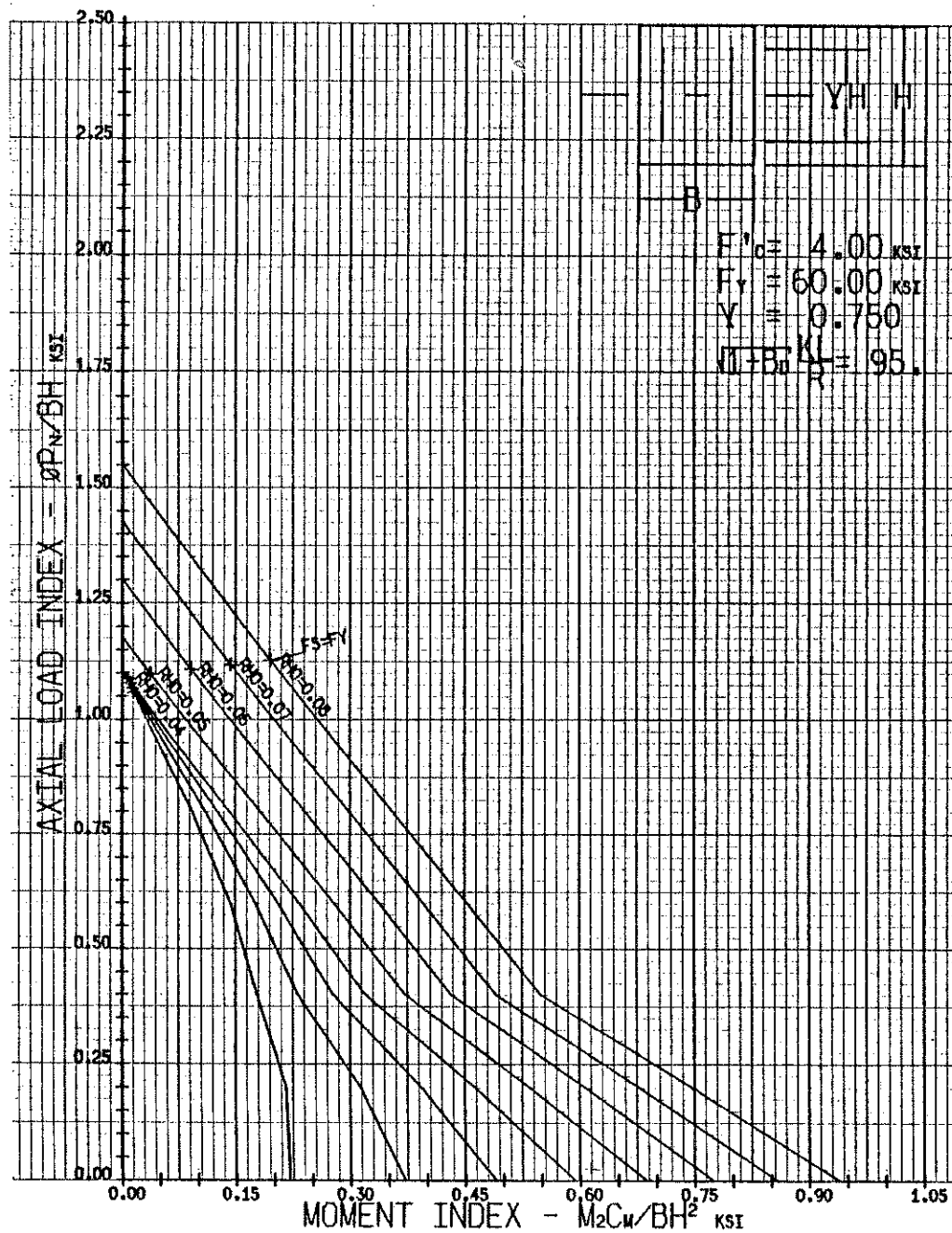


Fig. L4-60.75-95 - Interaction Diagram

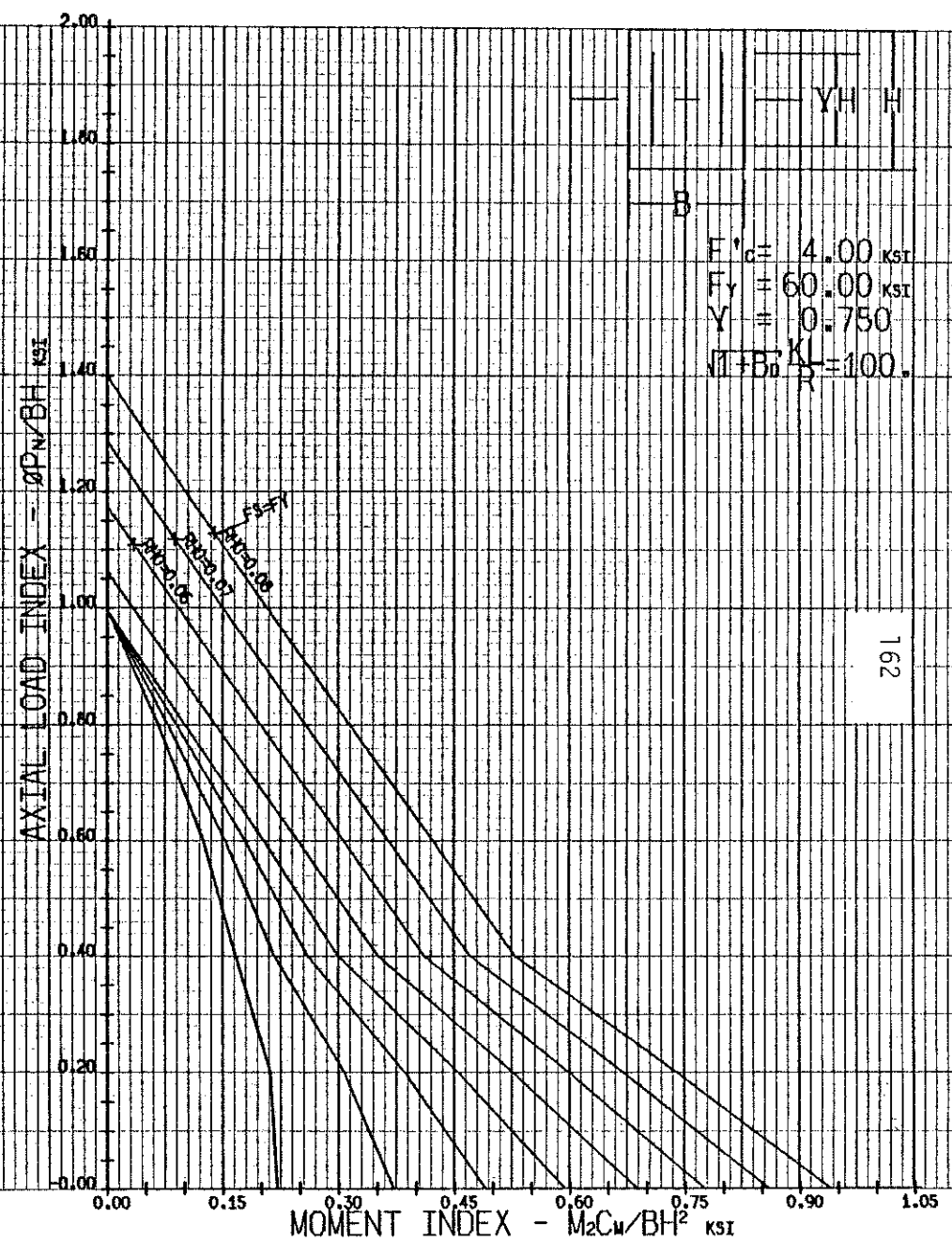


Fig. L4-60.75-100 - Interaction Diagram



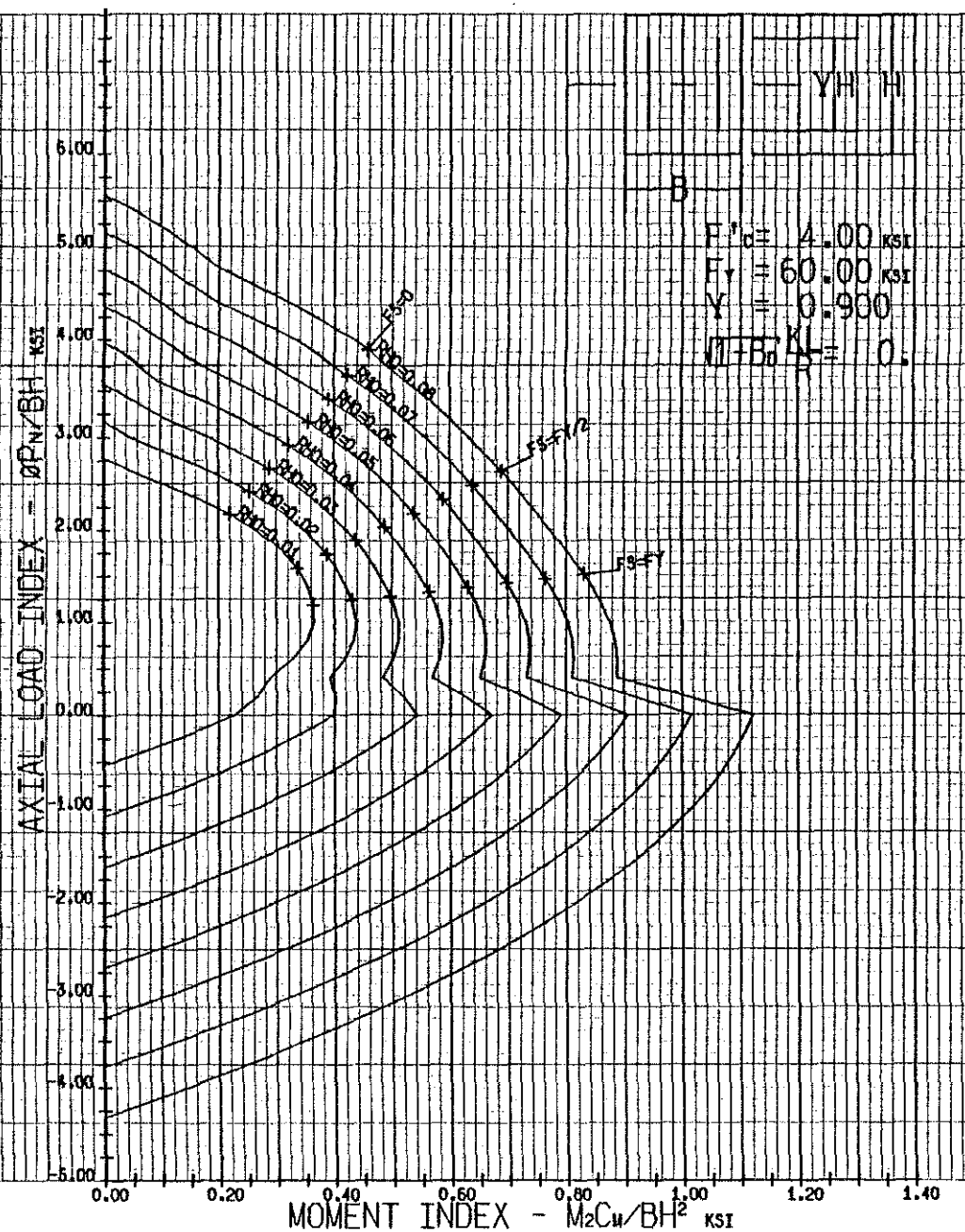


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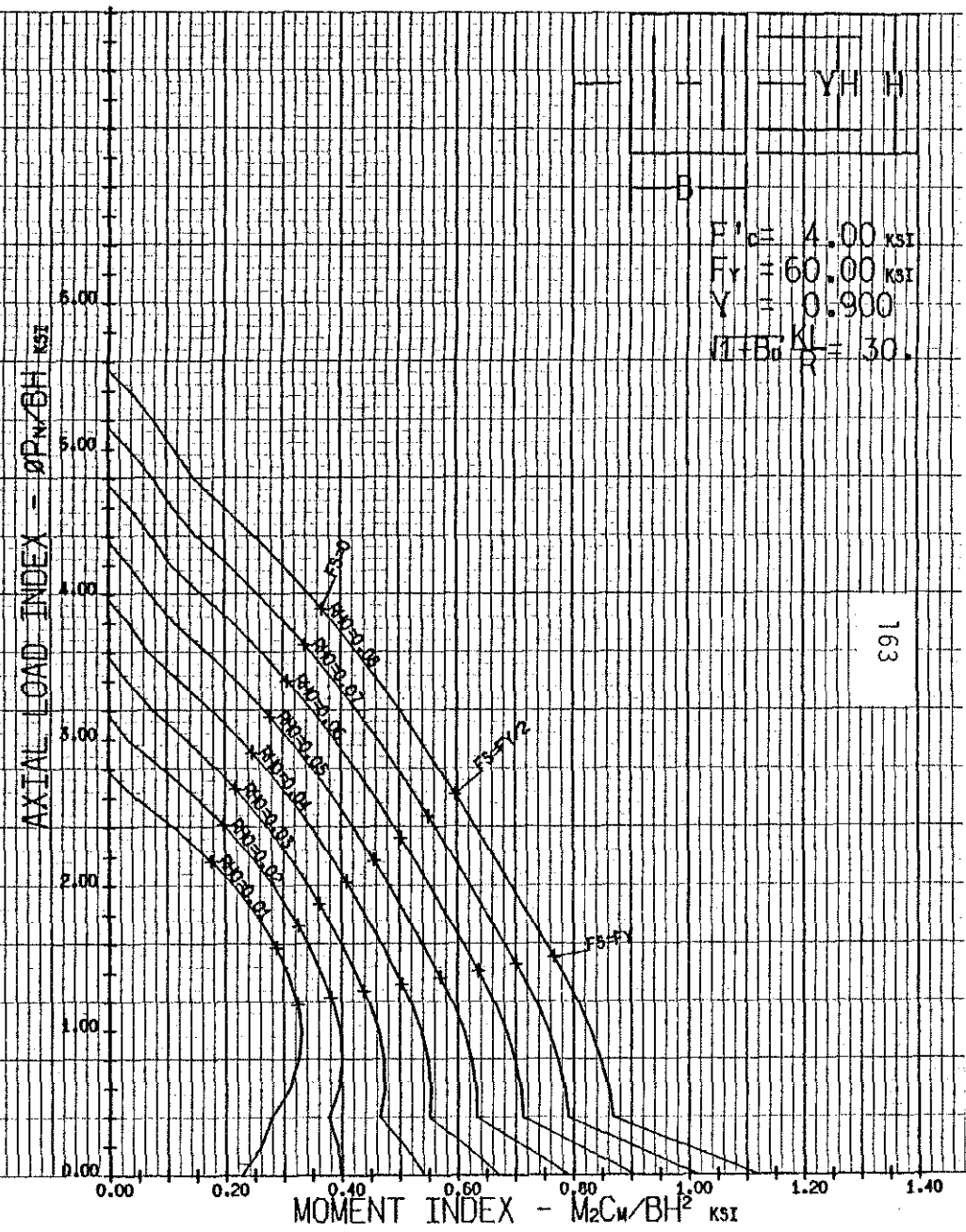


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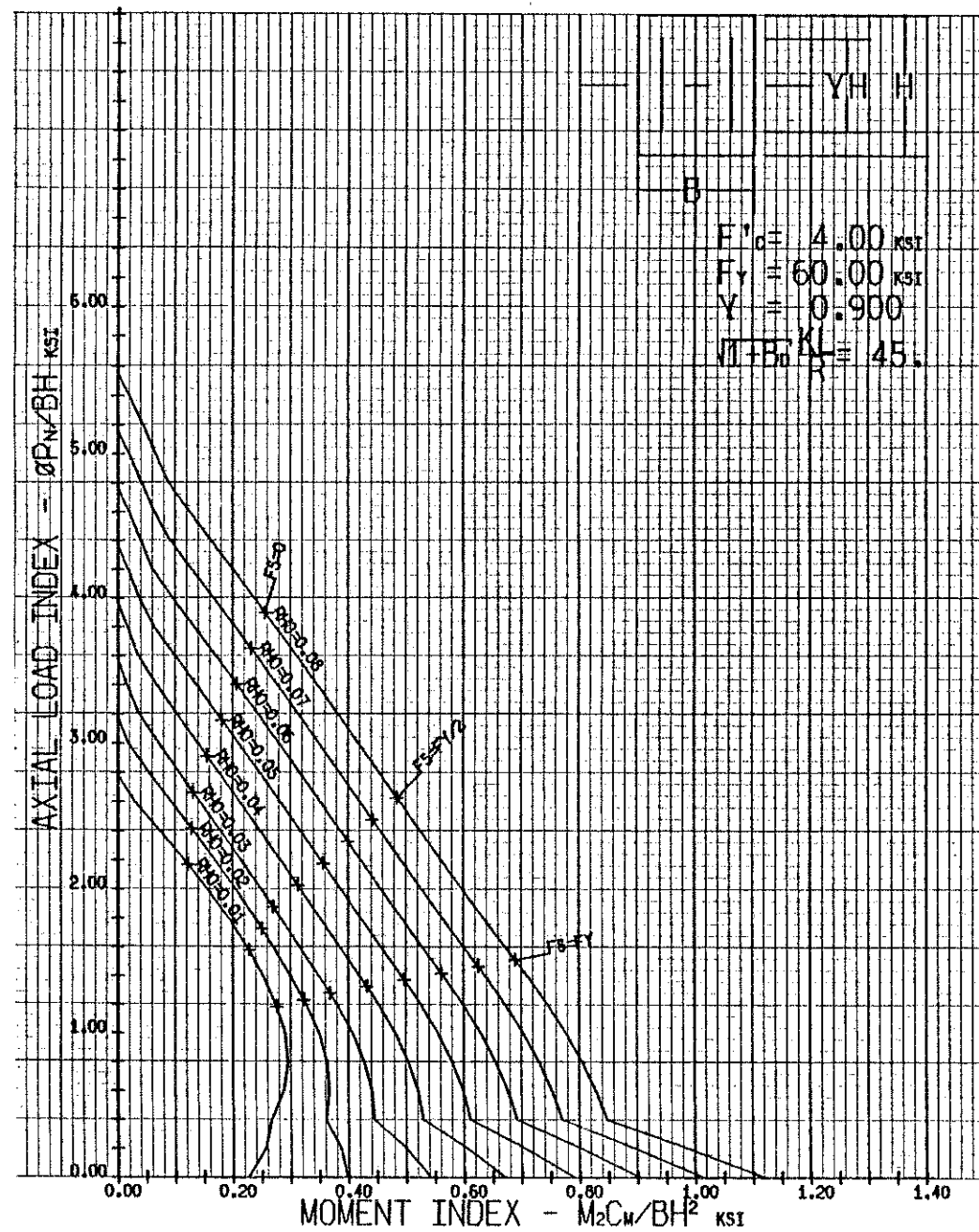


Fig. L4-60.90-45 - Interaction Diagram

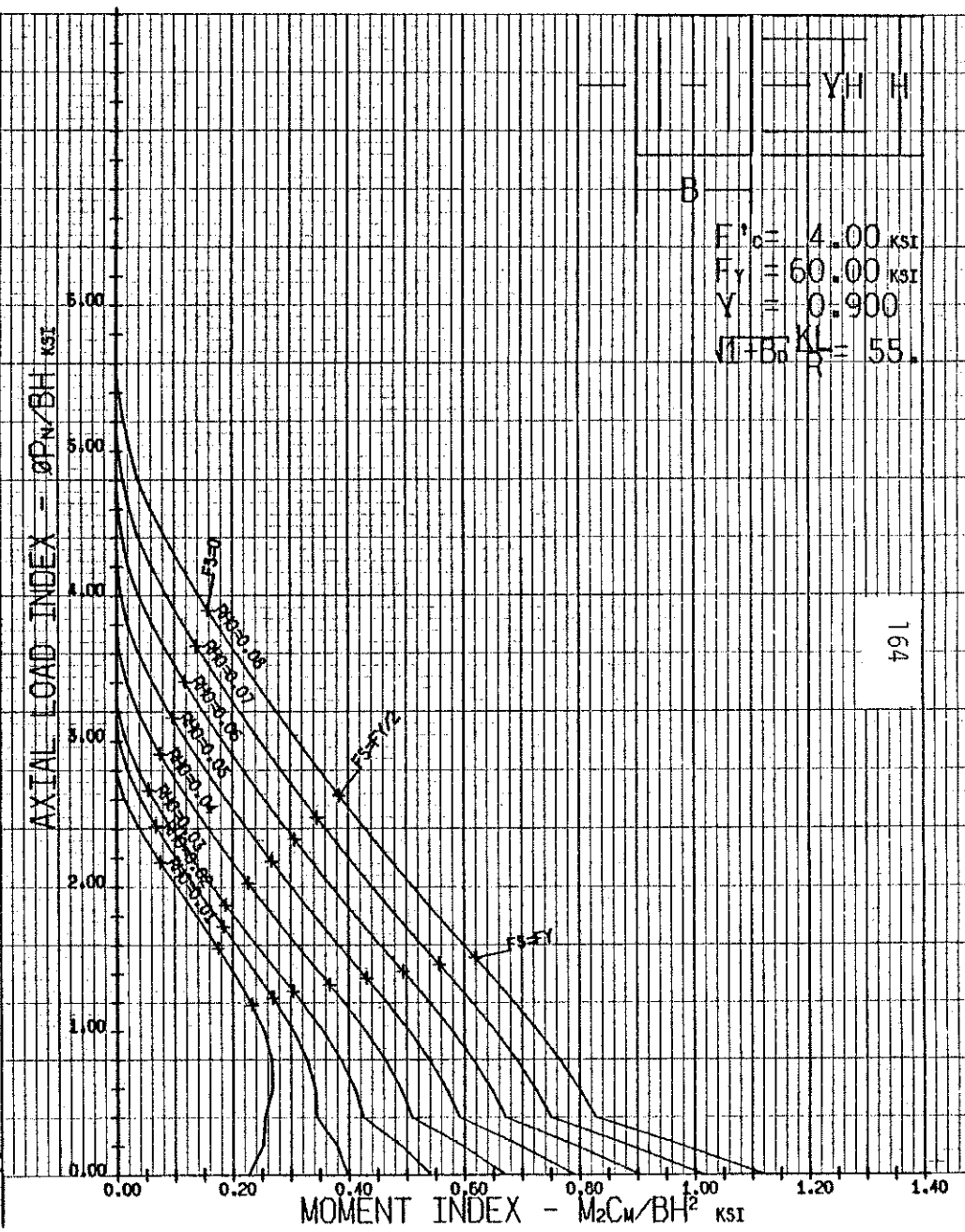


Fig. L4-60.90-55 - Interaction Diagram

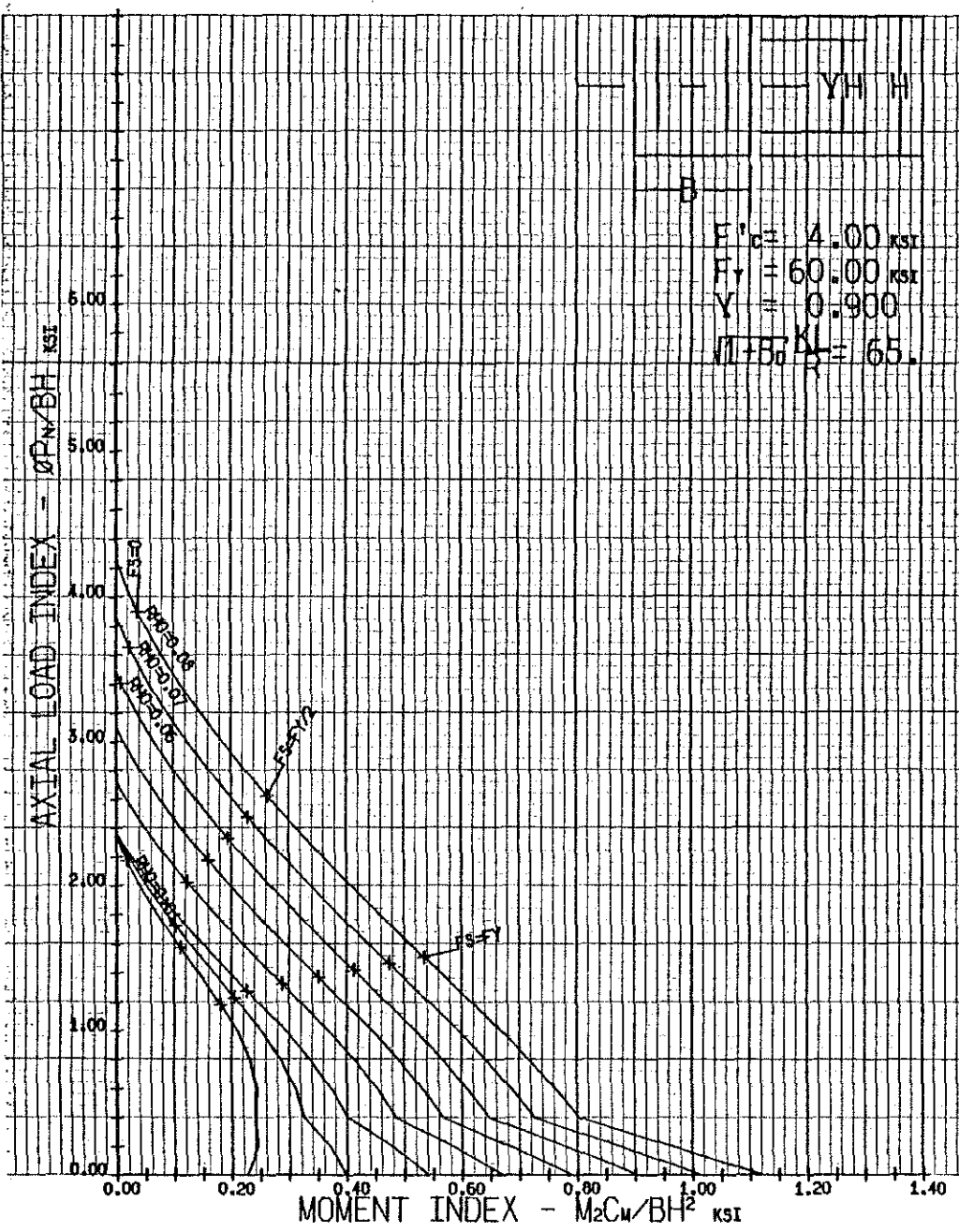


Fig. L4-60.90-65 - Interaction Diagram

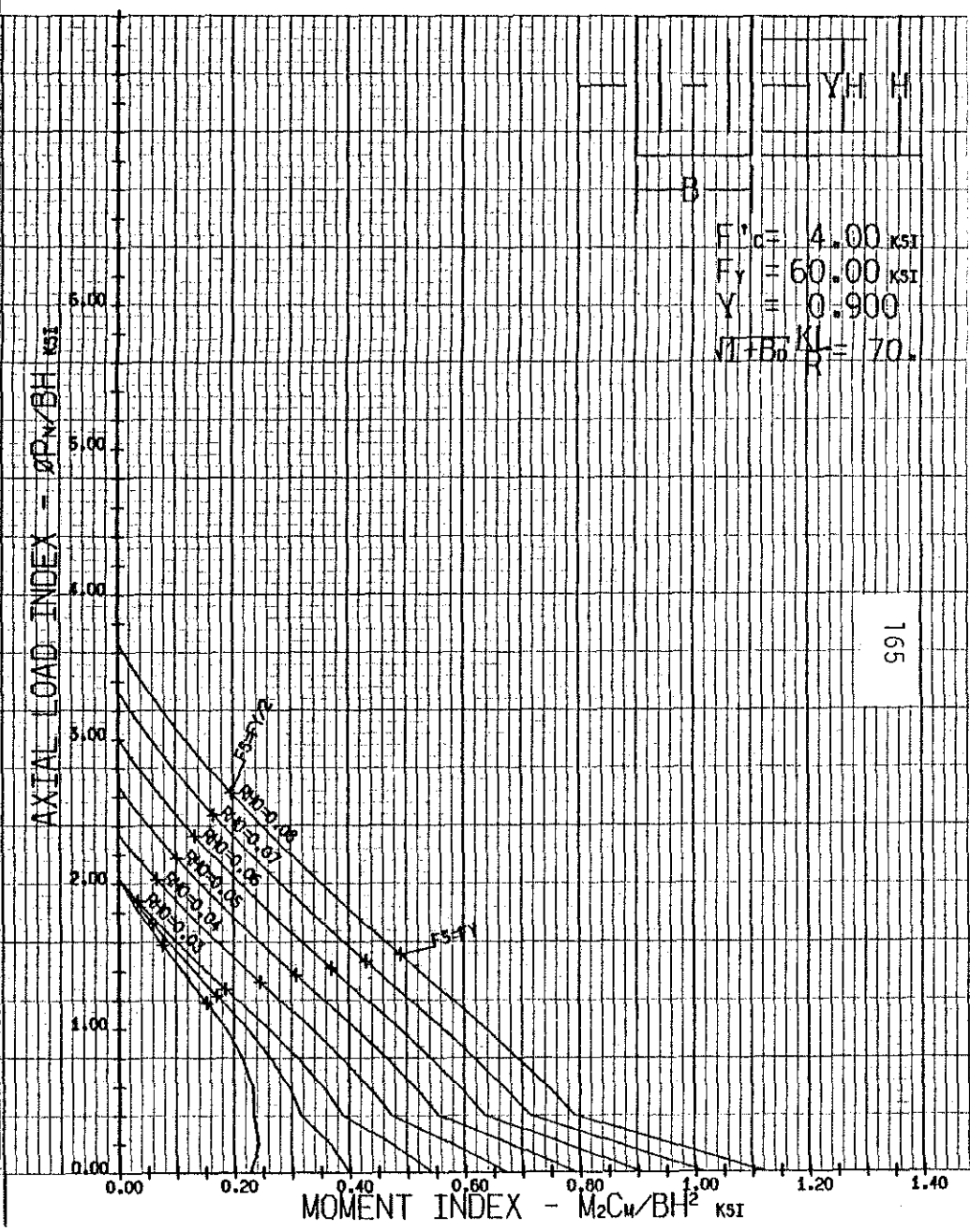
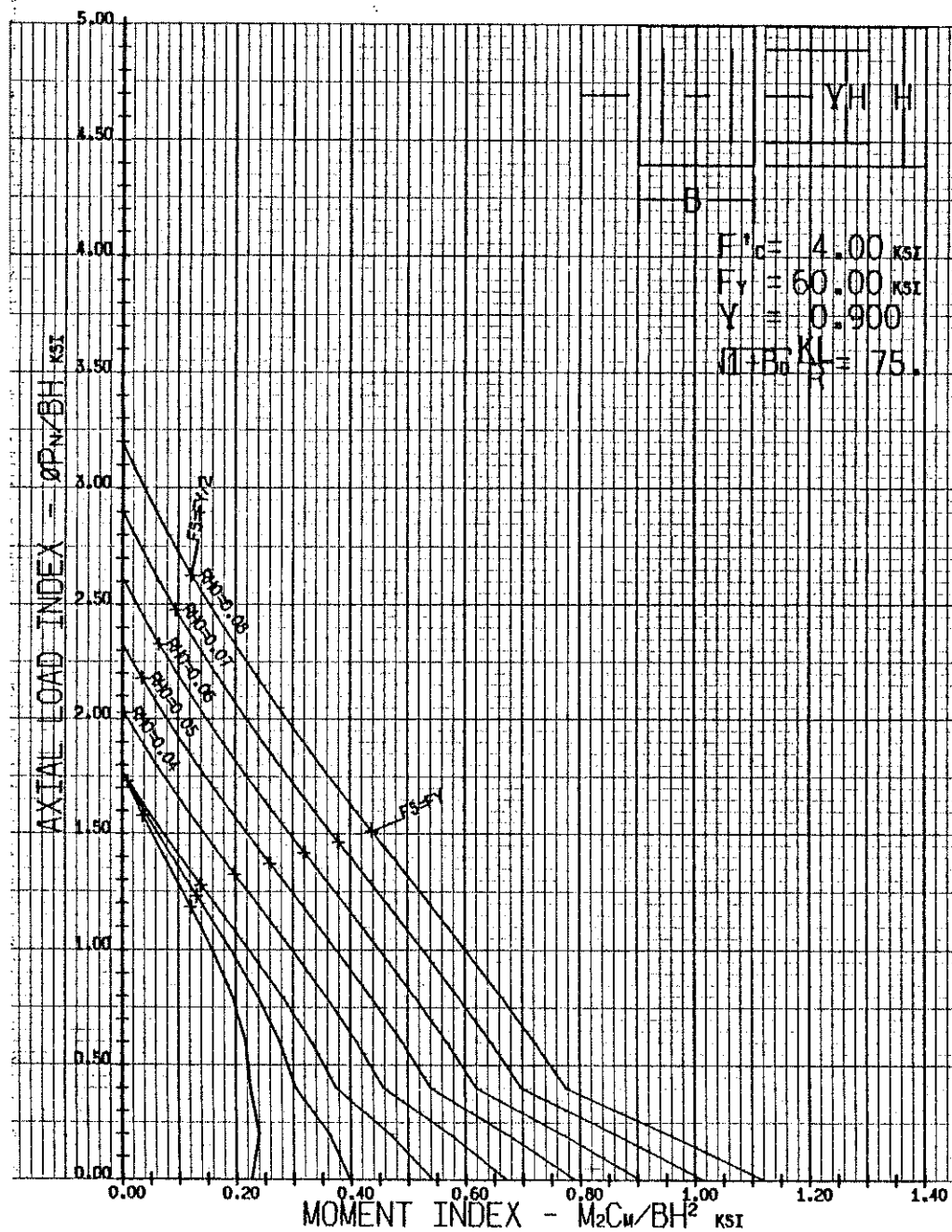


Fig. L4-60.90-70 - Interaction Diagram



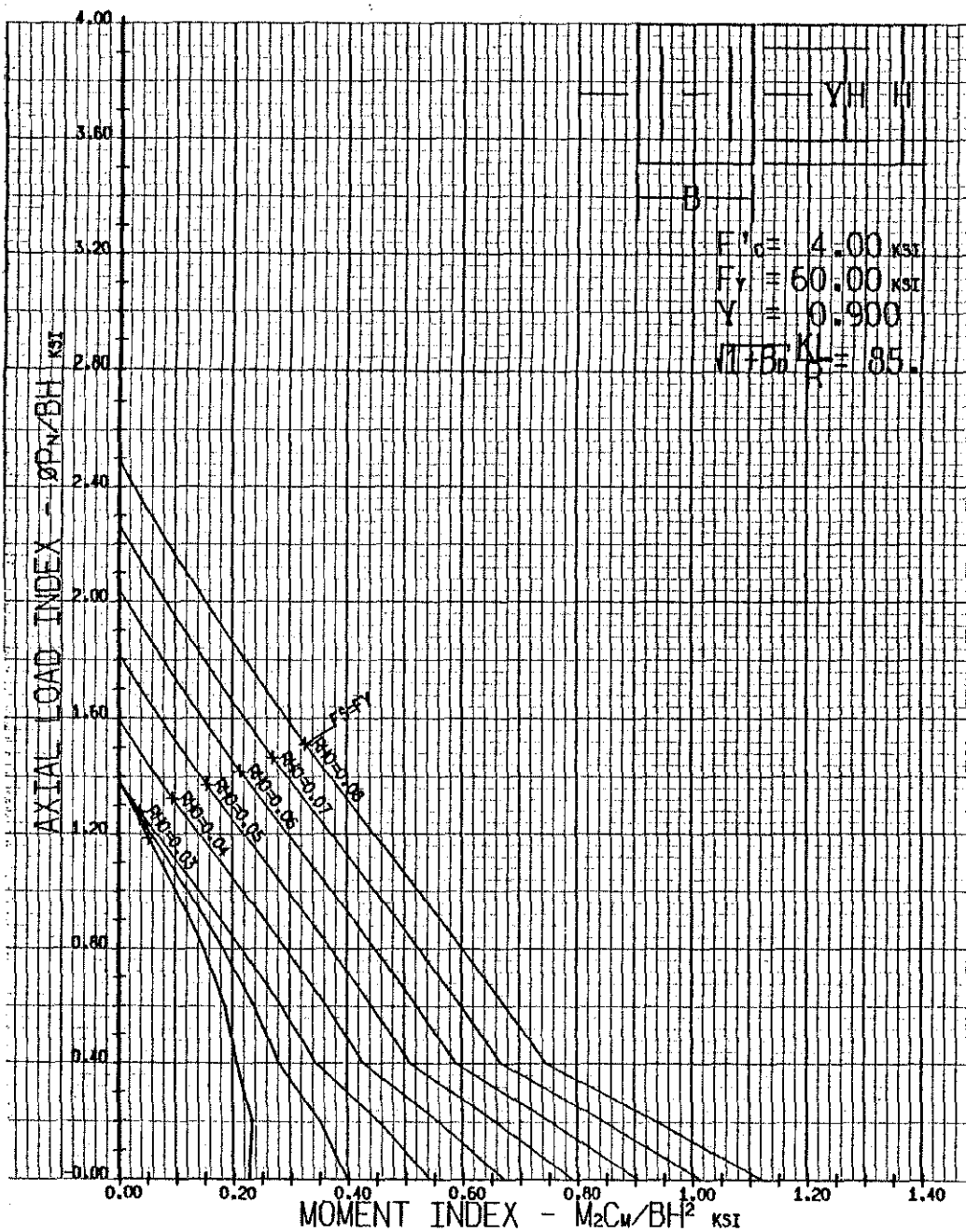


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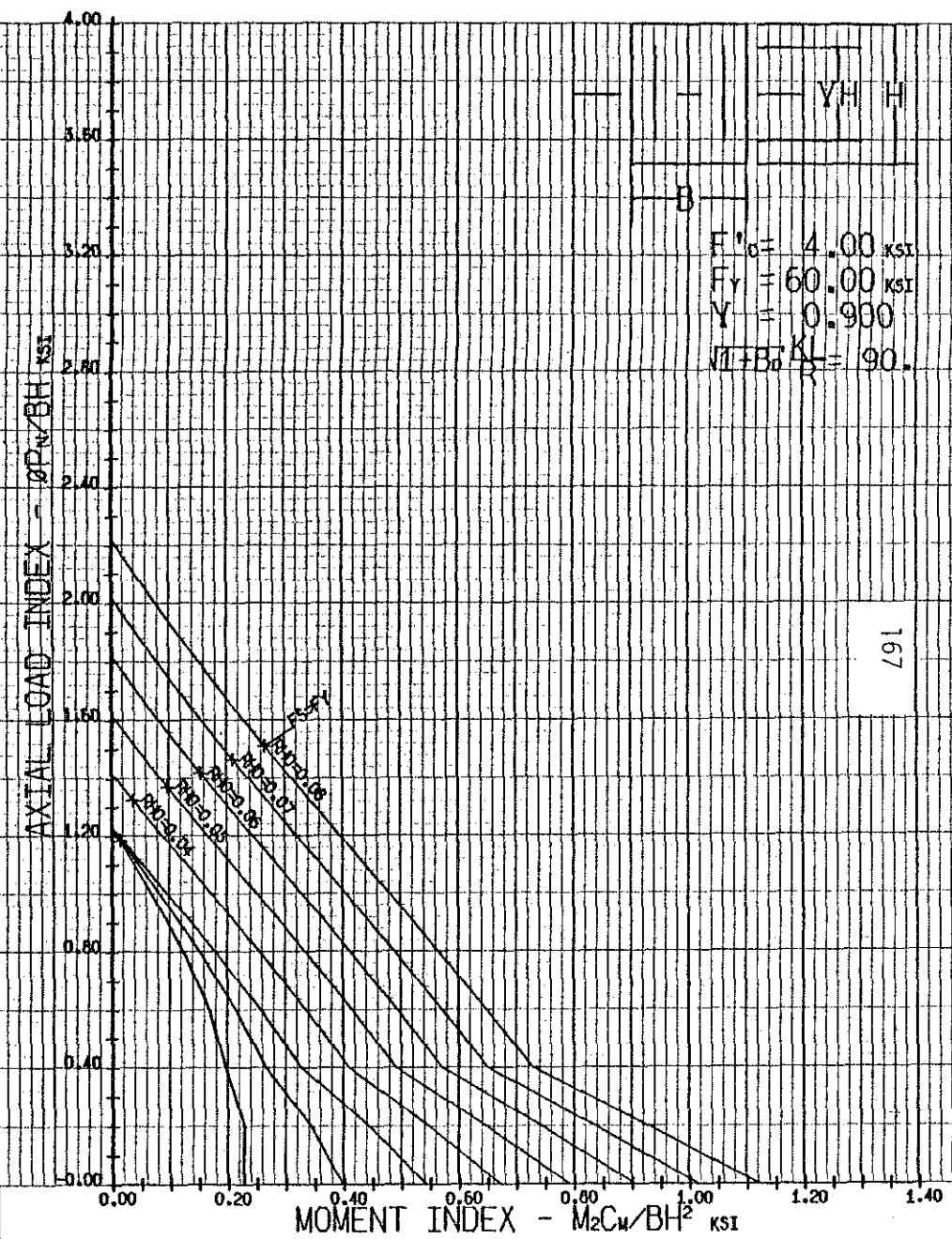


Fig. L4-60.90-90 - Interaction Diagram



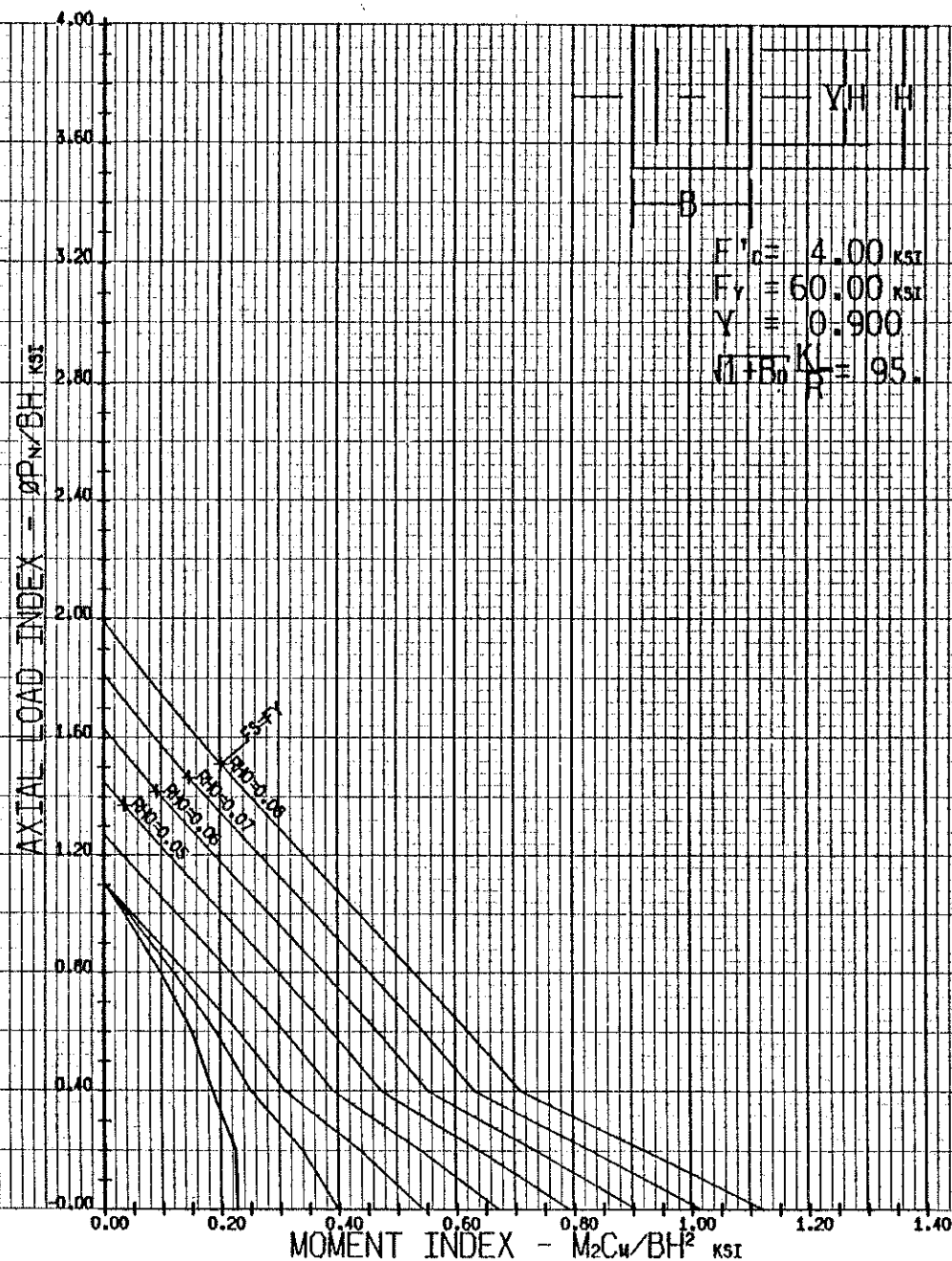


Fig. L4-60.90-95 - Interaction Diagram

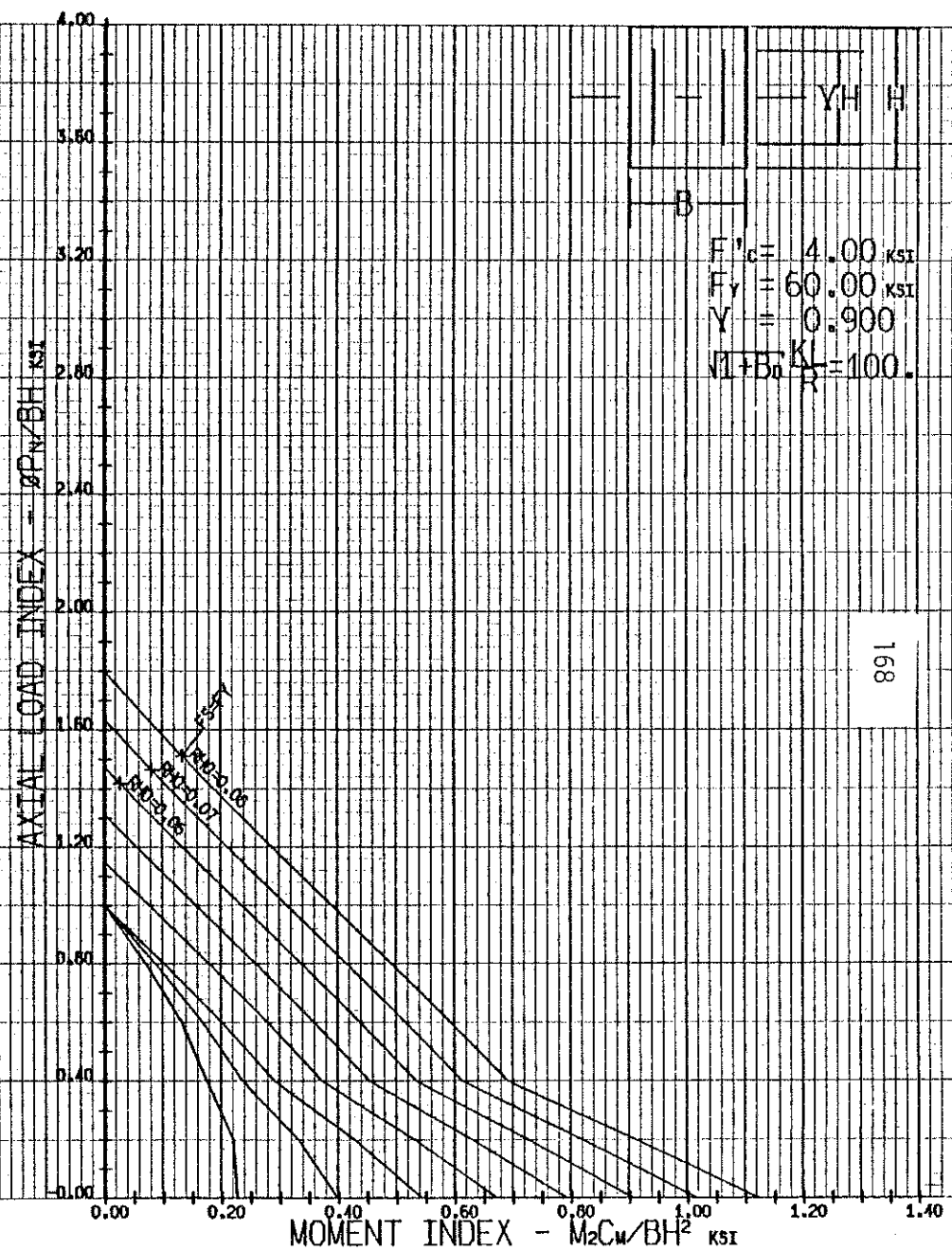


Fig. L4-60.90-100 - Interaction Diagram

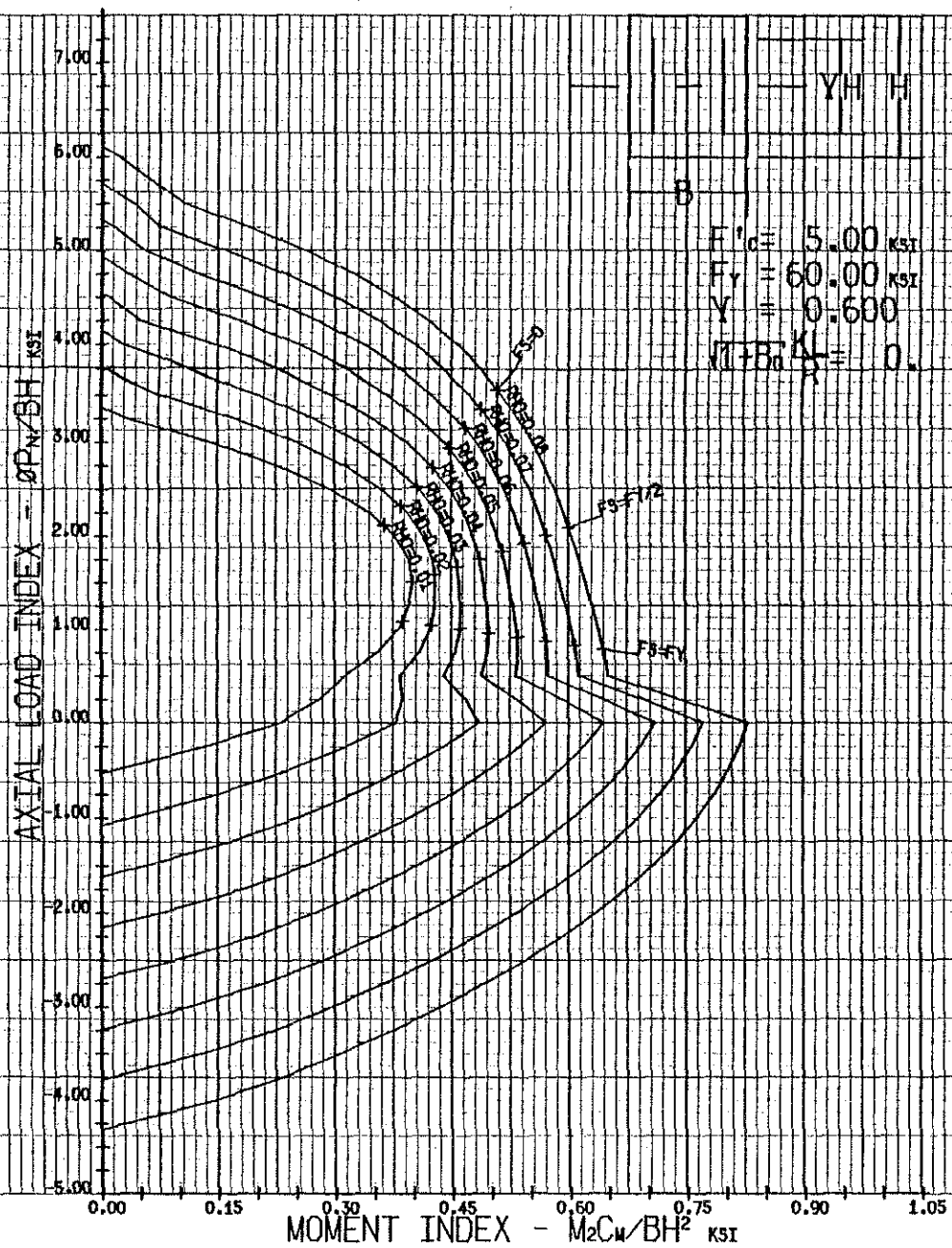


Fig. L5-60.60-0 - Interaction Diagram

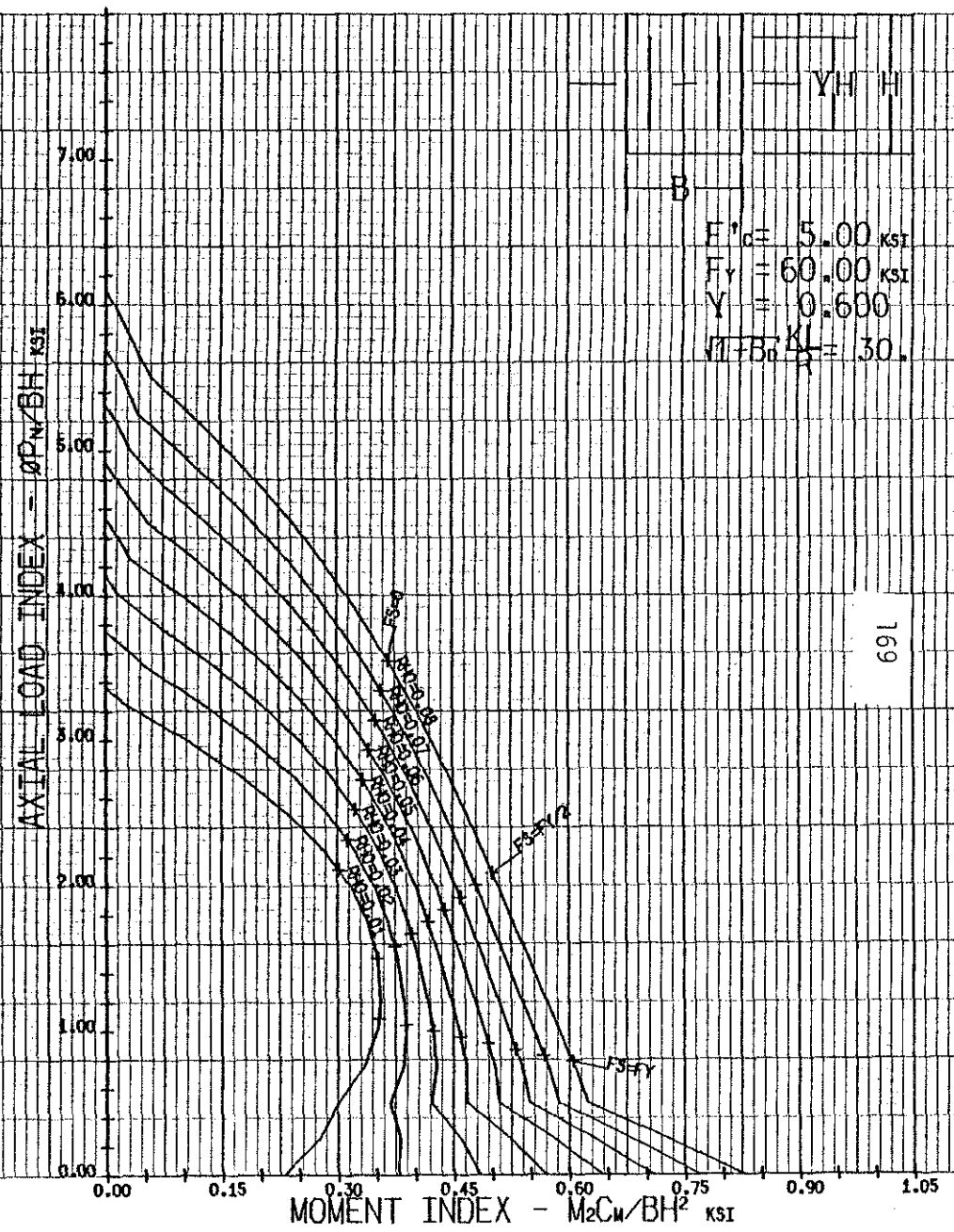


Fig. L5-60.60-30 - Interaction Diagram



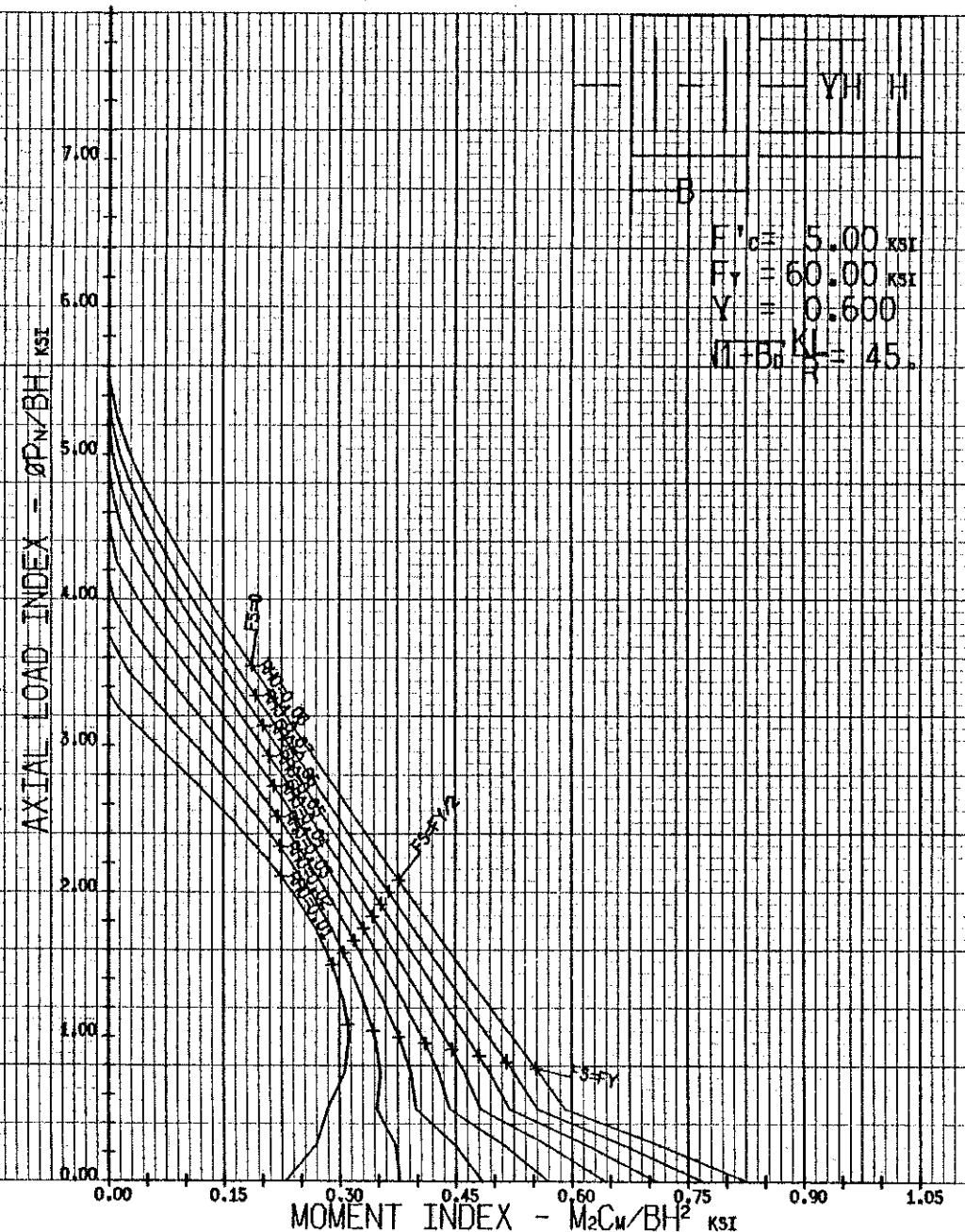


Fig. L5-60.60-45 - Interaction Diagram

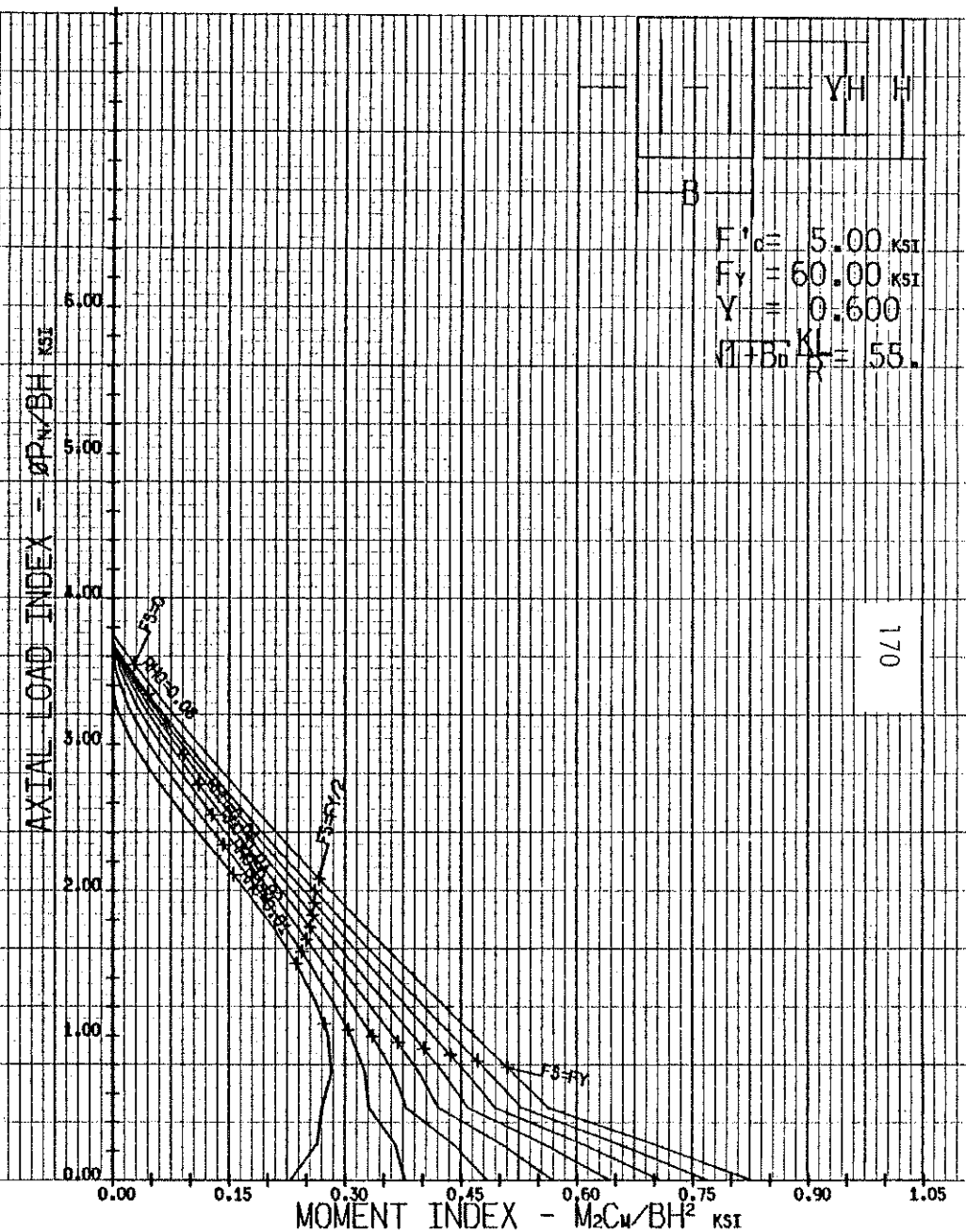


Fig. L5-60.60-55 - Interaction Diagram

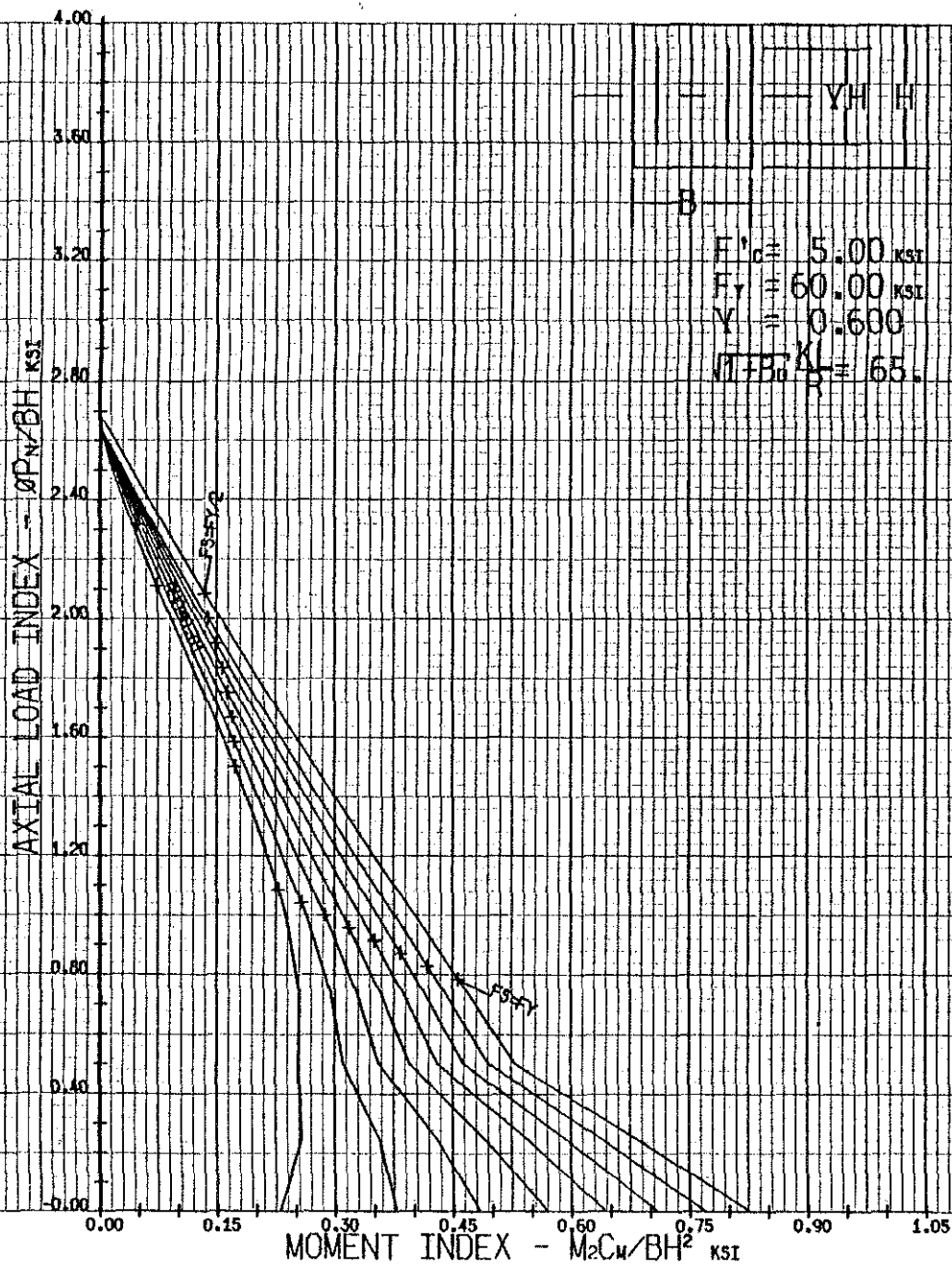


Fig. L5-60.60-65 - Interaction Diagram

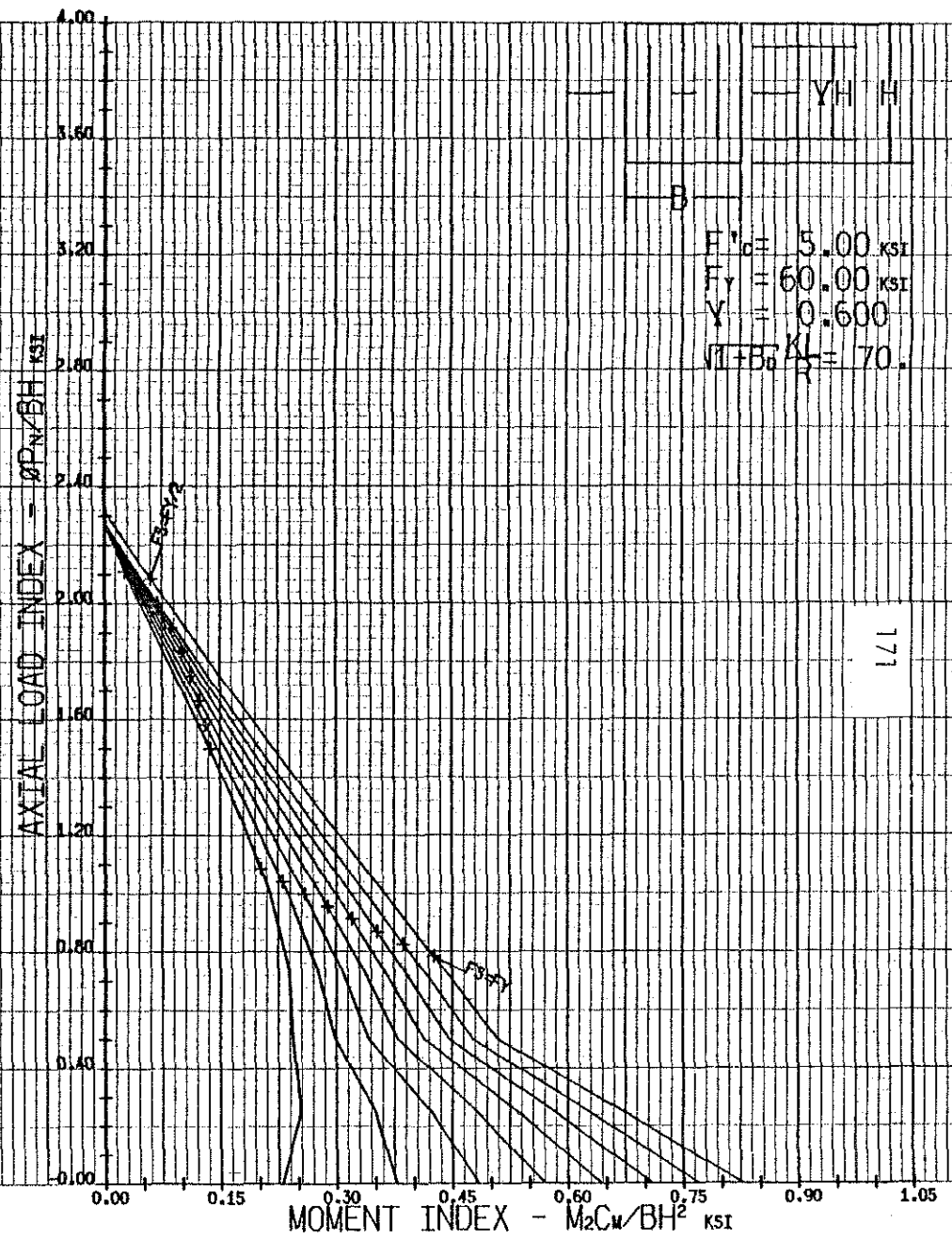


Fig. L5-60.60-70 - Interaction Diagram

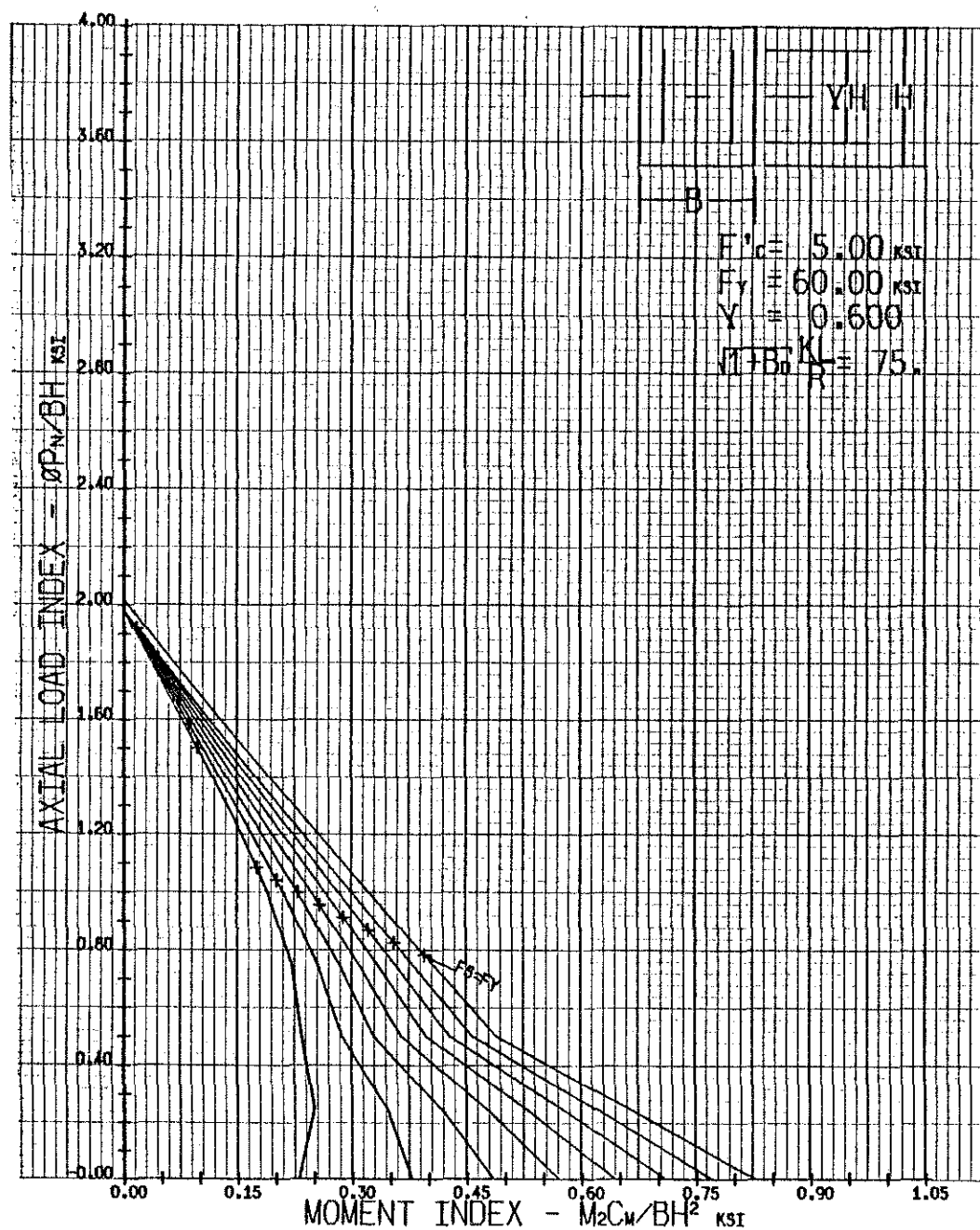


Fig. L5-60.60-75 - Interaction Diagram

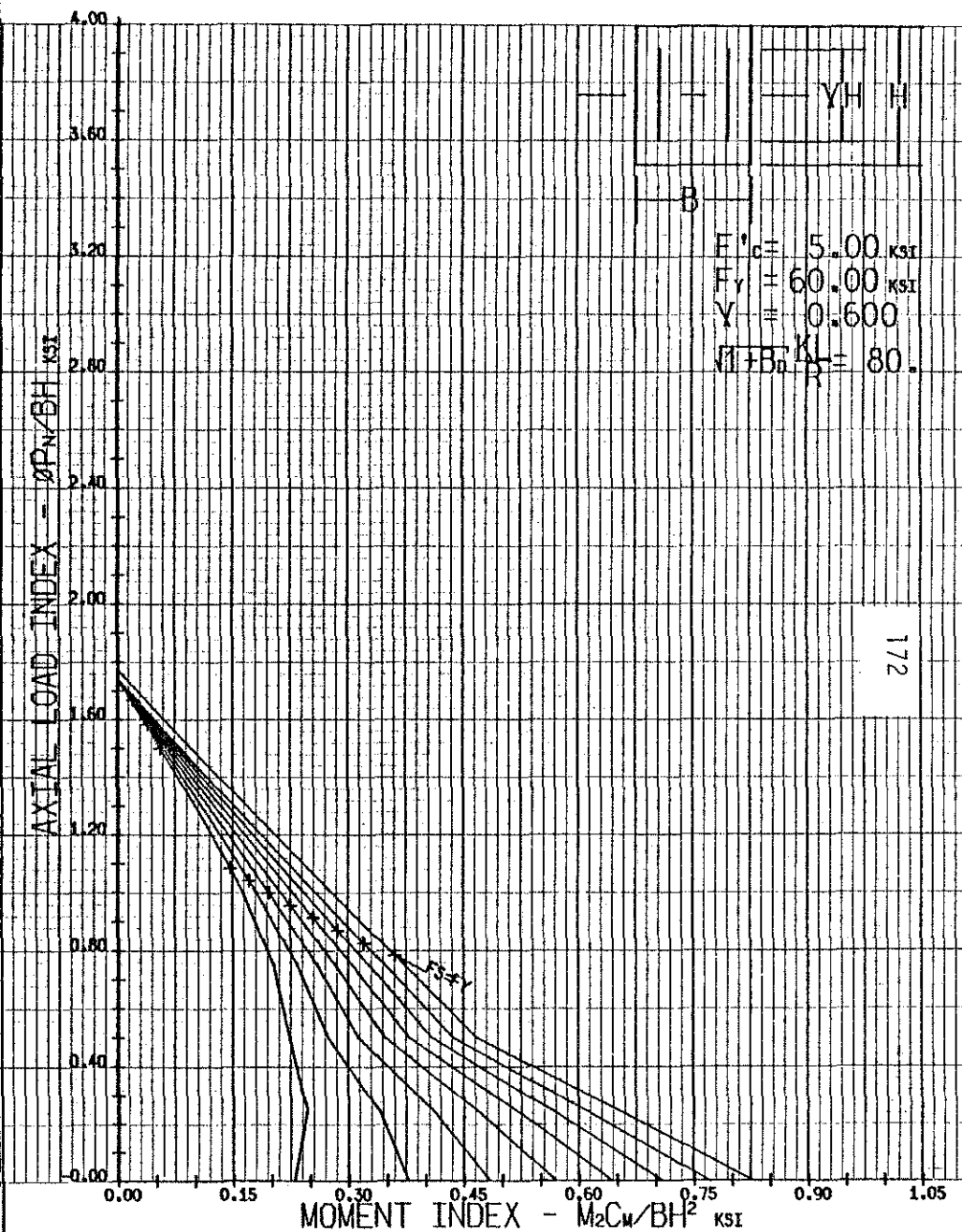


Fig. L5-60.60-80 - Interaction Diagram

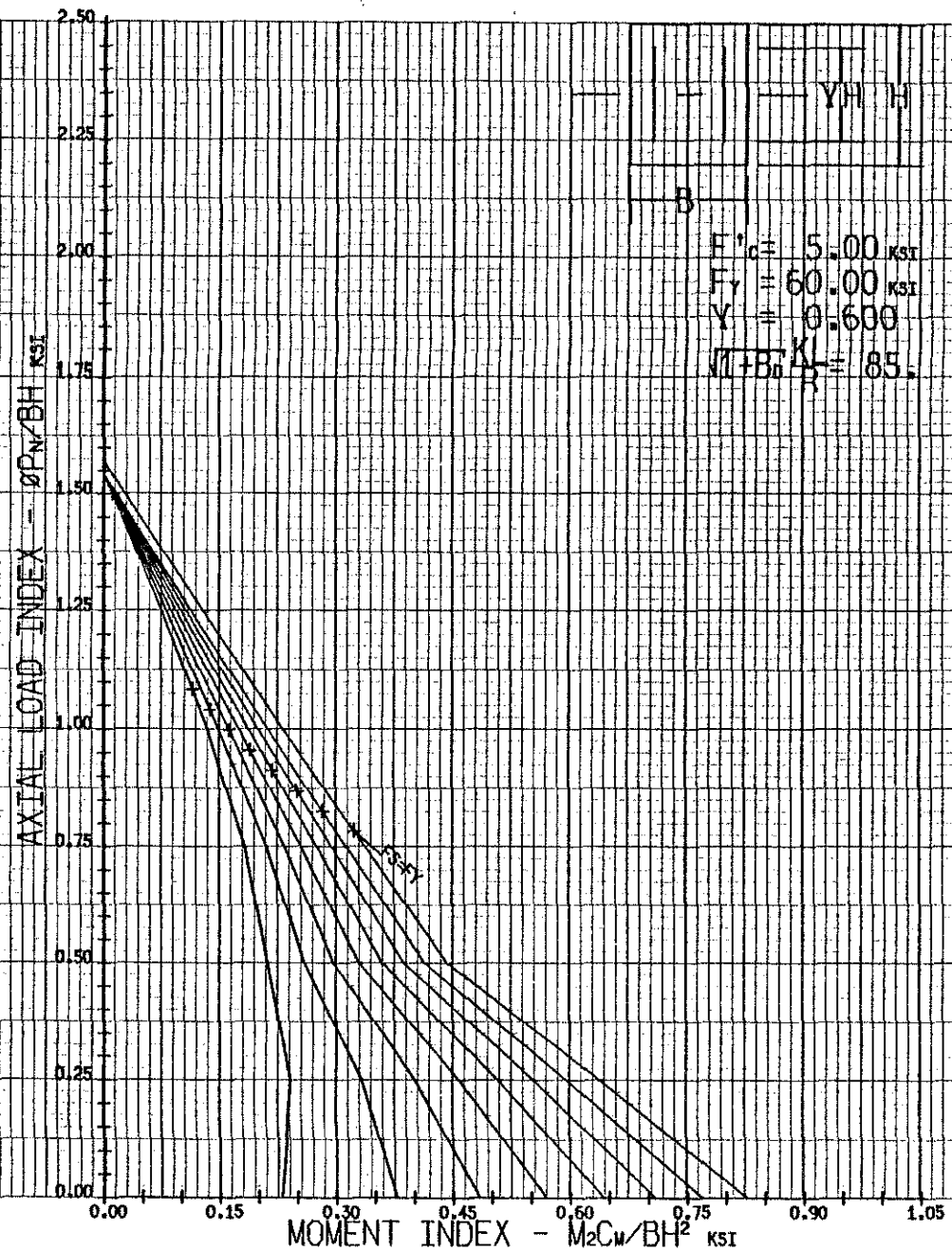


Fig. L5-60.60-85 - Interaction Diagram

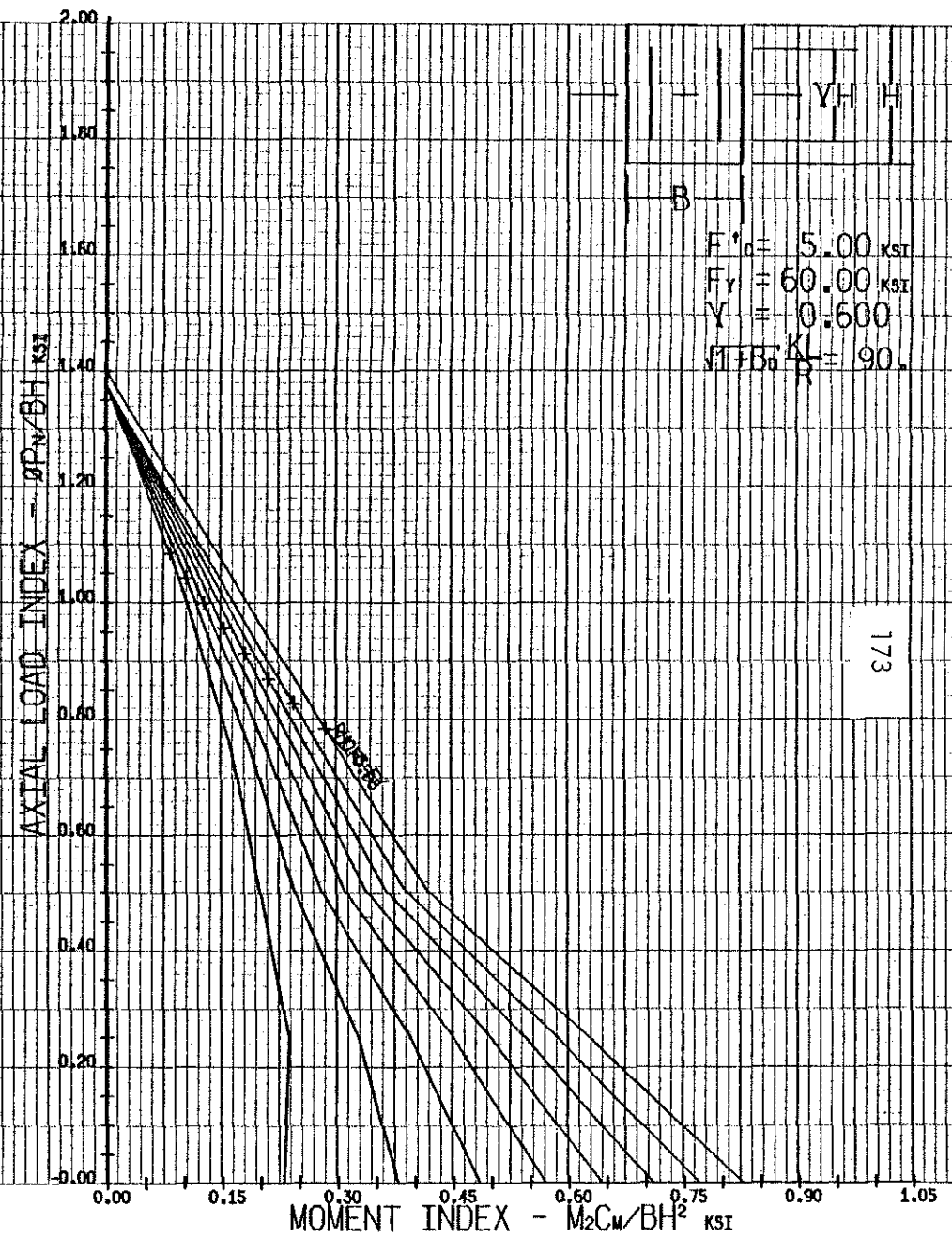


Fig. L5-60.60-90 - Interaction Diagram

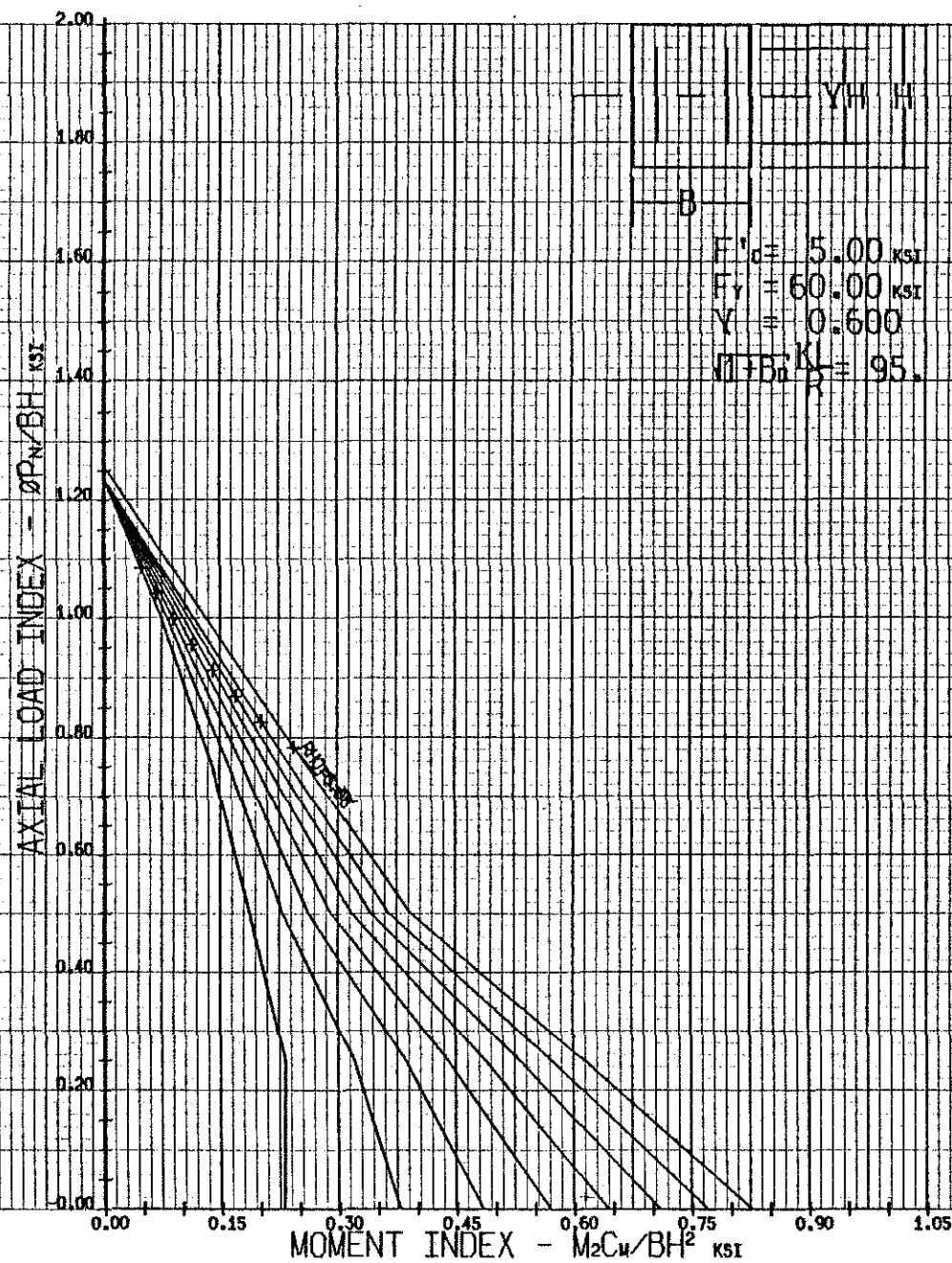


Fig. L5-60.60-95 - Interaction Diagram

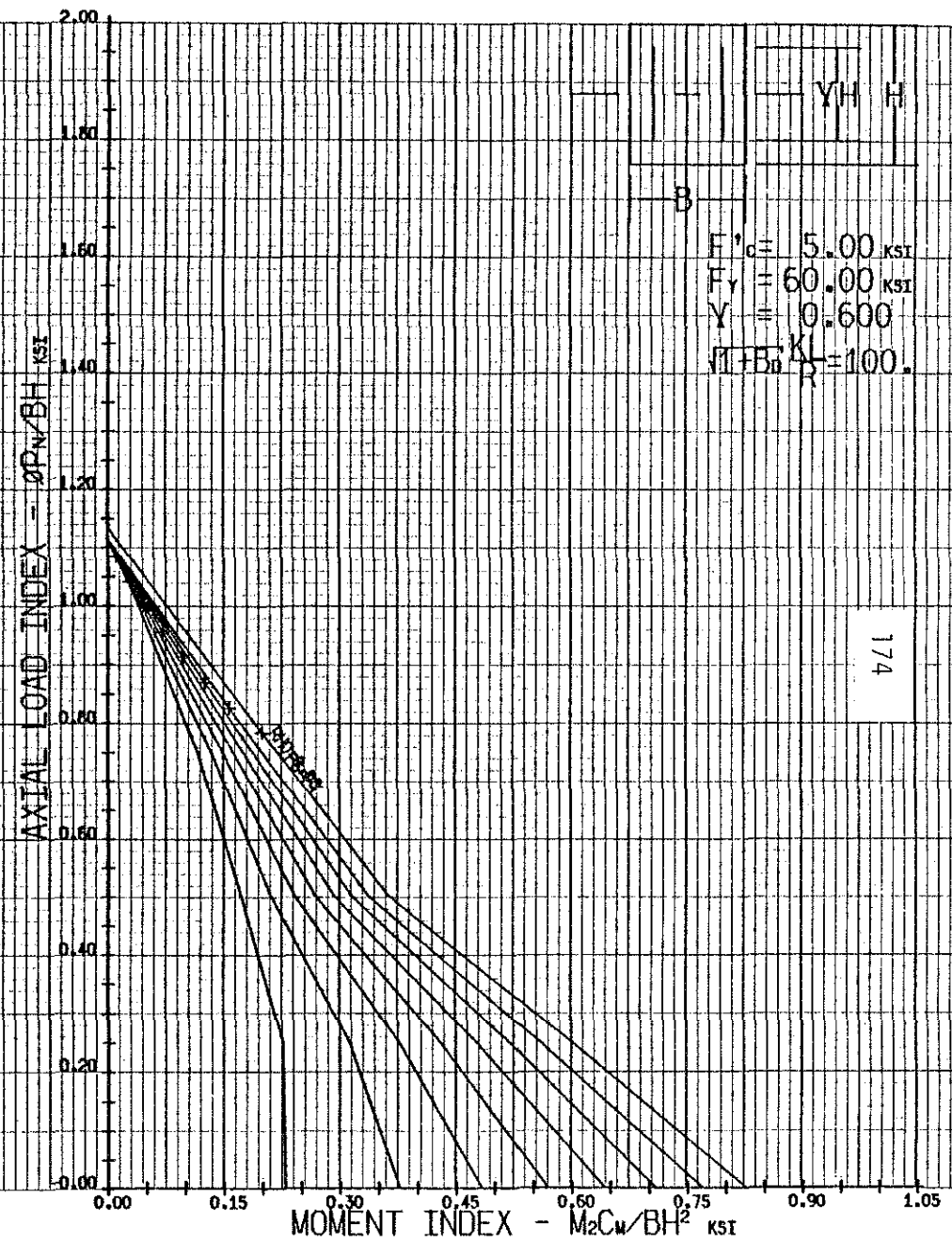


Fig. L5-60.60-100 - Interaction Diagram



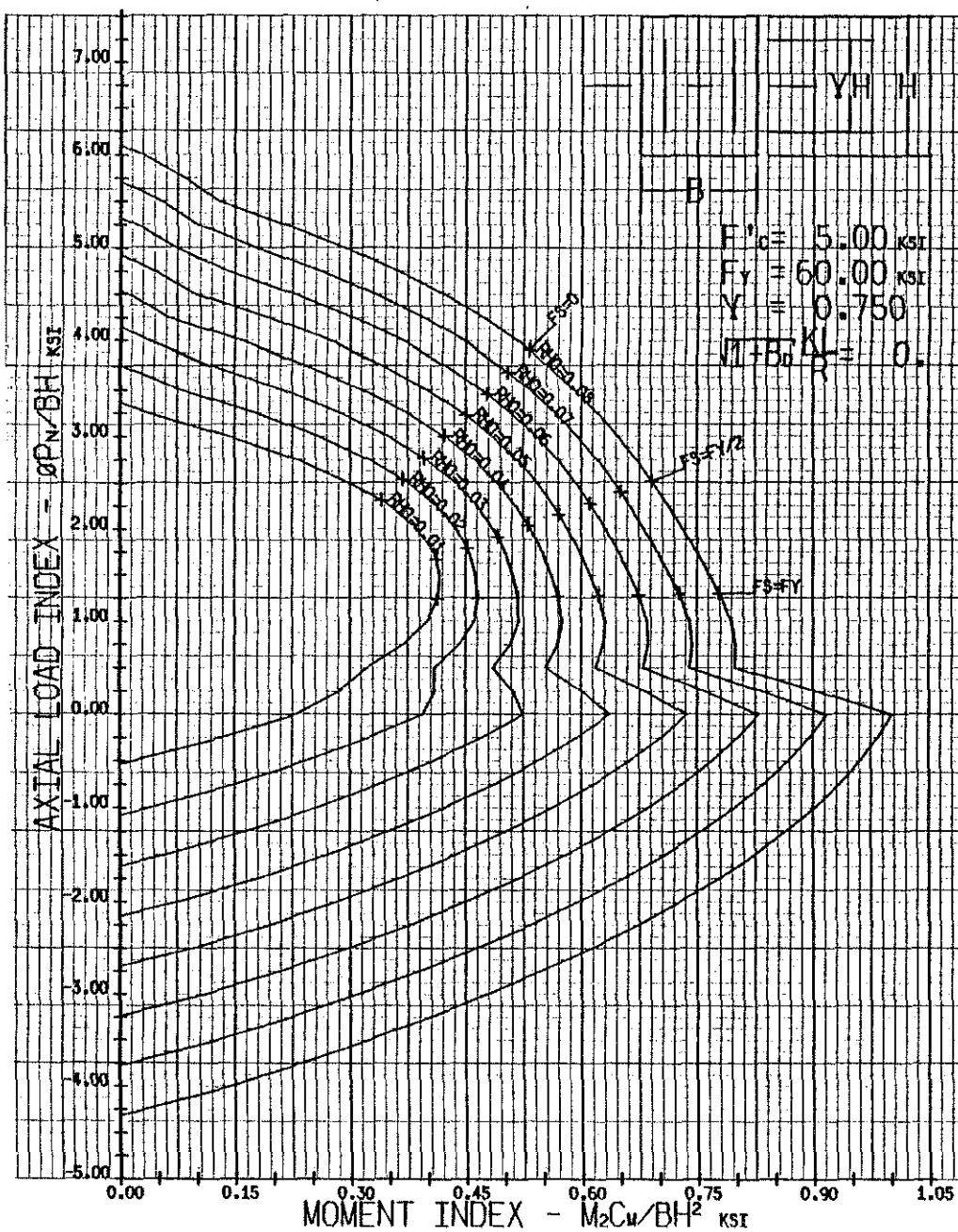


Fig. L5-60.75-0 - Interaction Diagram

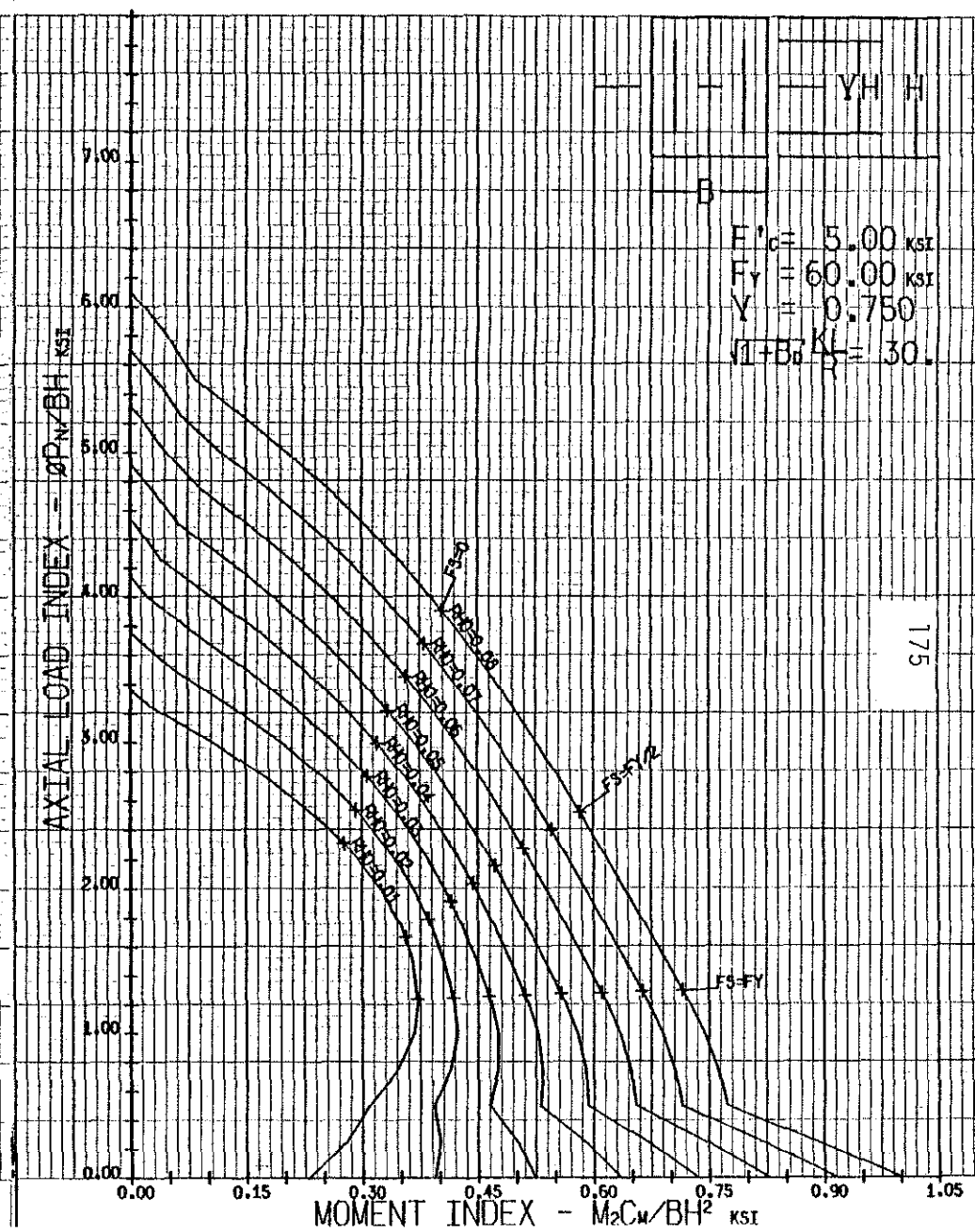


Fig. L5-60.75-30 - Interaction Diagram

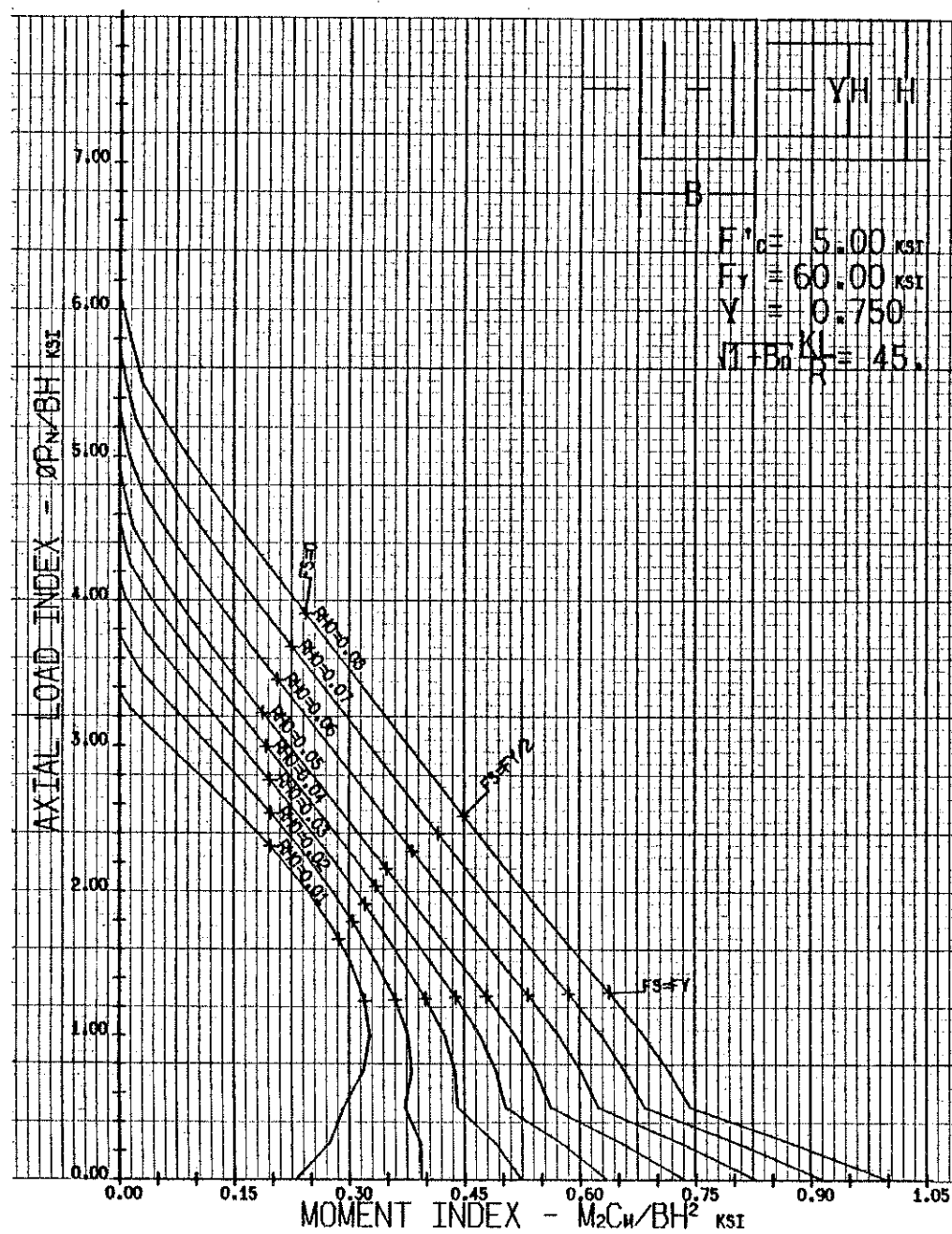


Fig. L5-60.75-45 - Interaction Diagram

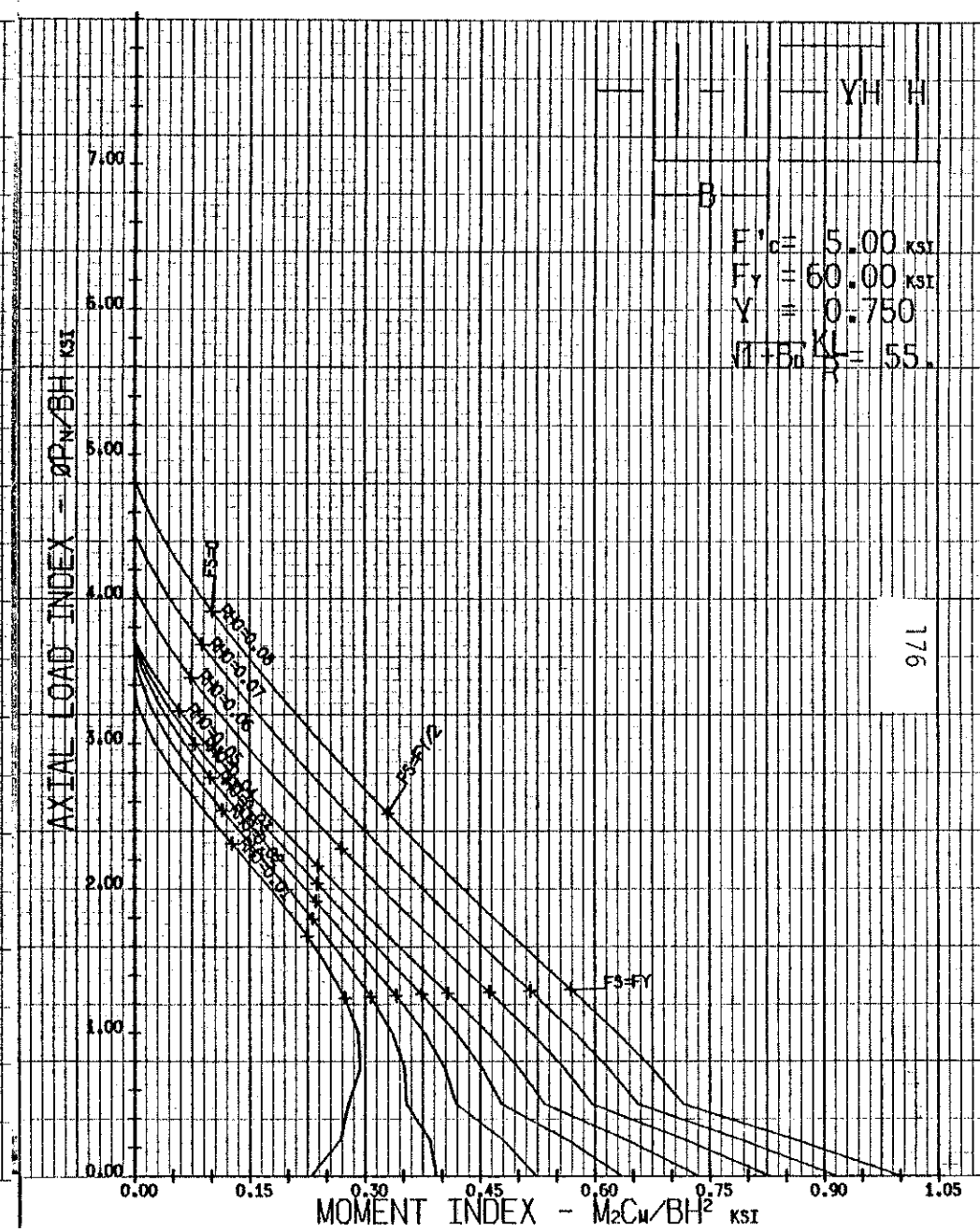


Fig. L5-60.75-55 - Interaction Diagram



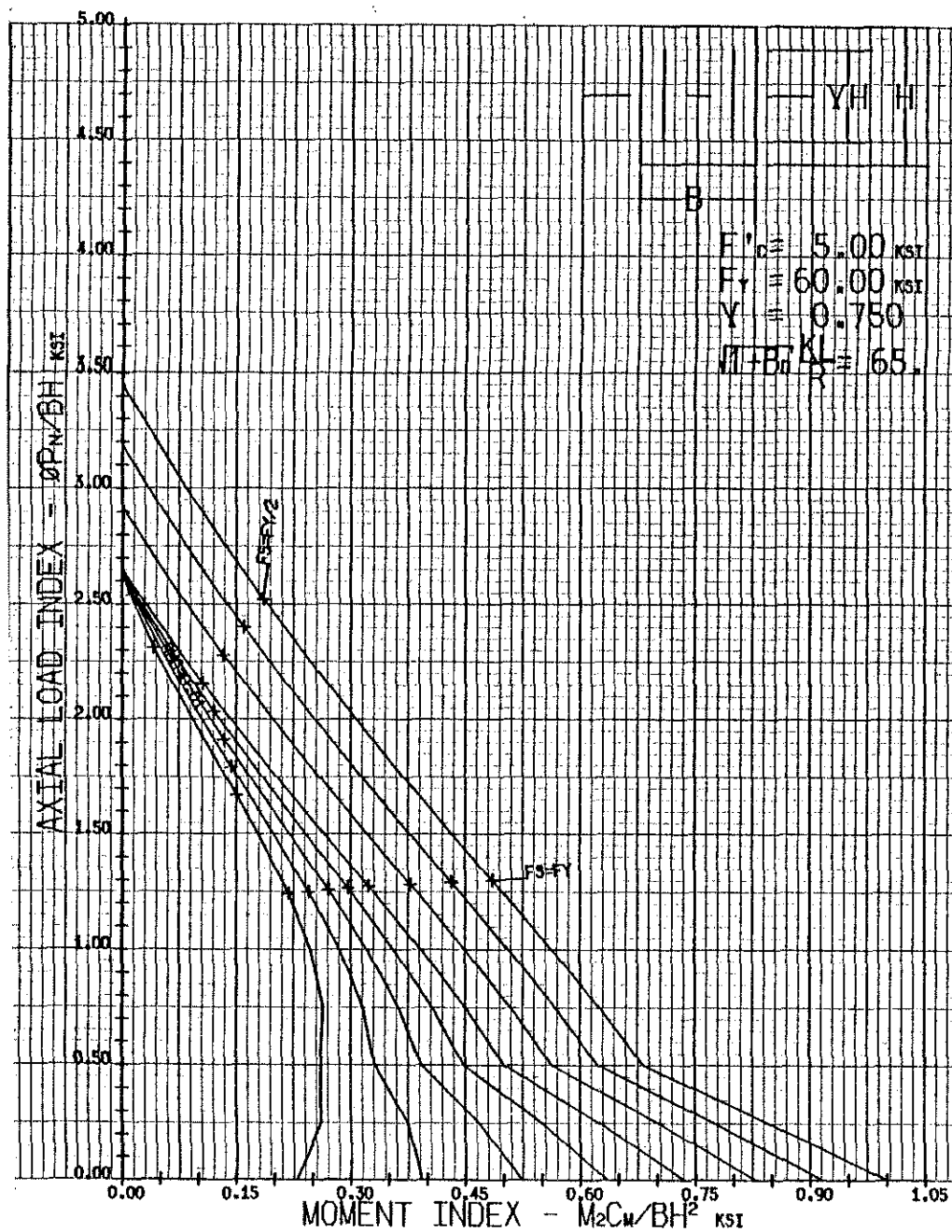


Fig. L5-60.75-65 - Interaction Diagram

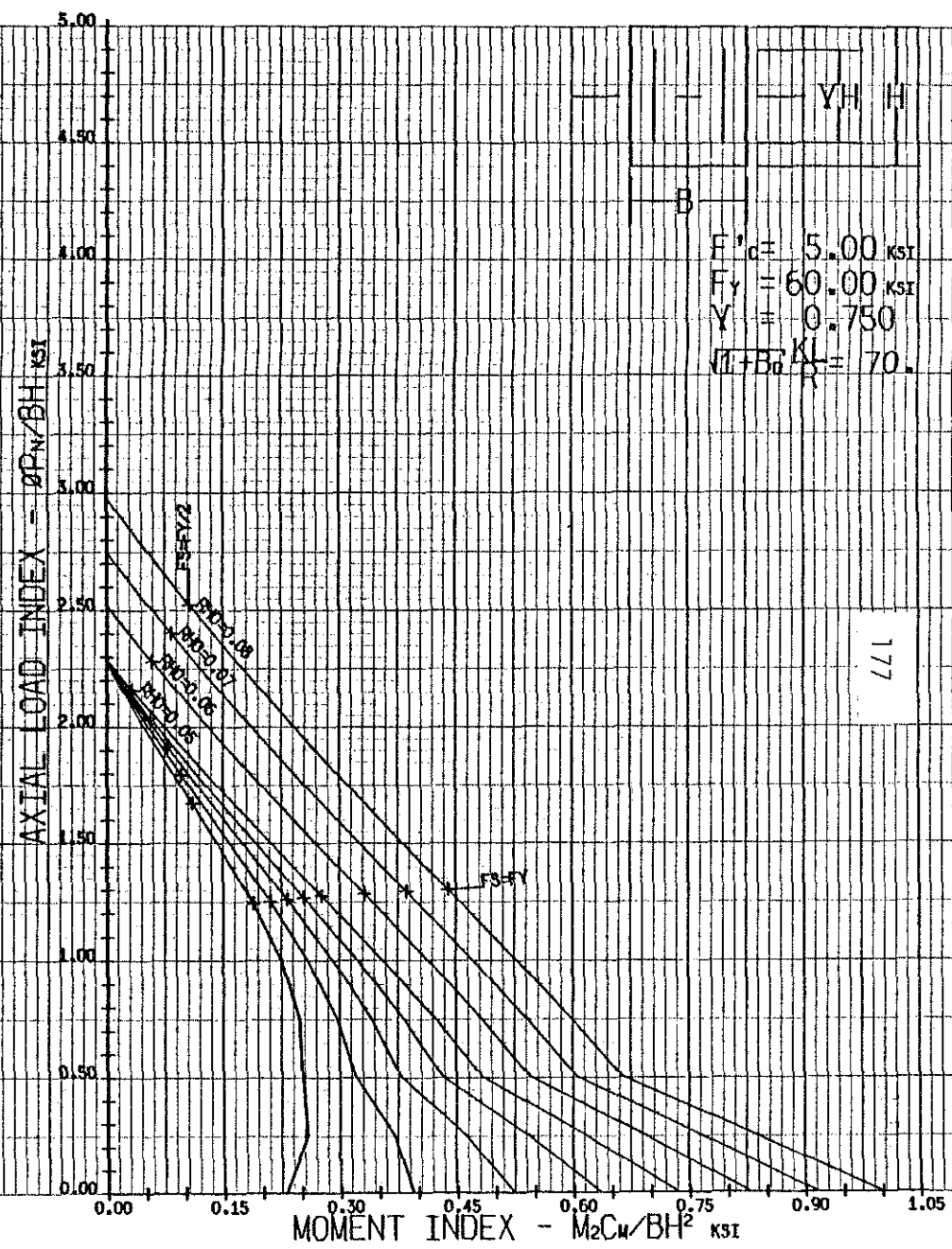
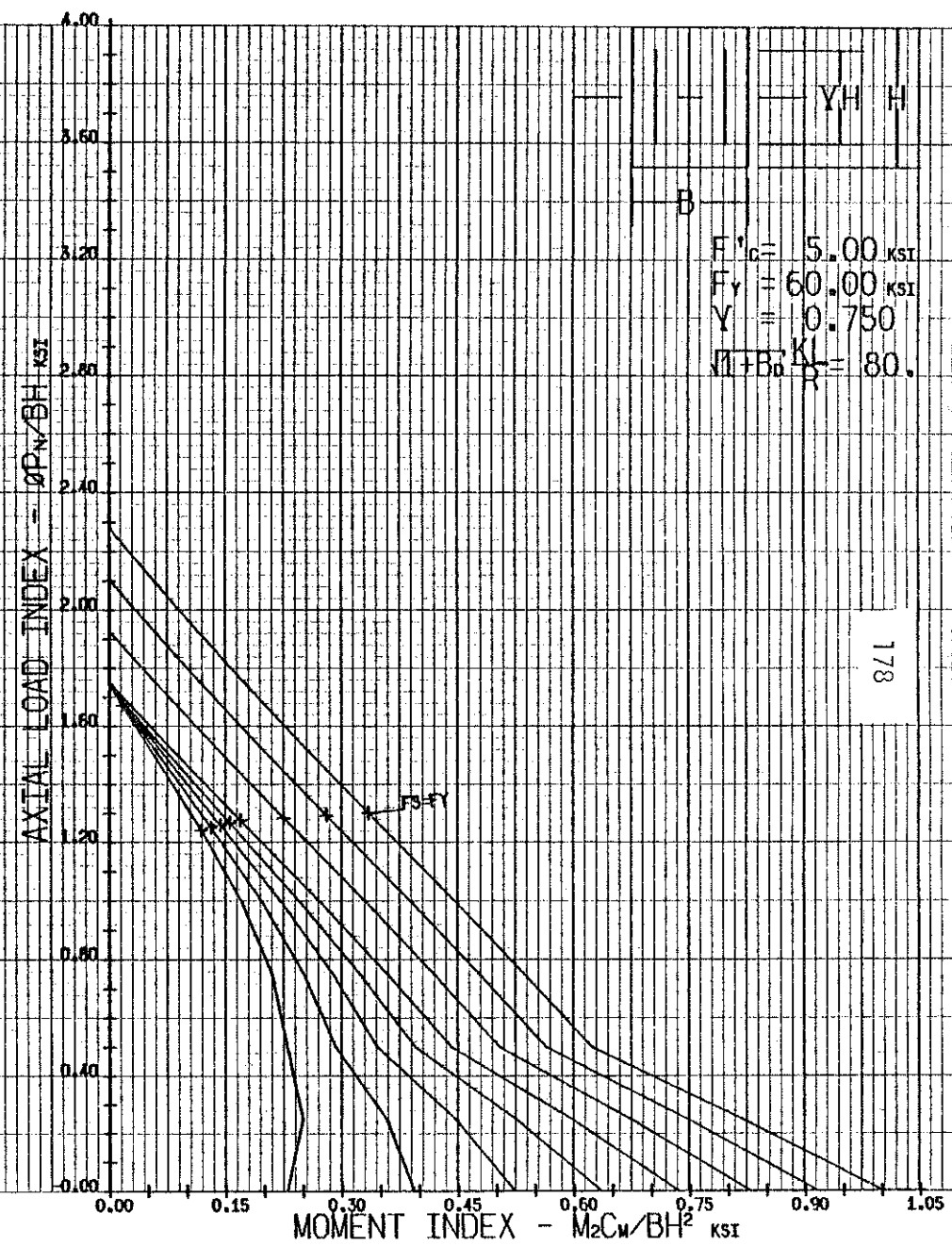
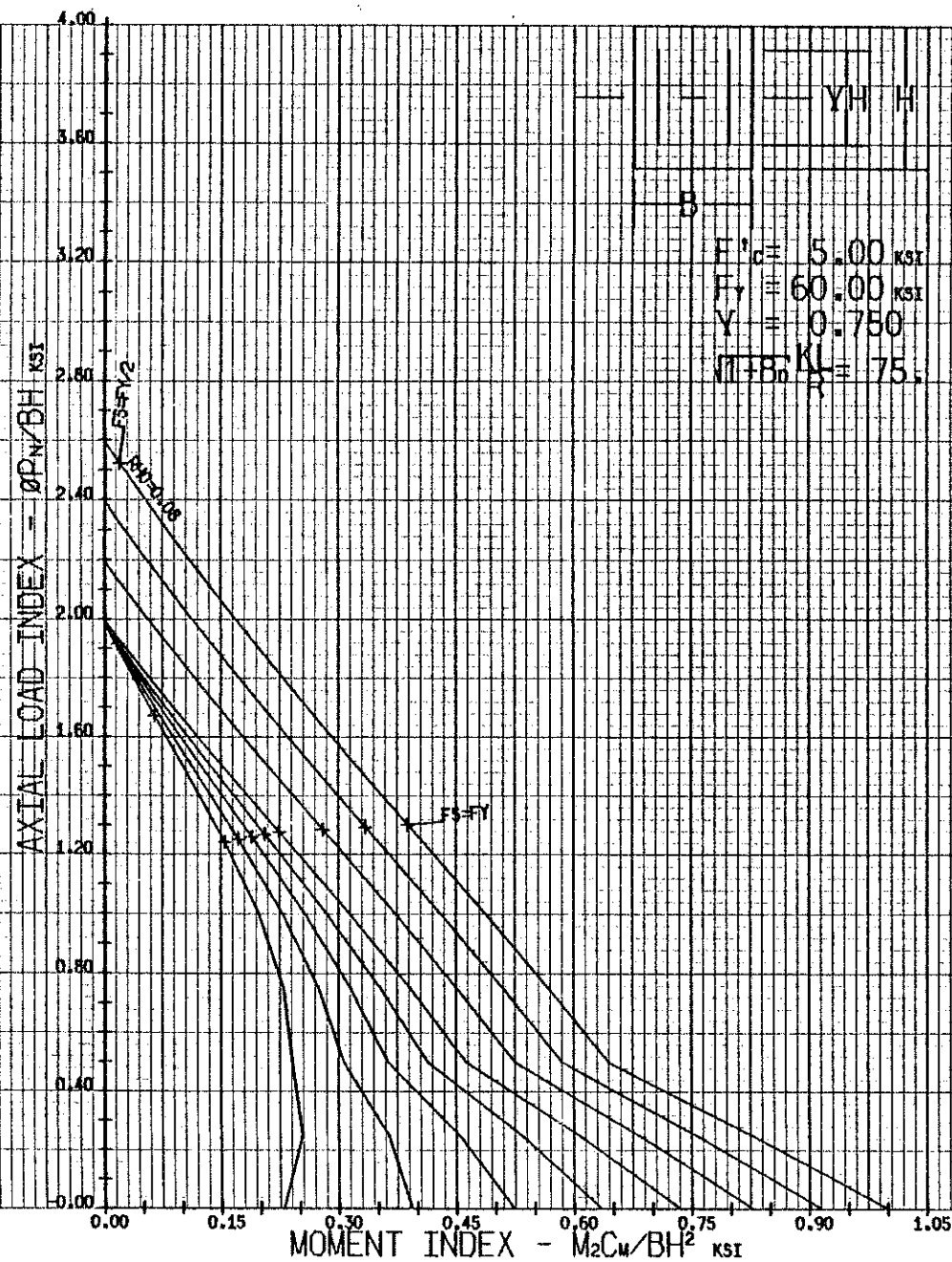


Fig. L5-60.75-70 - Interaction Diagram



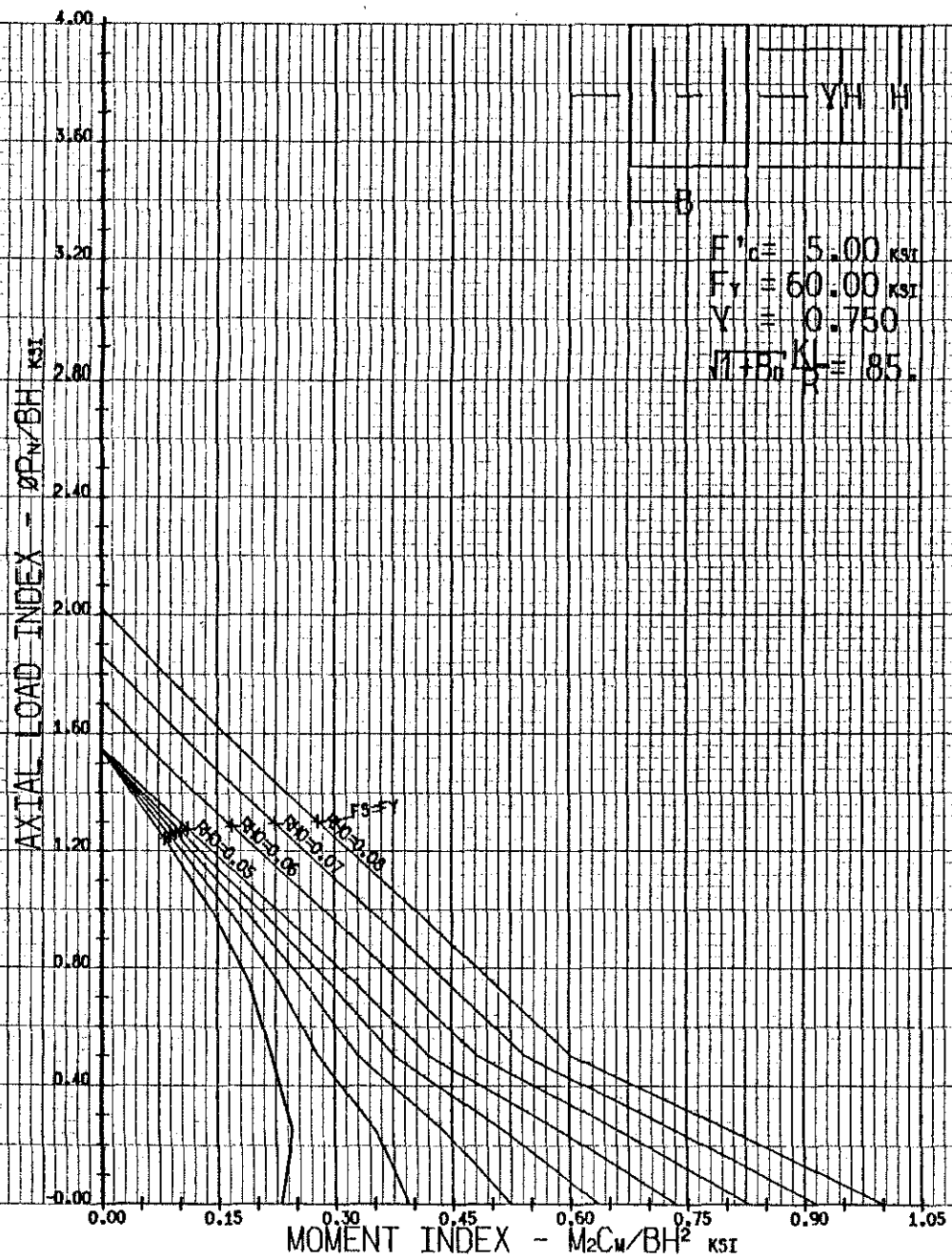


Fig. L5-60.75-85 - Interaction Diagram

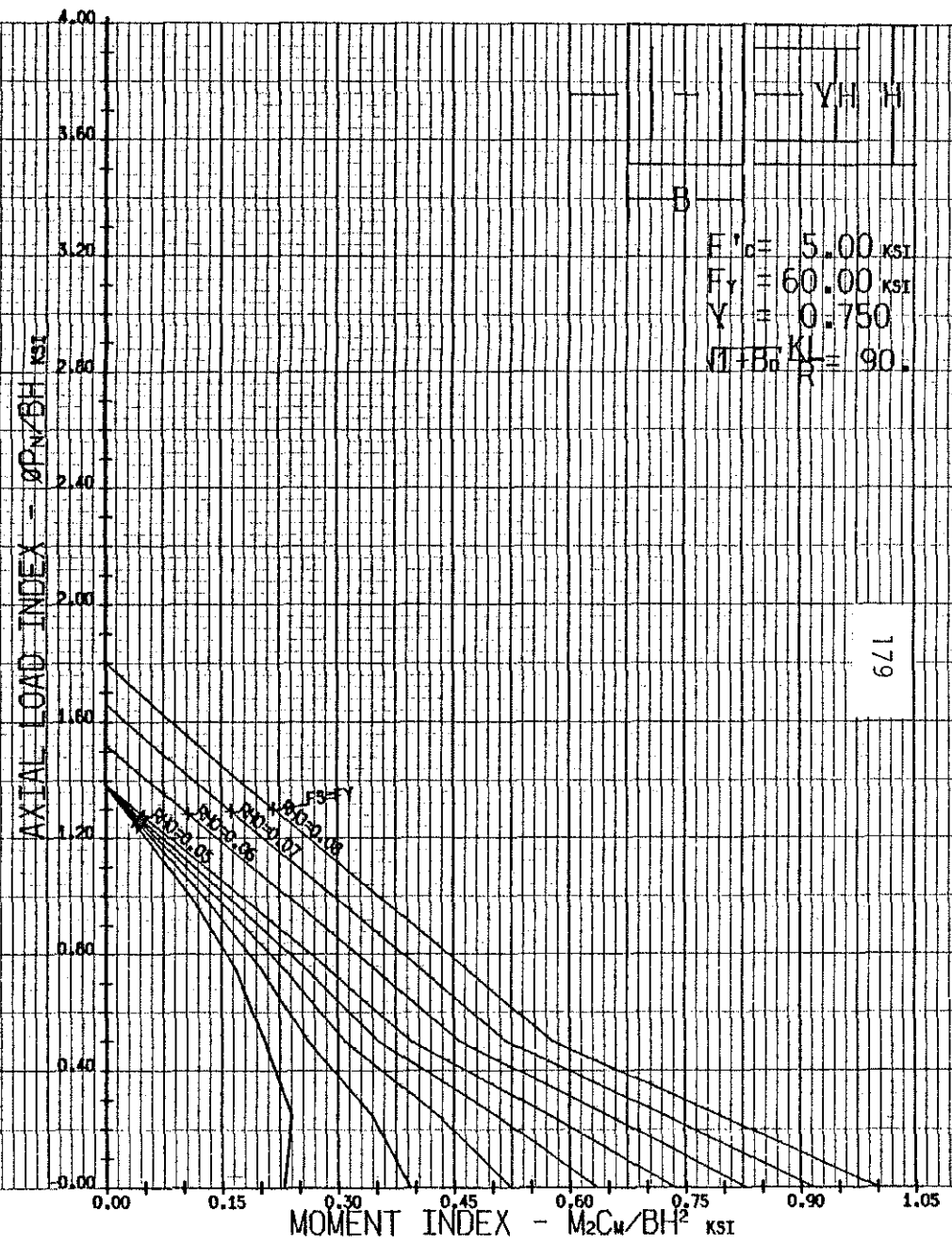


Fig. L5-60.75-90 - Interaction Diagram

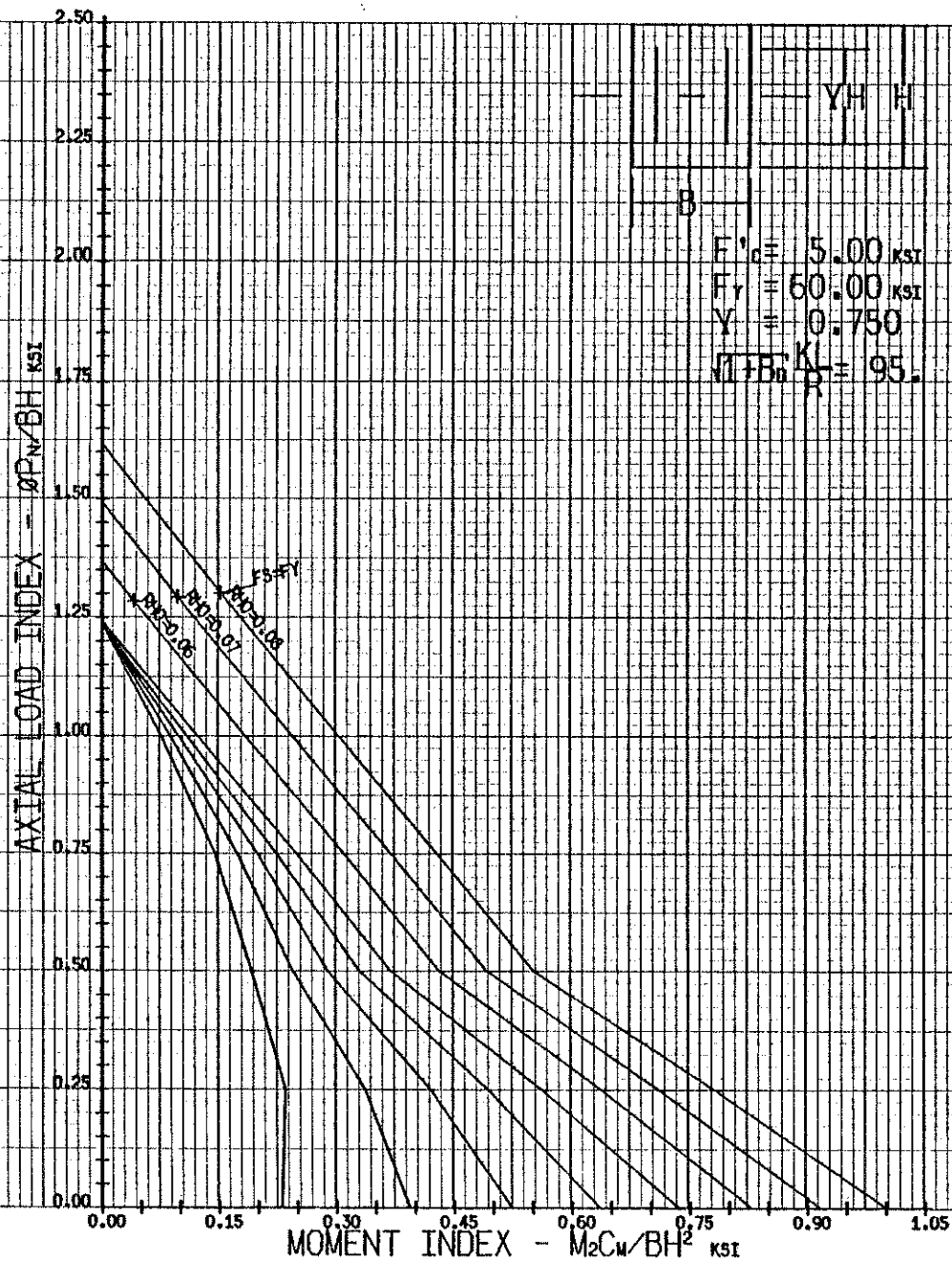


Fig. L5-60.75-95 - Interaction Diagram

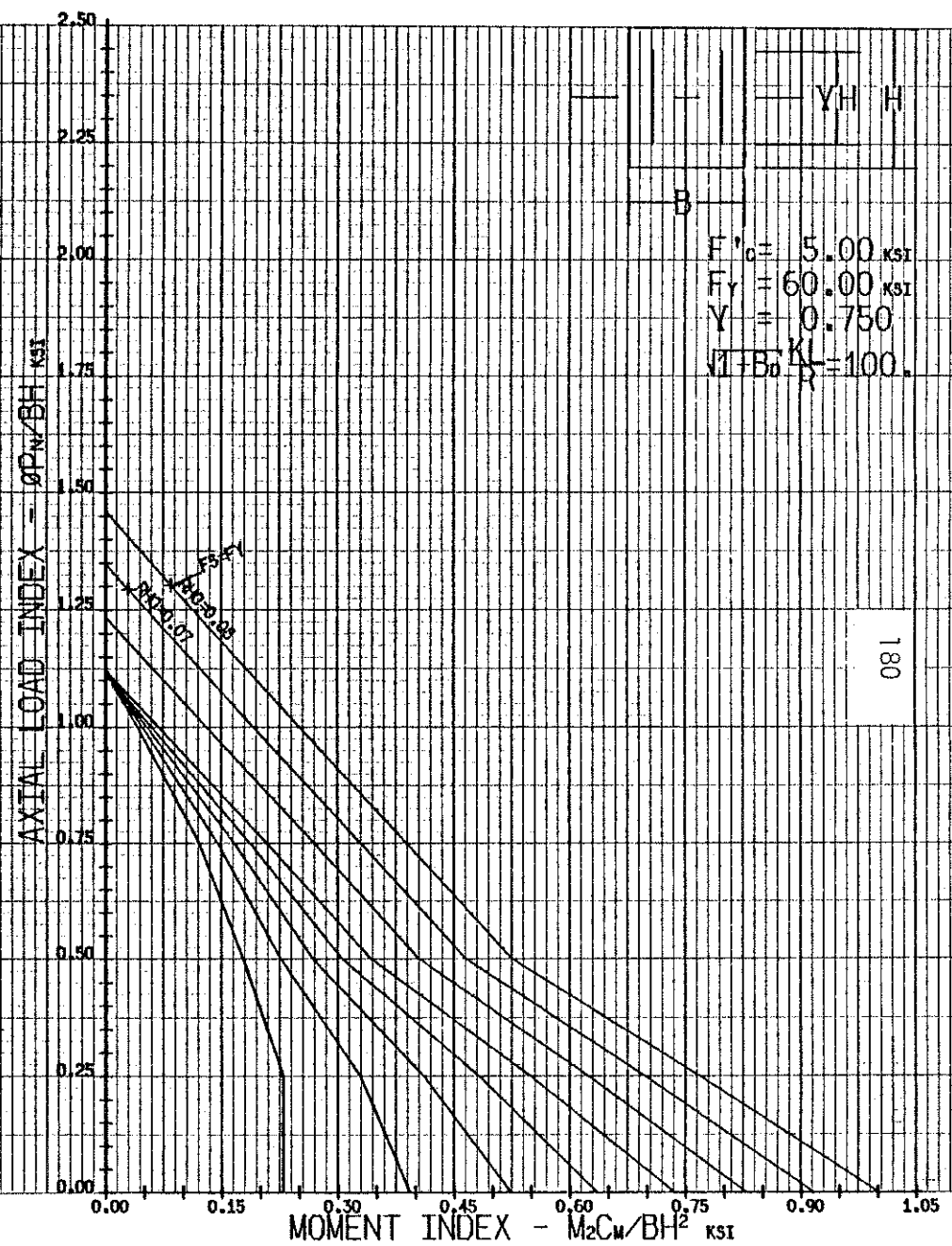


Fig. L5-60.75-100 - Interaction Diagram

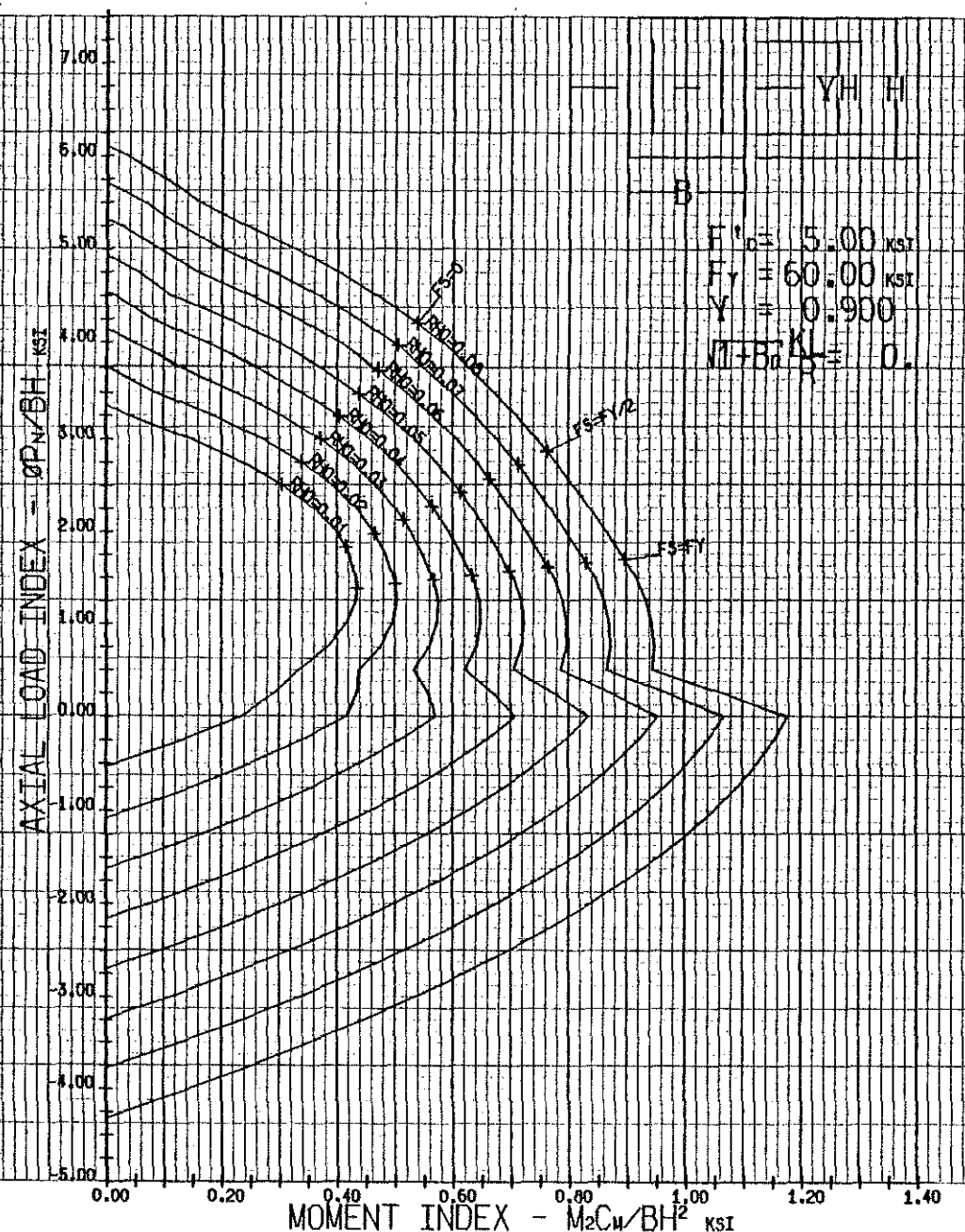


Fig. L5-60.90-0 - Interaction Diagram

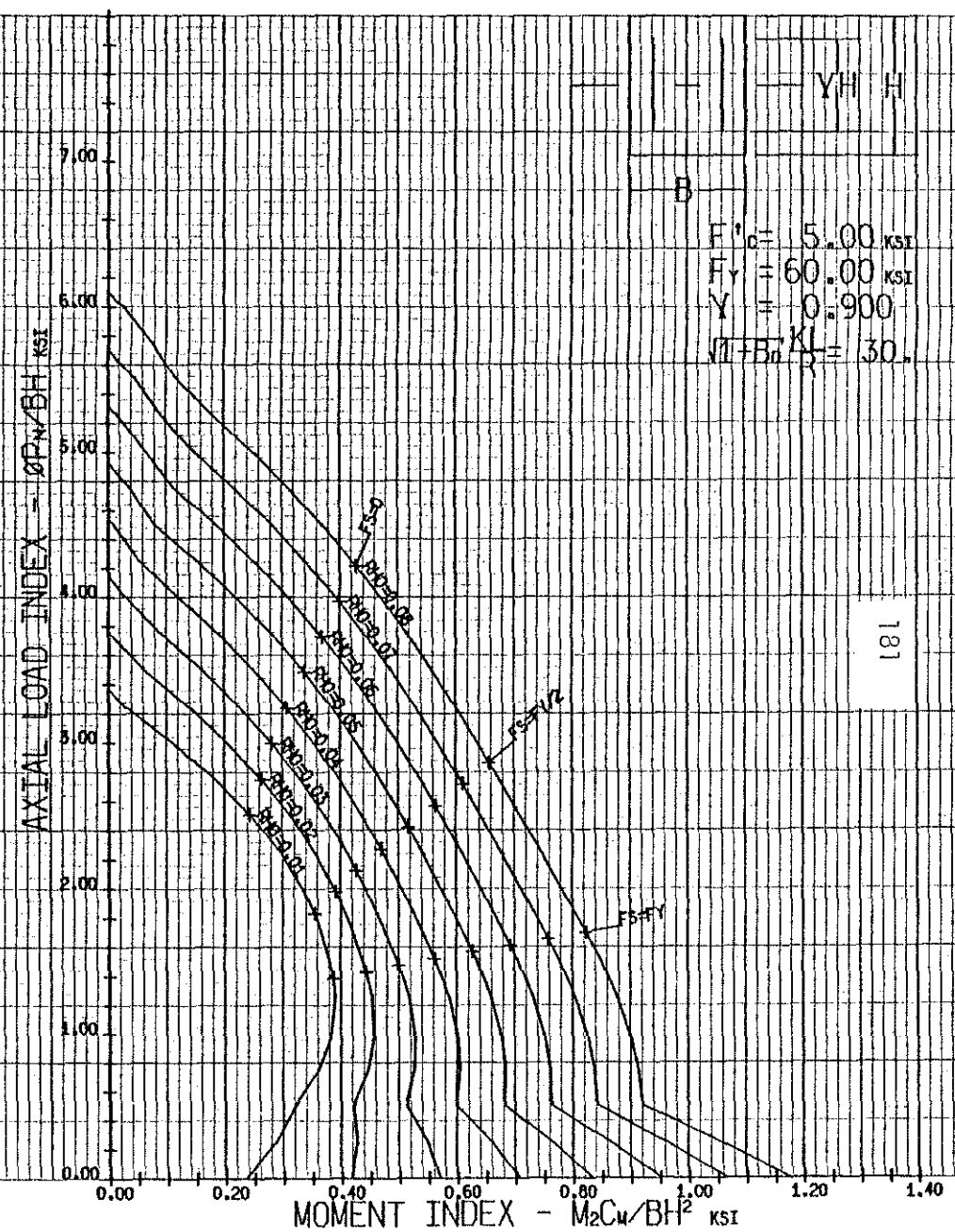


Fig. L5-60.90-30 - Interaction Diagram



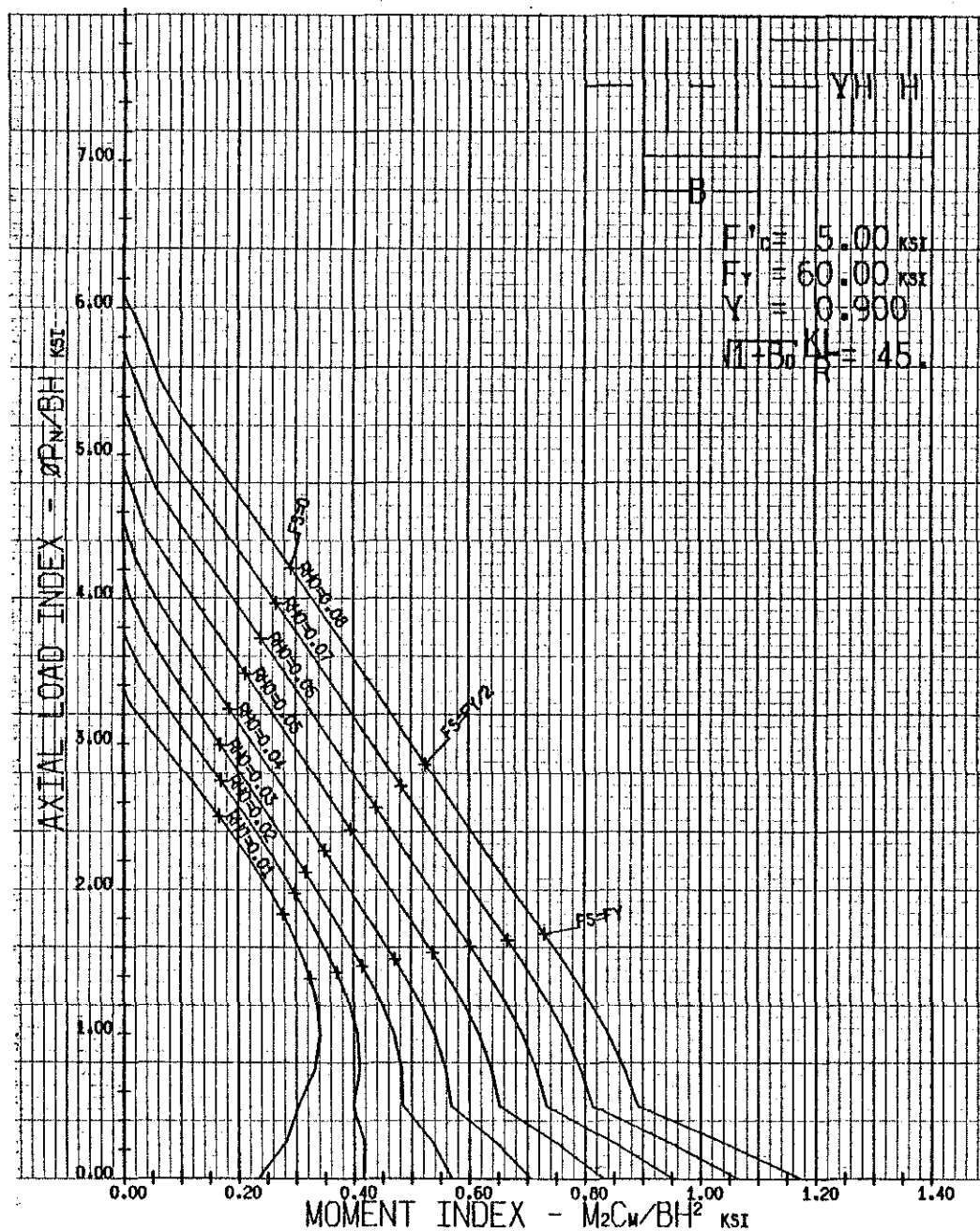


Fig. L5-60.90-45 - Interaction Diagram

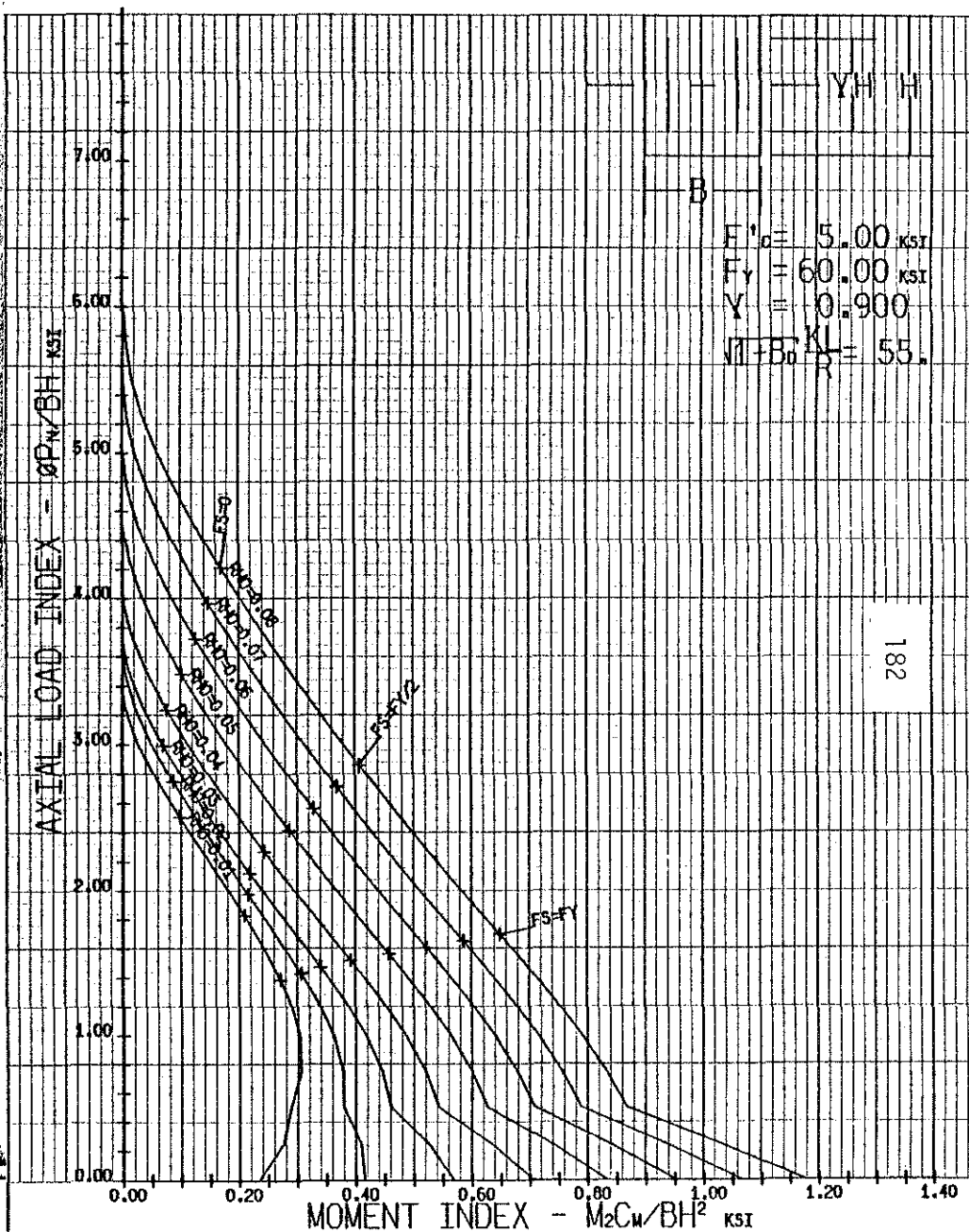


Fig. L5-60.90-55 - Interaction Diagram

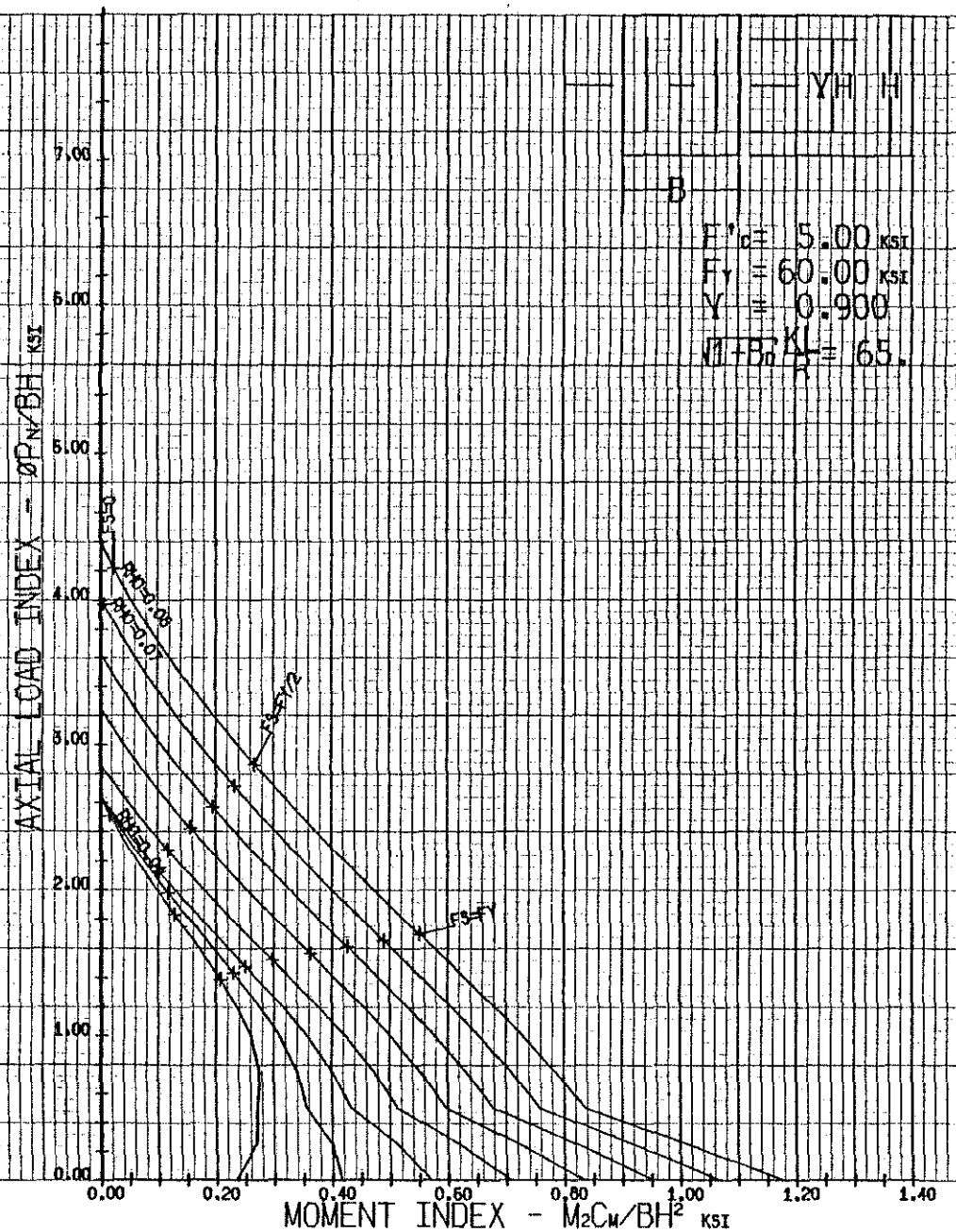


Fig. L5-60.90-65 - Interaction Diagram

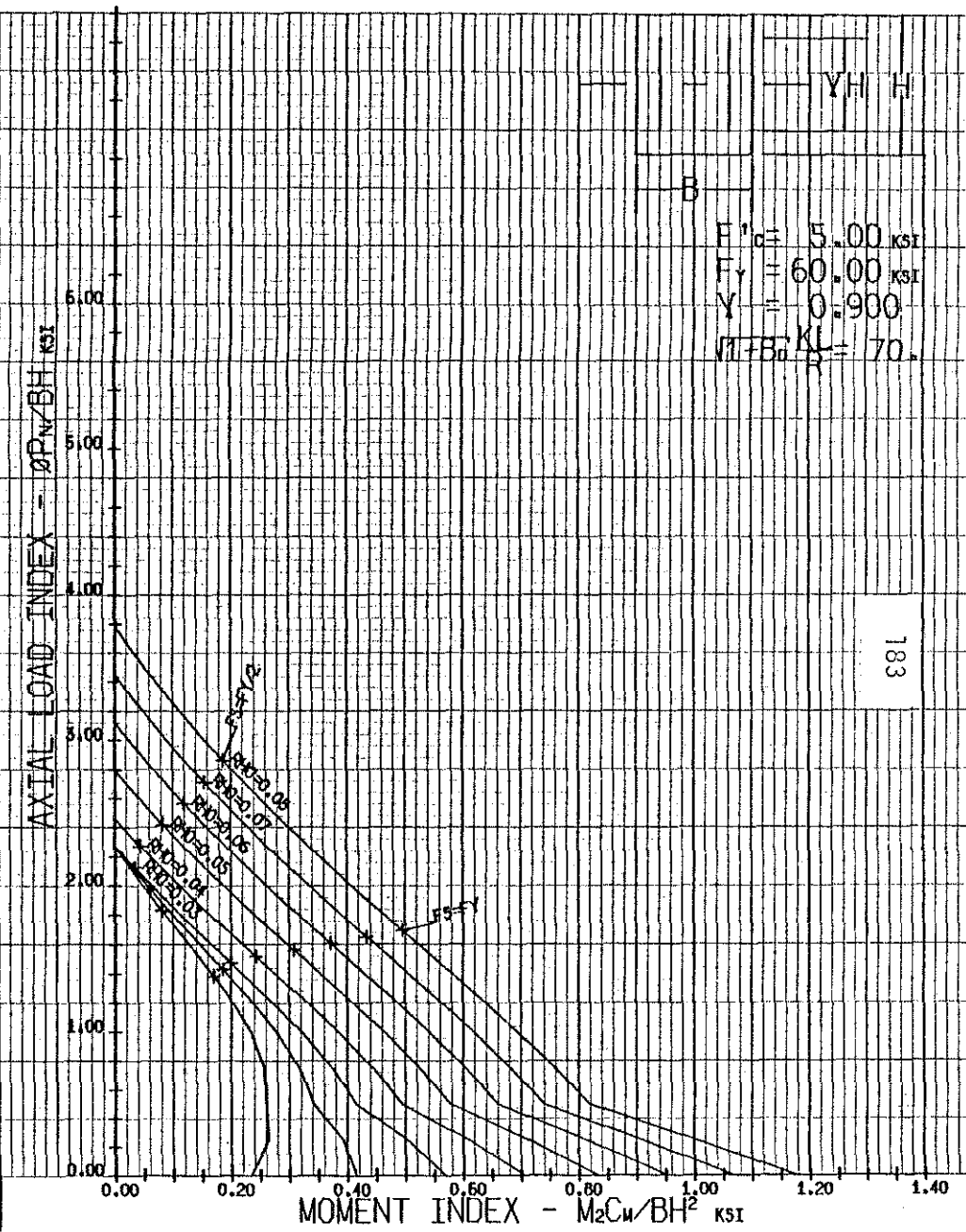


Fig. L5-60.90-70 - Interaction Diagram



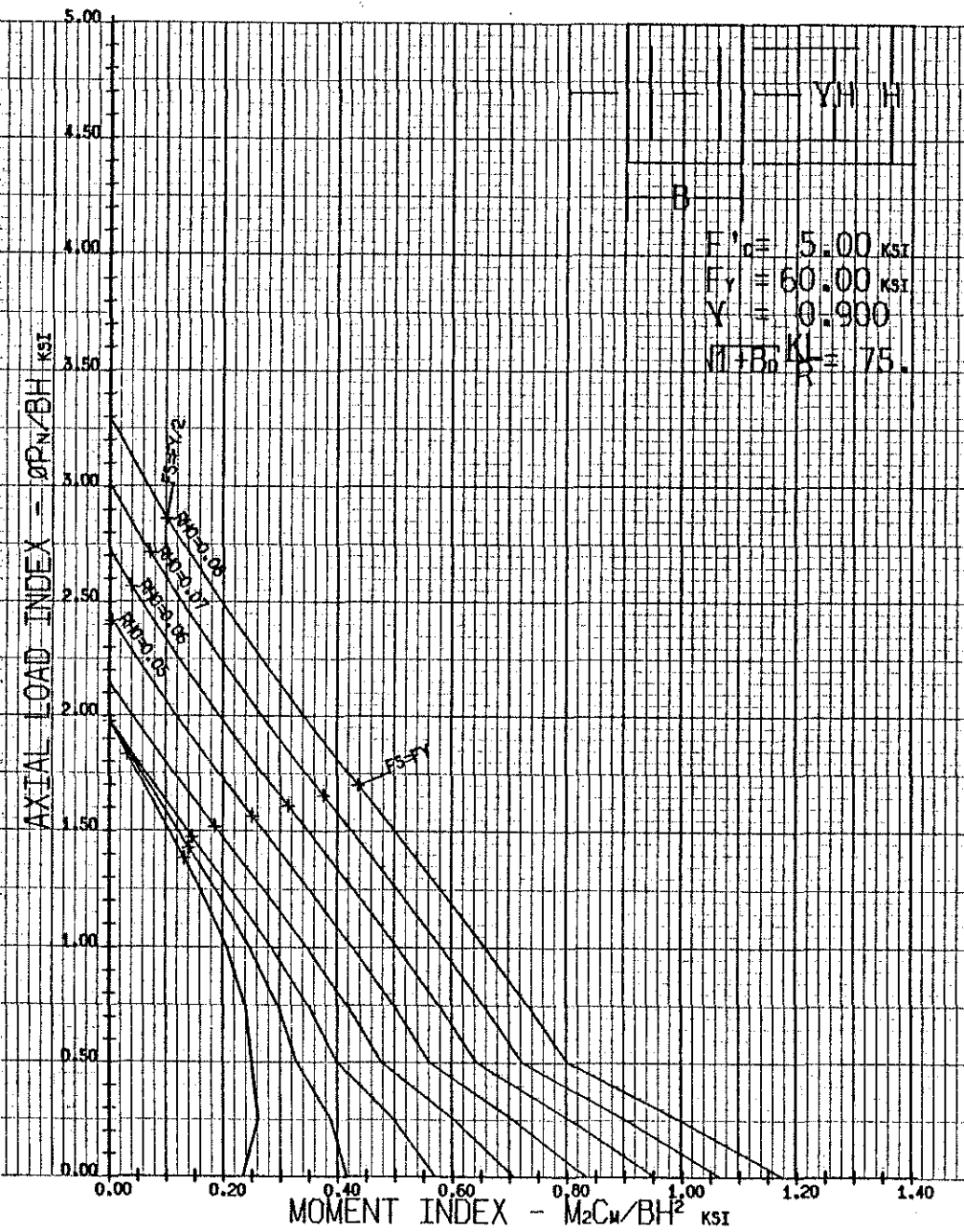


Fig. L5-60.90-75 - Interaction Diagram

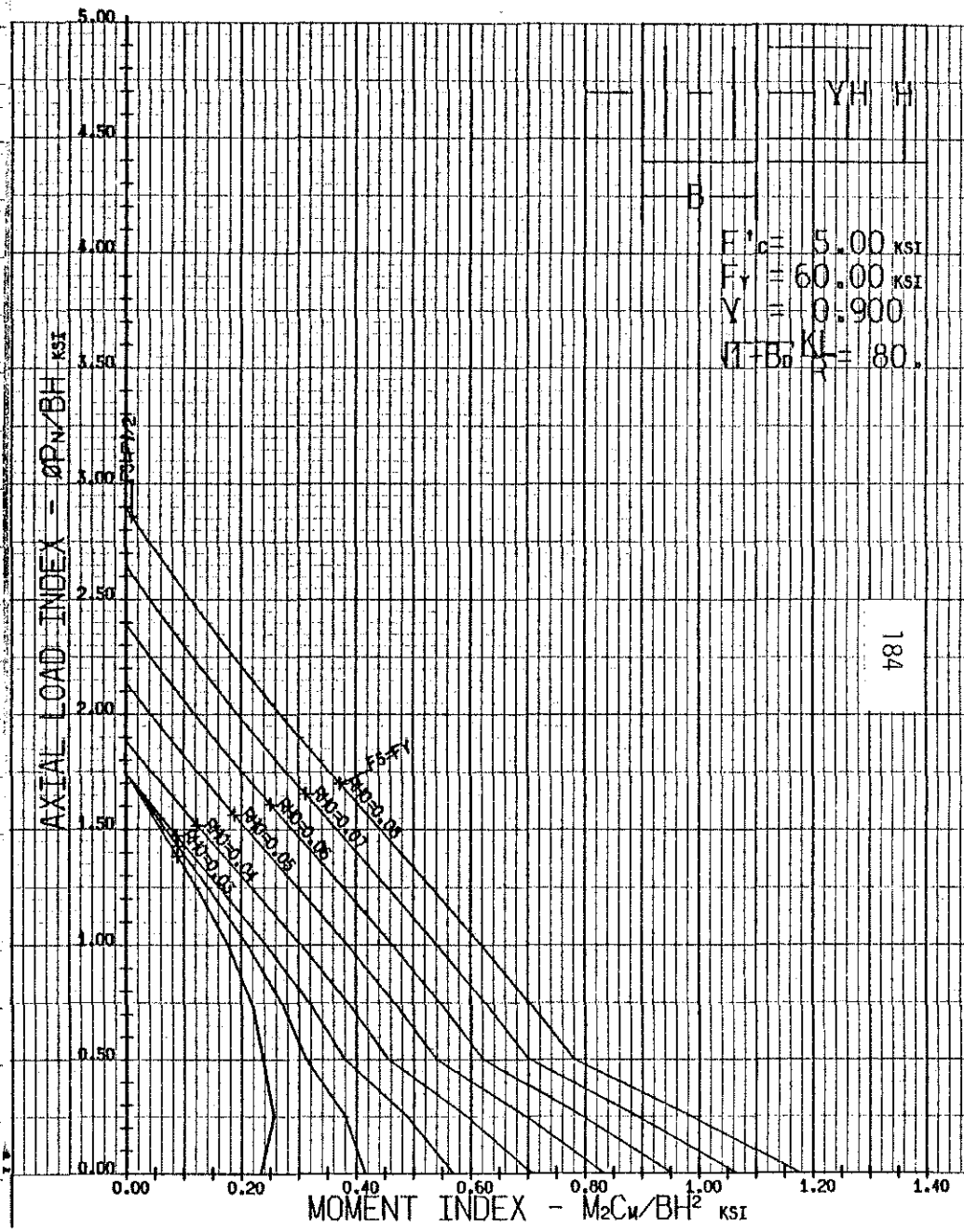


Fig. L5-60.90-80 - Interaction Diagram

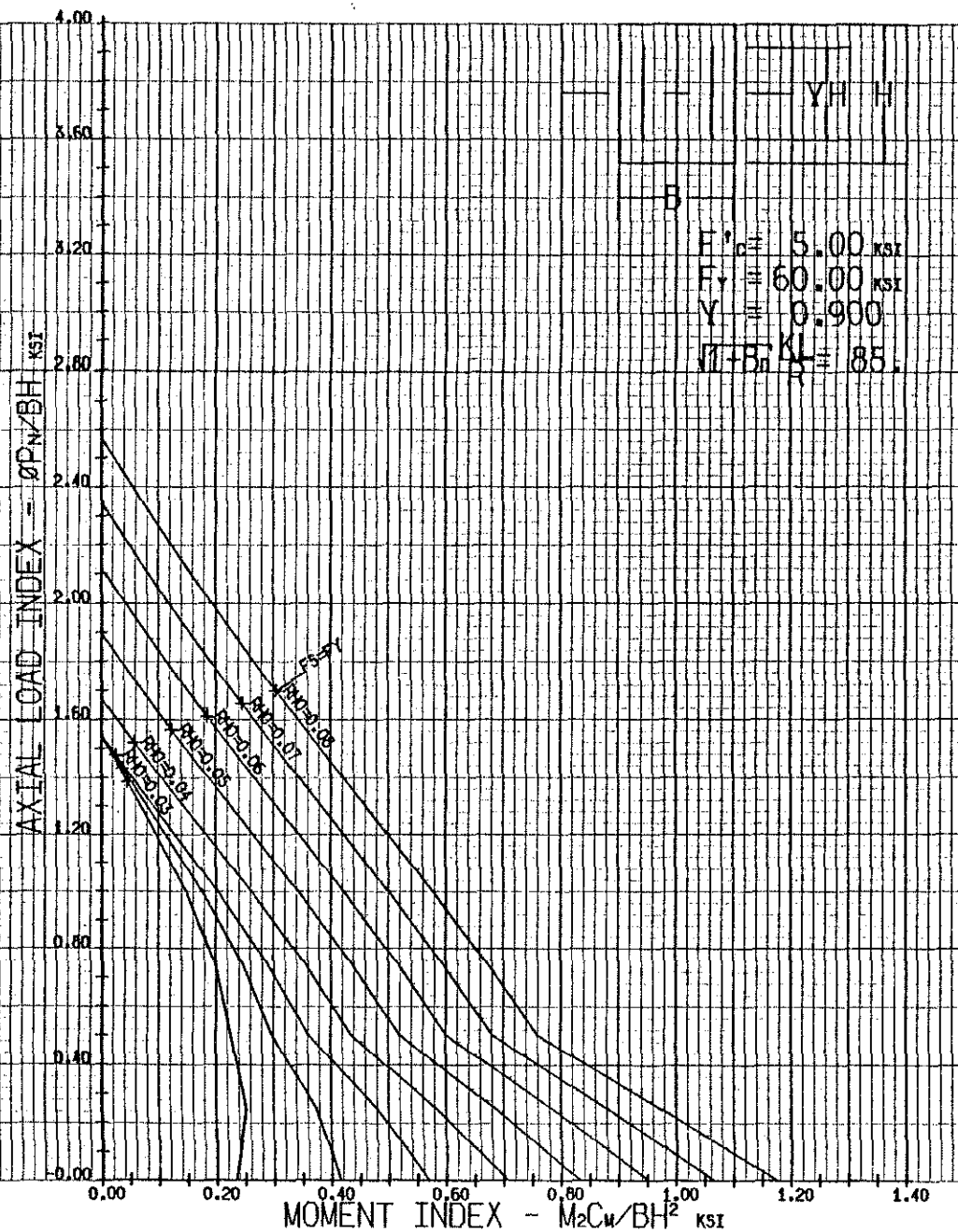


Fig. L5-60.90-85 - Interaction Diagram

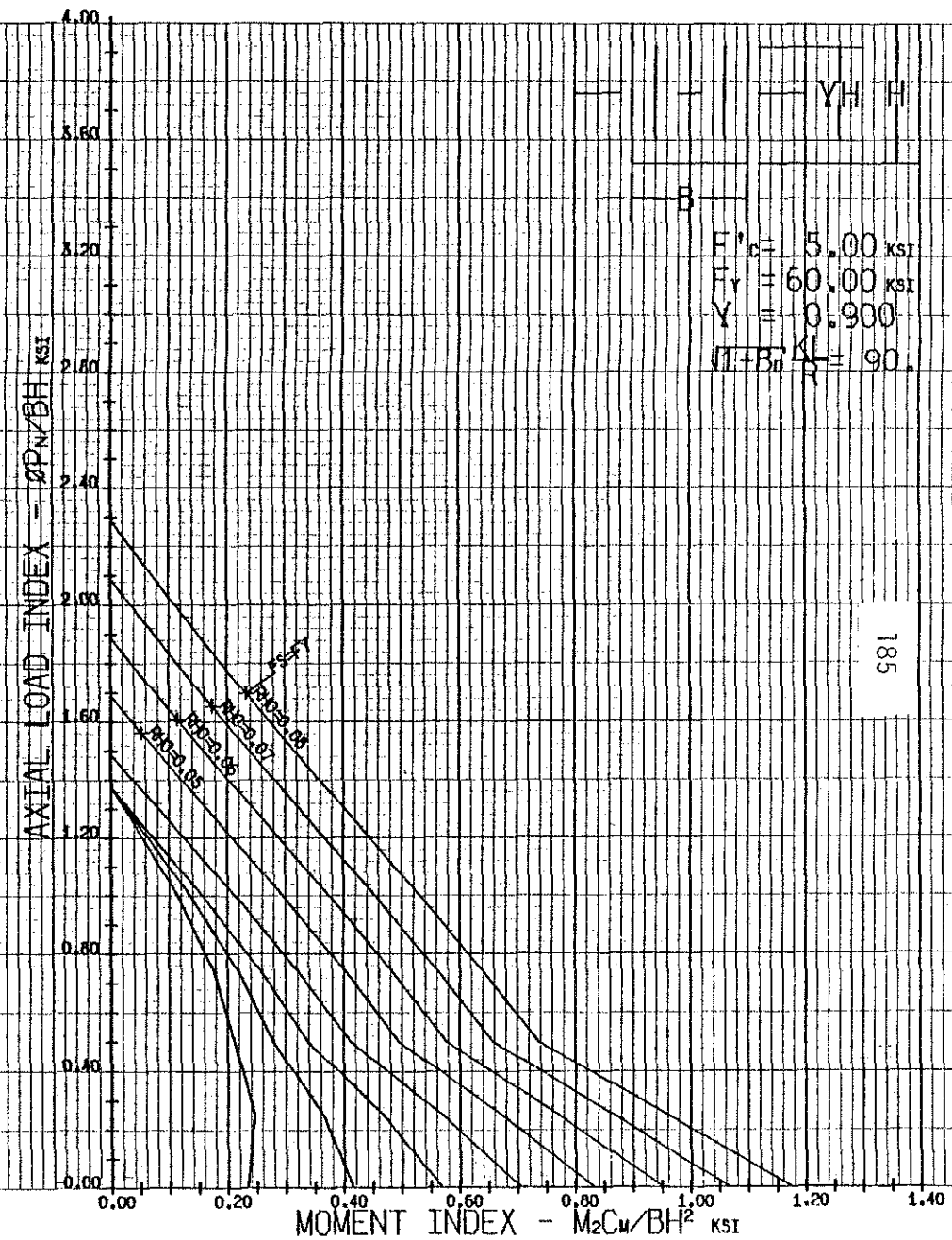


Fig. L5-60.90-90 - Interaction Diagram

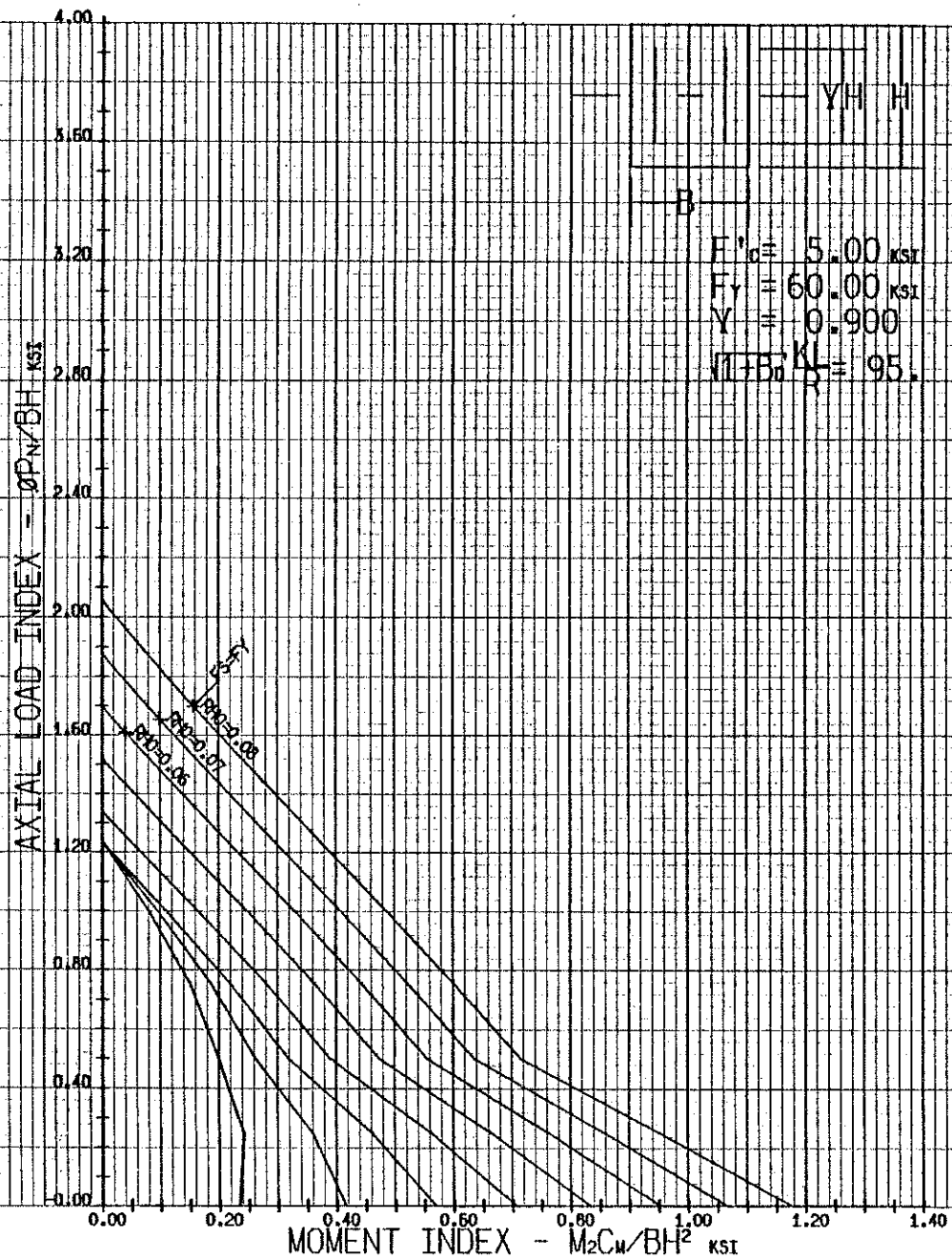


Fig. L5-60.90-95 - Interaction Diagram

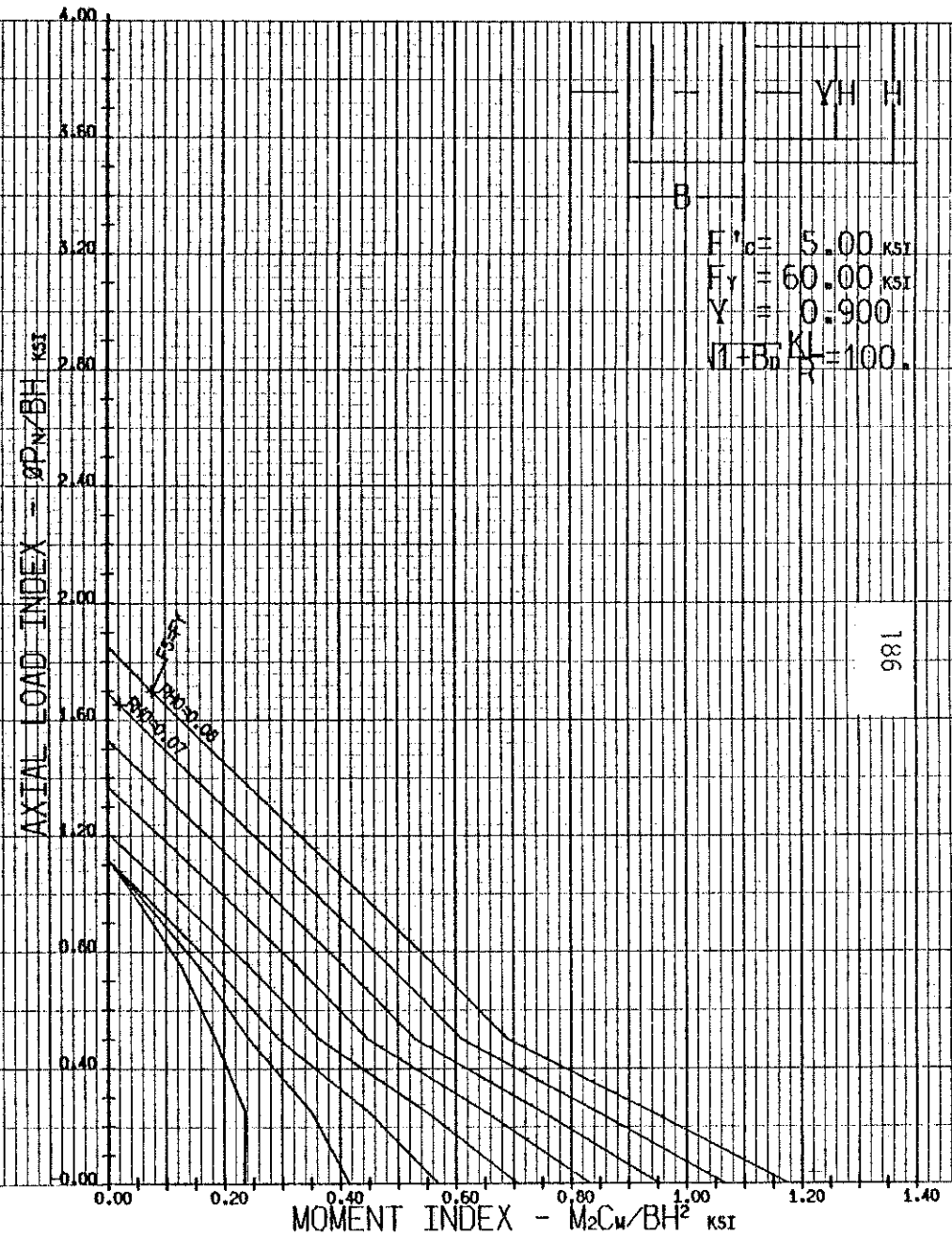
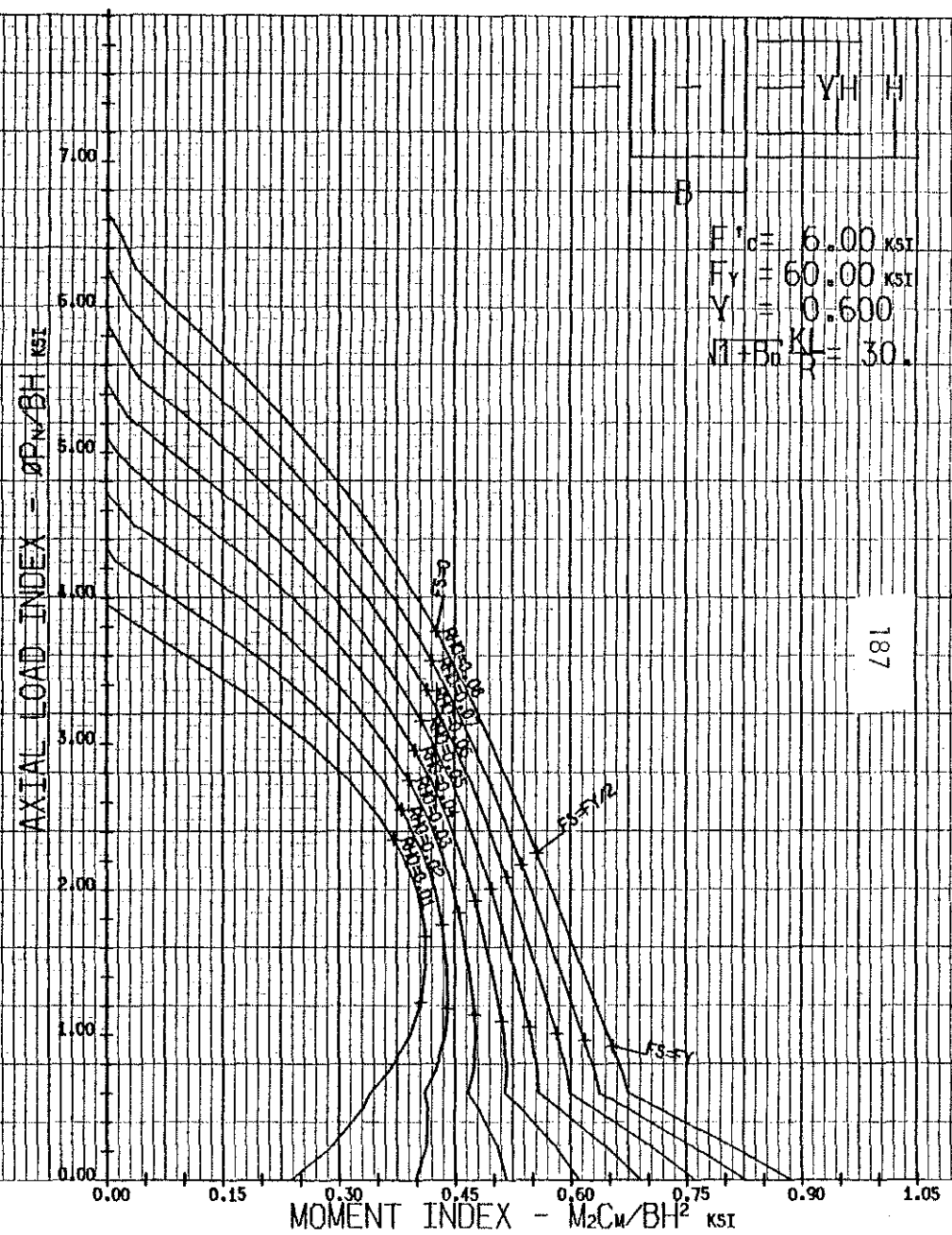
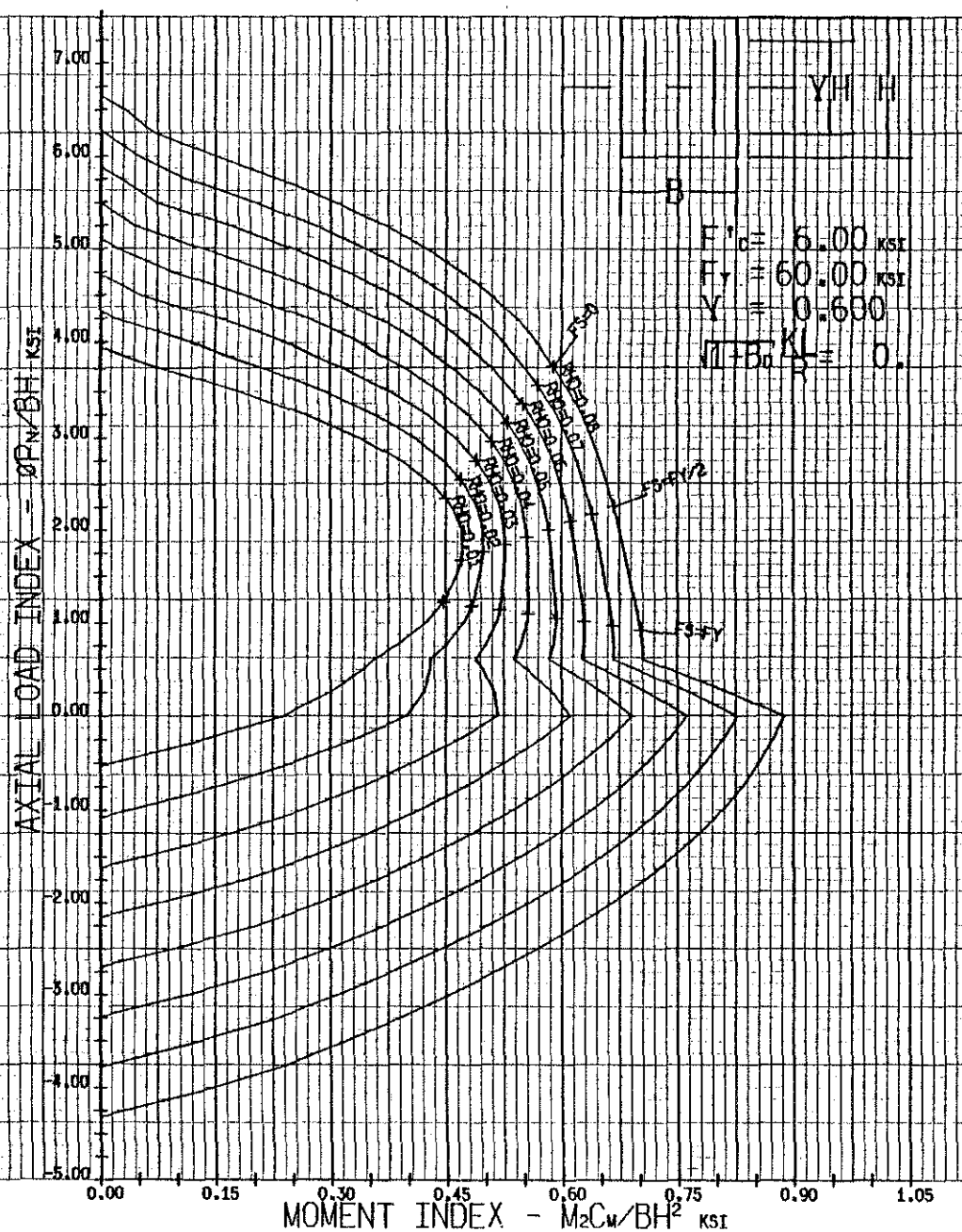


Fig. L5-60.90-100 - Interaction Diagram



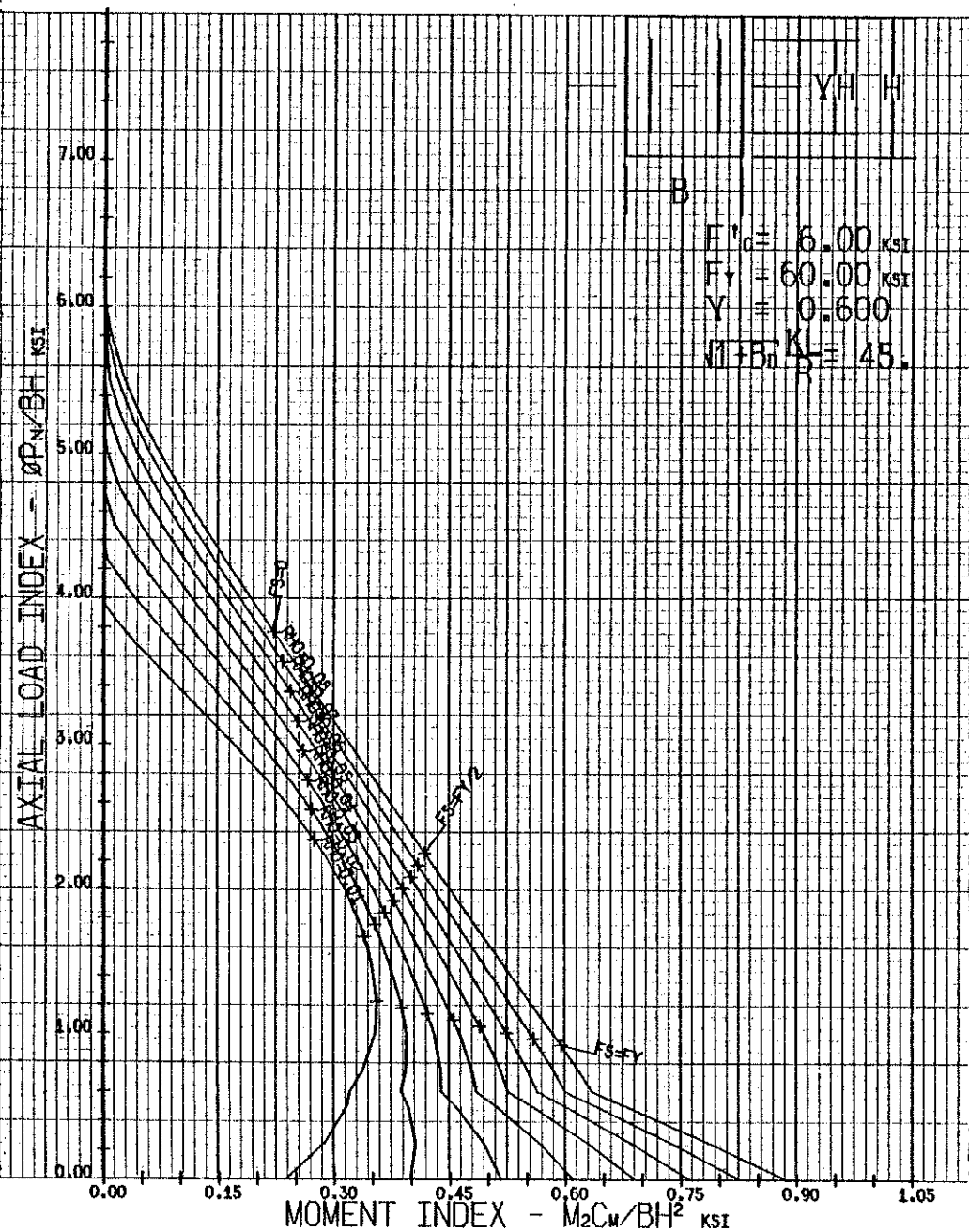


Fig. L6-60.60-45 - Interaction Diagram

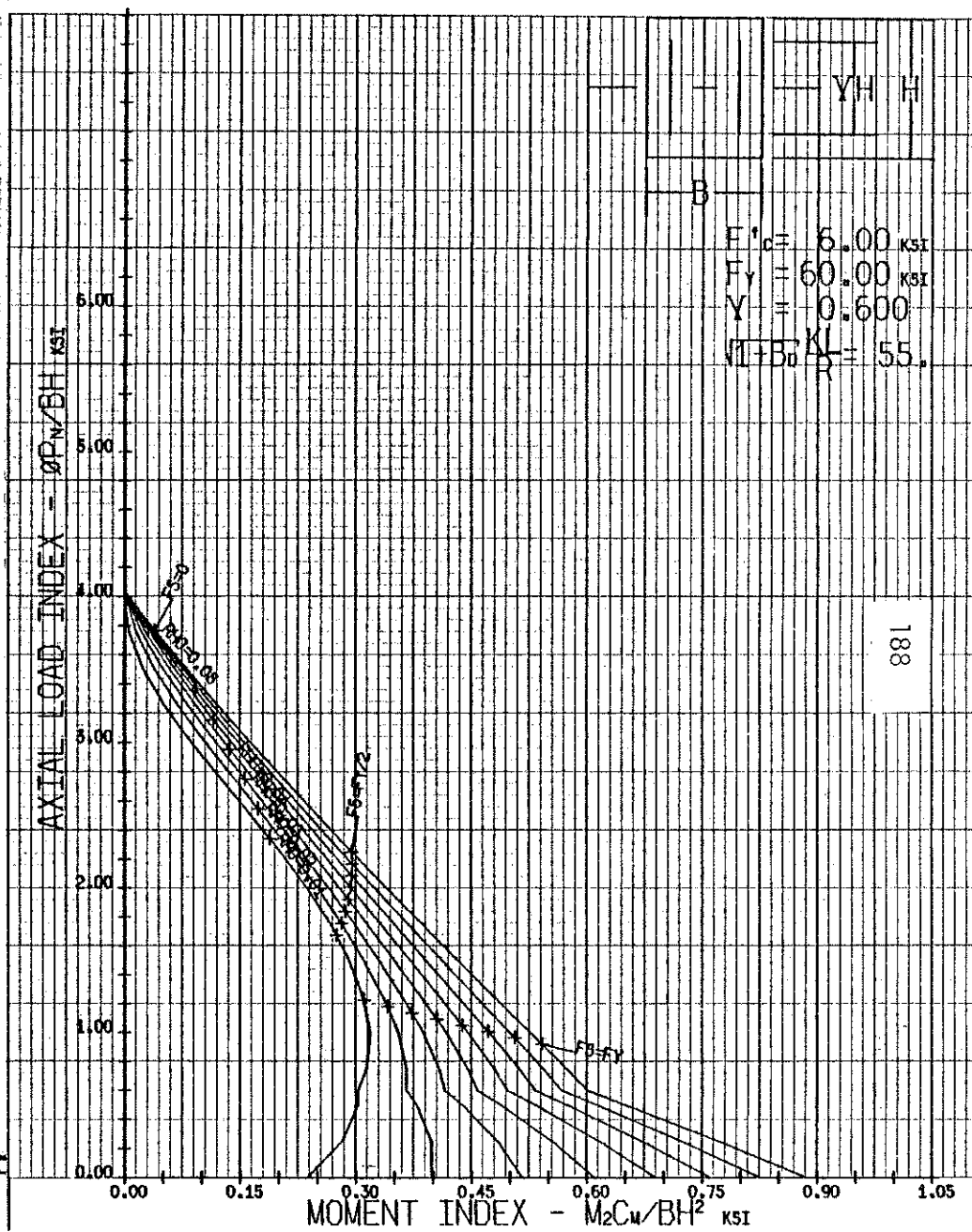


Fig. L6-60.60-55 - Interaction Diagram



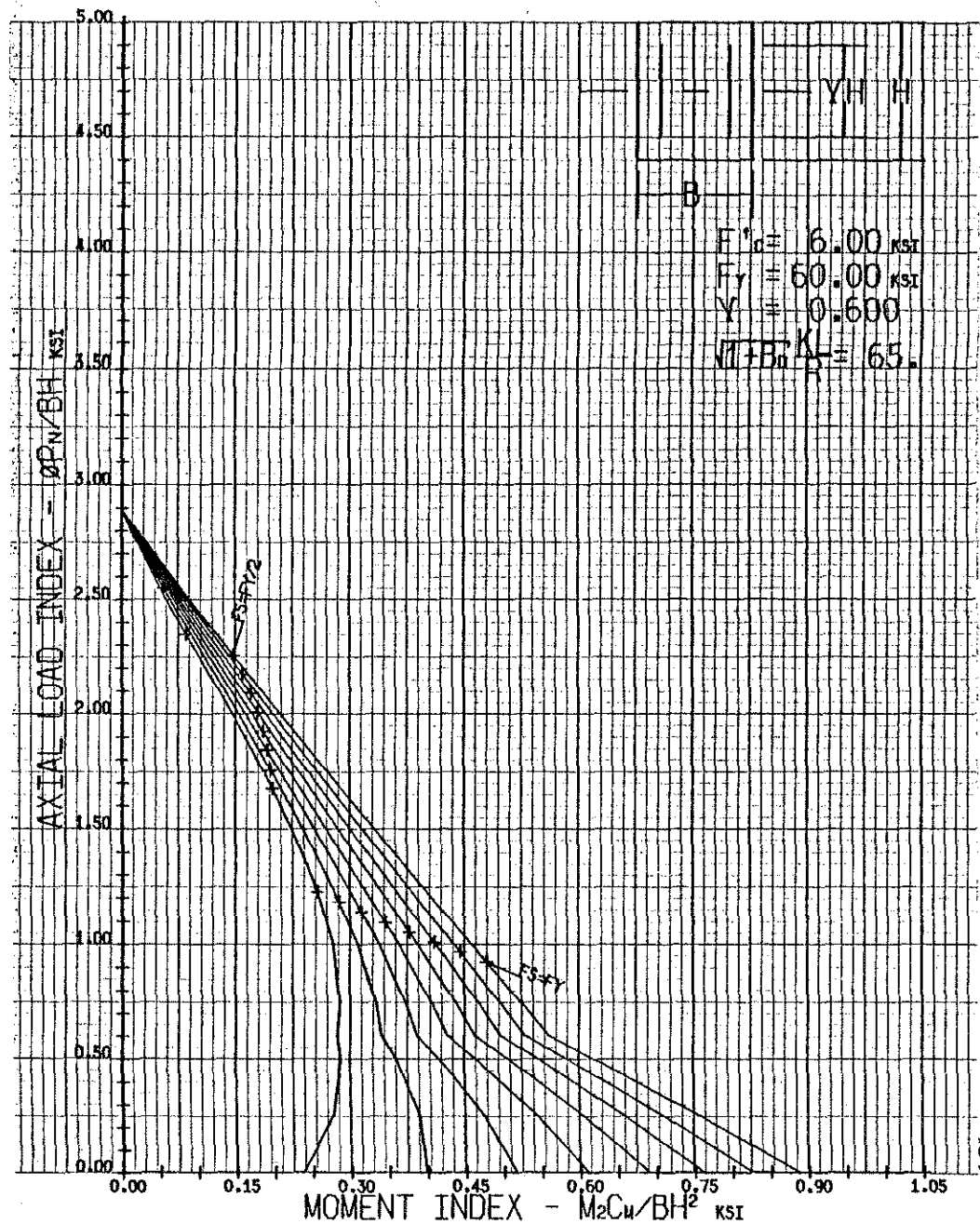


Fig. L6-60.60-65 - Interaction Diagram

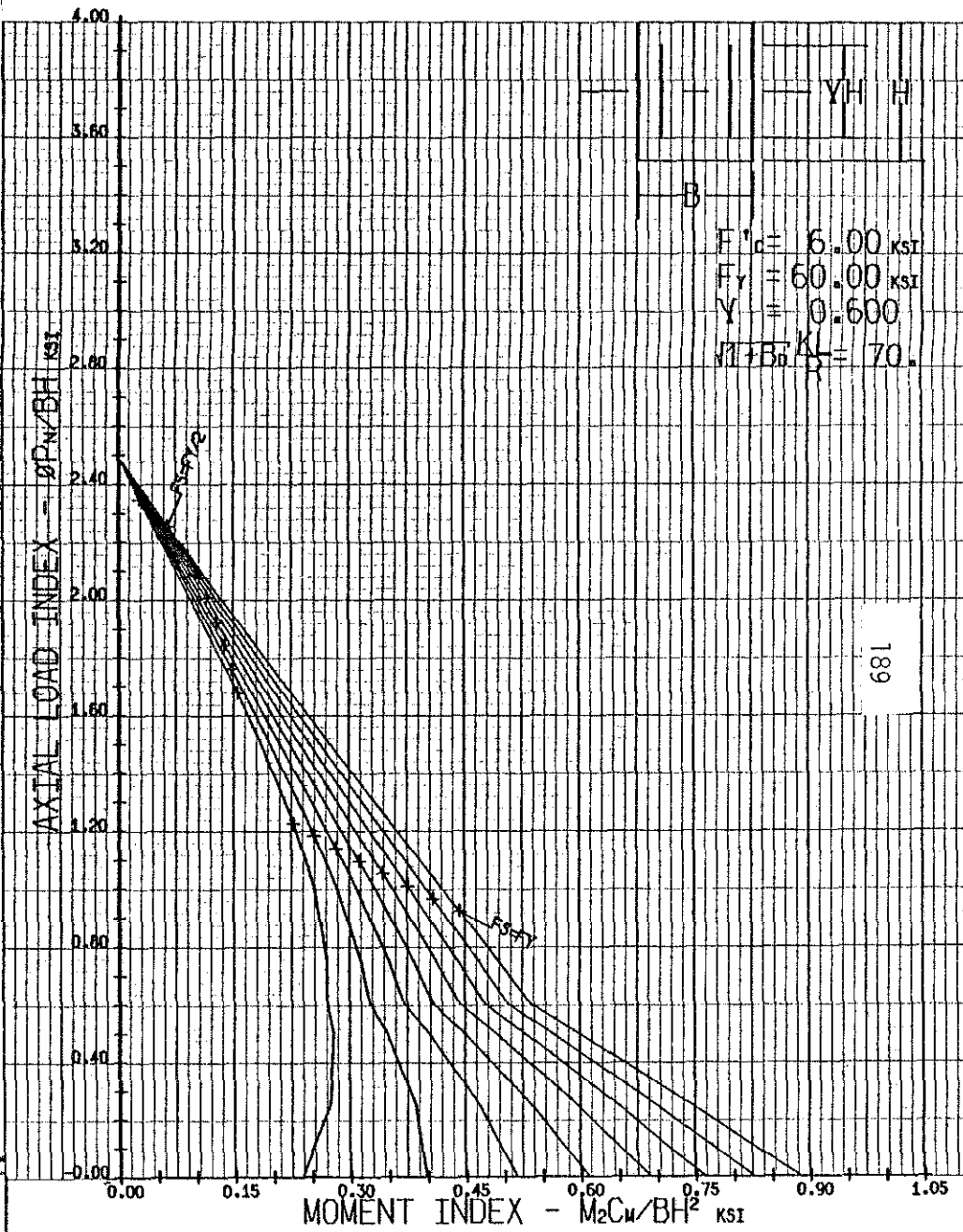


Fig. L6-60.60-70 - Interaction Diagram

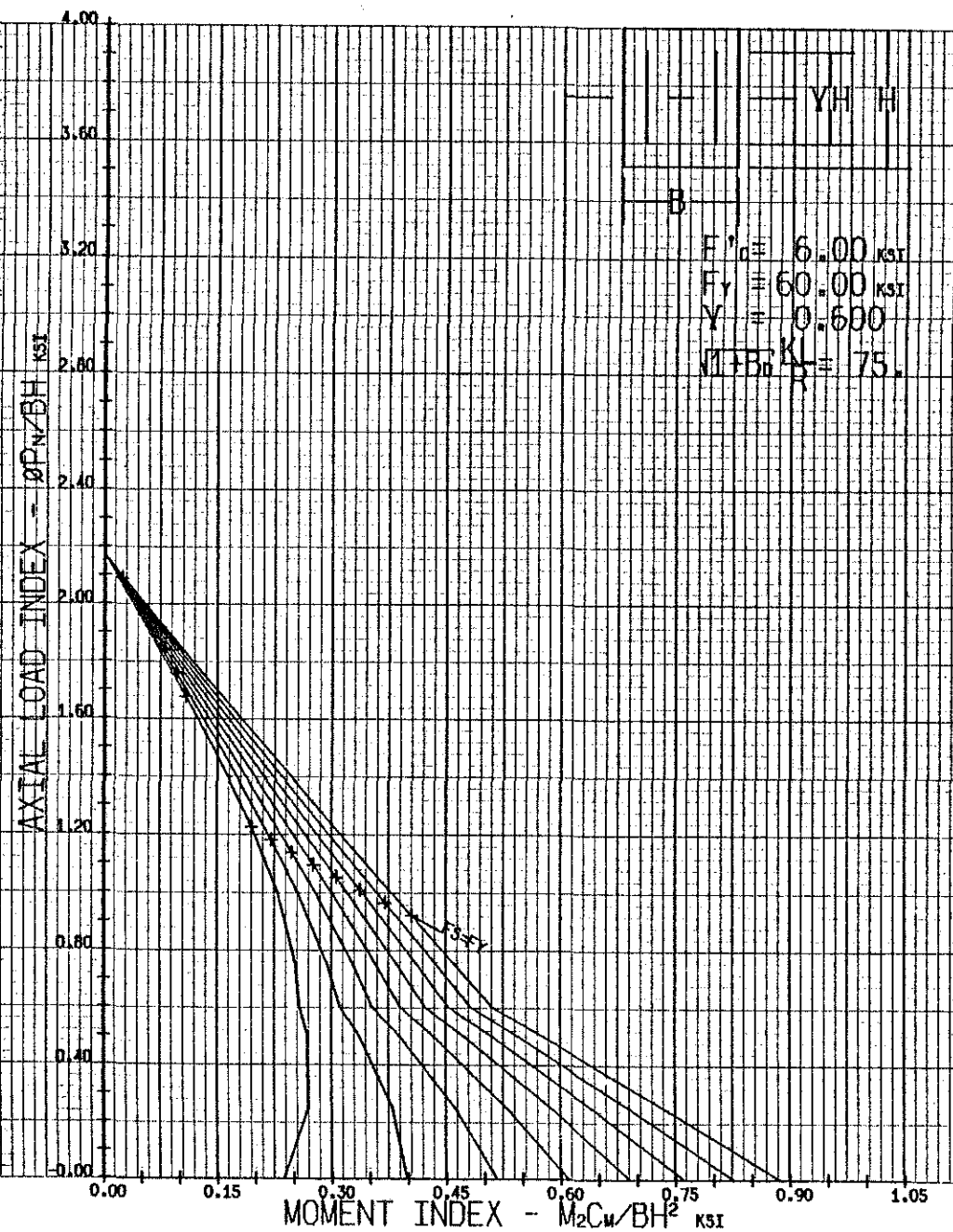


Fig. L6-60.60-75 - Interaction Diagram

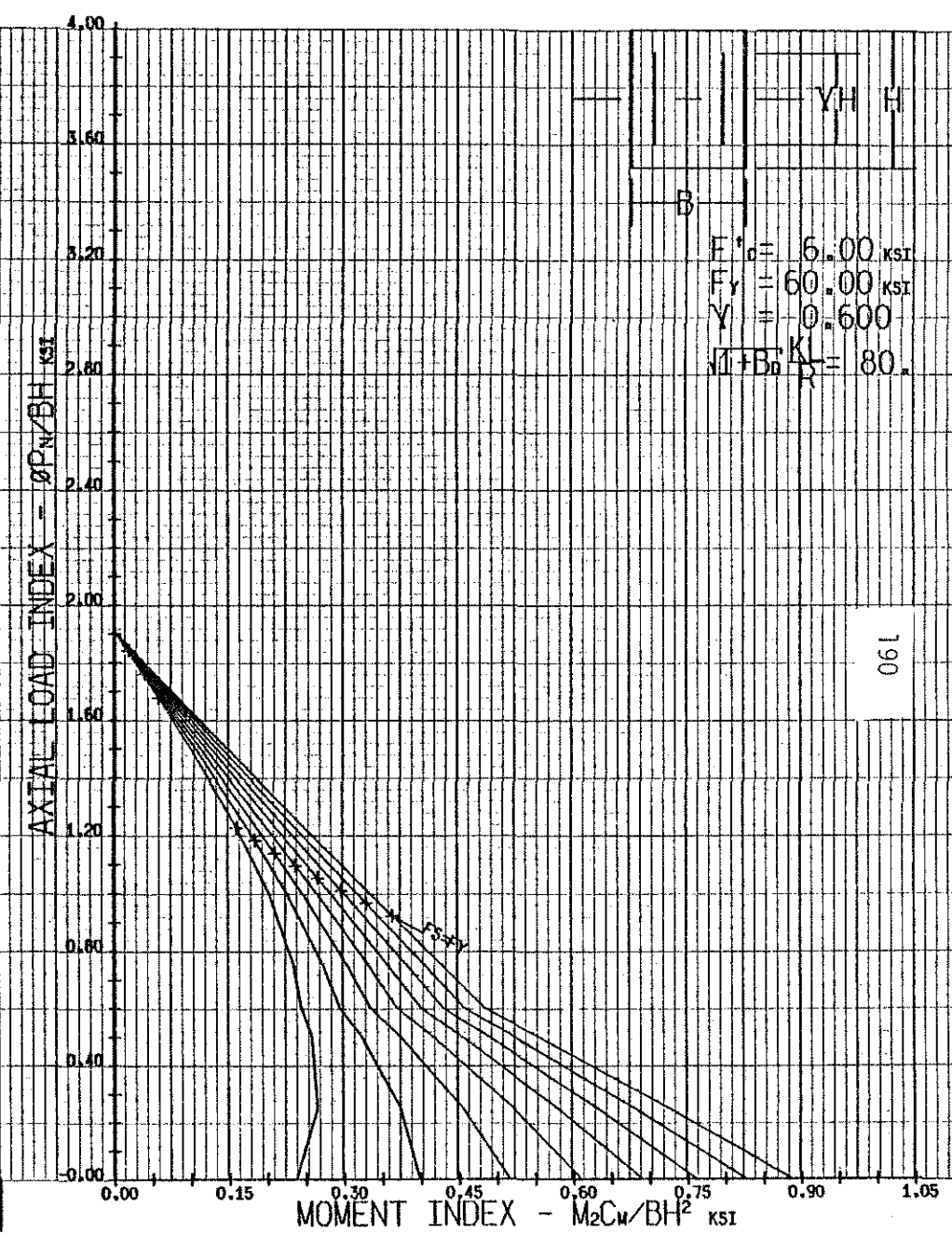


Fig. L6-60.60-80 - Interaction Diagram



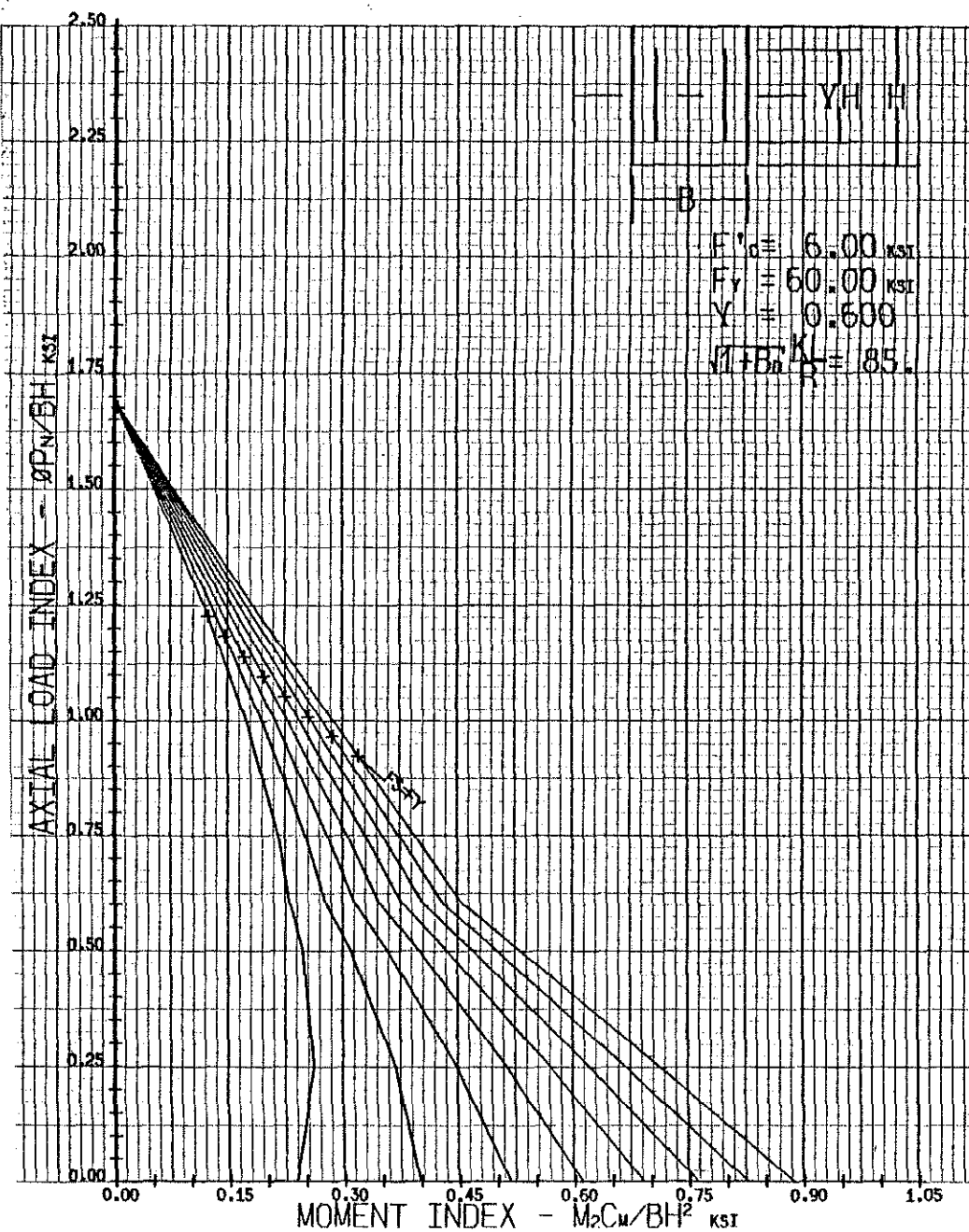


Fig. L6-60.60-85 - Interaction Diagram

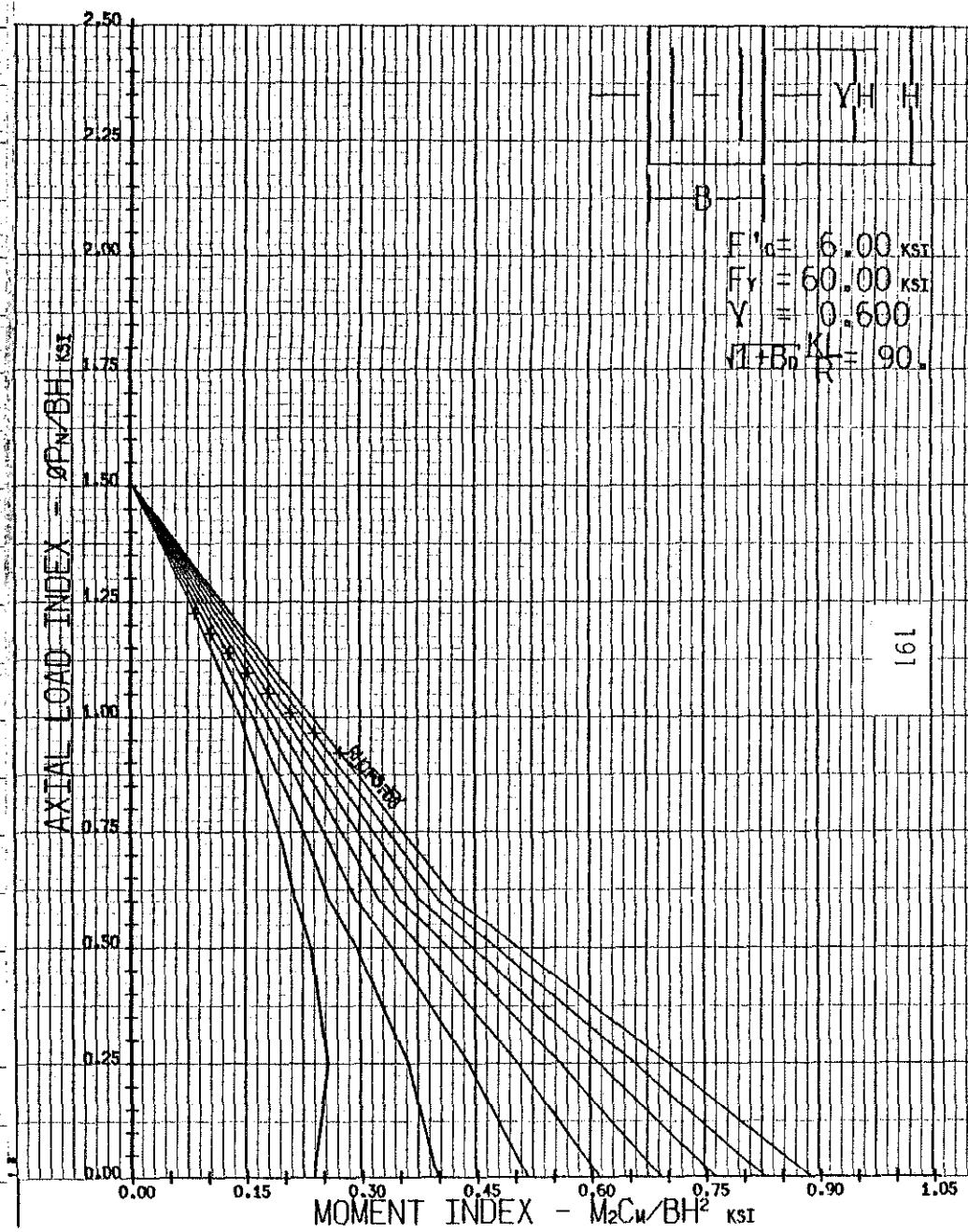


Fig. L6-60.60-90 - Interaction Diagram

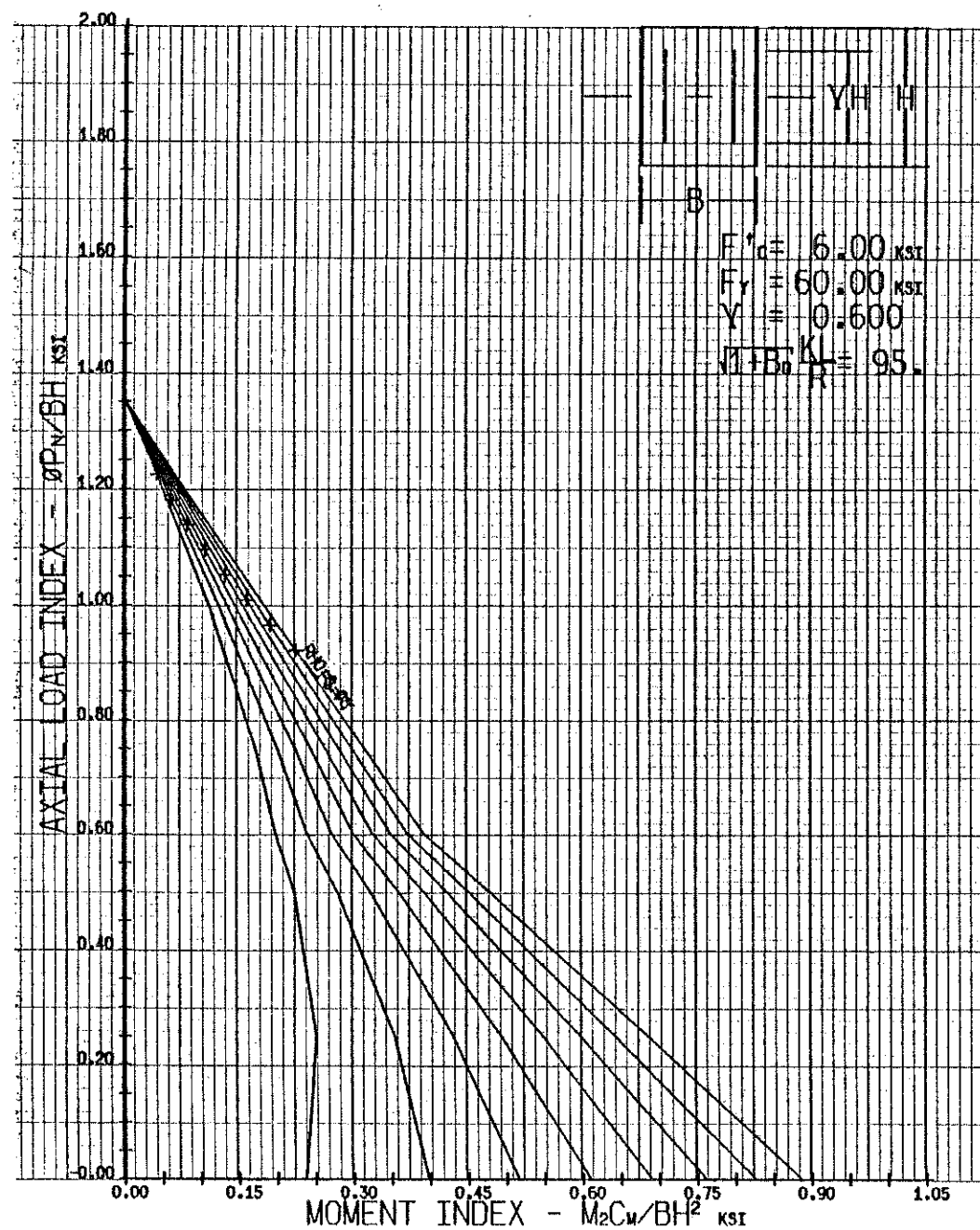


Fig. L6-60.60-95 - Interaction Diagram

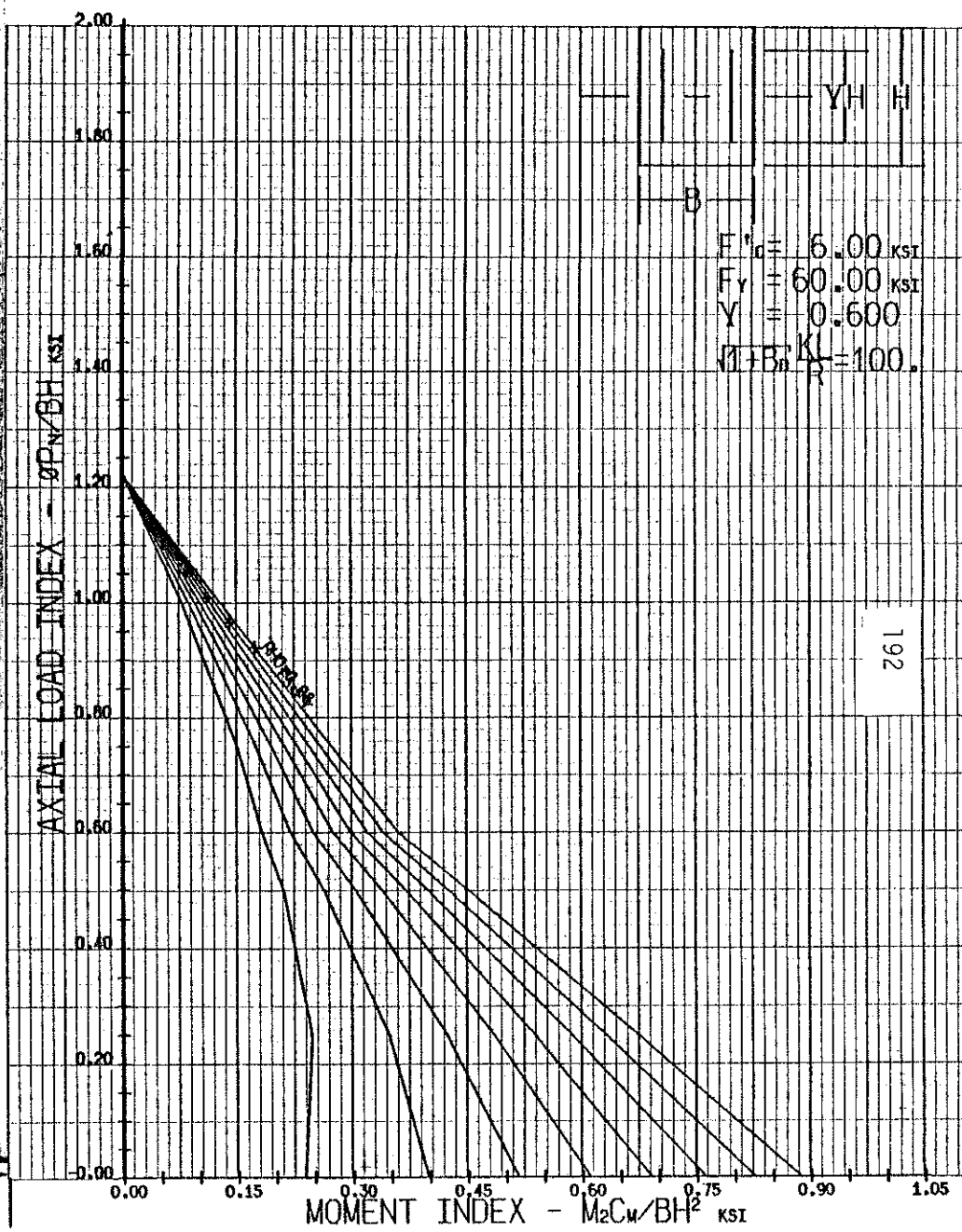


Fig. L6-60.60-100 - Interaction Diagram

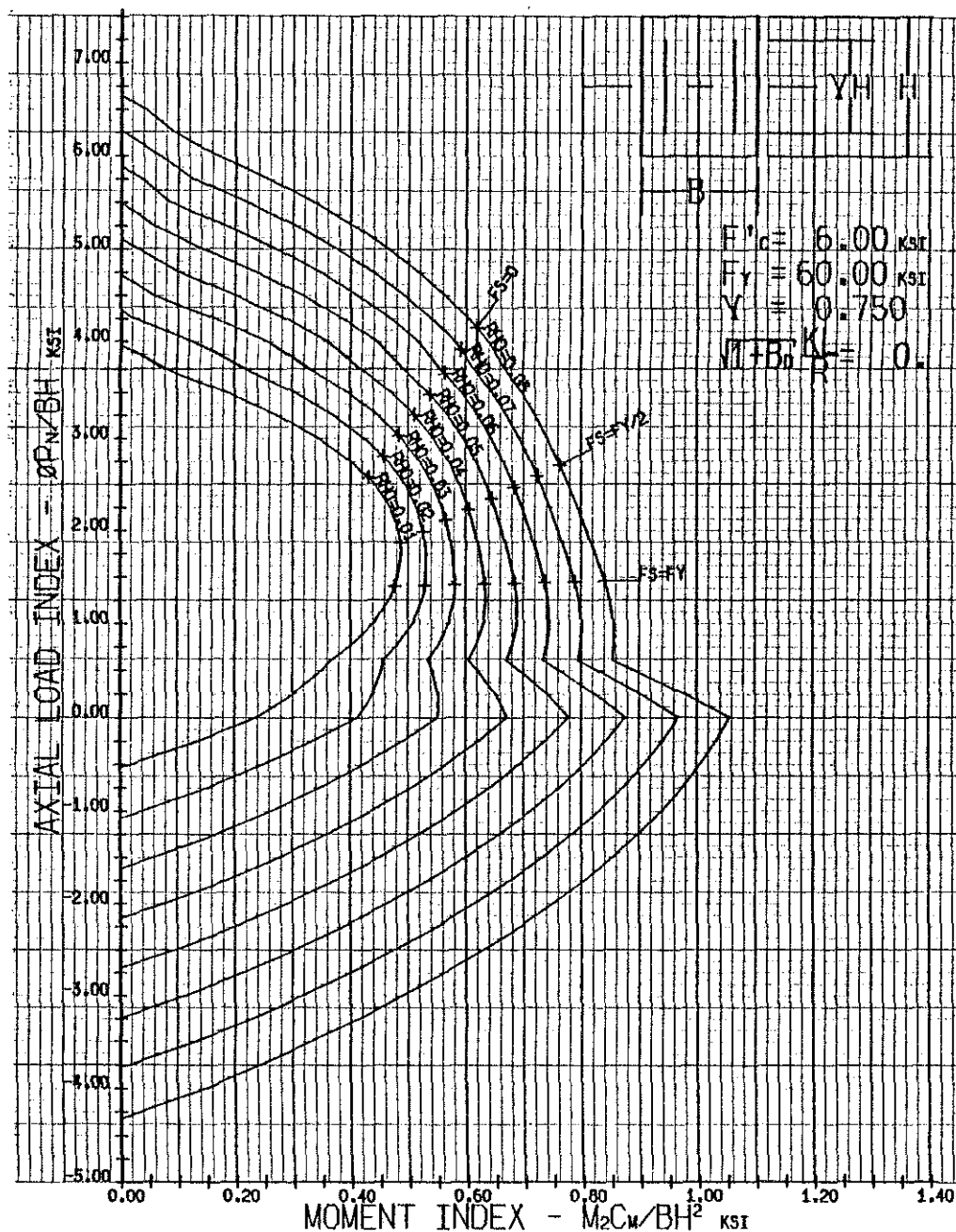


Fig. L6-60.75-0 - Interaction Diagram

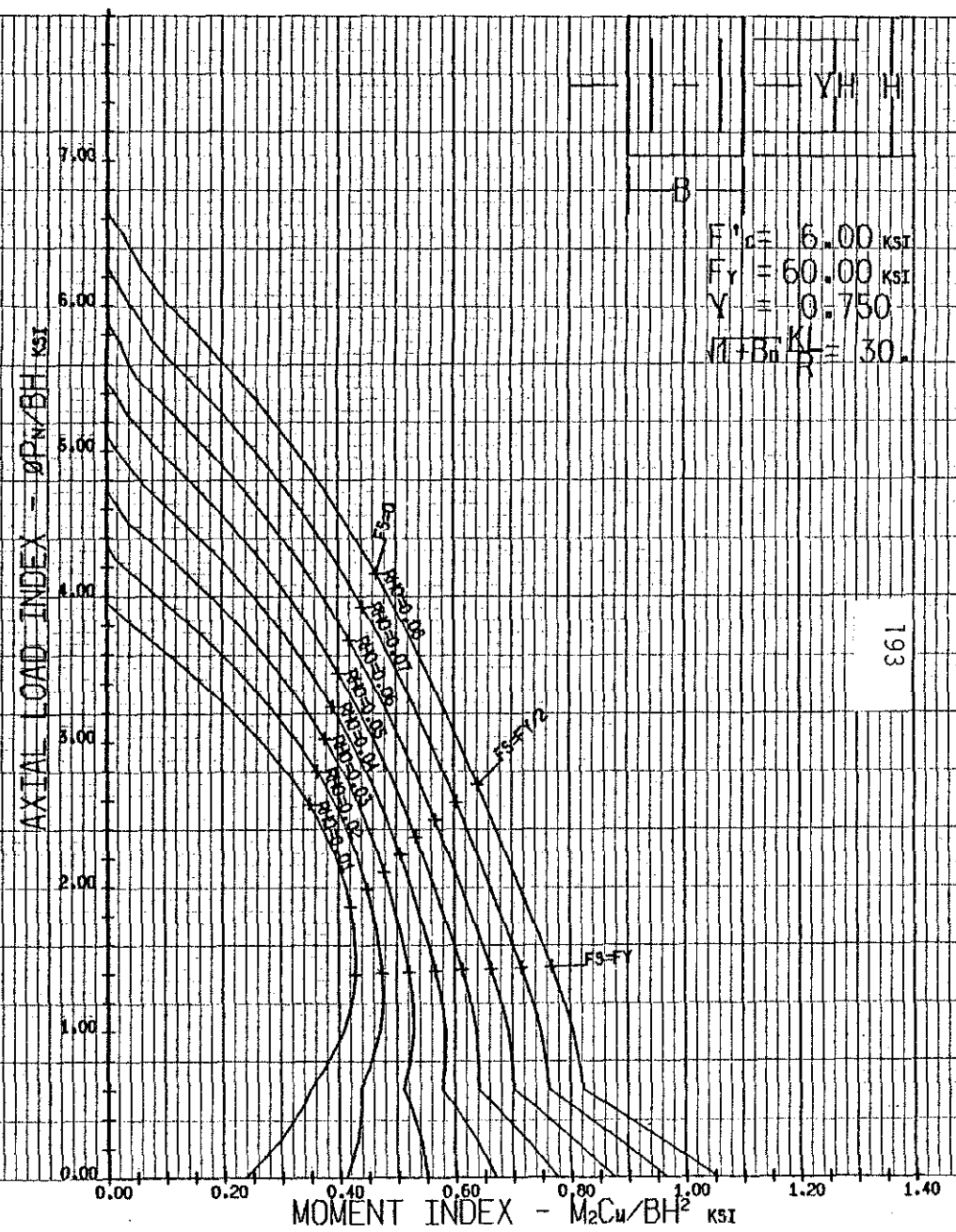


Fig. L6-60.75-30 - Interaction Diagram

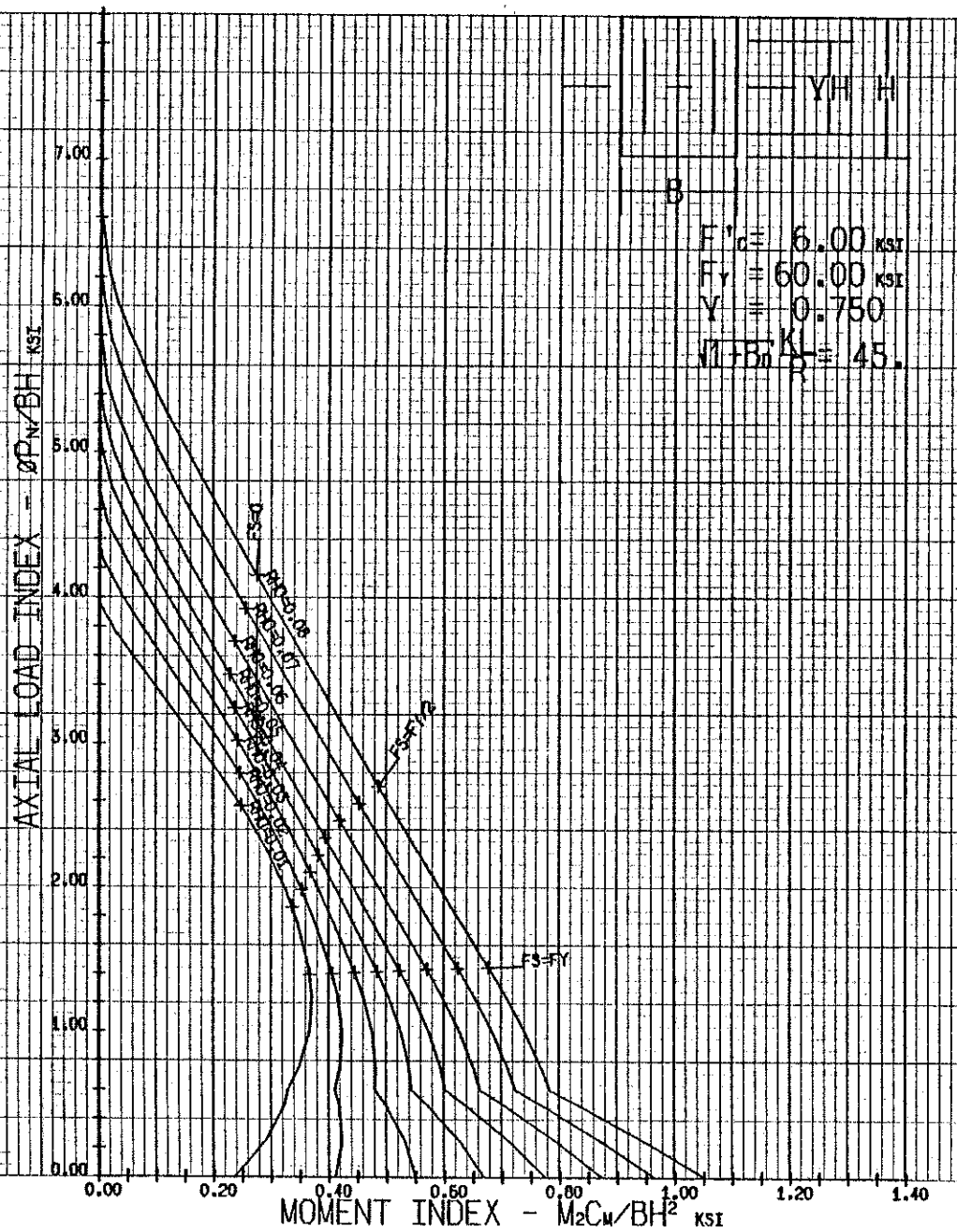


Fig. L6-60.75-45 - Interaction Diagram

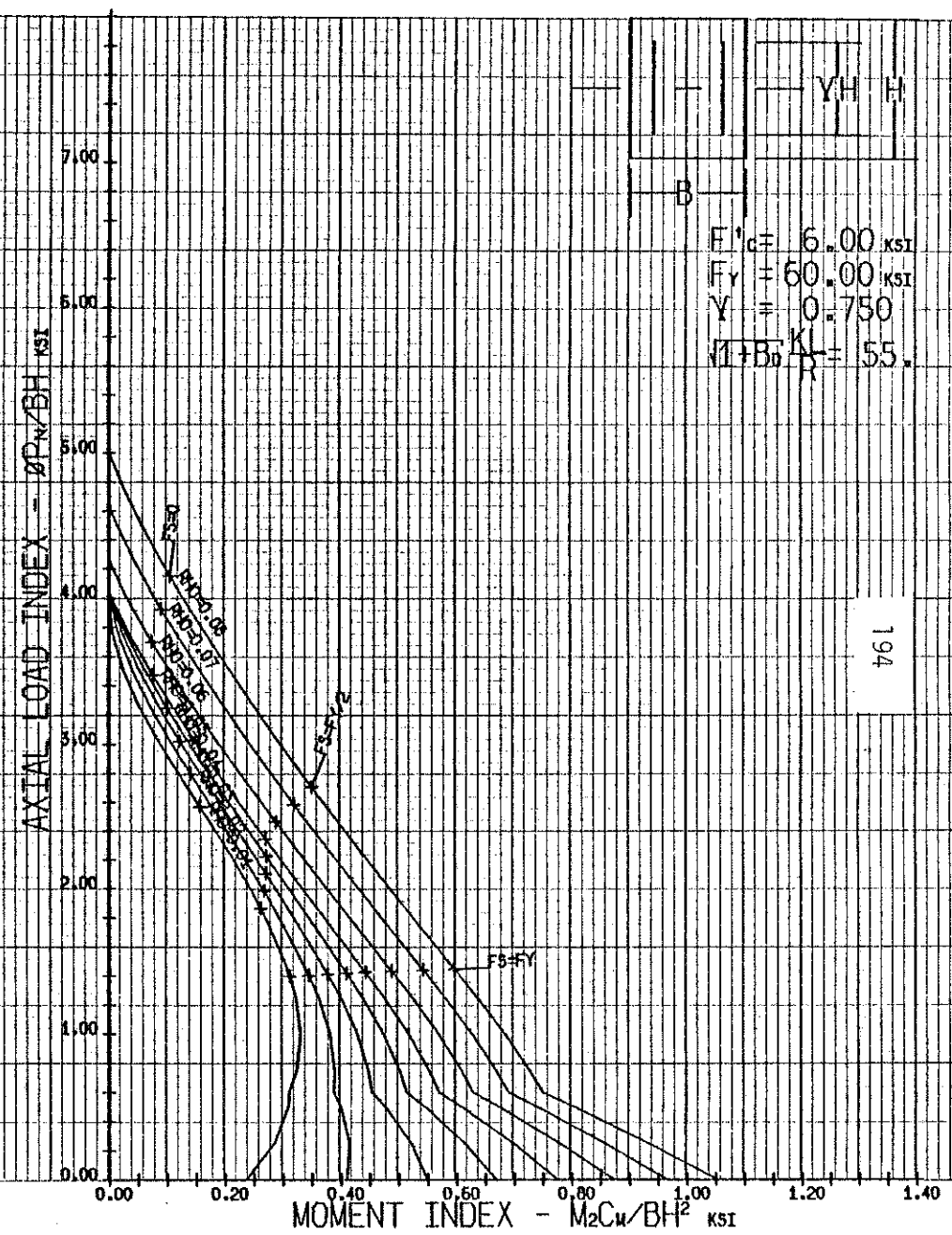


Fig. L6-60.75-55 - Interaction Diagram

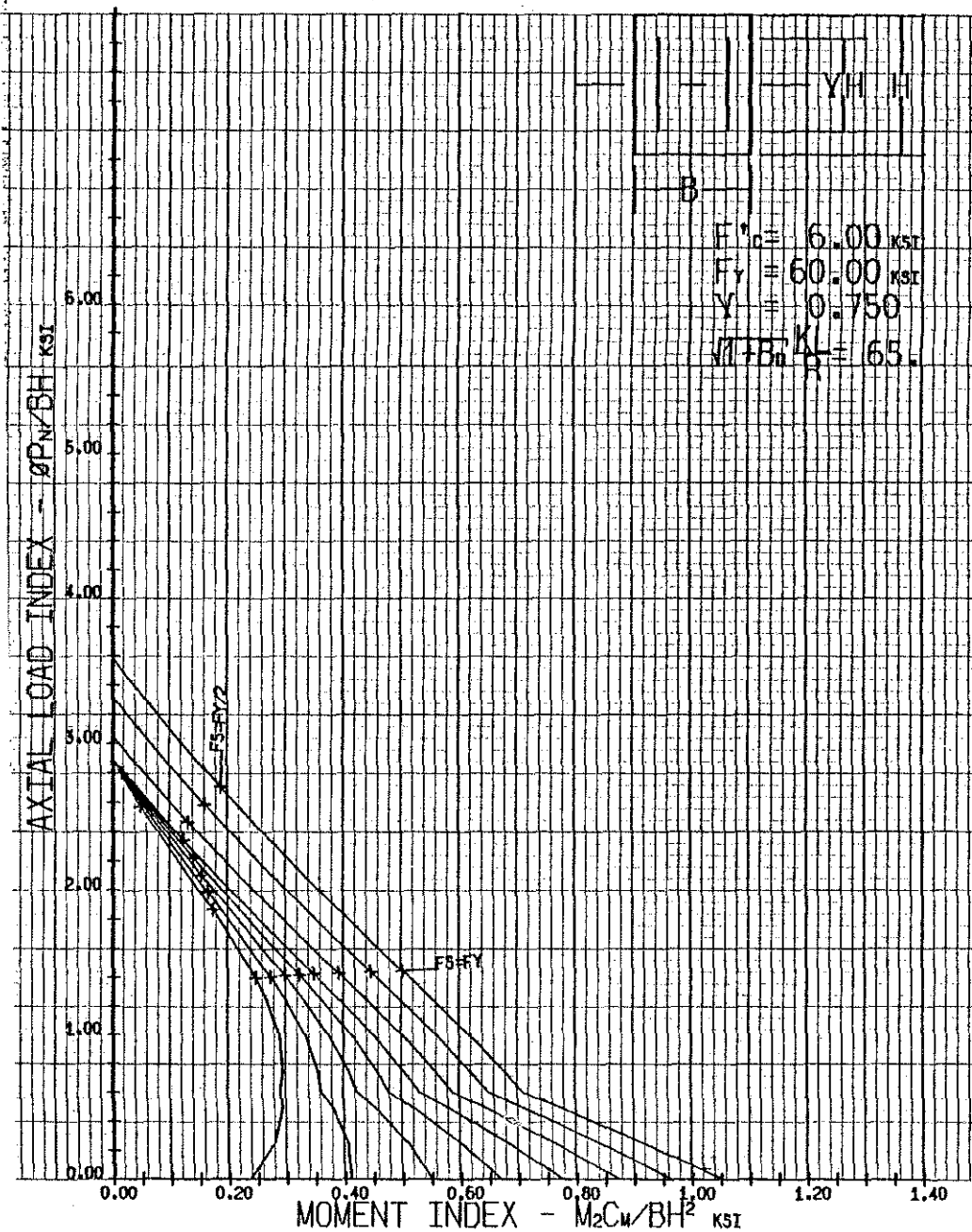


Fig. L6-60.75-65 - Interaction Diagram

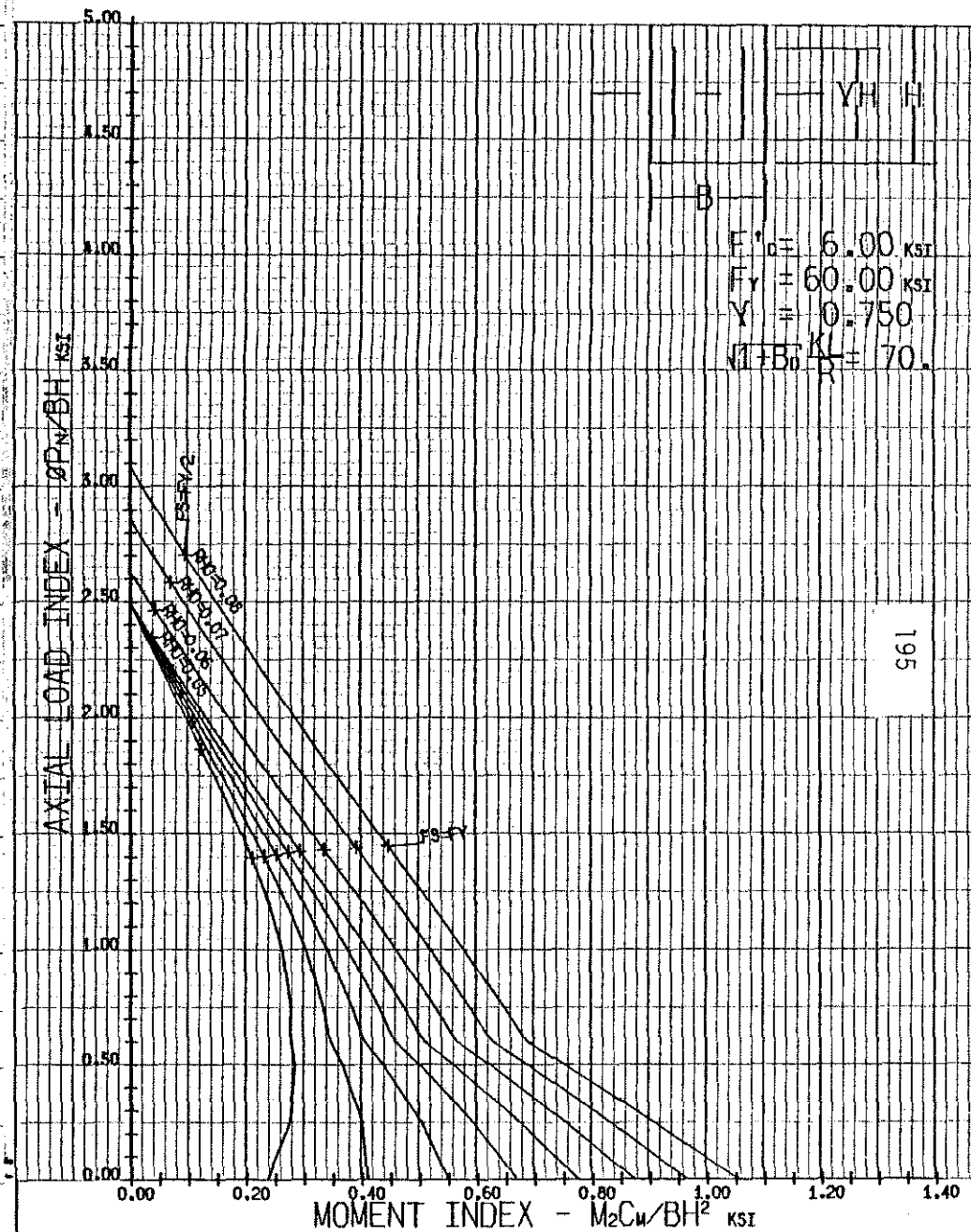


Fig. L6-60.75-70 - Interaction Diagram



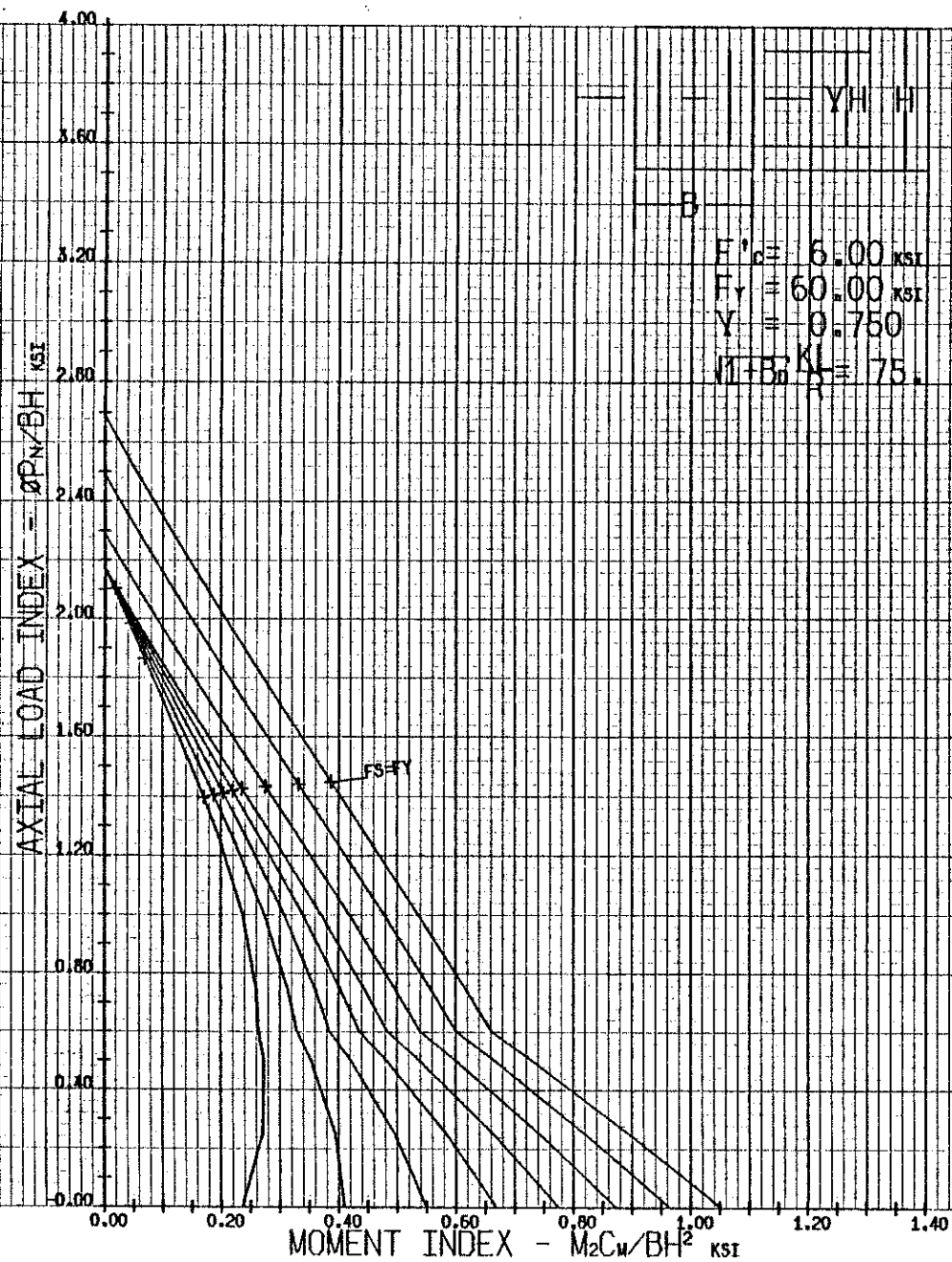


Fig. L6-60.75-75 - Interaction Diagram

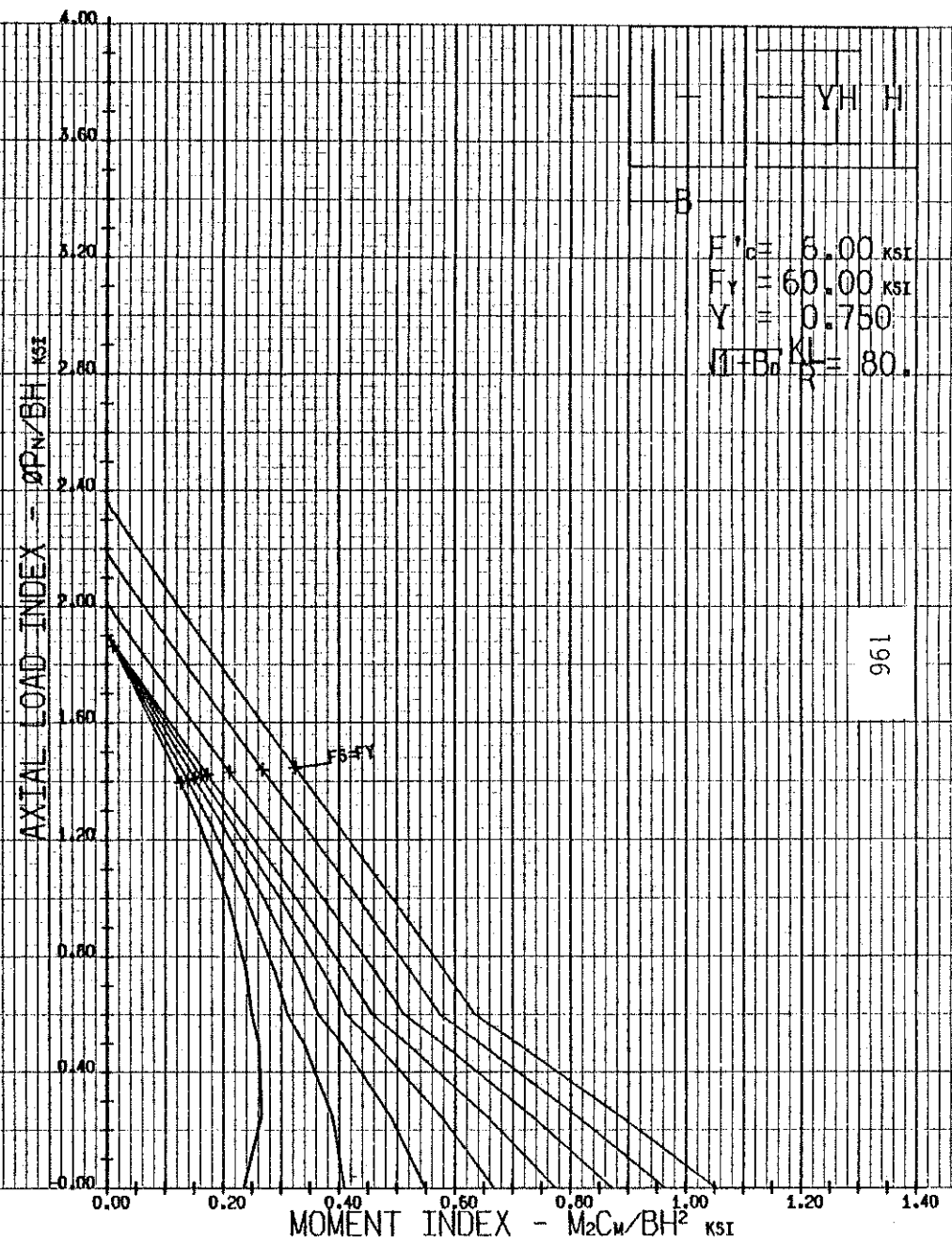


Fig. L6-60.75-80 - Interaction Diagram

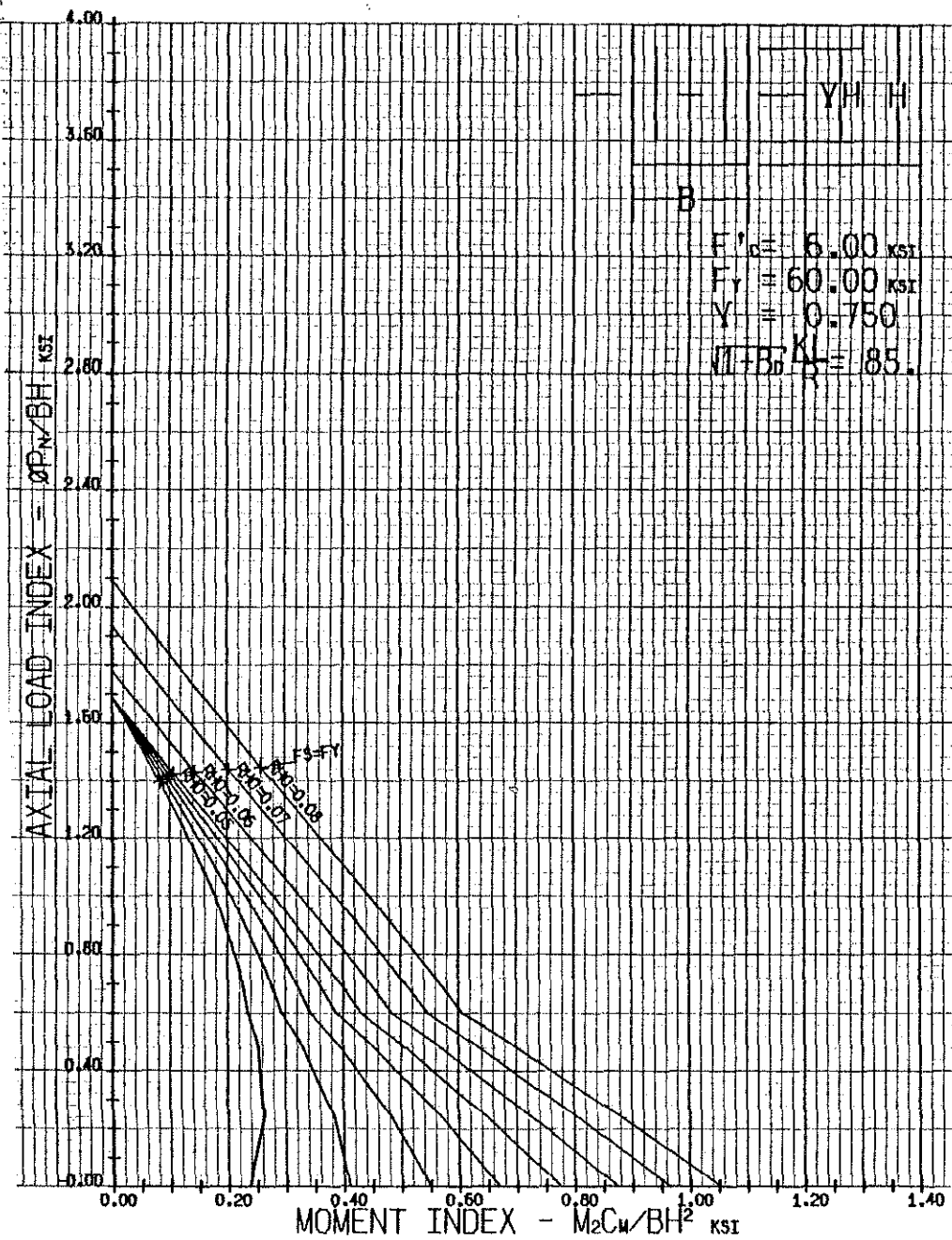


Fig. L6-60.75-85 - Interaction Diagram

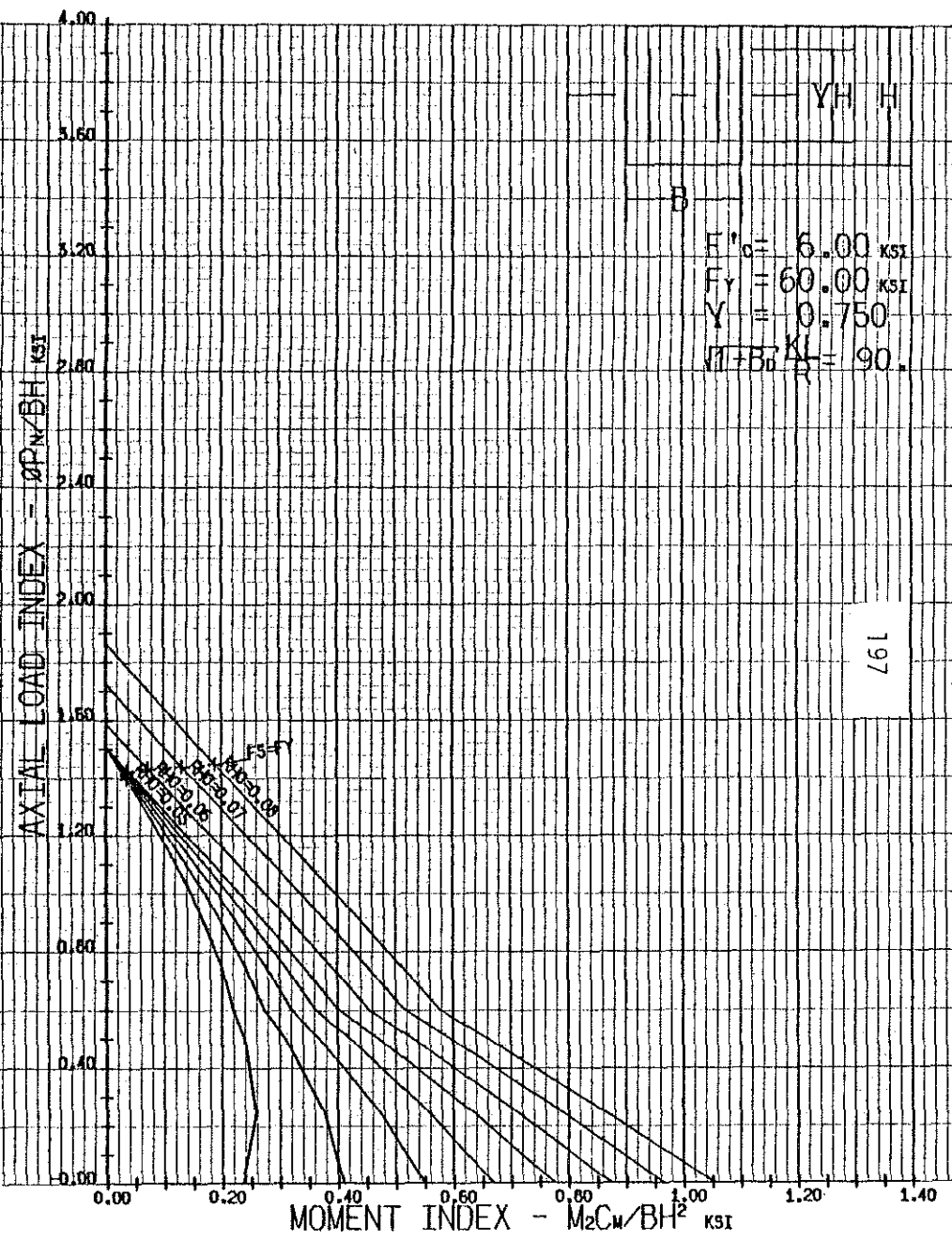


Fig. L6-60.75-90 - Interaction Diagram



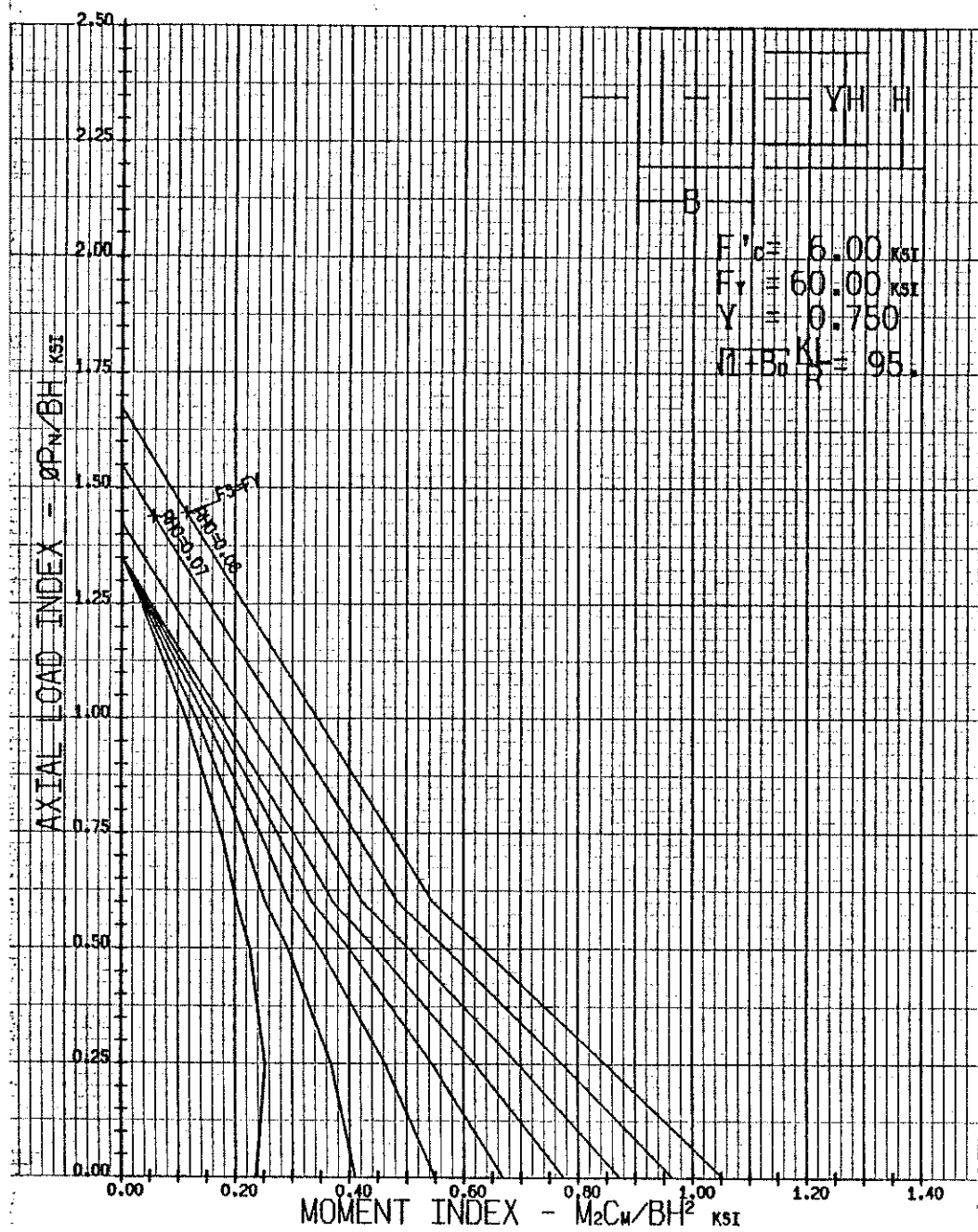


Fig. L6-60.75-95 - Interaction Diagram

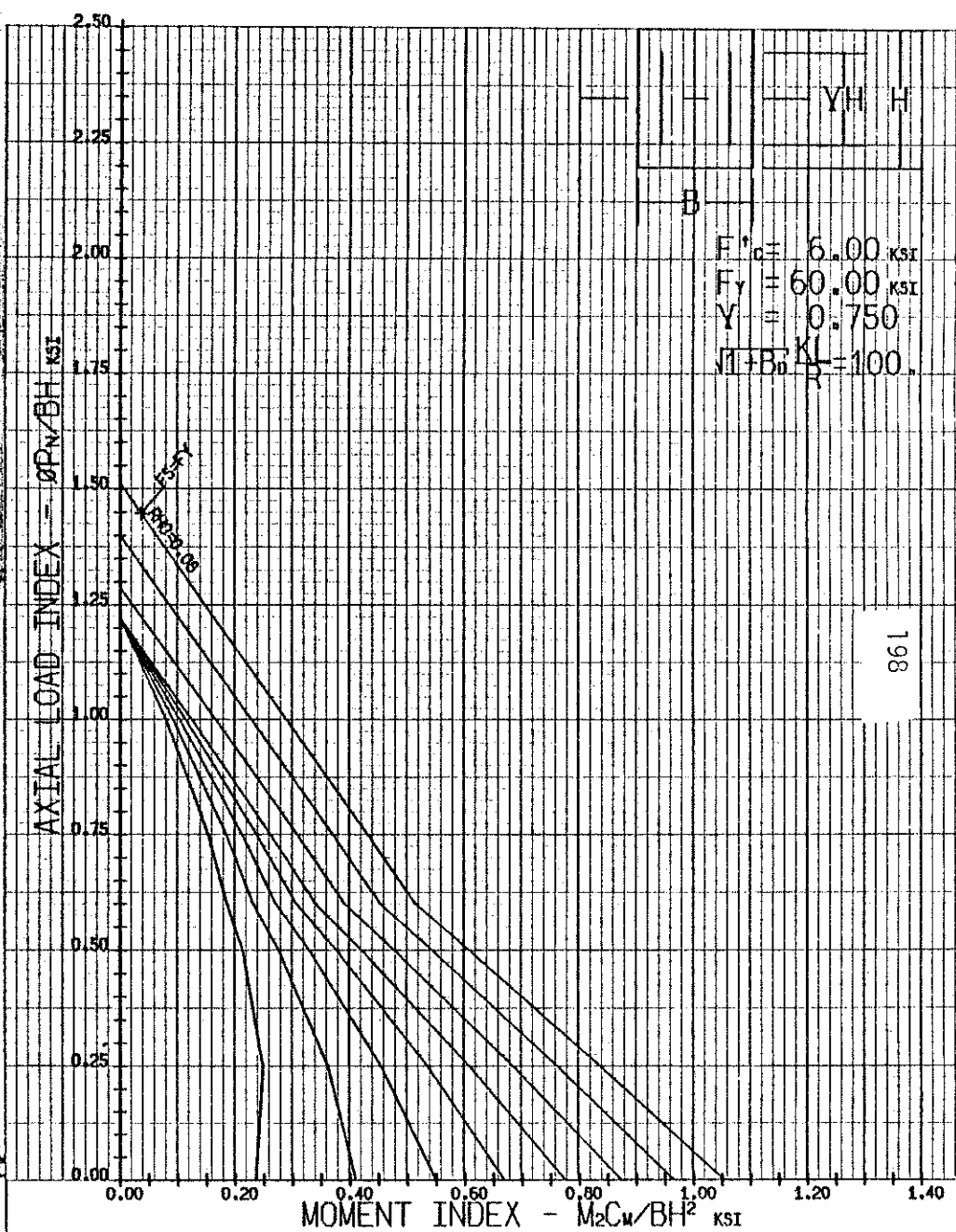


Fig. L6-60.75-100 - Interaction Diagram

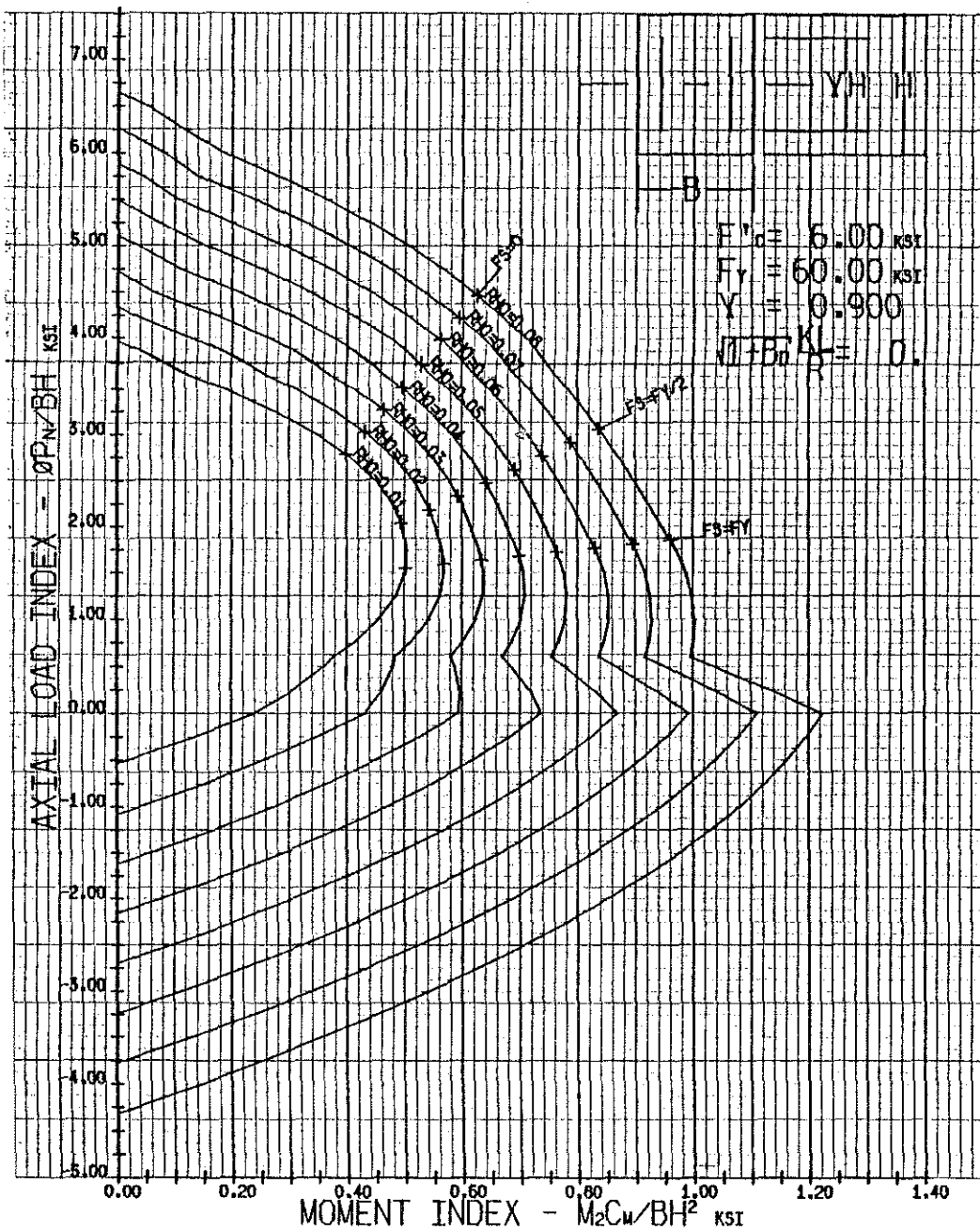


Fig. L6-60.90-0 - Interaction Diagram

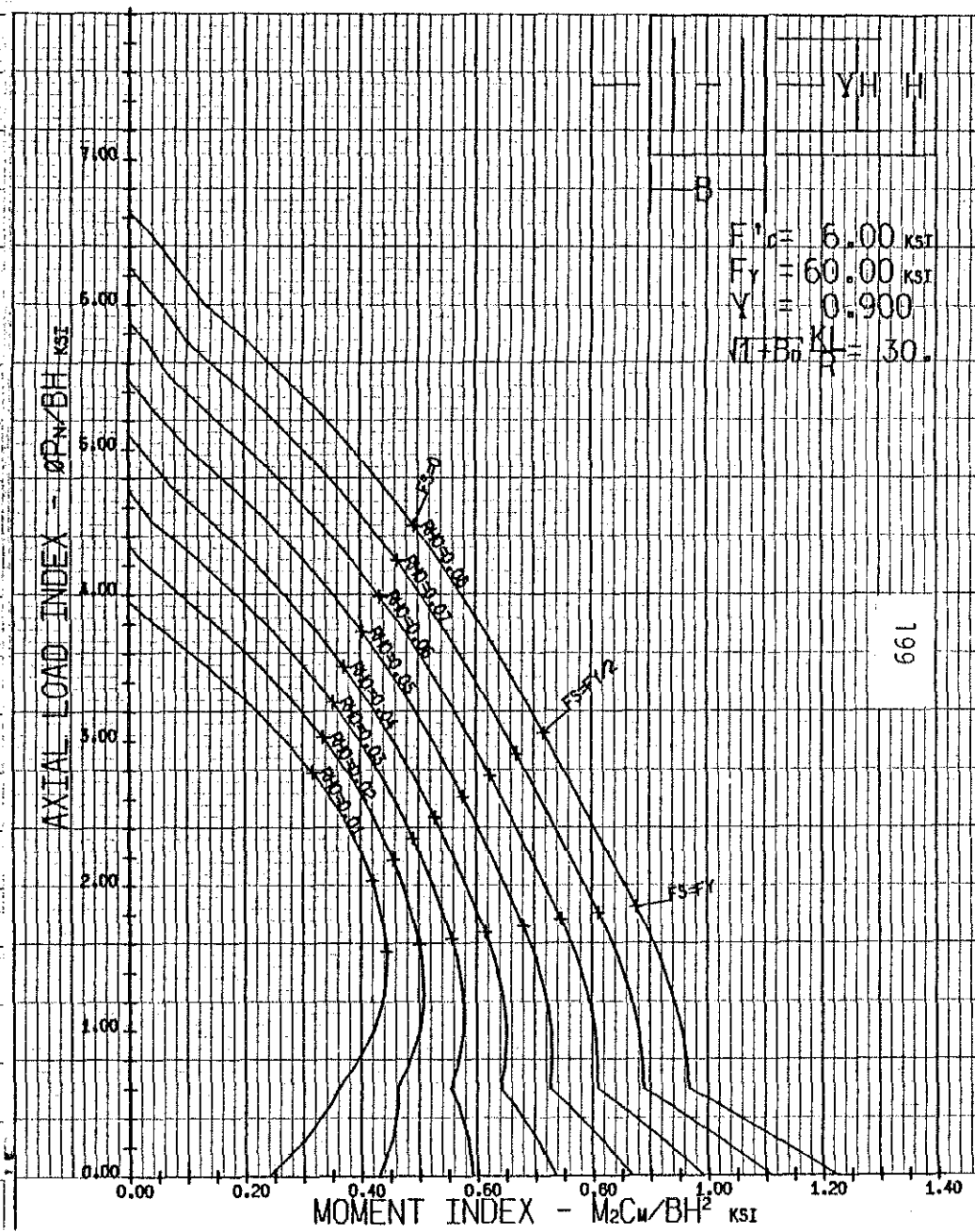


Fig. L6-60.90-30 - Interaction Diagram

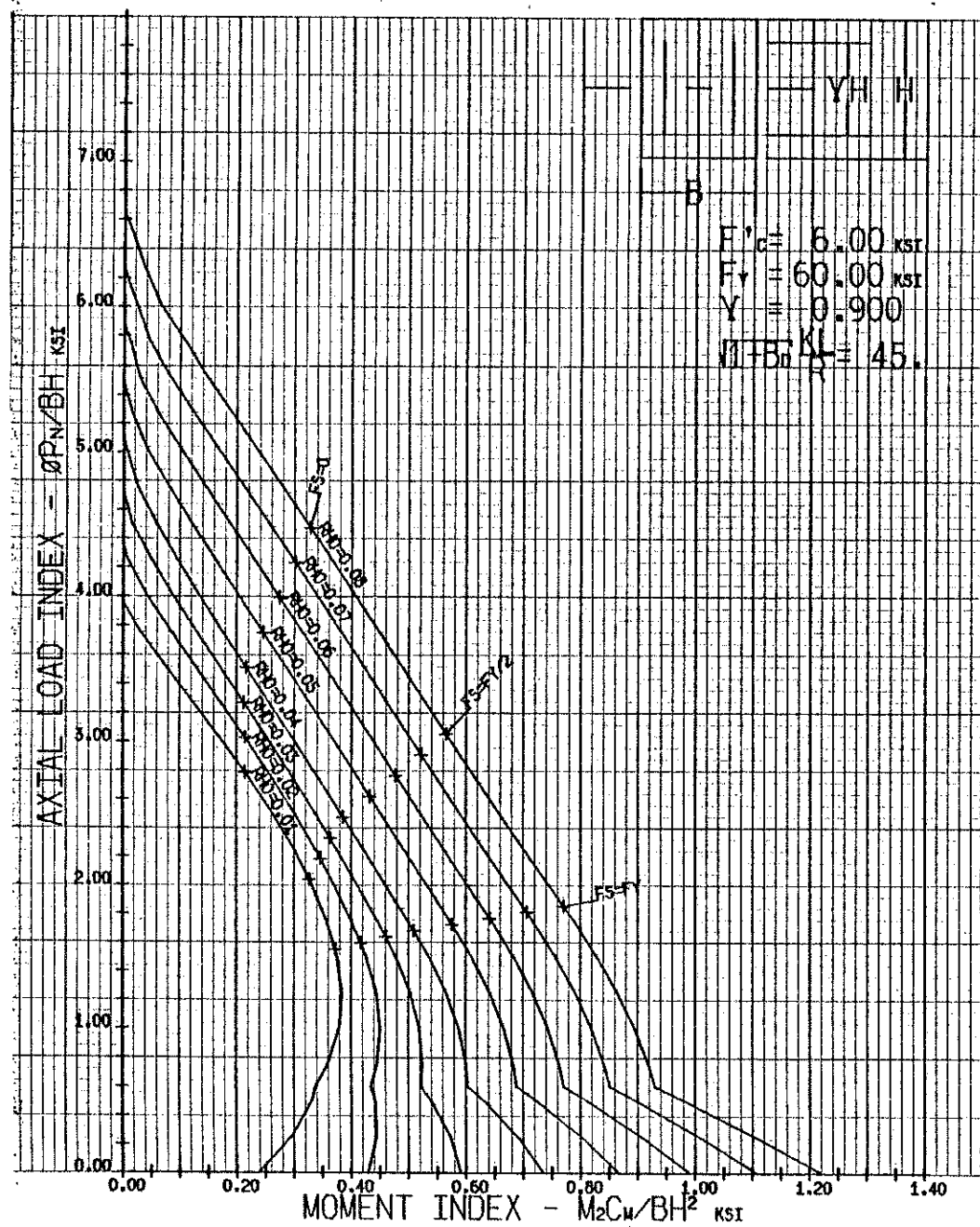


Fig. L6-60.90-45 - Interaction Diagram

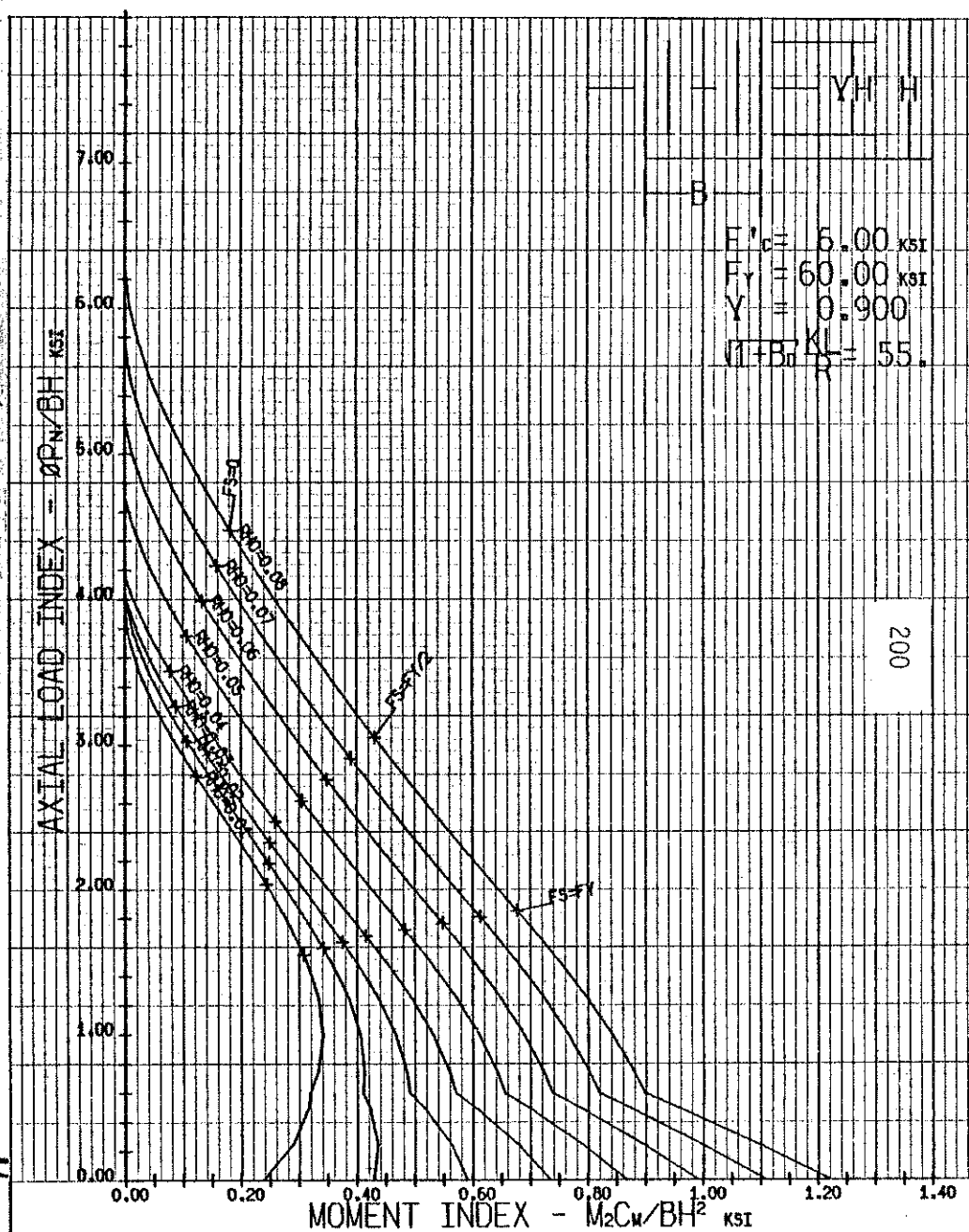


Fig. L6-60.90-55 - Interaction Diagram

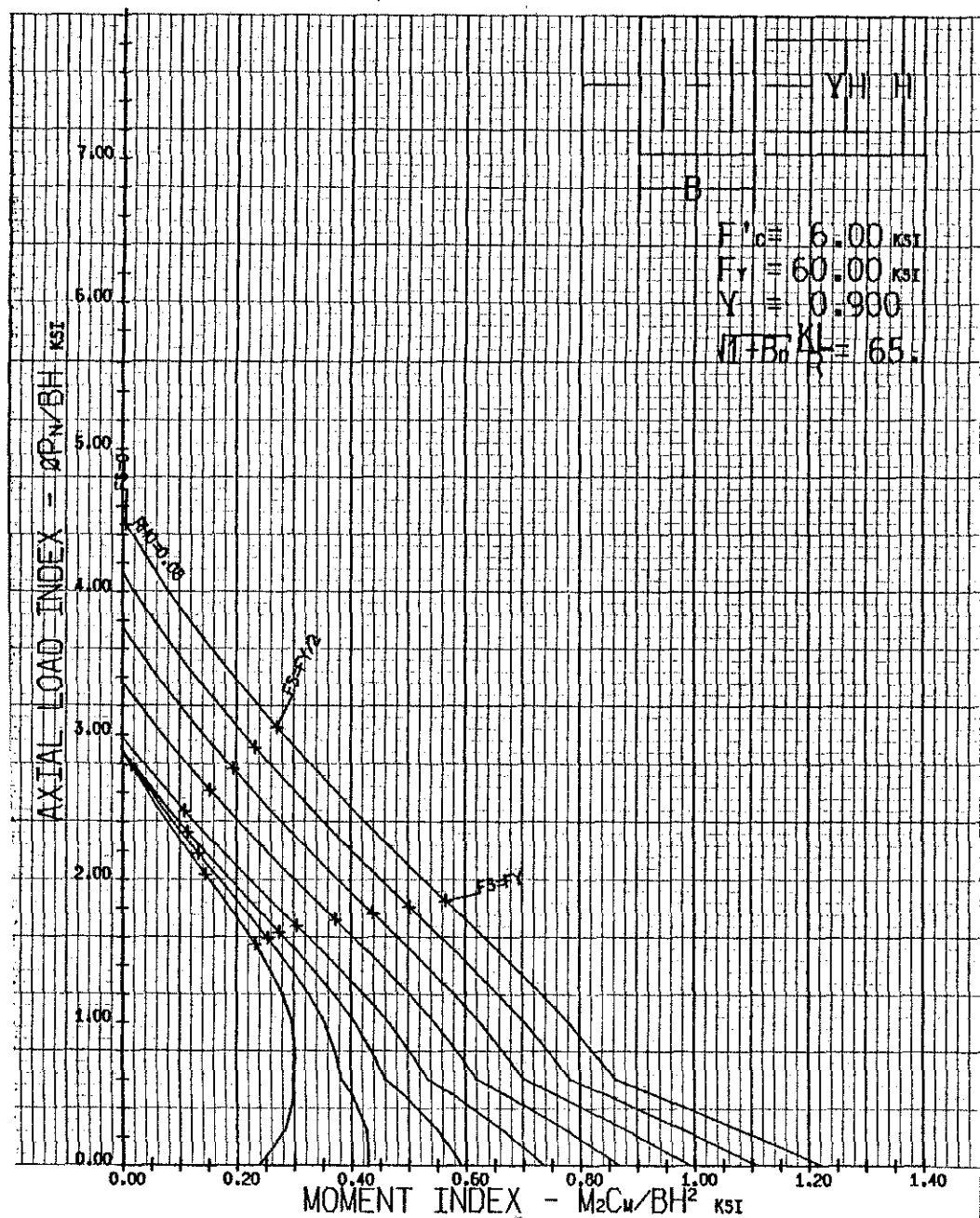


Fig. L6-60.90-65 - Interaction Diagram

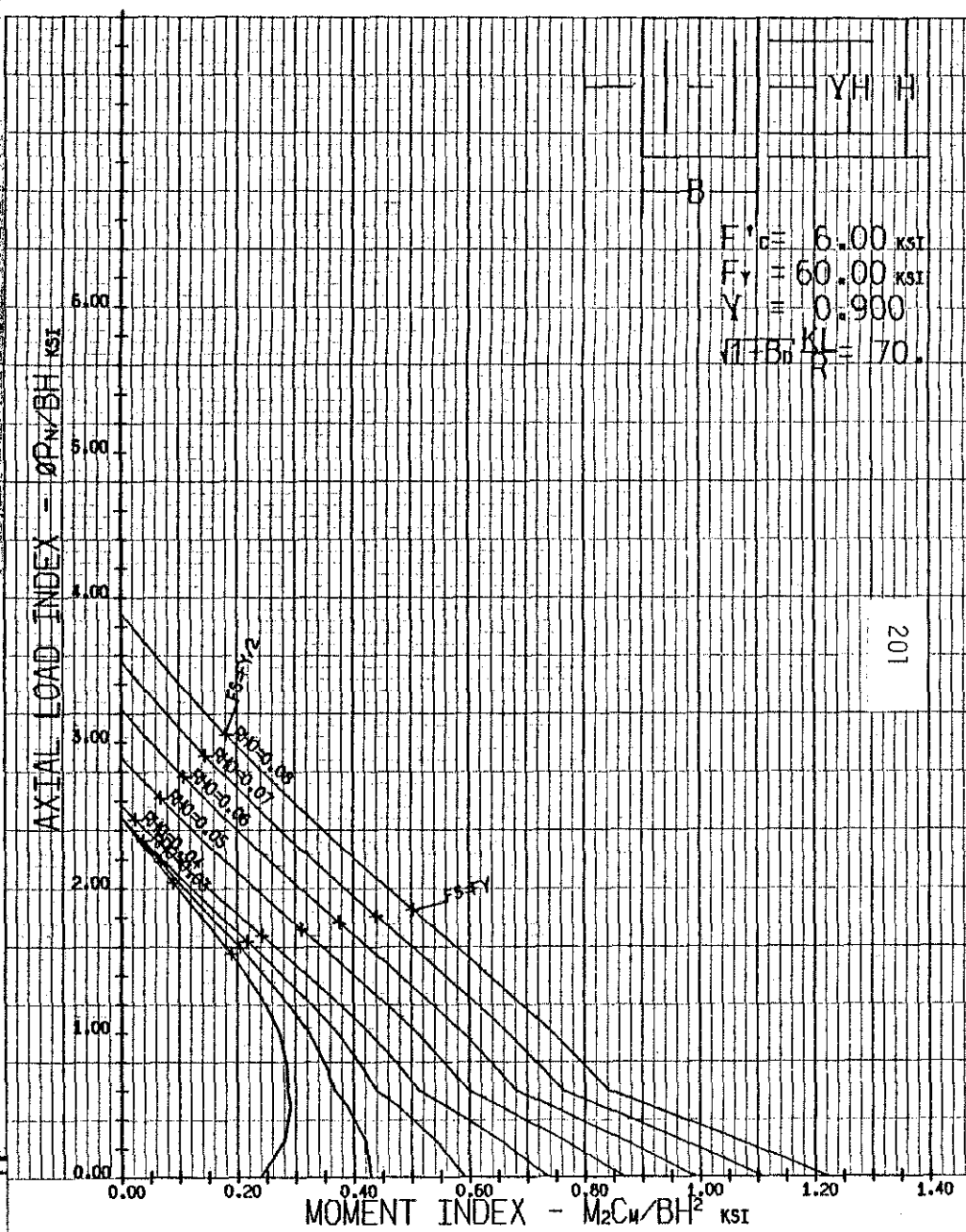


Fig. L6-60.90-70 - Interaction Diagram

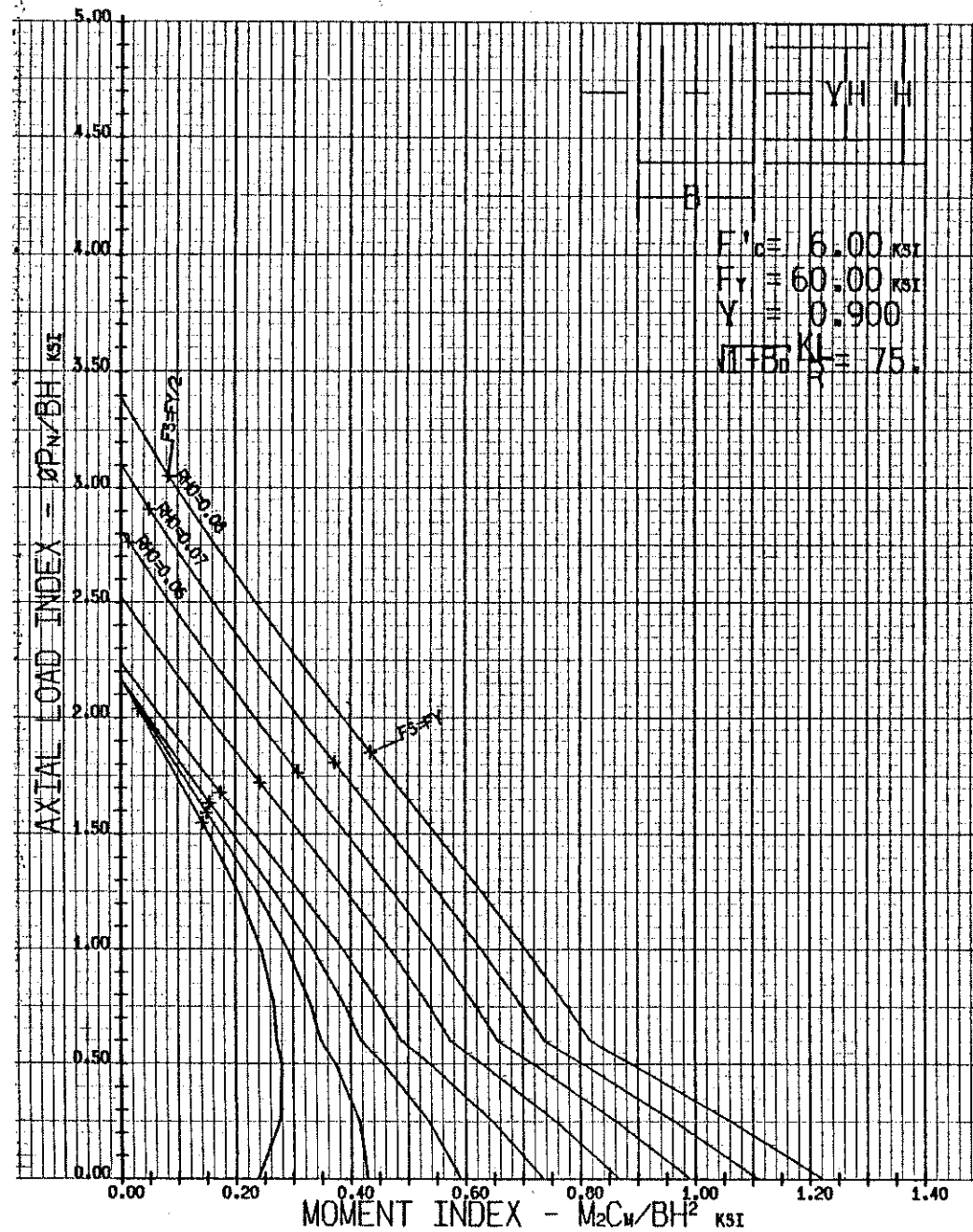


Fig. L6-60.90-75 - Interaction Diagram

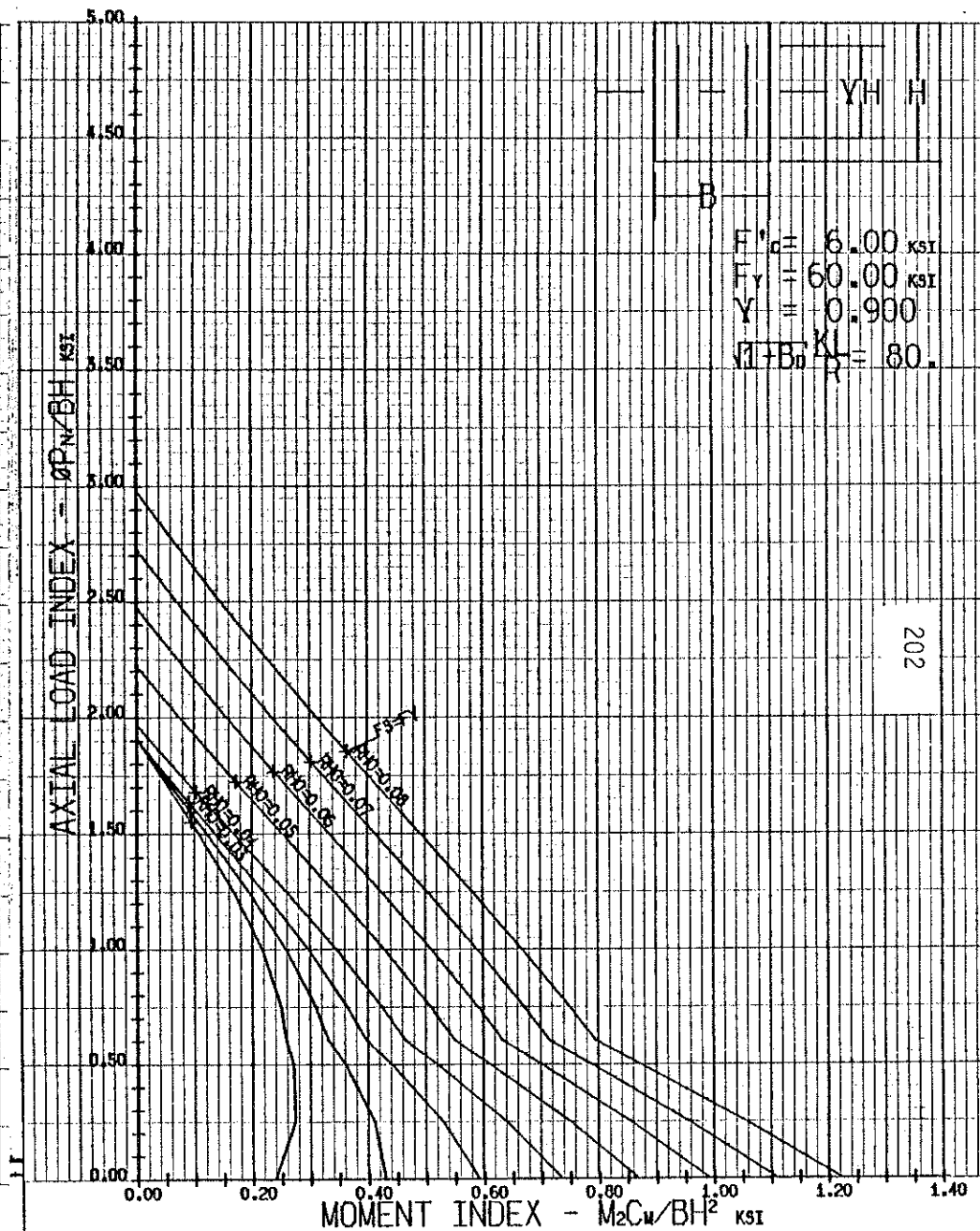


Fig. L6-60.90-80 - Interaction Diagram



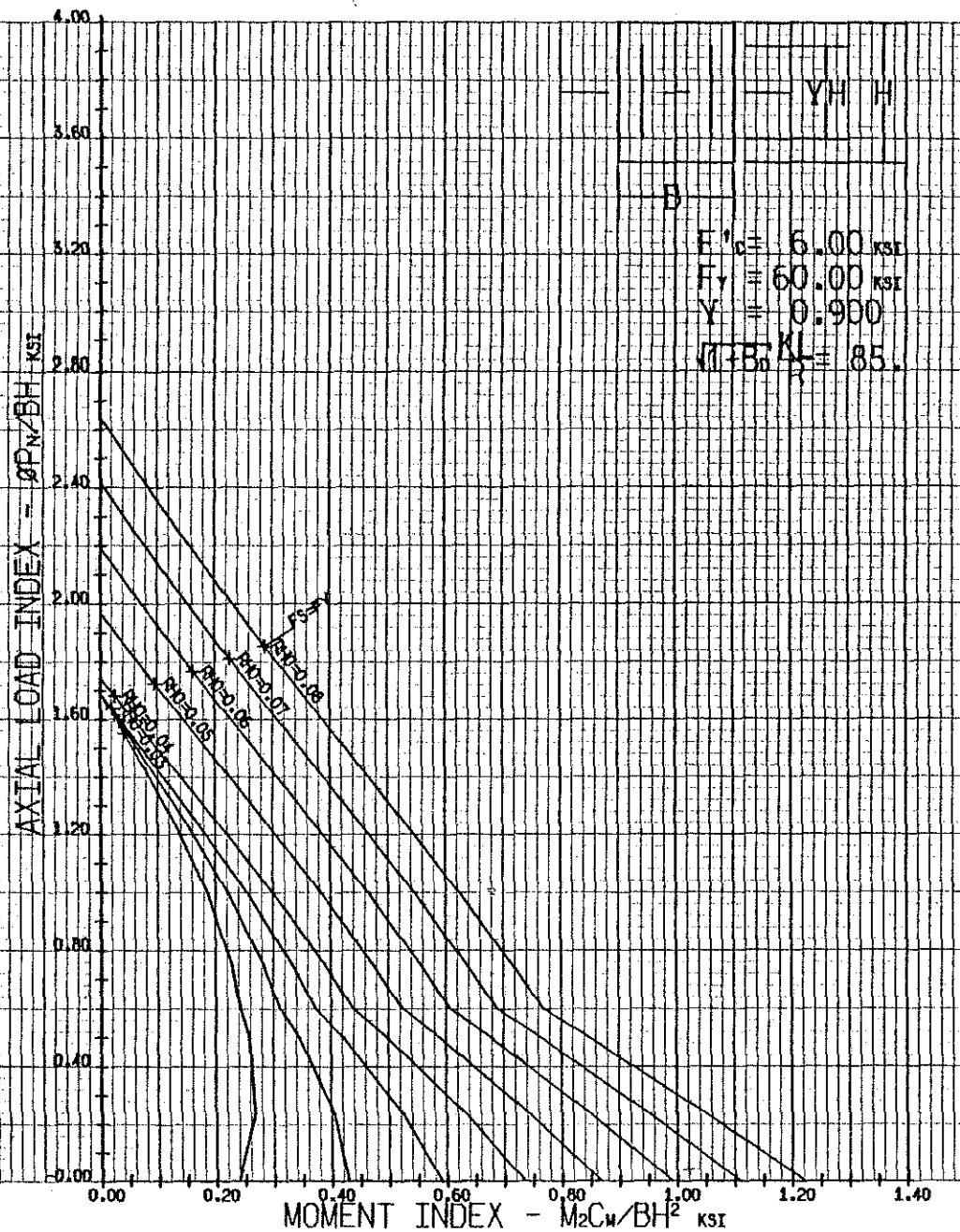


Fig. L6-60.90-85 - Interaction Diagram

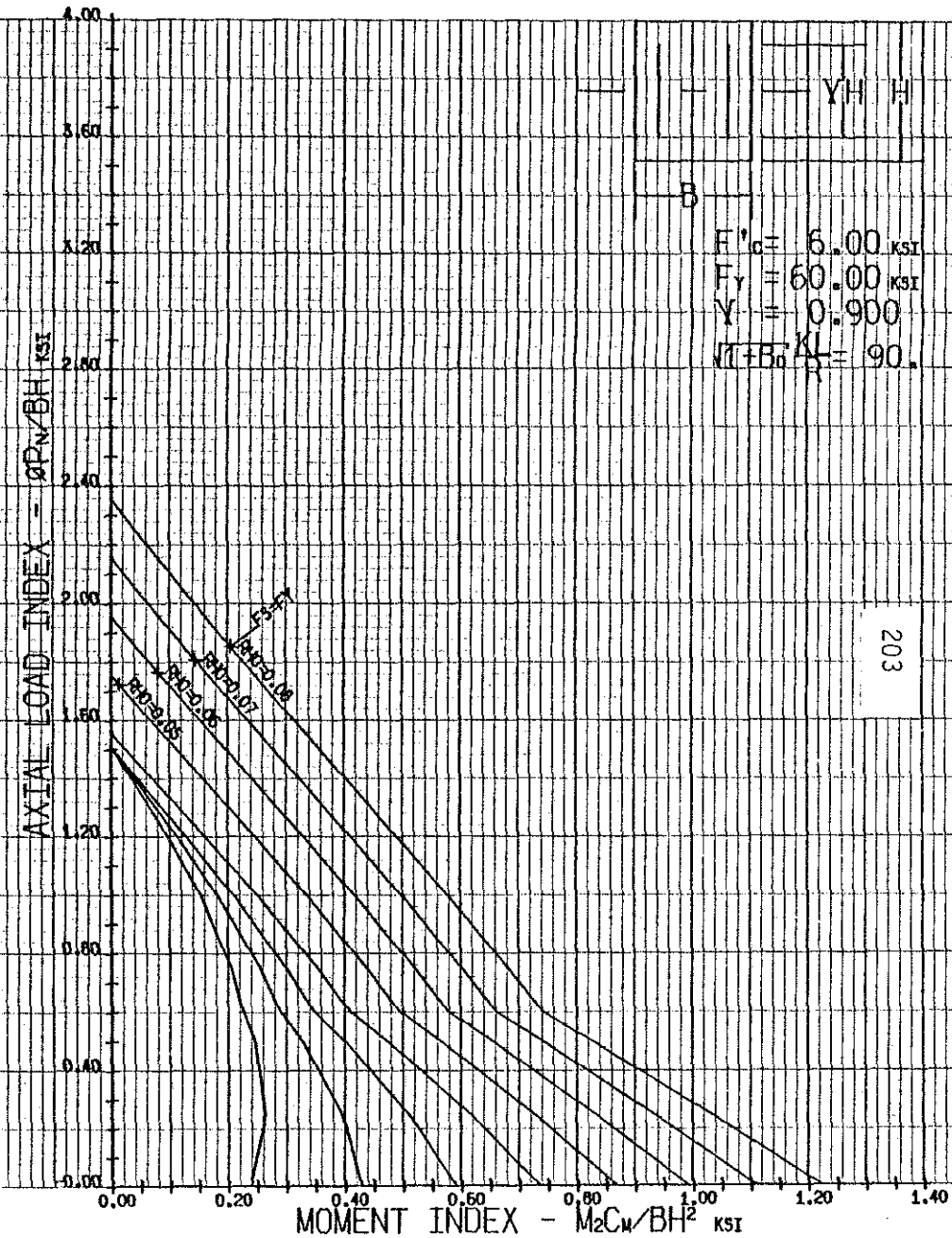


Fig. L6-60.90-90 - Interaction Diagram



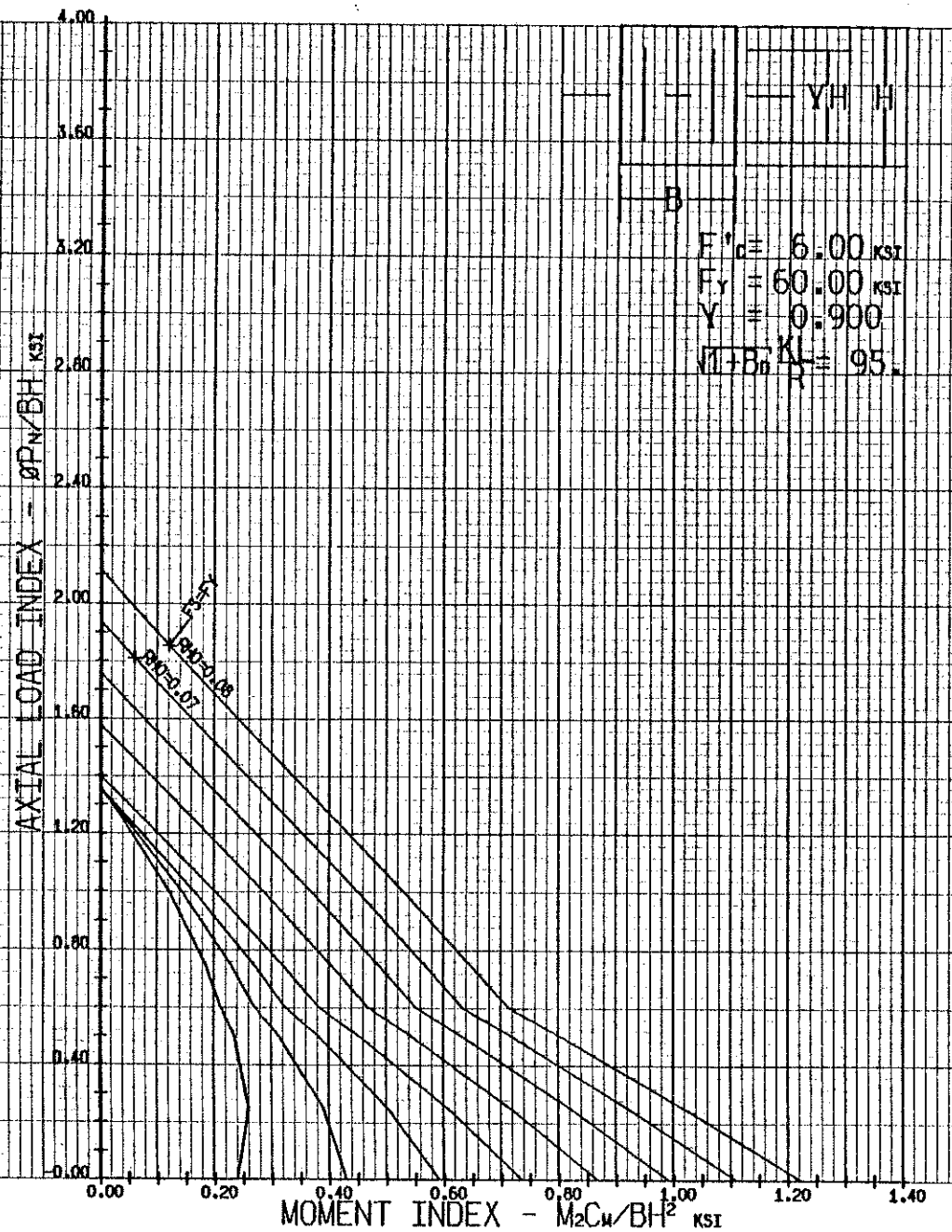


Fig. L6-60.90-95 - Interaction Diagram

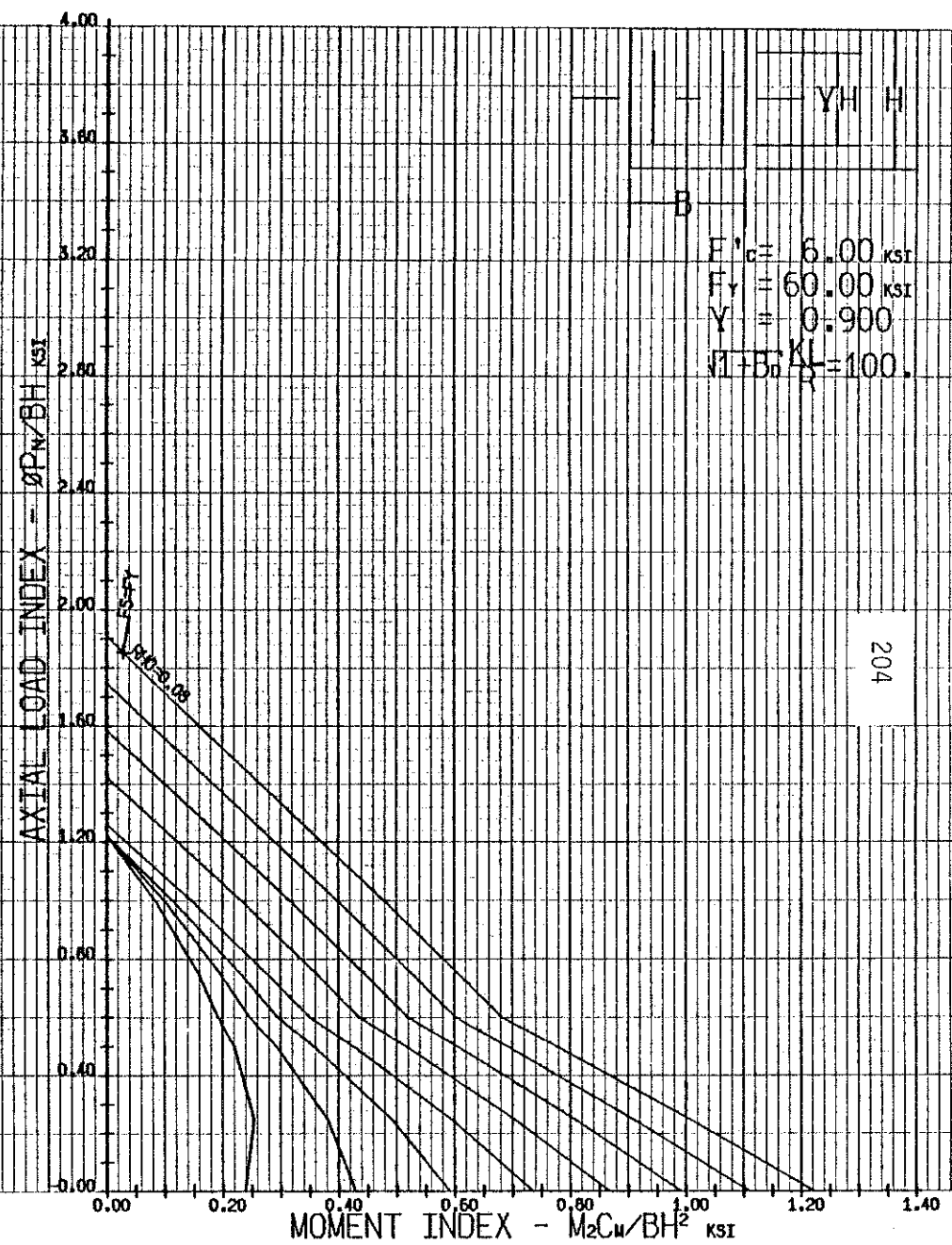


Fig. L6-60.90-100 - Interaction Diagram

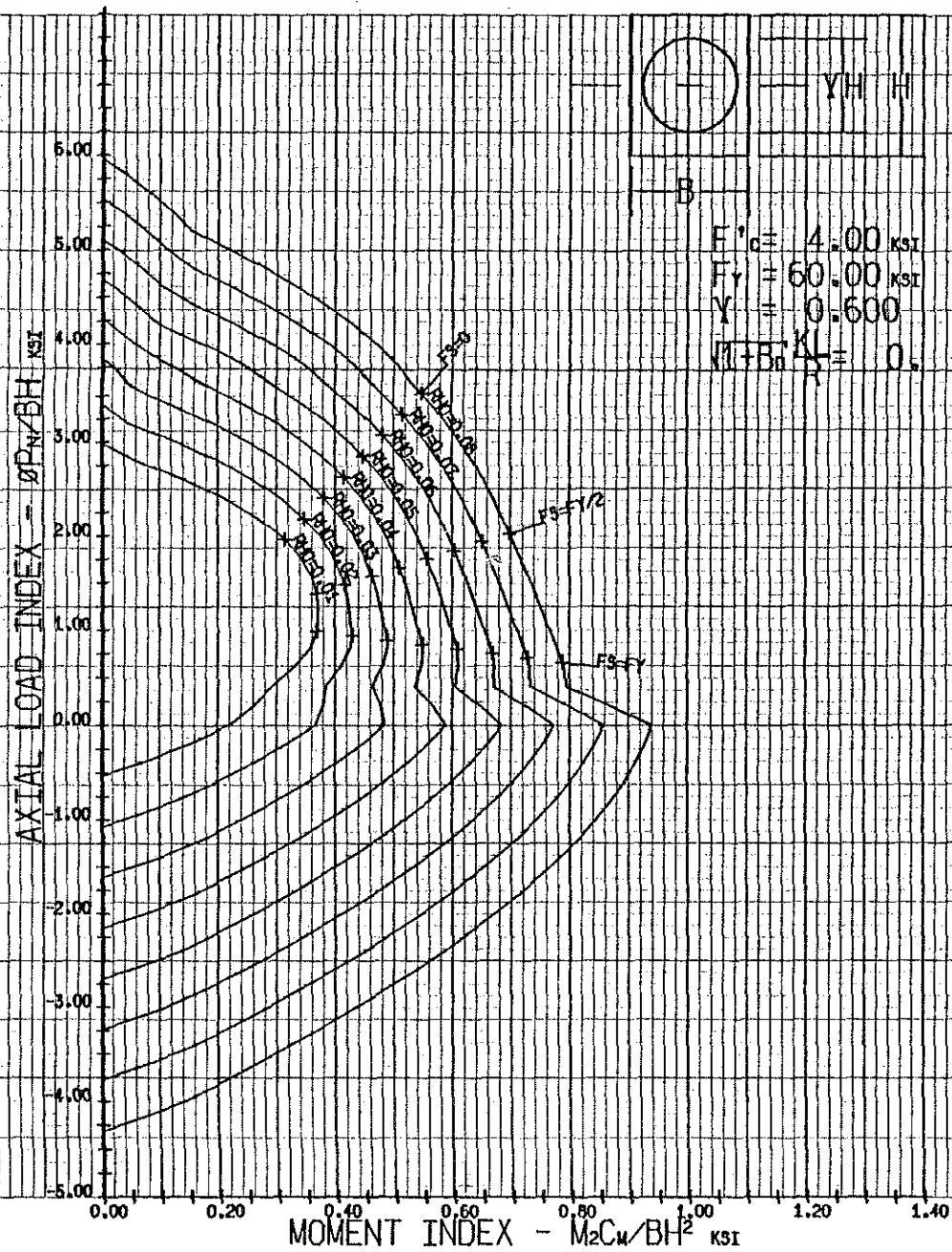


Fig. S4-60.60-0 - Interaction Diagram

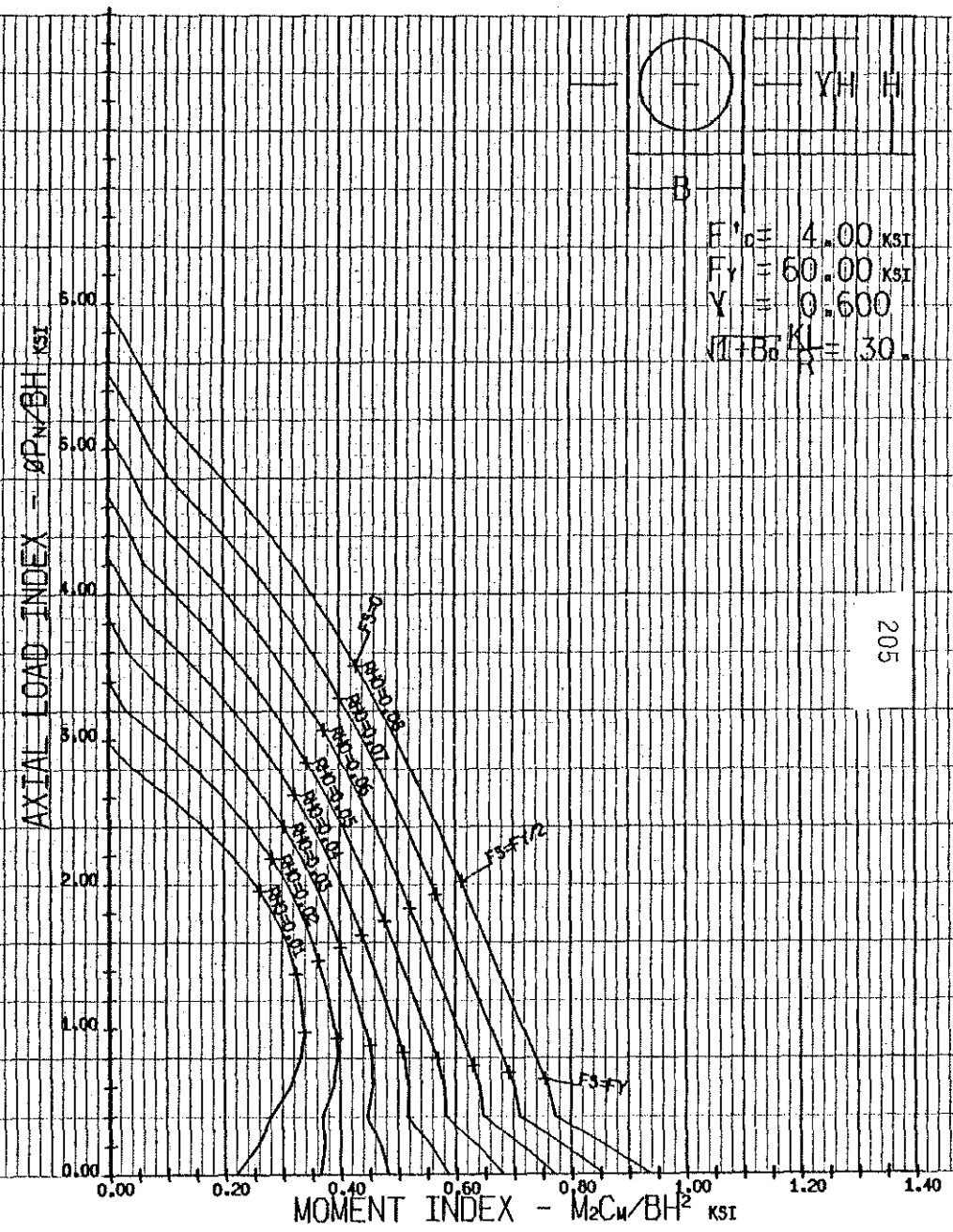


Fig. S4-60.60-30 - Interaction Diagram

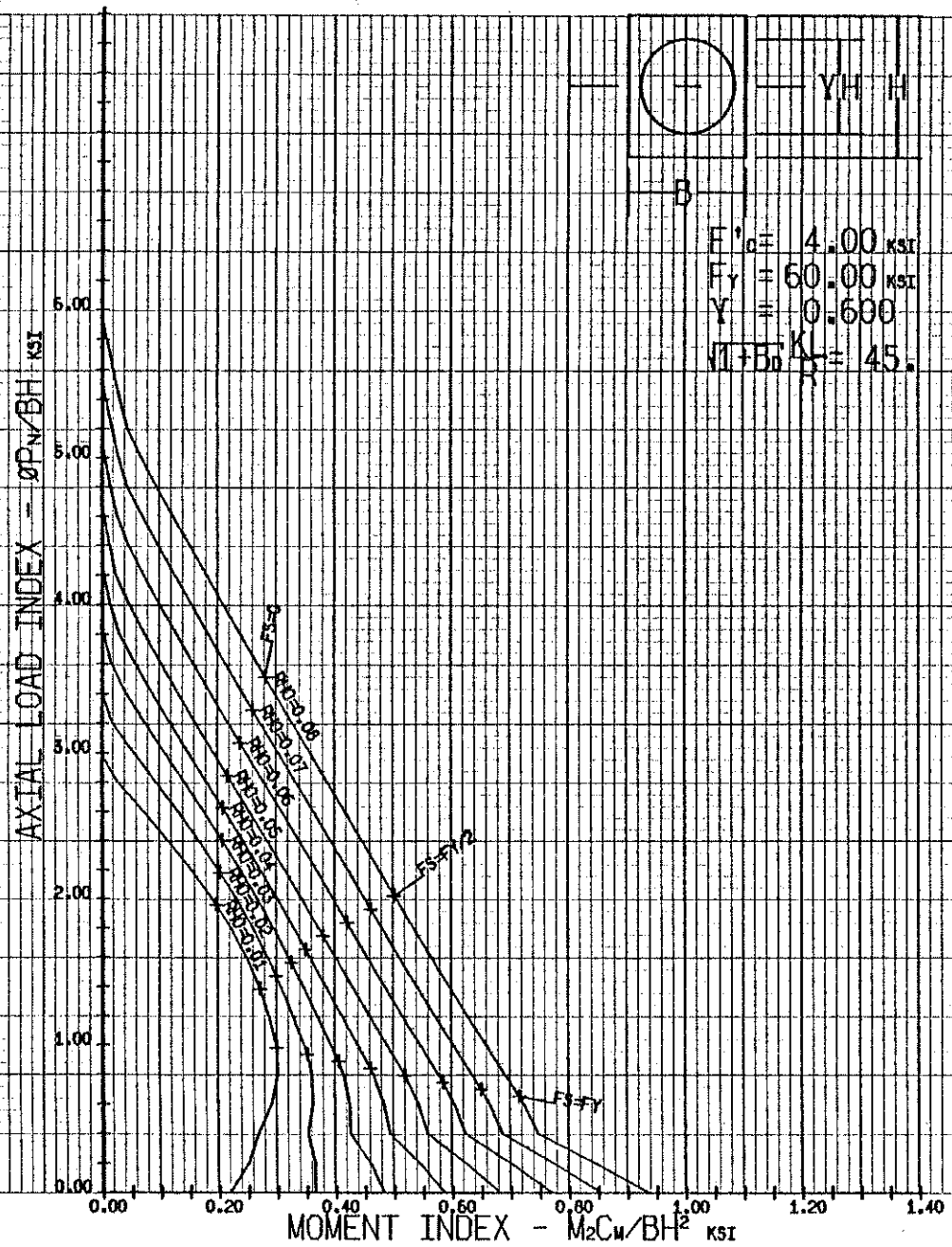


Fig. S4-60.60-45 - Interaction Diagram

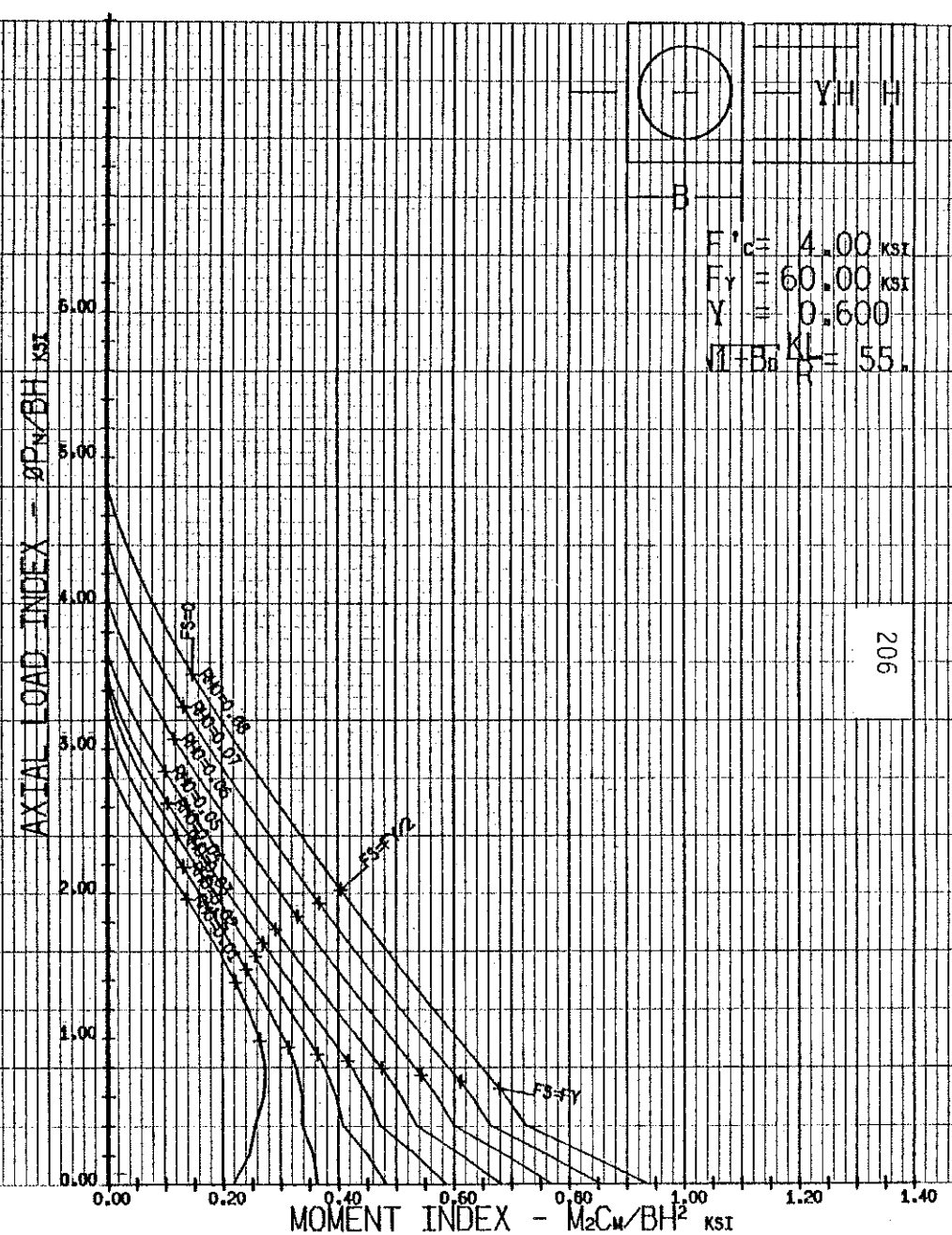


Fig. S4-60.60-55 - Interaction Diagram

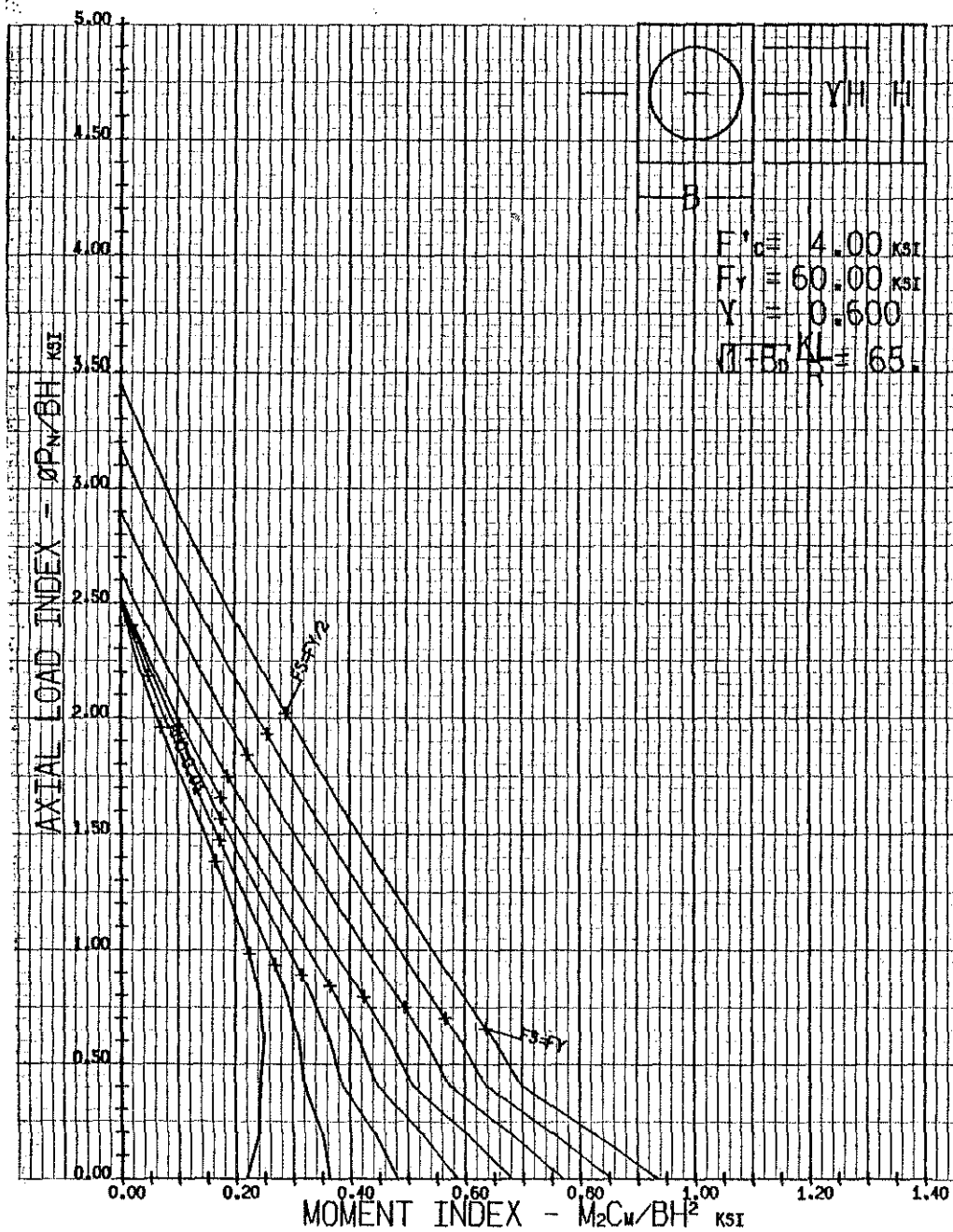


Fig. S4-60.60-65 - Interaction Diagram

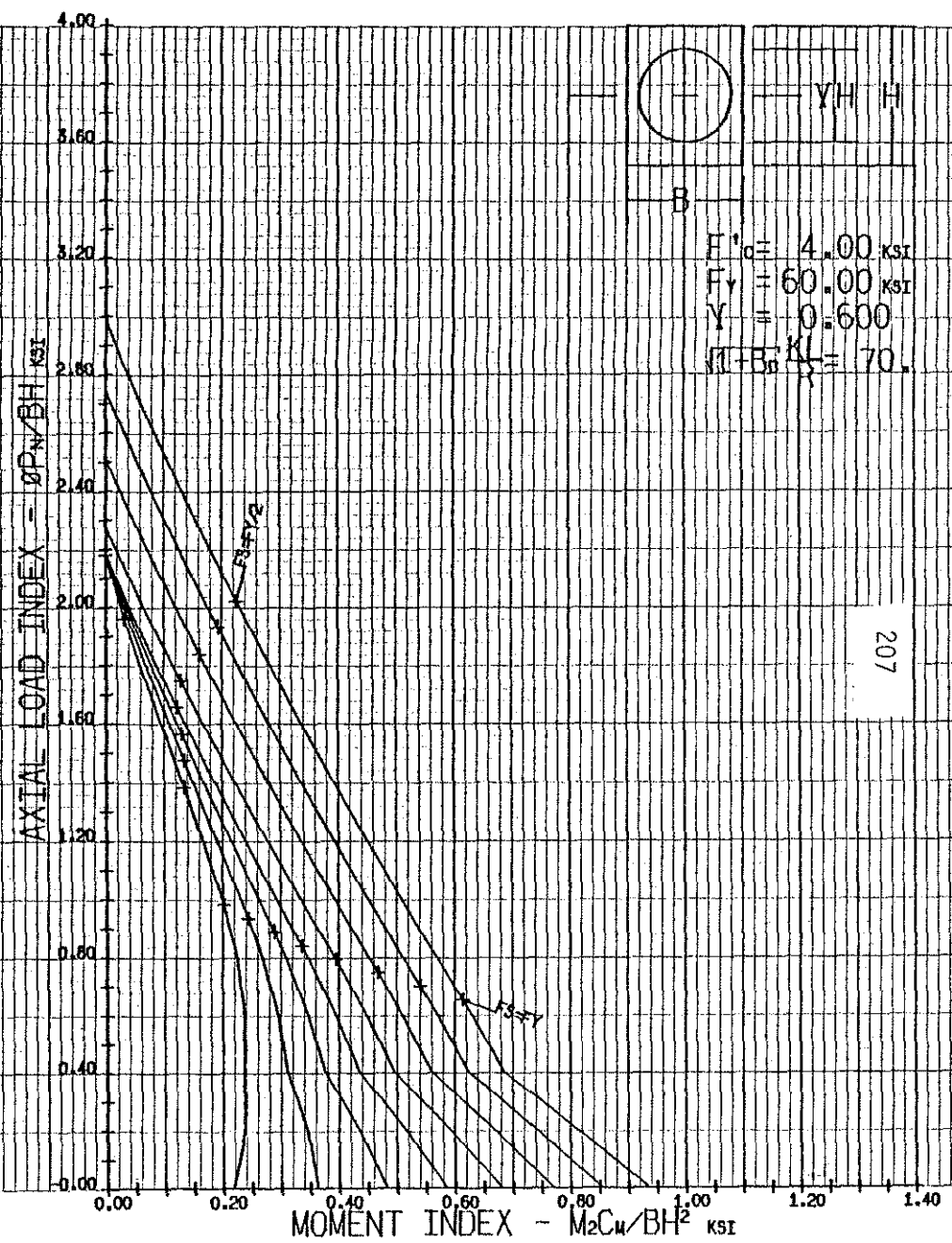


Fig. S4-60.60-70 - Interaction Diagram





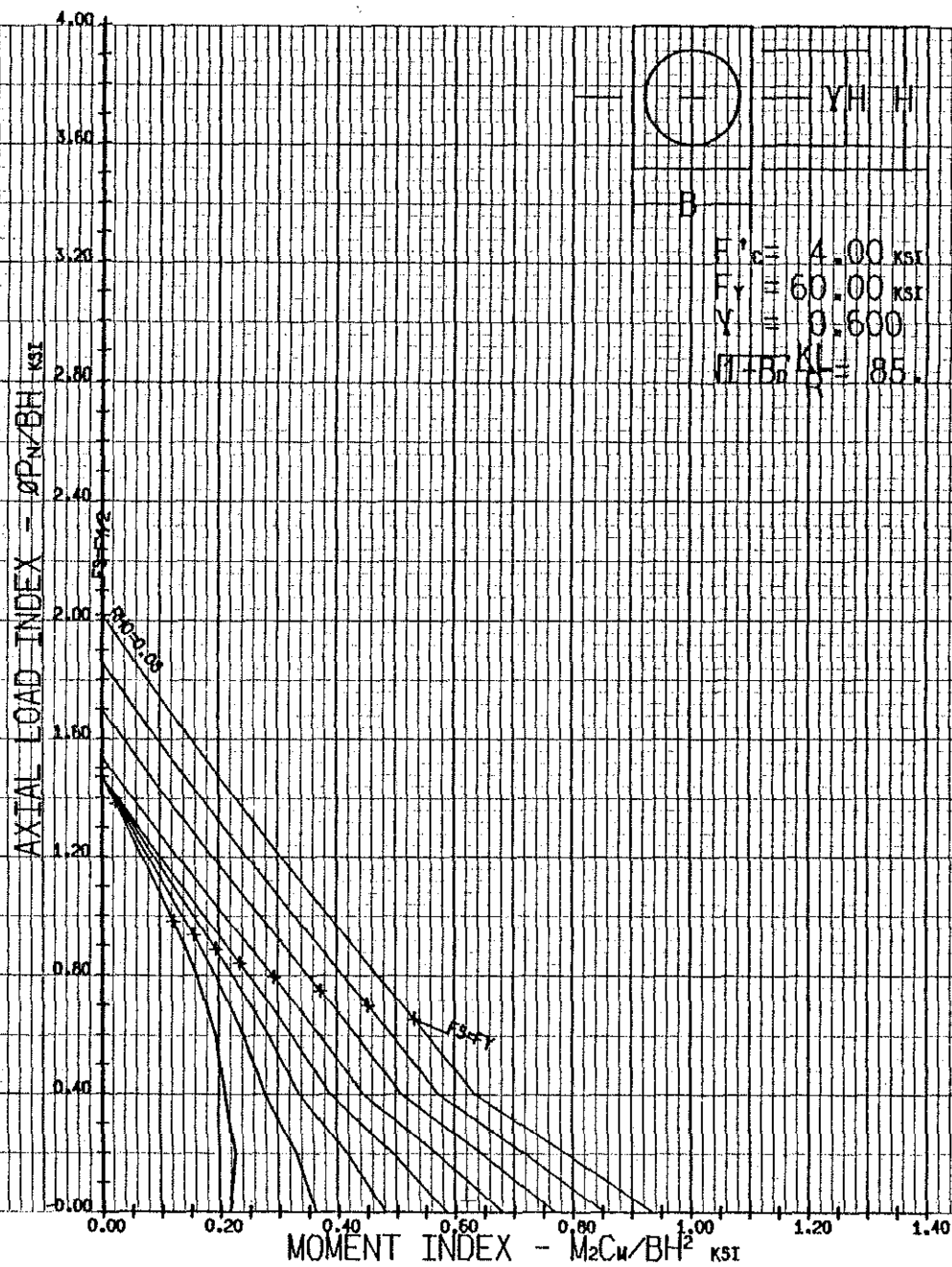


Fig. S4-60.60-85 - Interaction Diagram

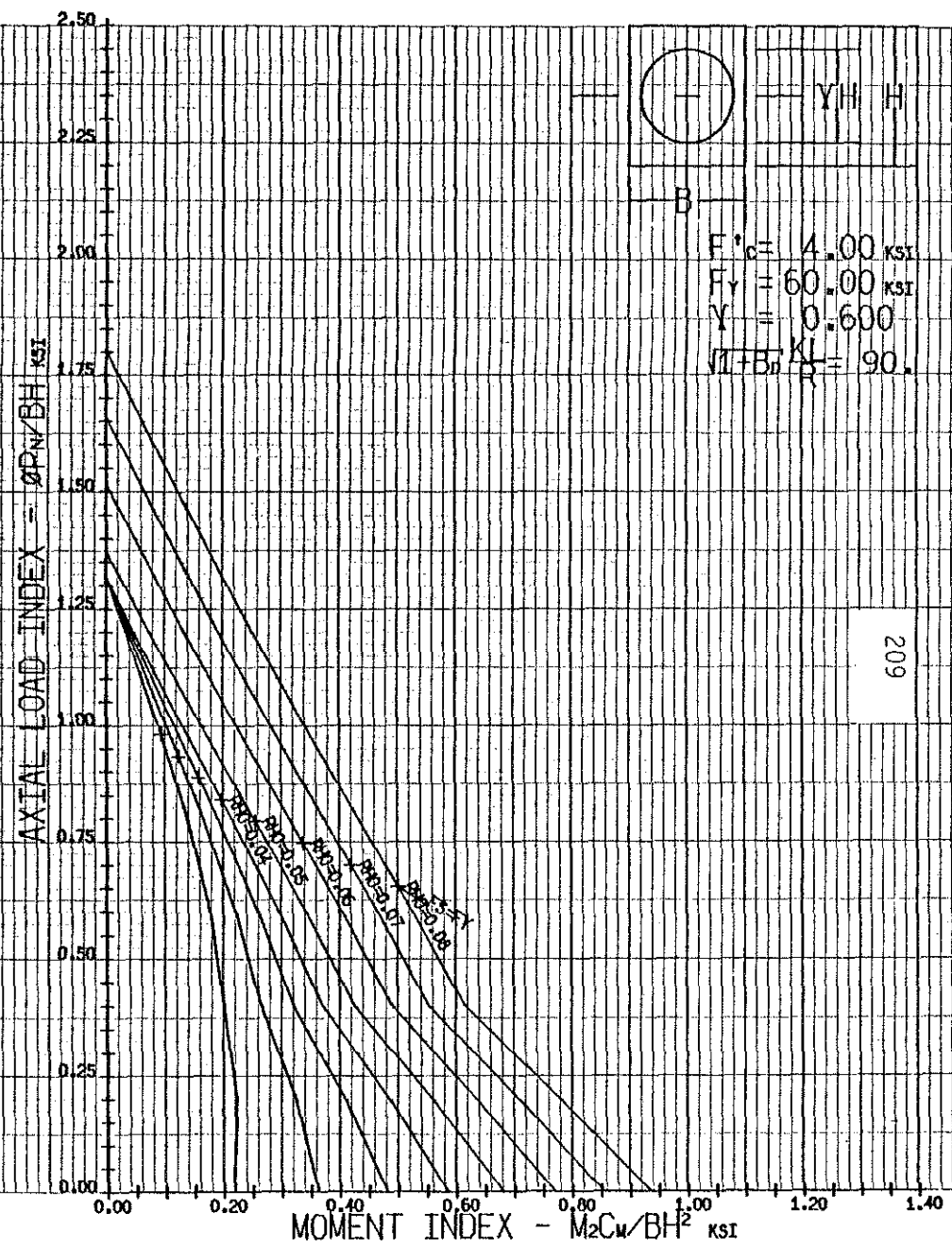


Fig. S4-60.60-90 - Interaction Diagram



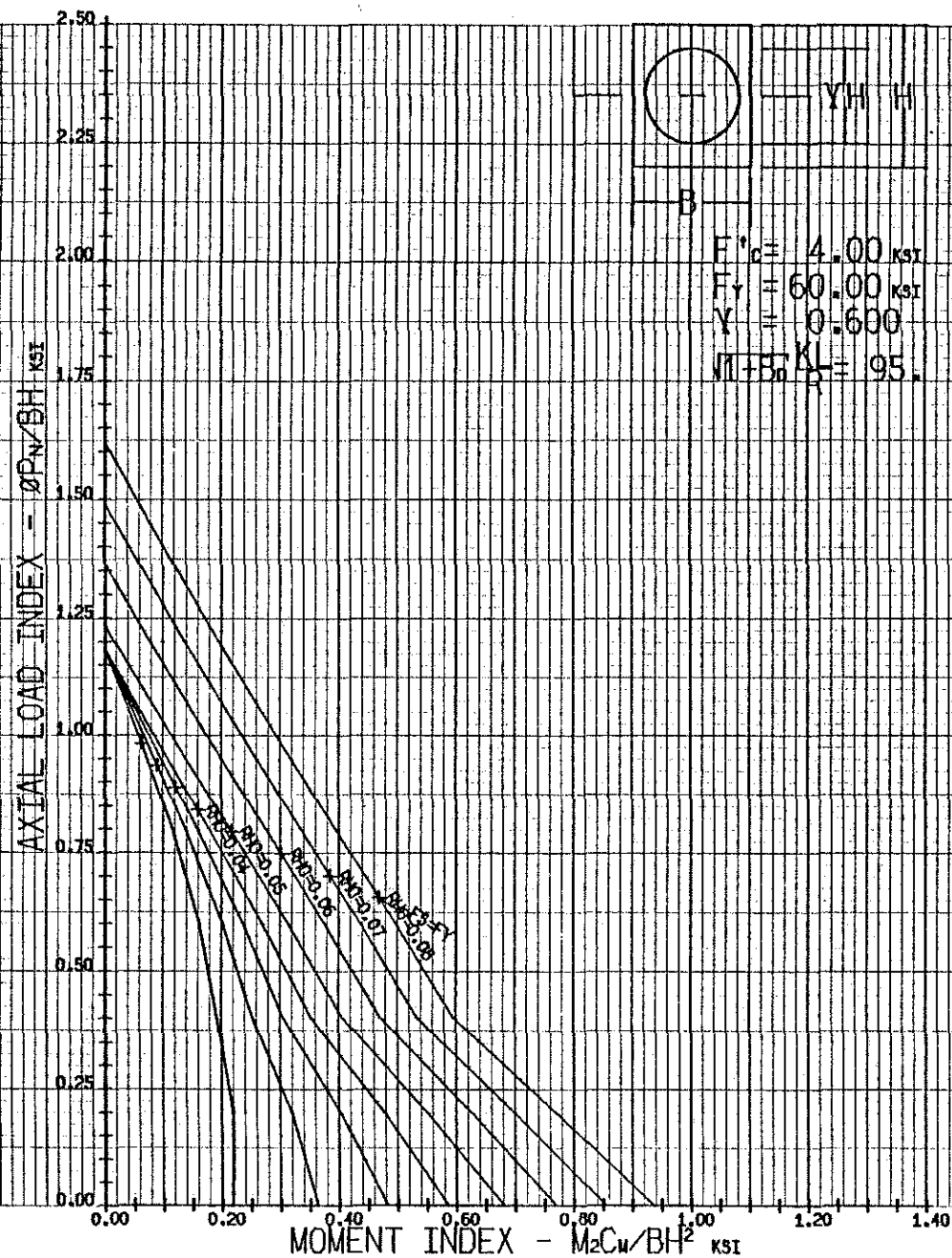


Fig. S4-60.60-95 - Interaction Diagram

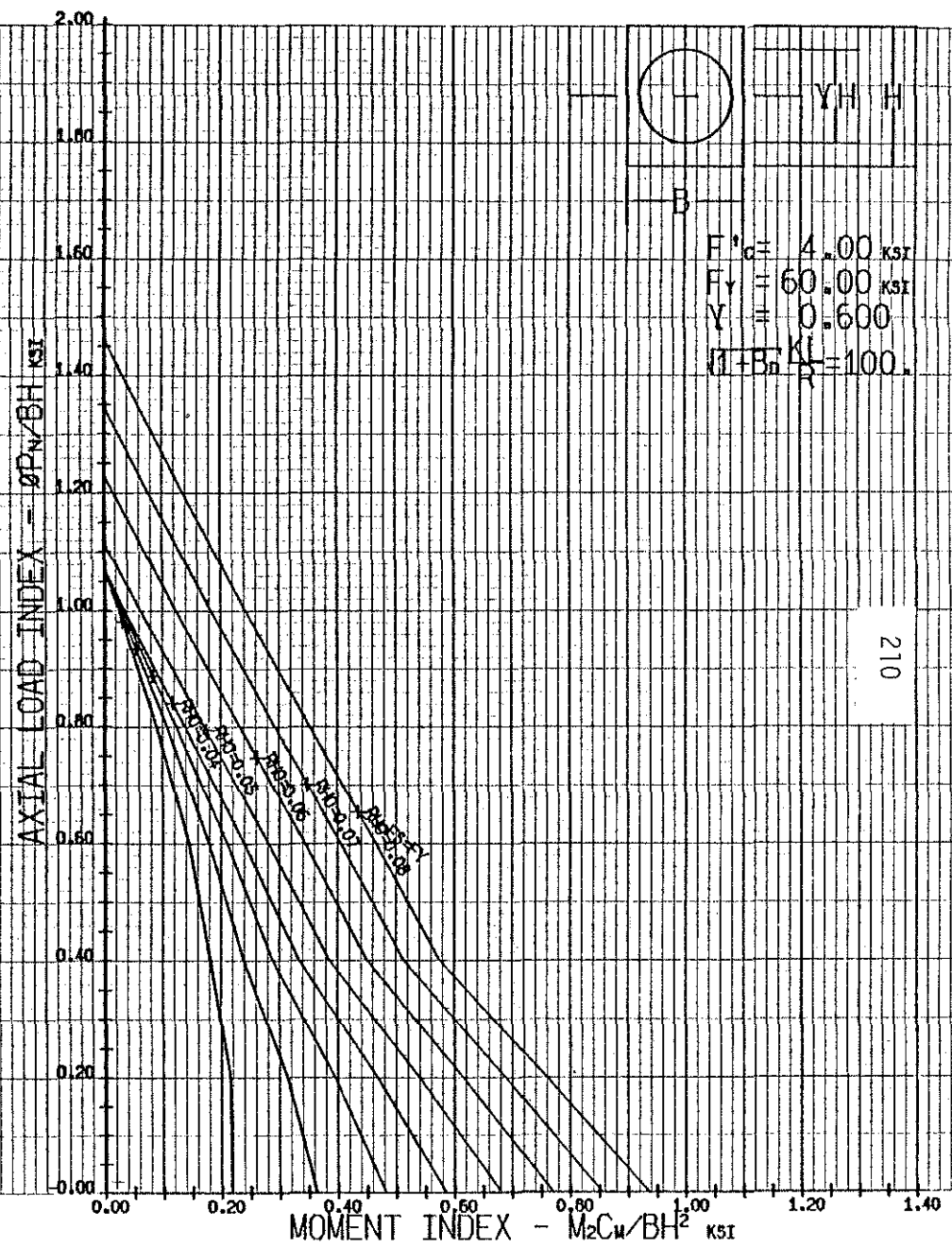


Fig. S4-60.60-100 - Interaction Diagram

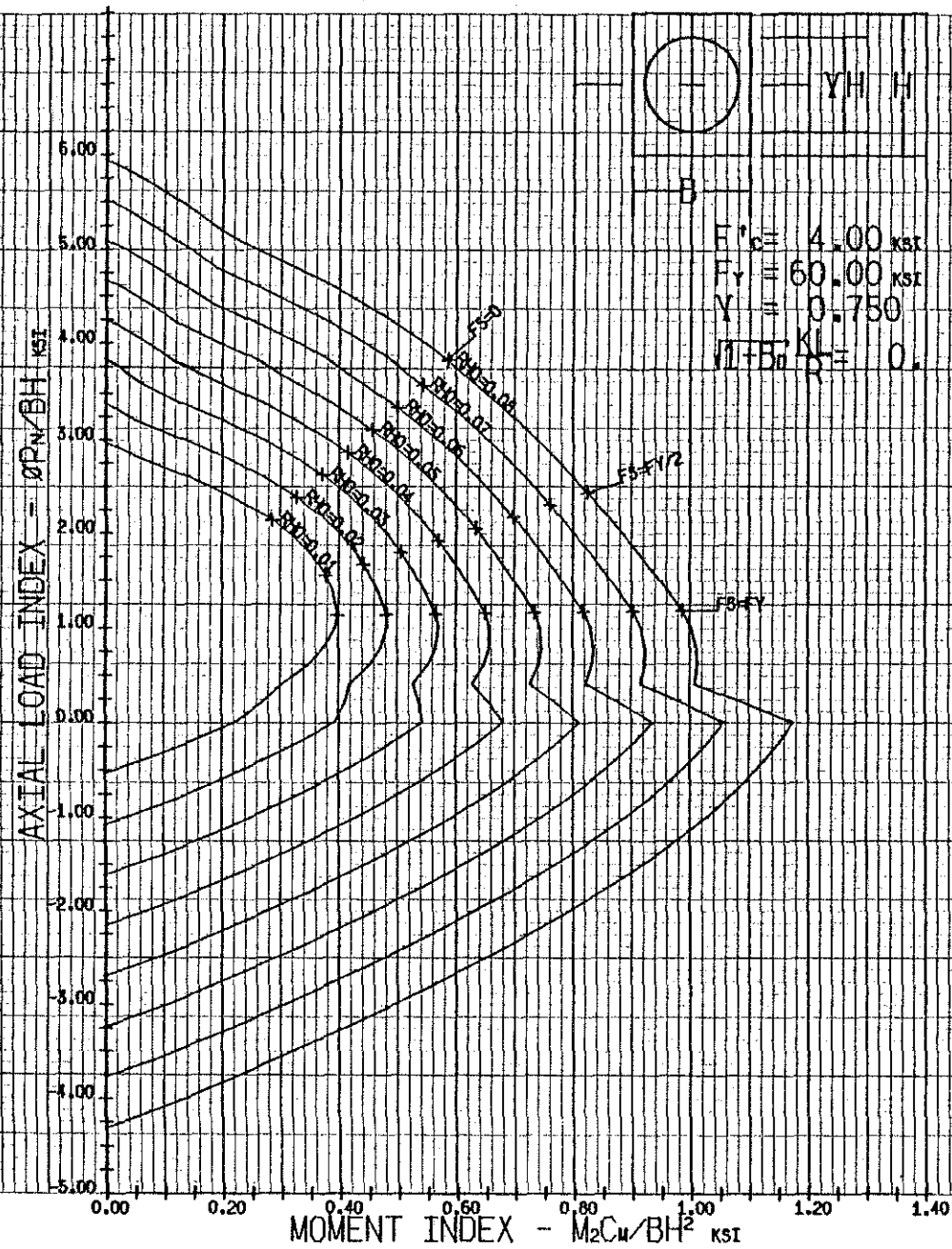


Fig. S4-60.75-0 - Interaction Diagram

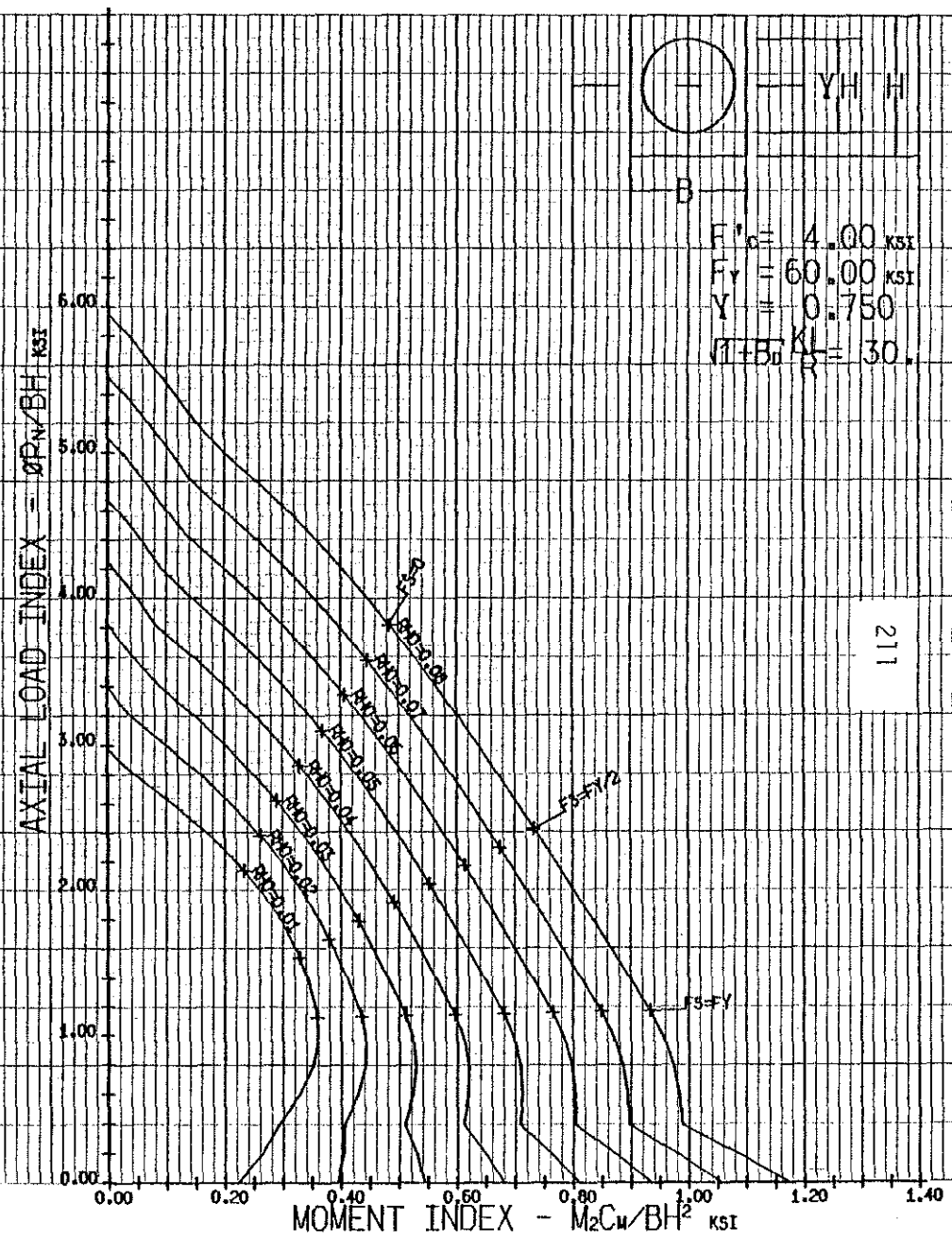
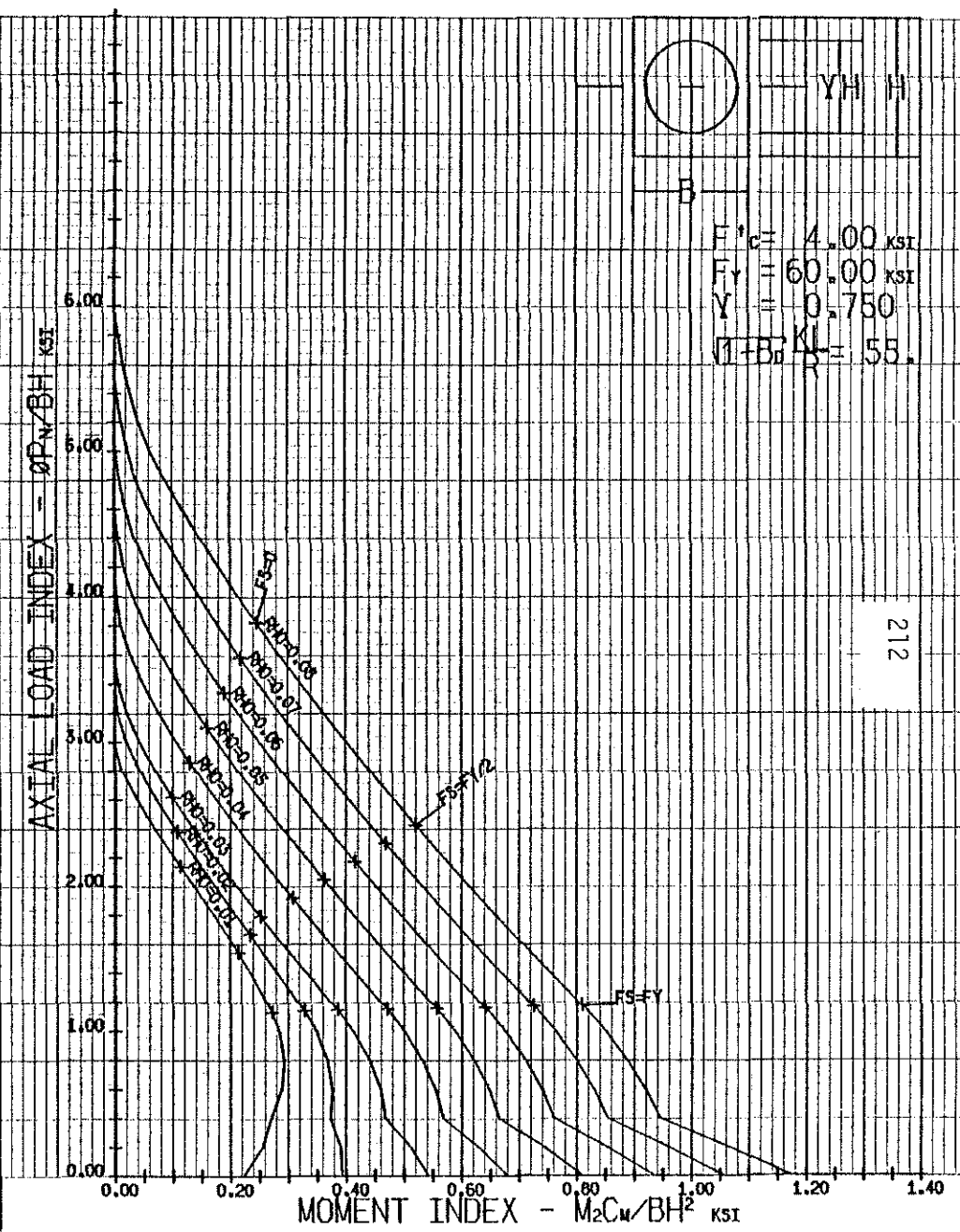
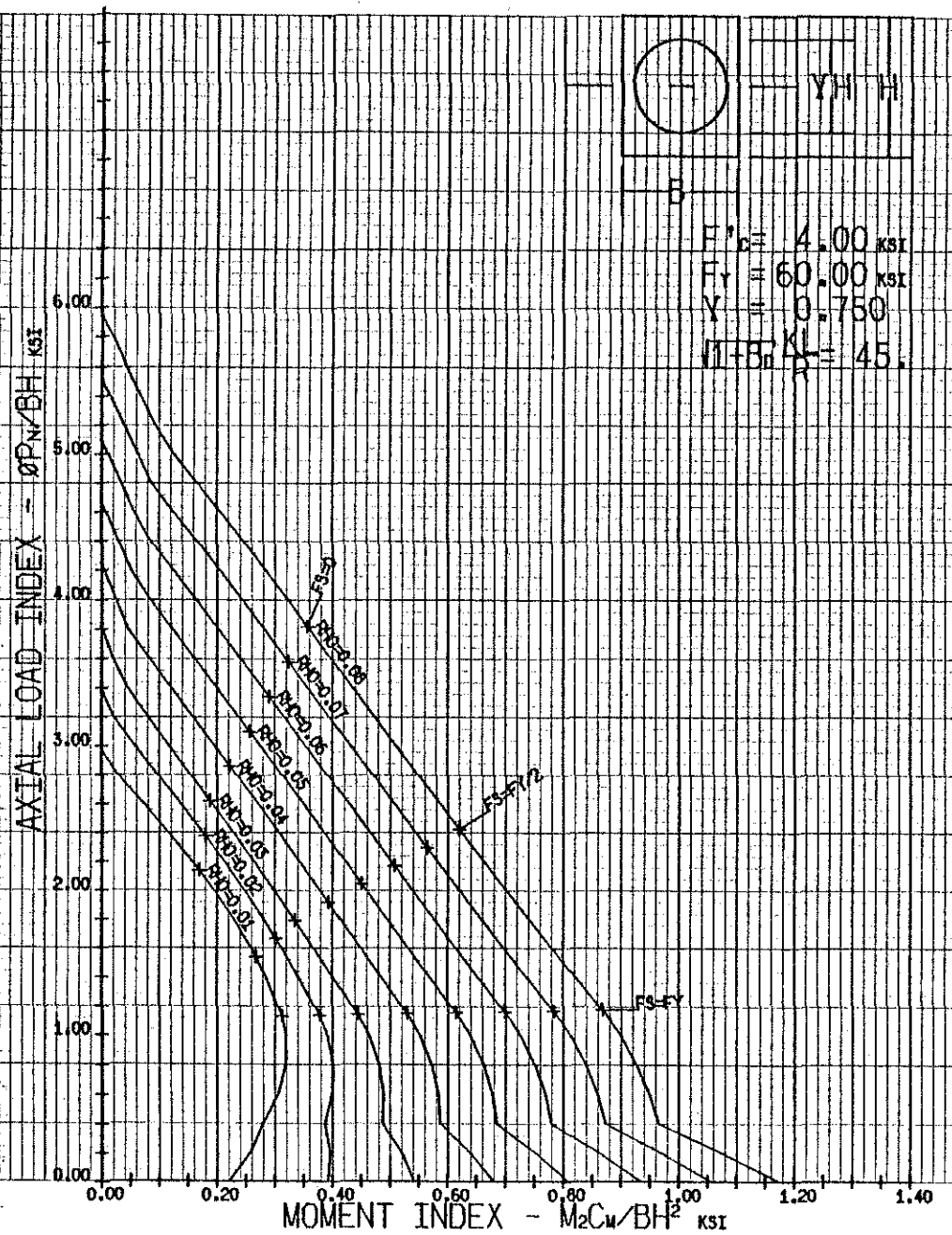


Fig. S4-60.75-30 - Interaction Diagram



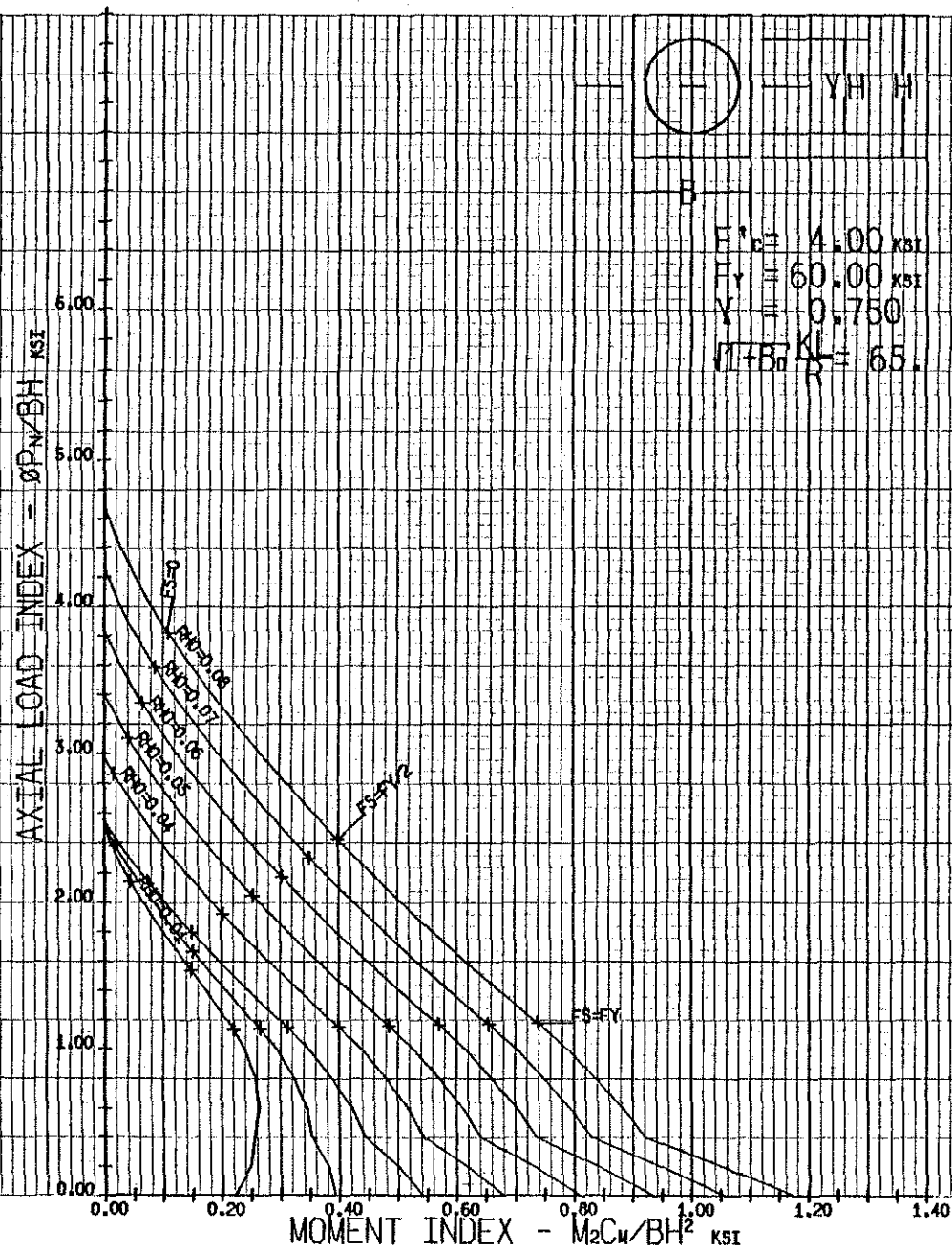


Fig. S4-60.75-65 - Interaction Diagram

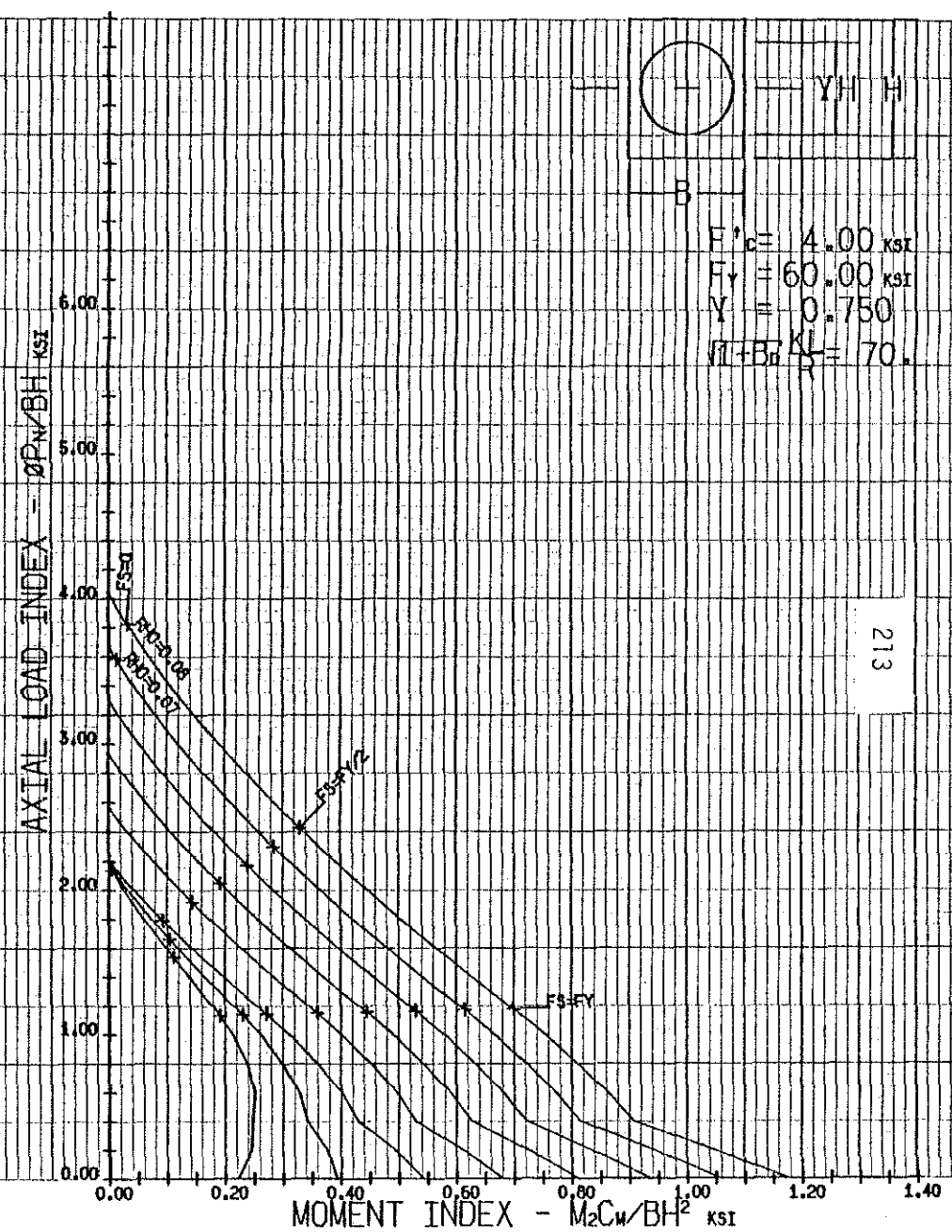


Fig. S4-60.75-70 - Interaction Diagram

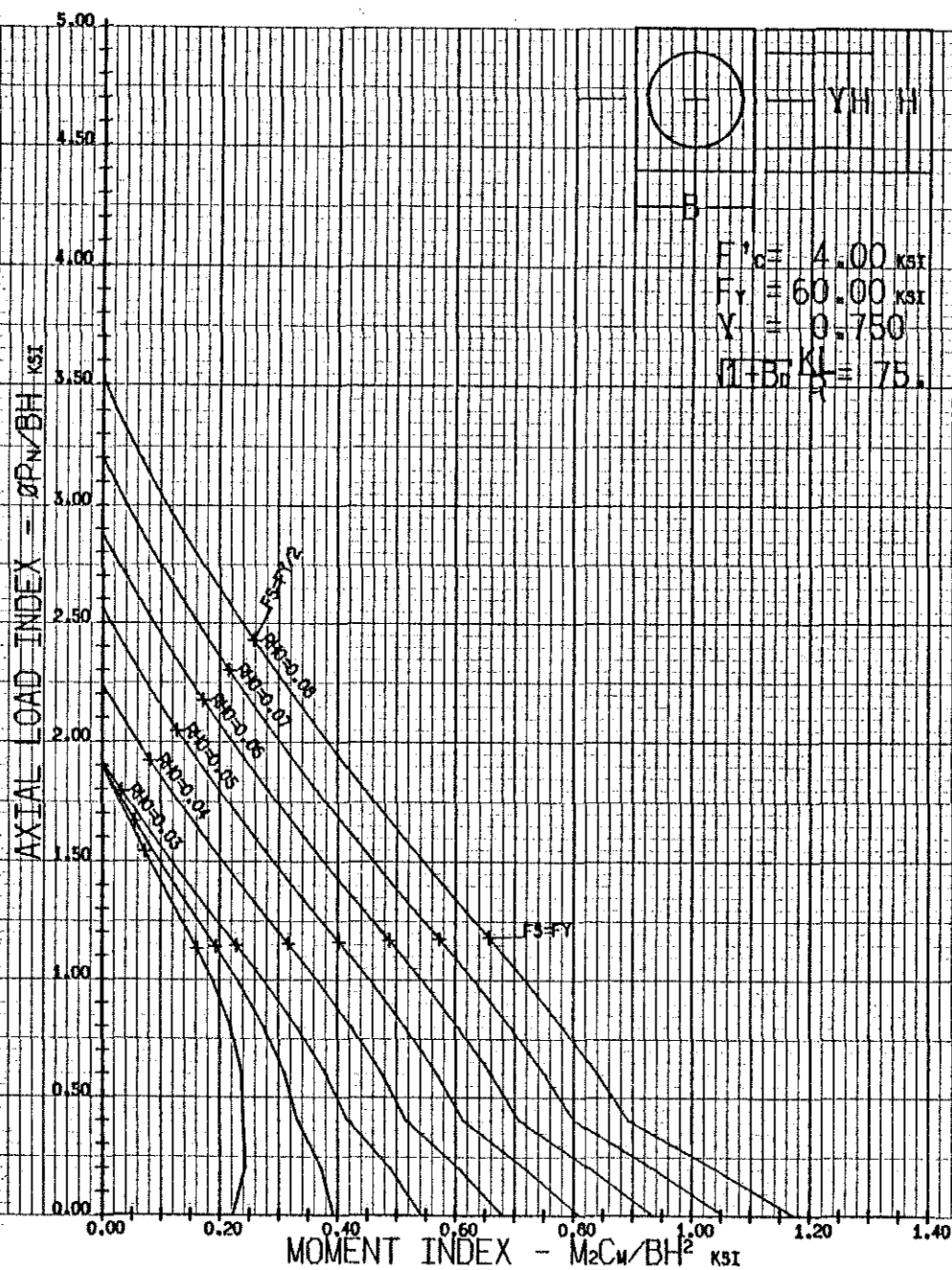


Fig. S4-60.75-75 - Interaction Diagram

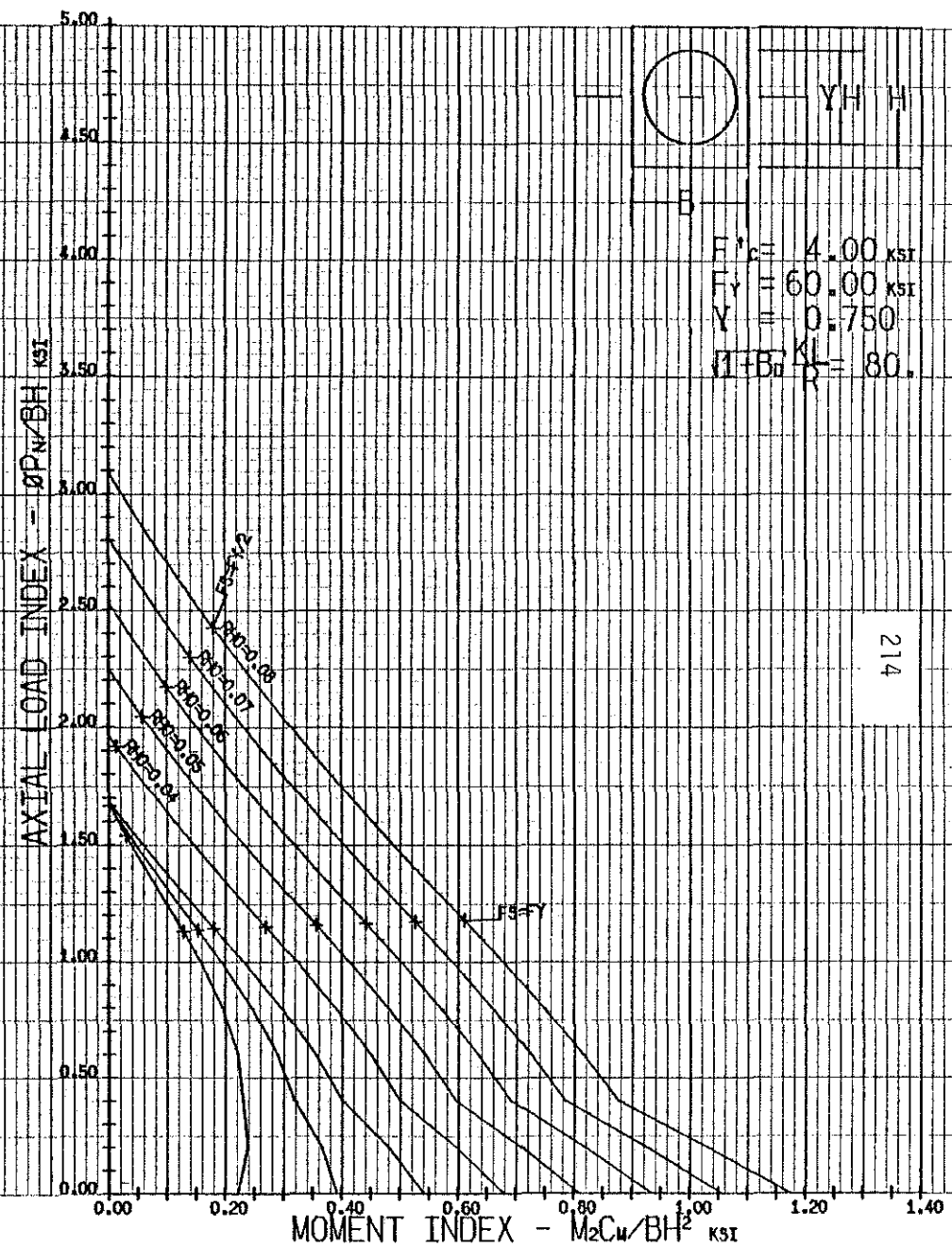


Fig. S4-60.75-80 - Interaction Diagram



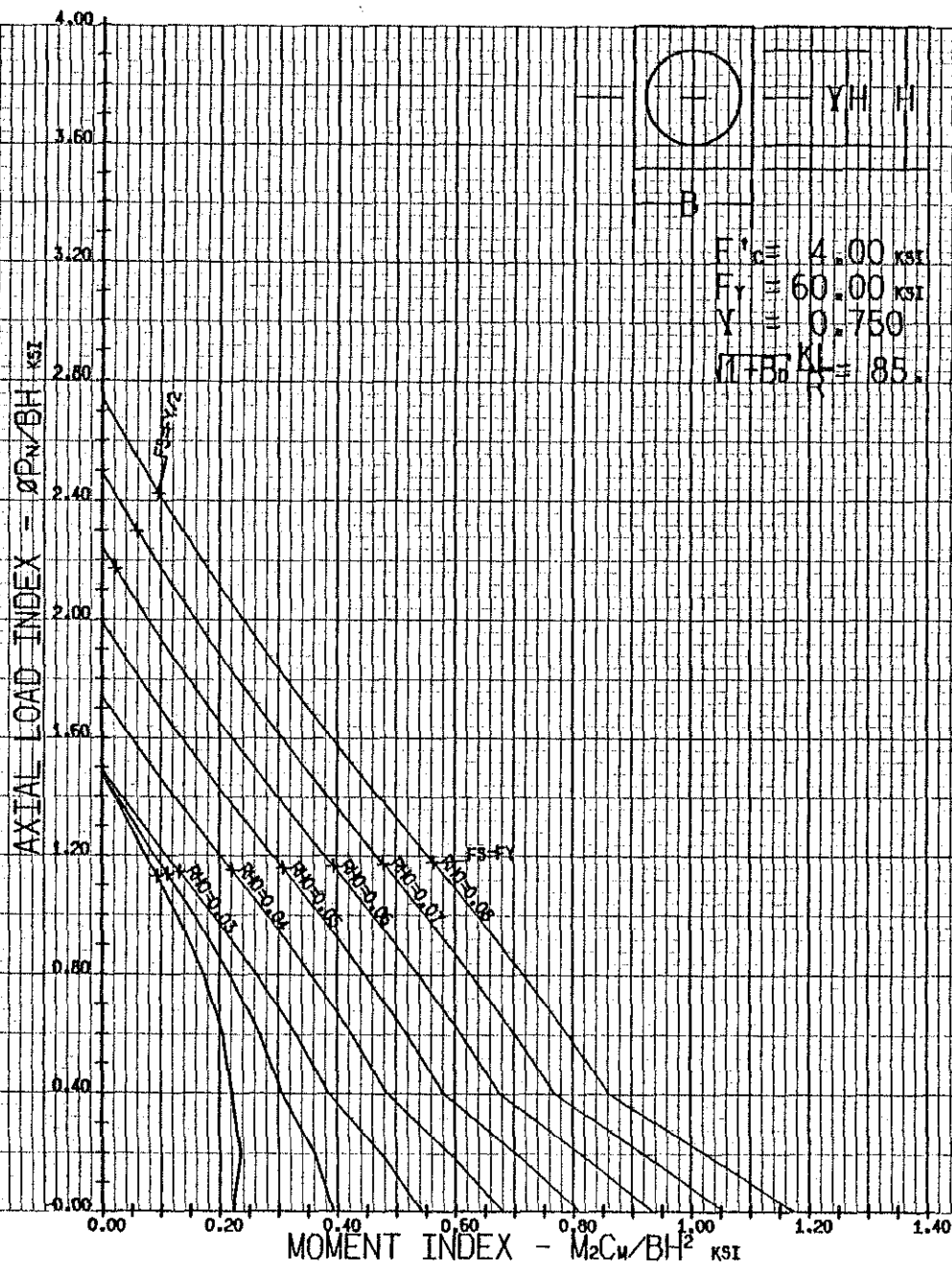


Fig. S4-60.75-85 - Interaction Diagram

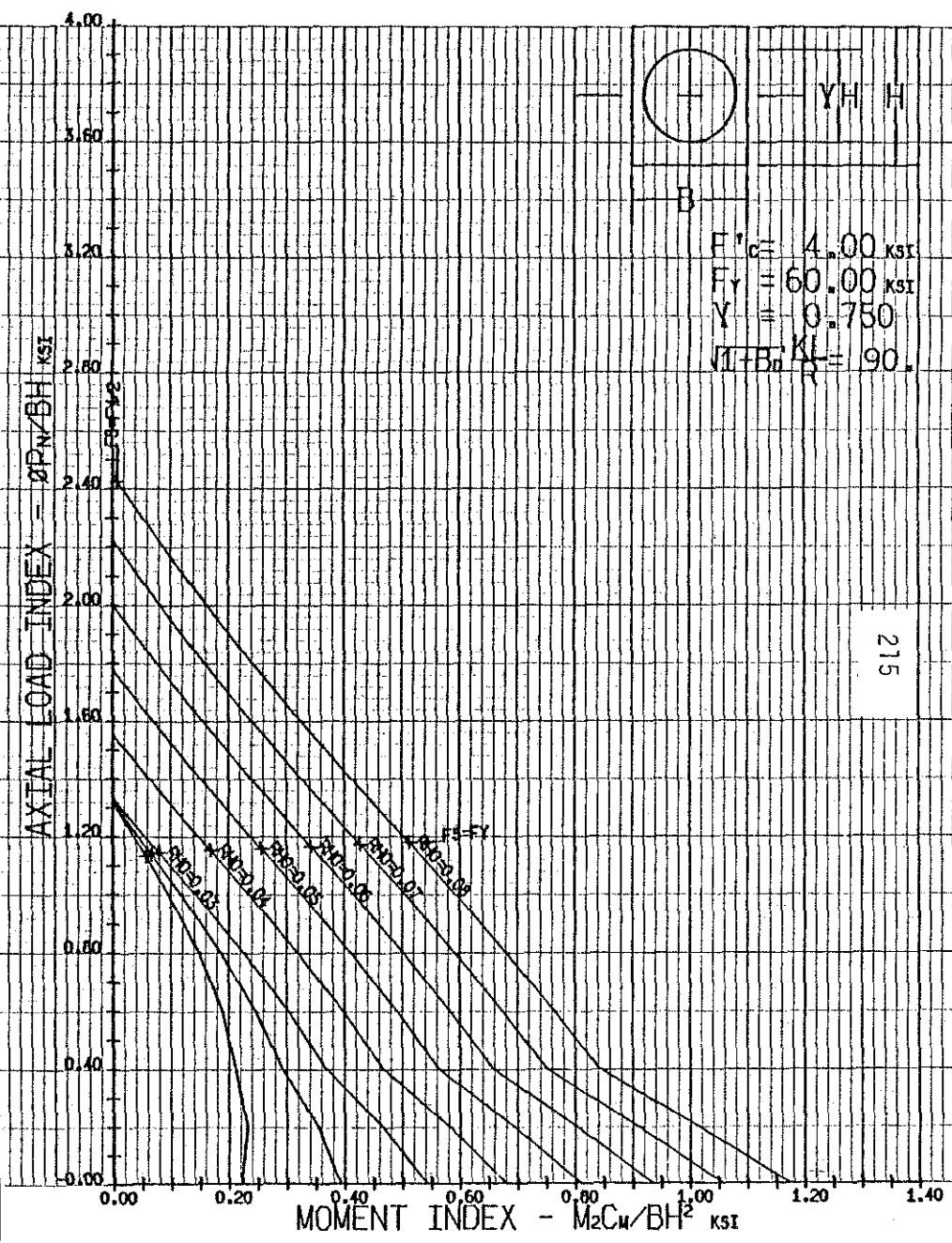
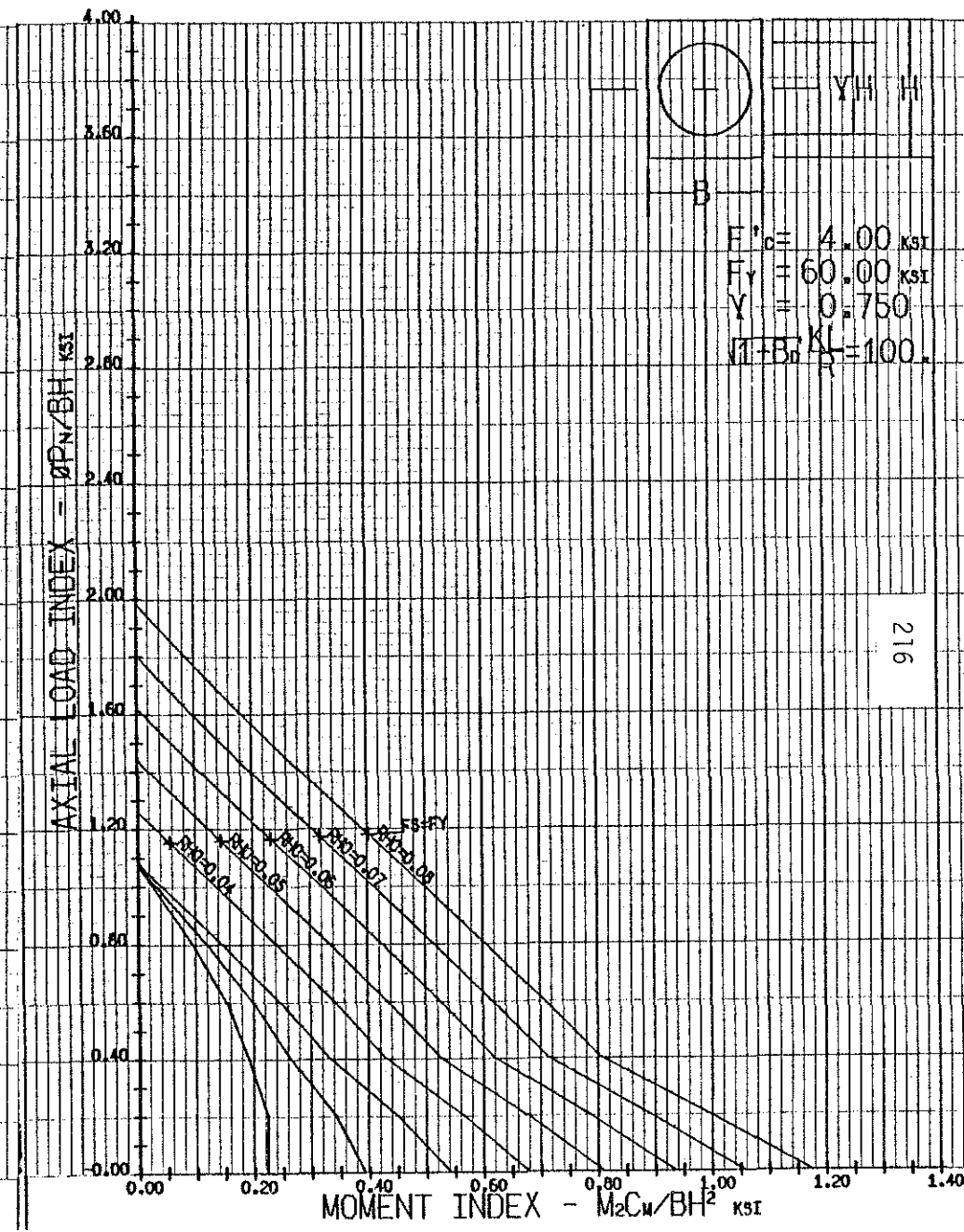
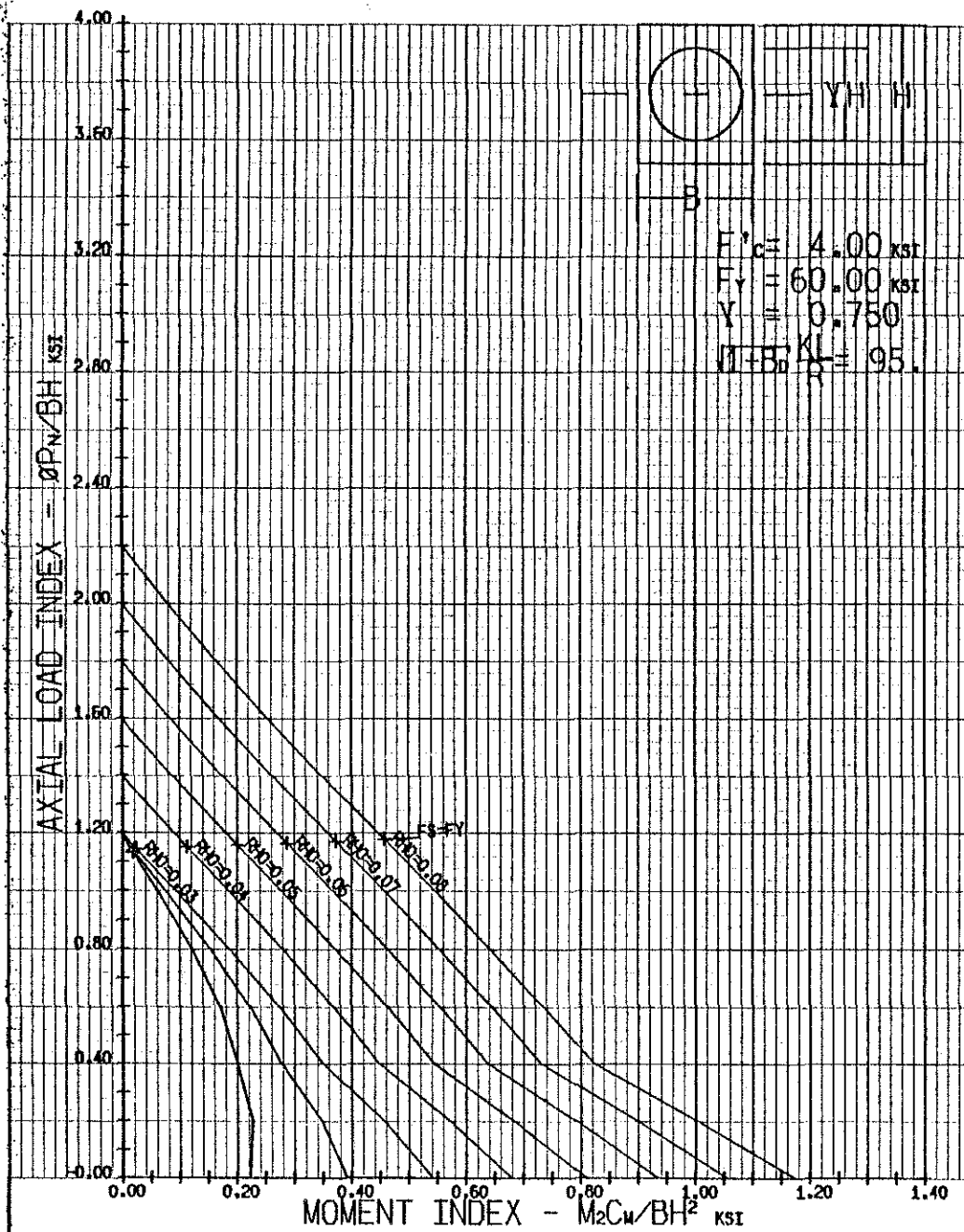


Fig. S4-60.75-90 - Interaction Diagram





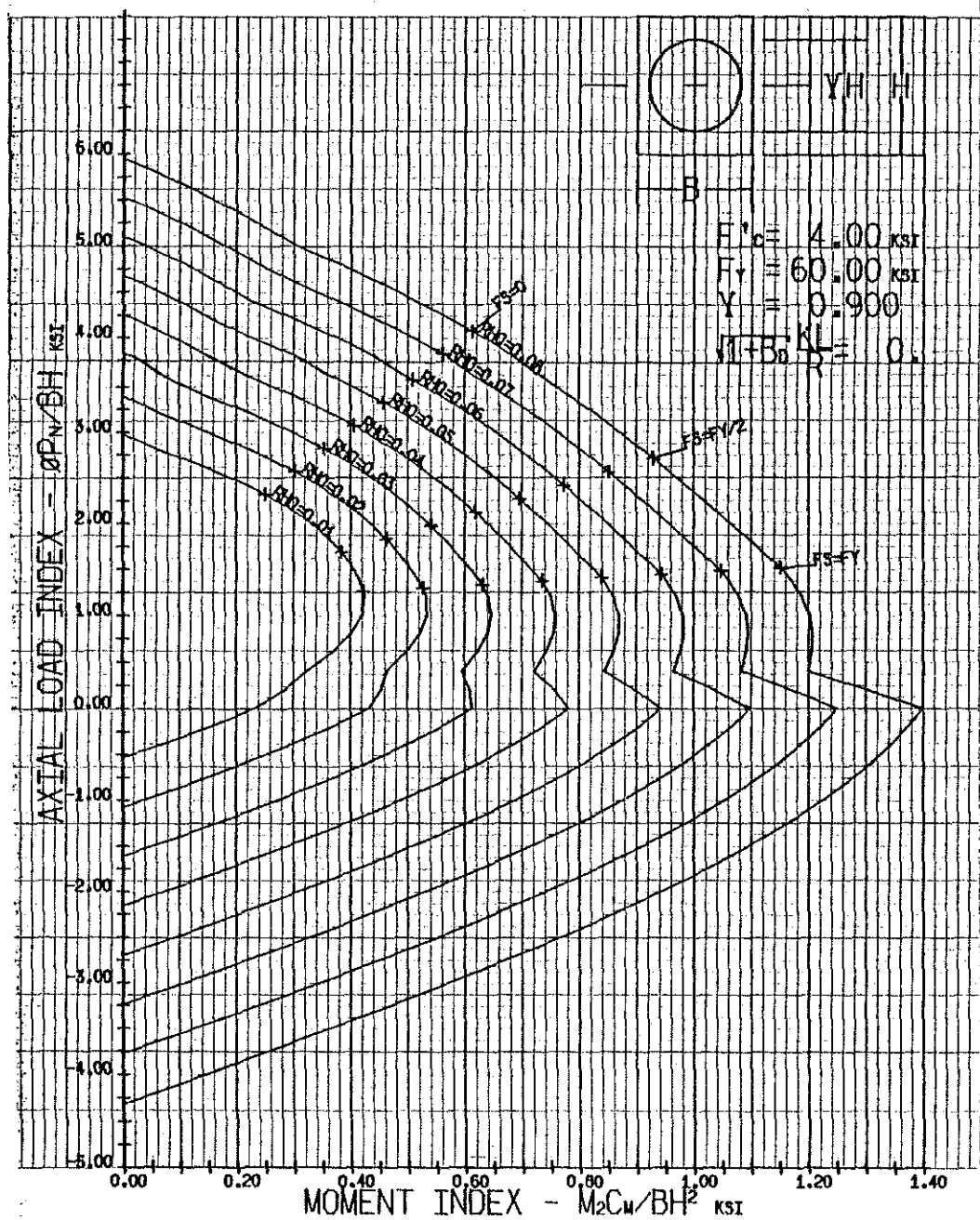


Fig. S4-60.90-0 - Interaction Diagram

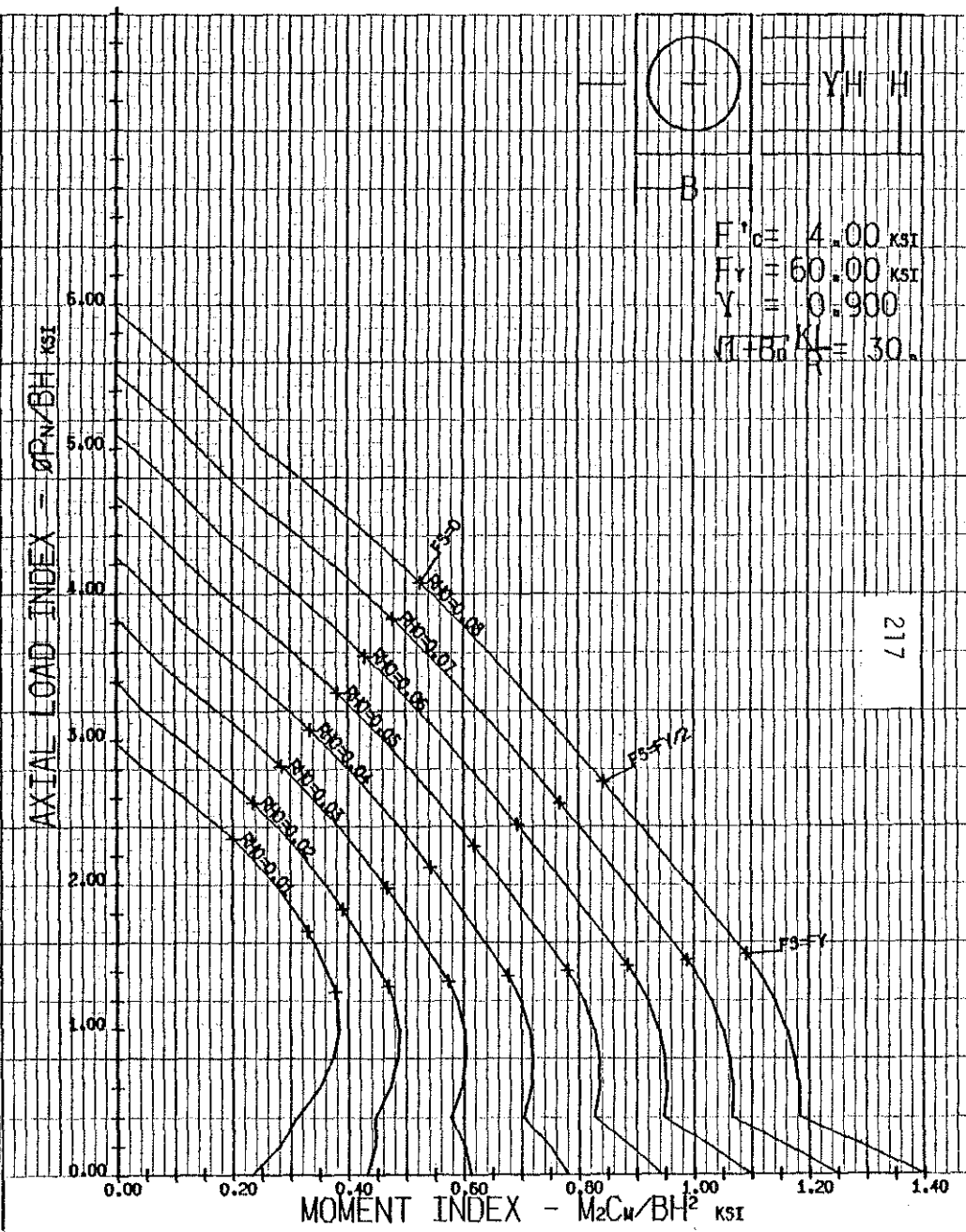
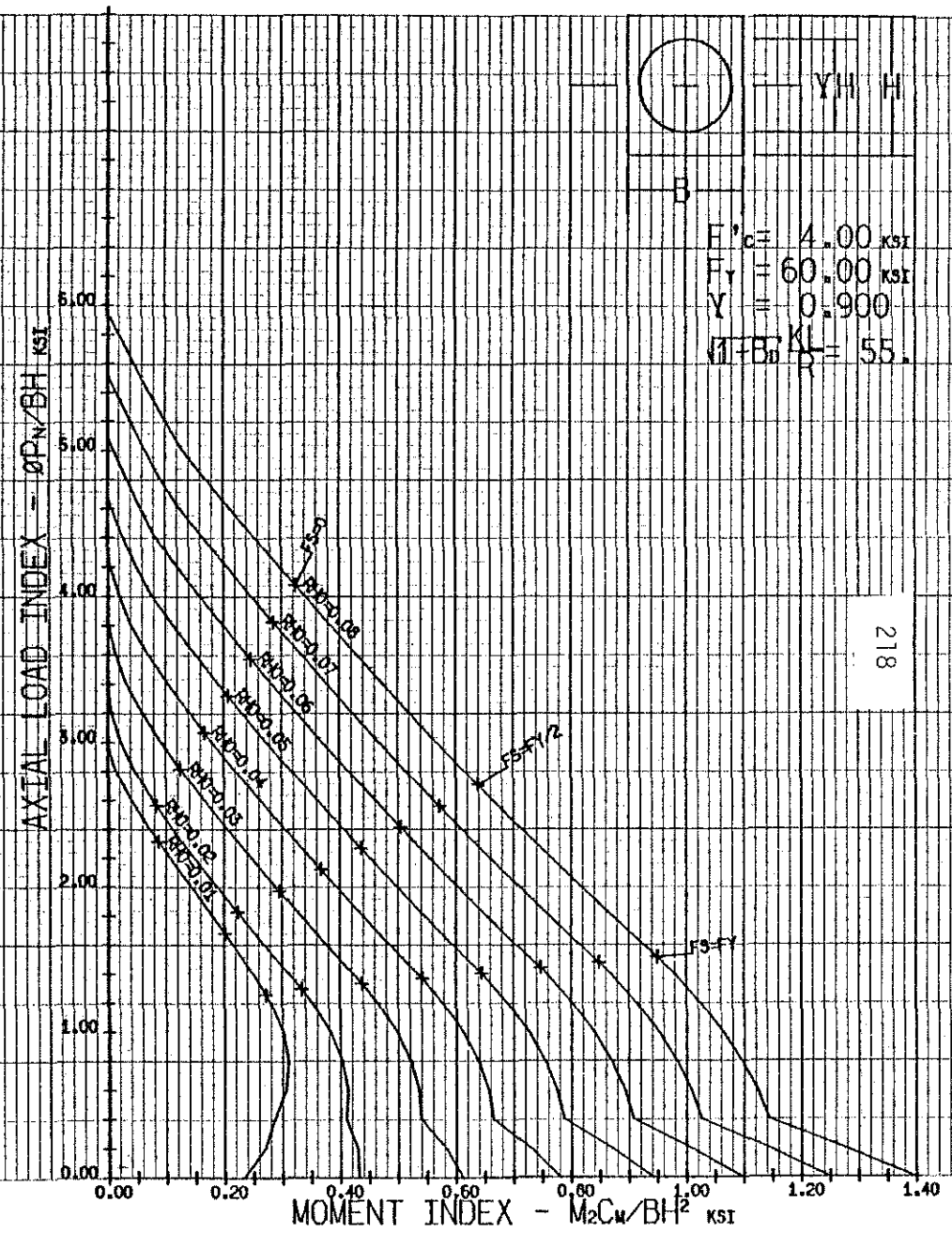
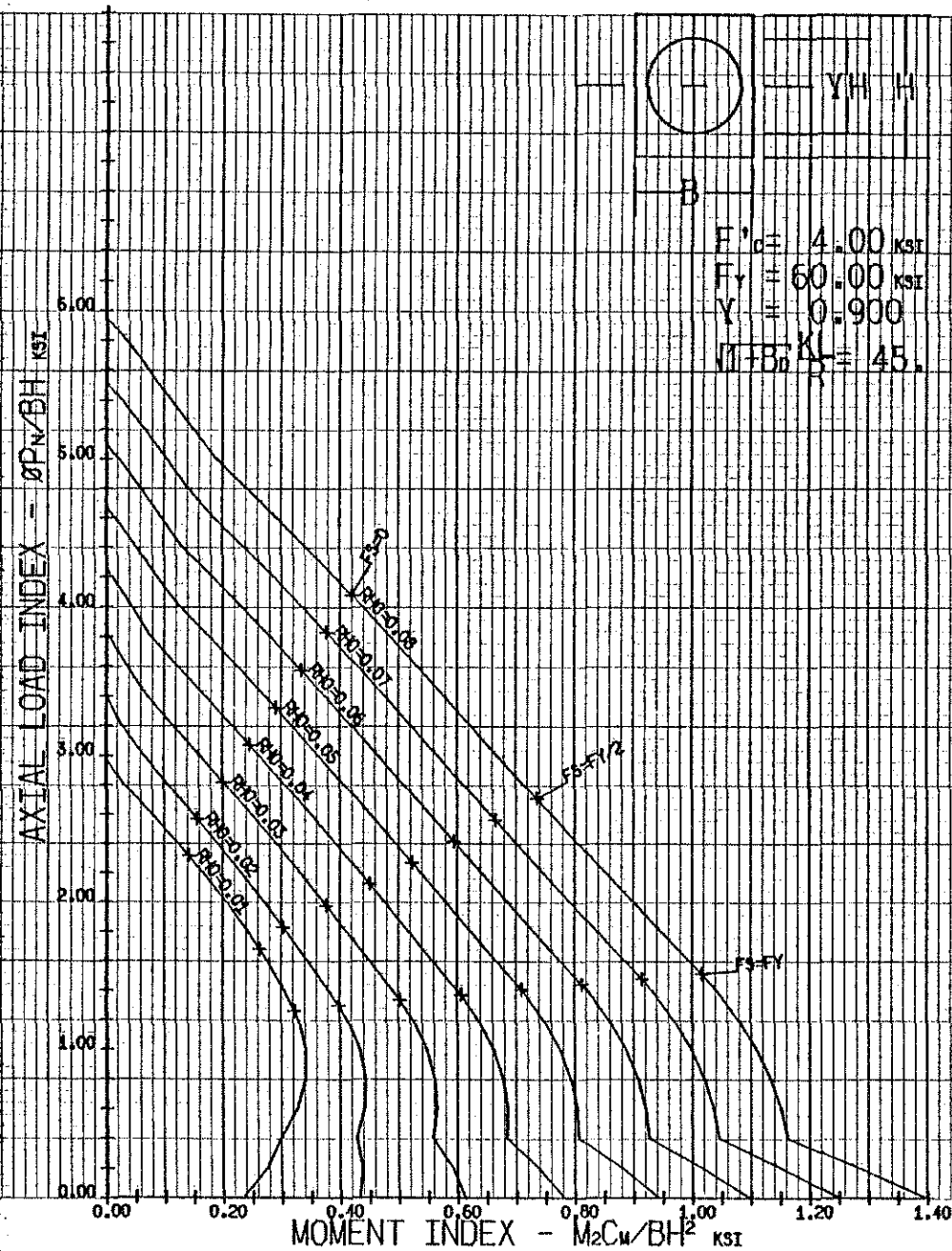


Fig. S4-60.90-30 - Interaction Diagram



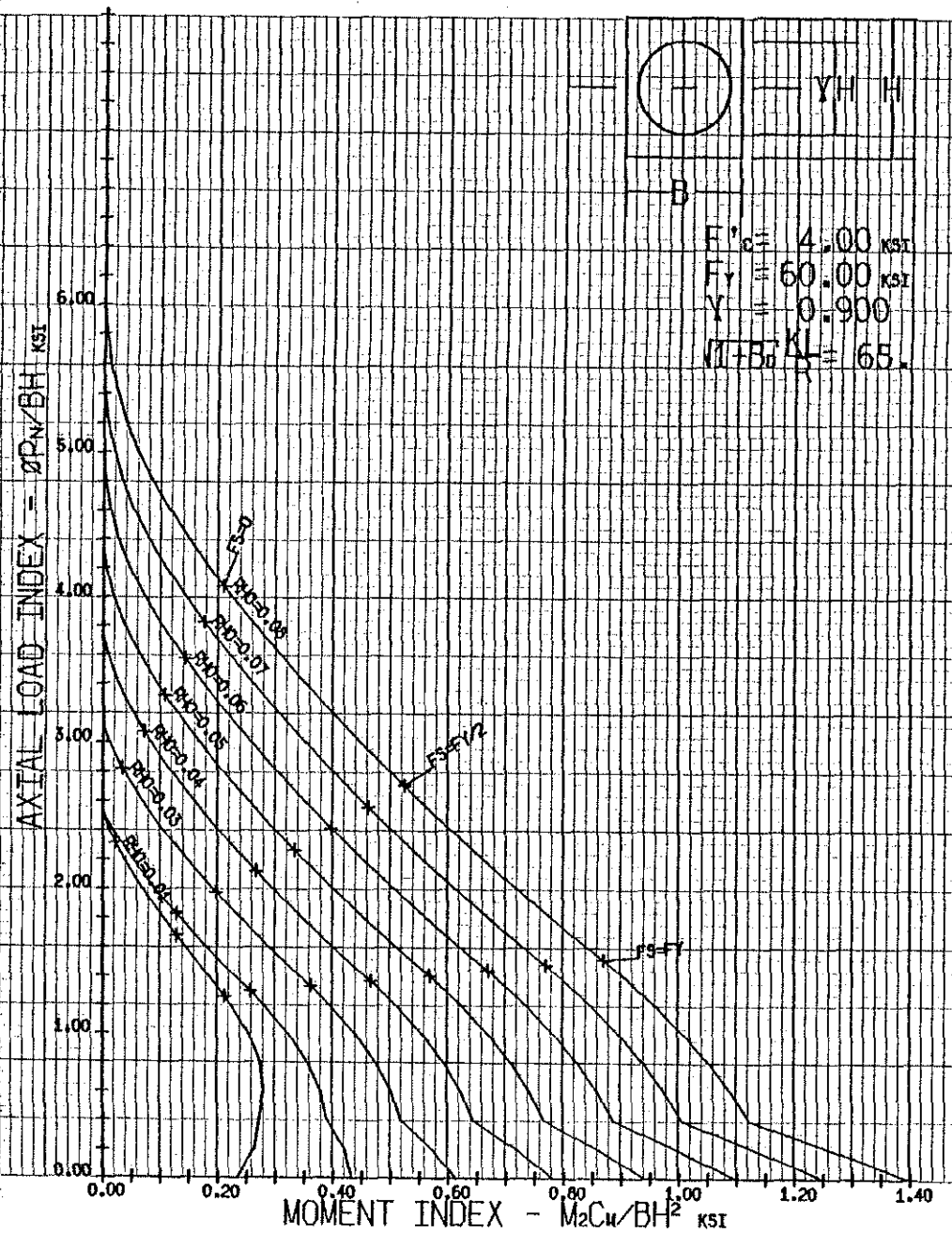


Fig. S4-60.90-65 - Interaction Diagram

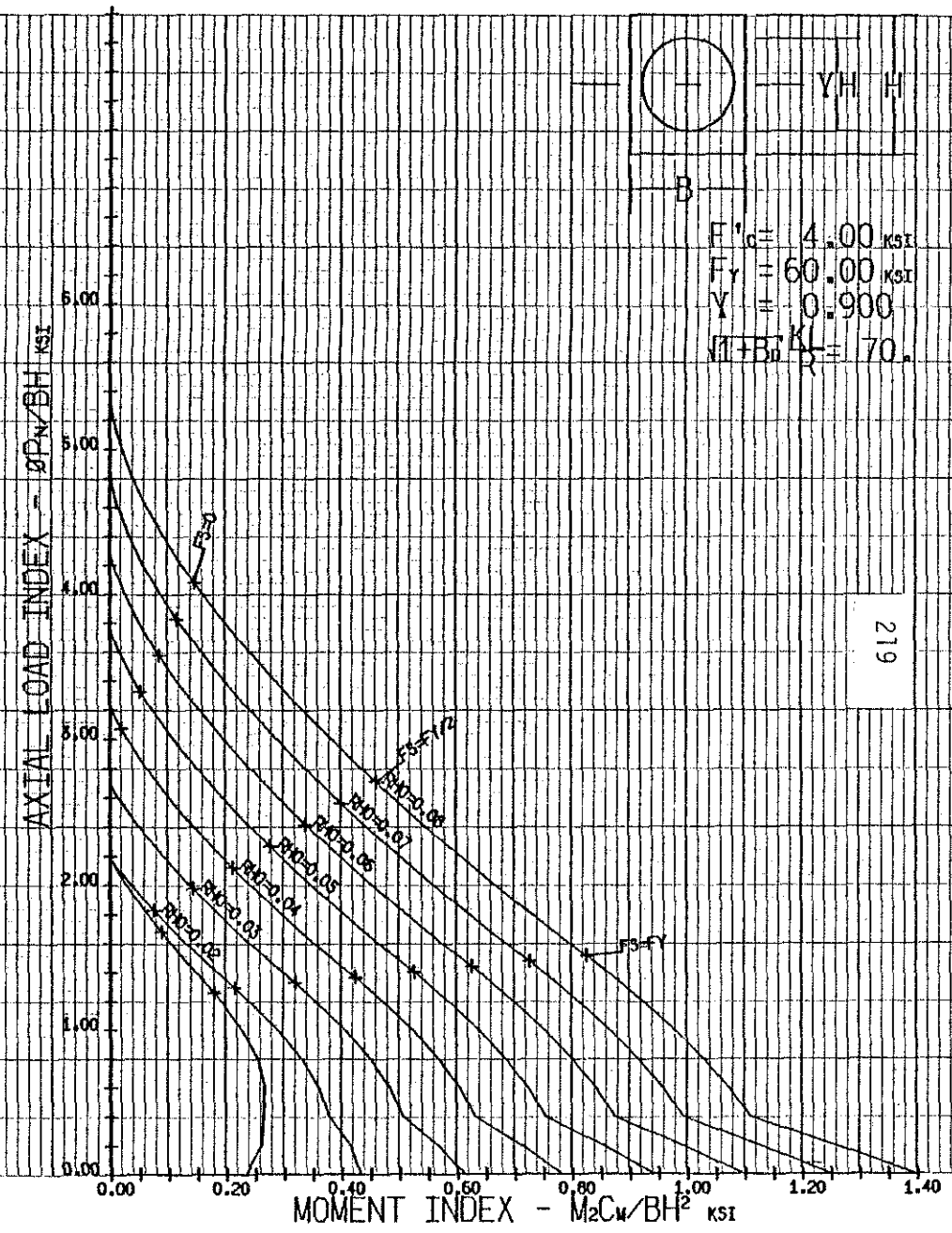


Fig. S4-60.90-70 - Interaction Diagram

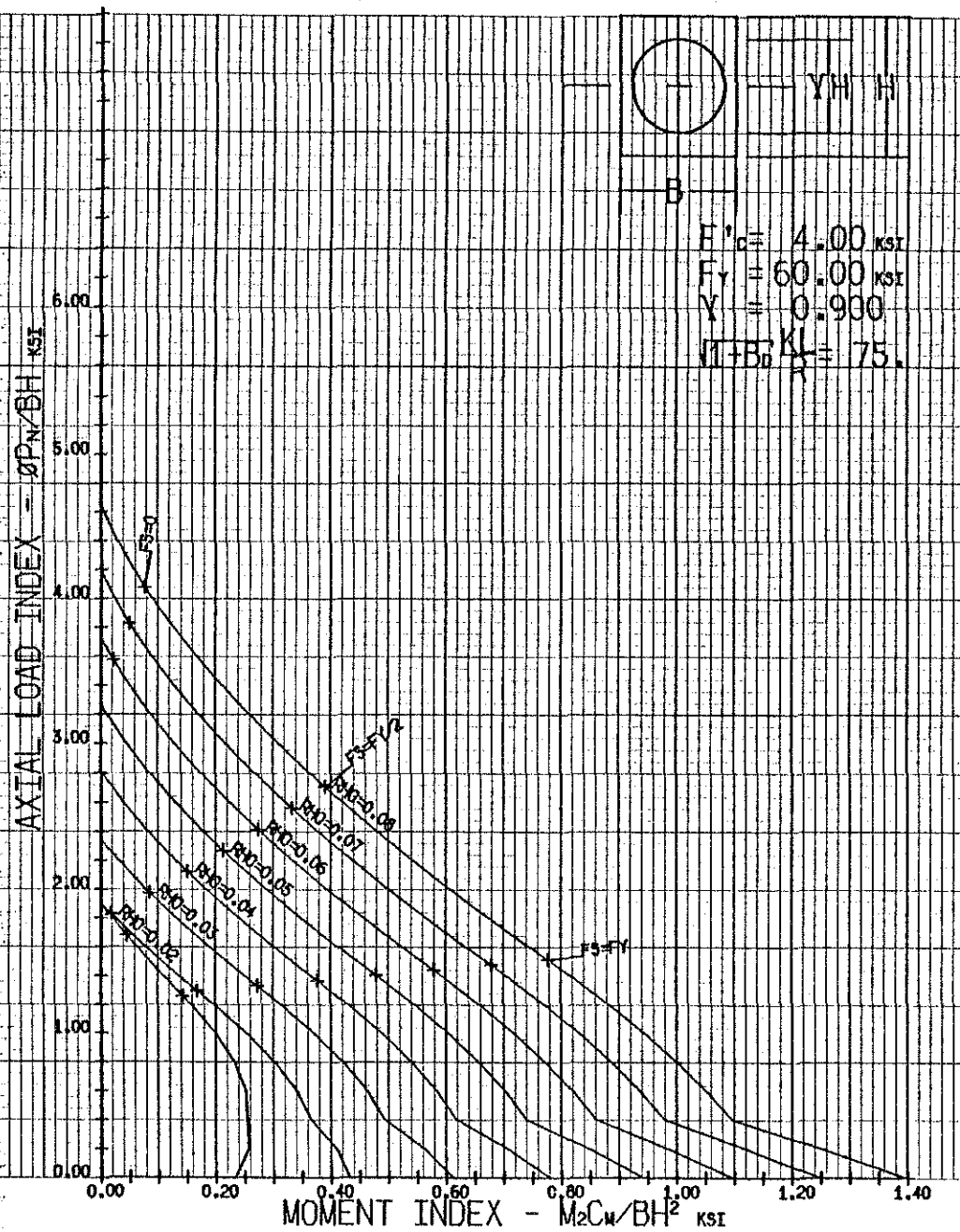


Fig. S4-60.90-75 - Interaction Diagram

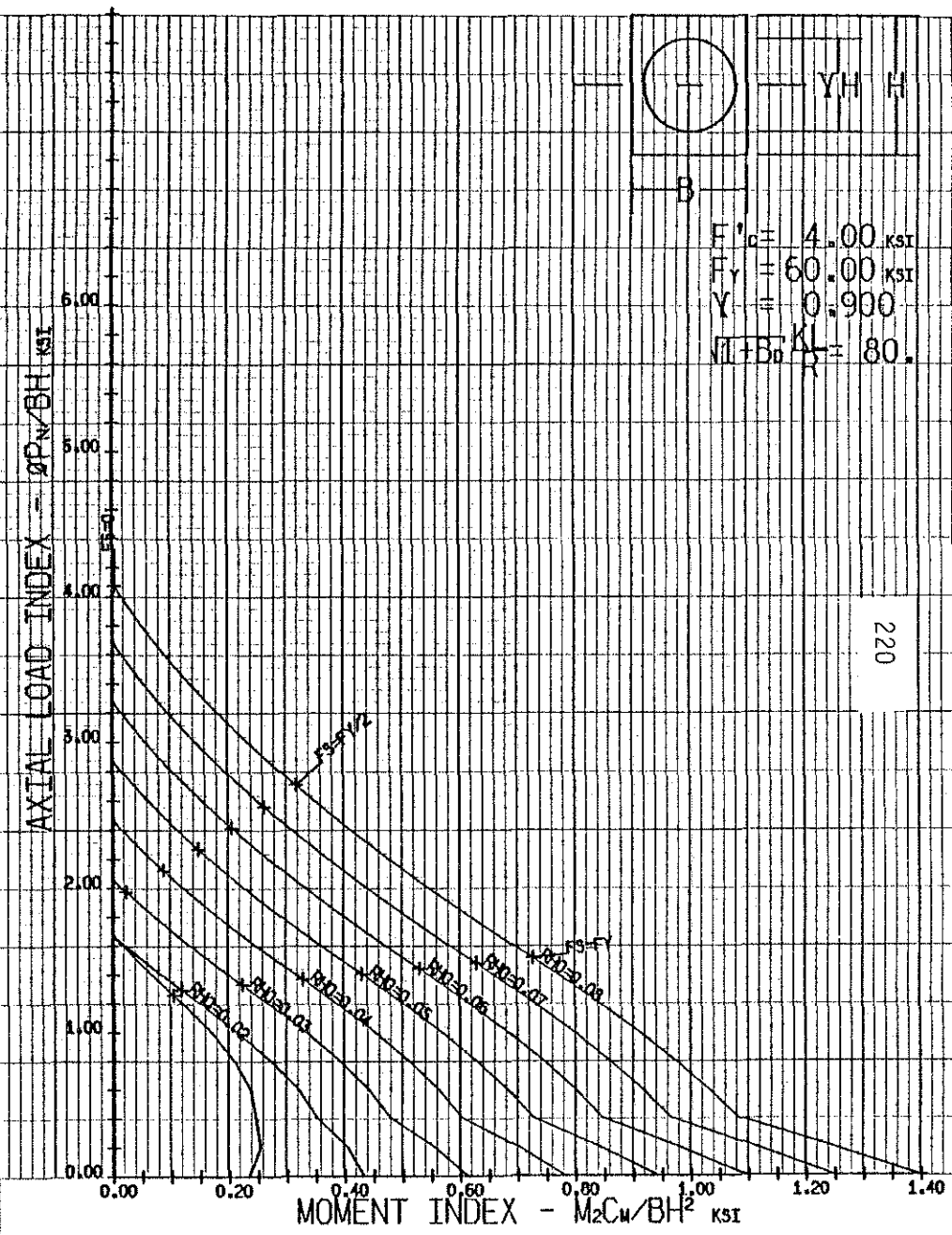


Fig. S4-60.90-80 - Interaction Diagram



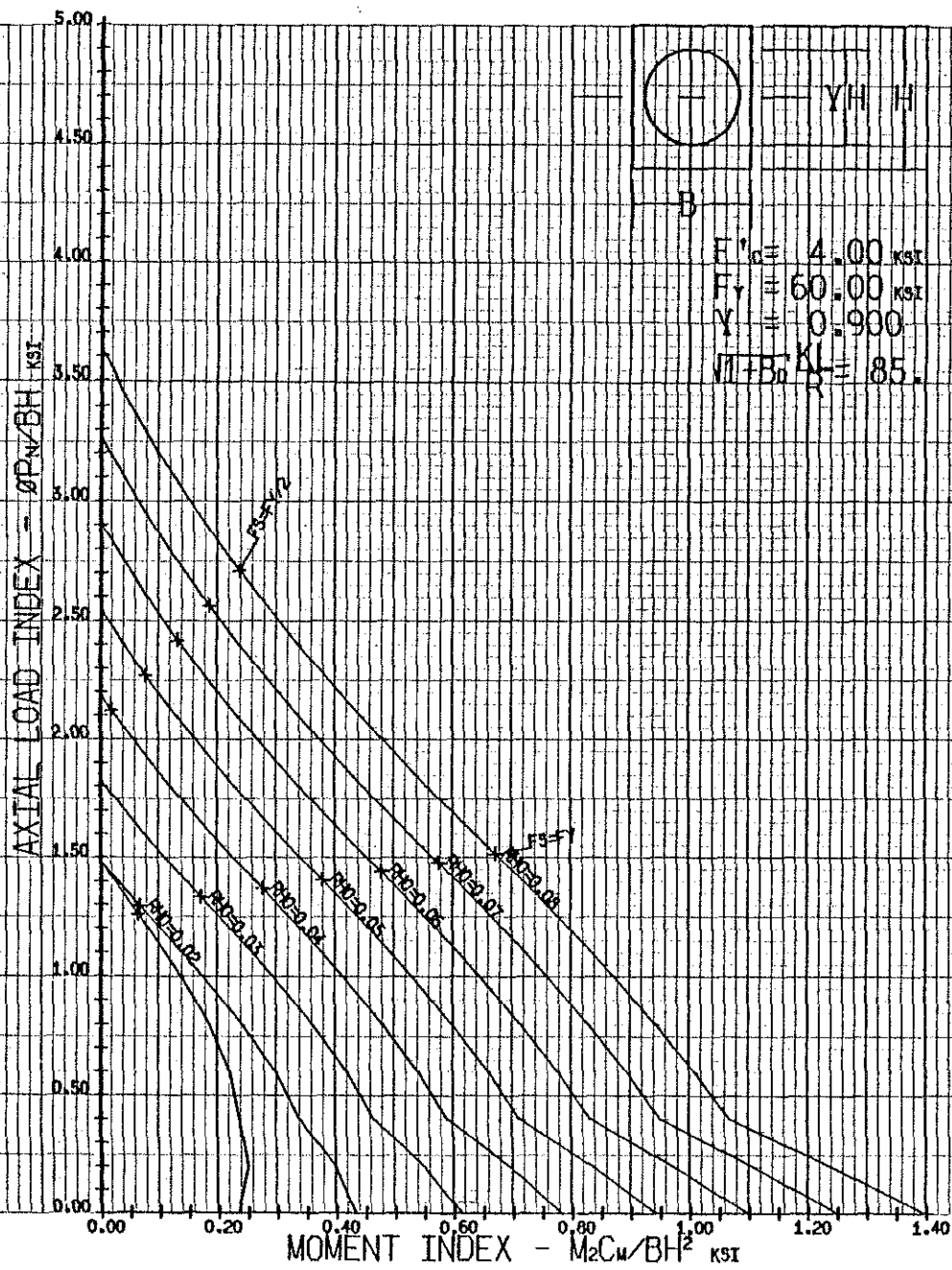


Fig. S4-60.90-85 - Interaction Diagram

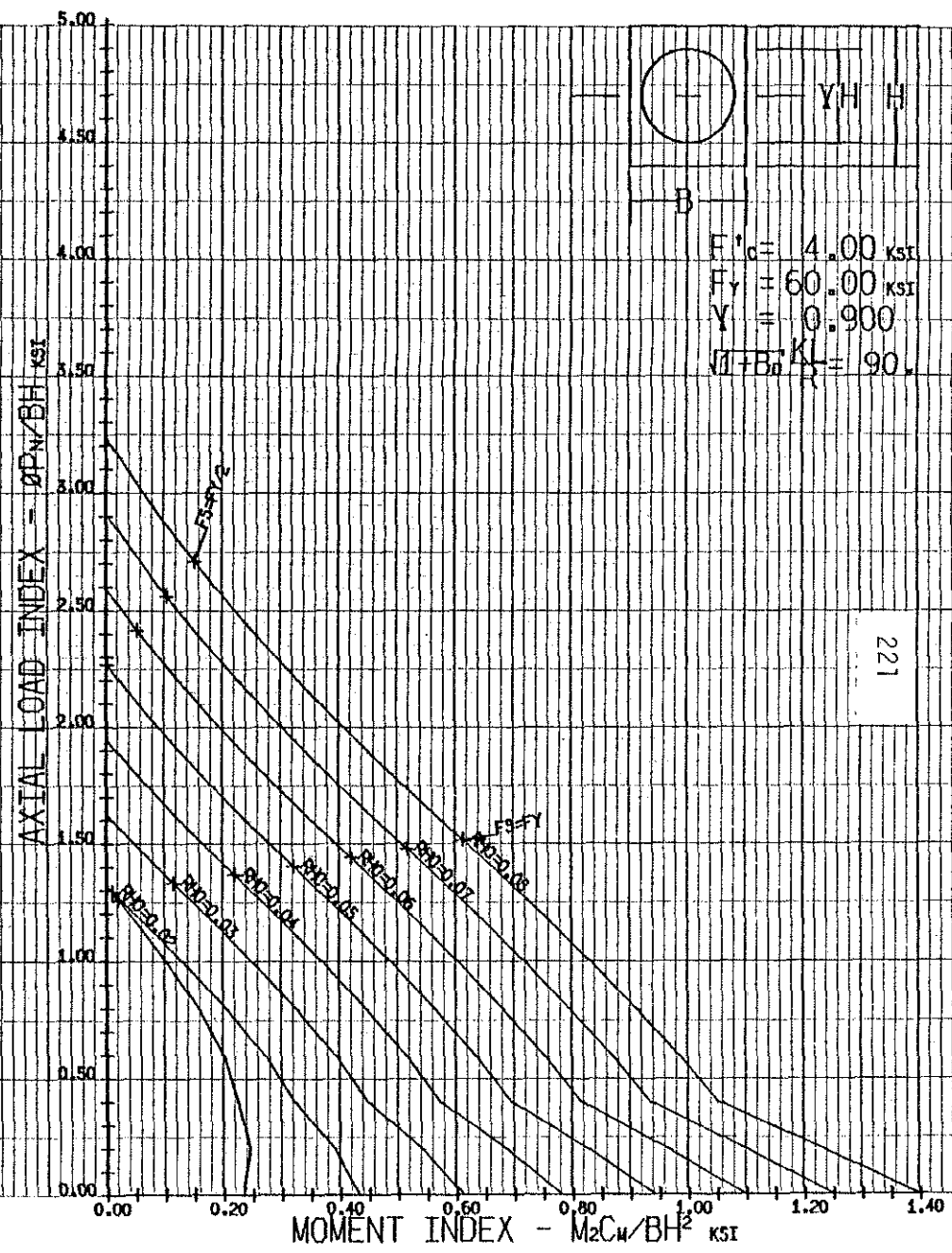


Fig. S4-60.90-90 - Interaction Diagram



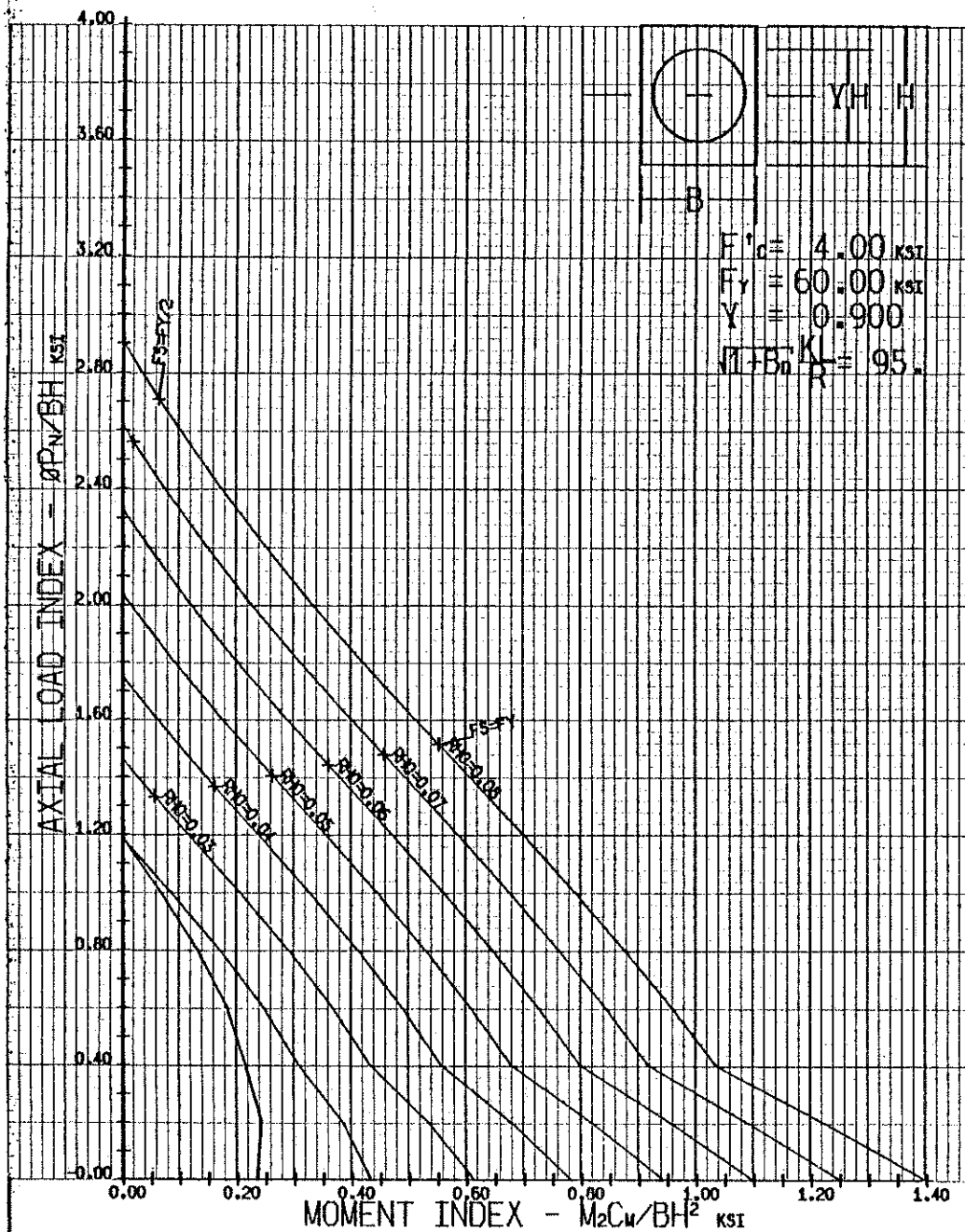


Fig. S4-60.90-95 - Interaction Diagram

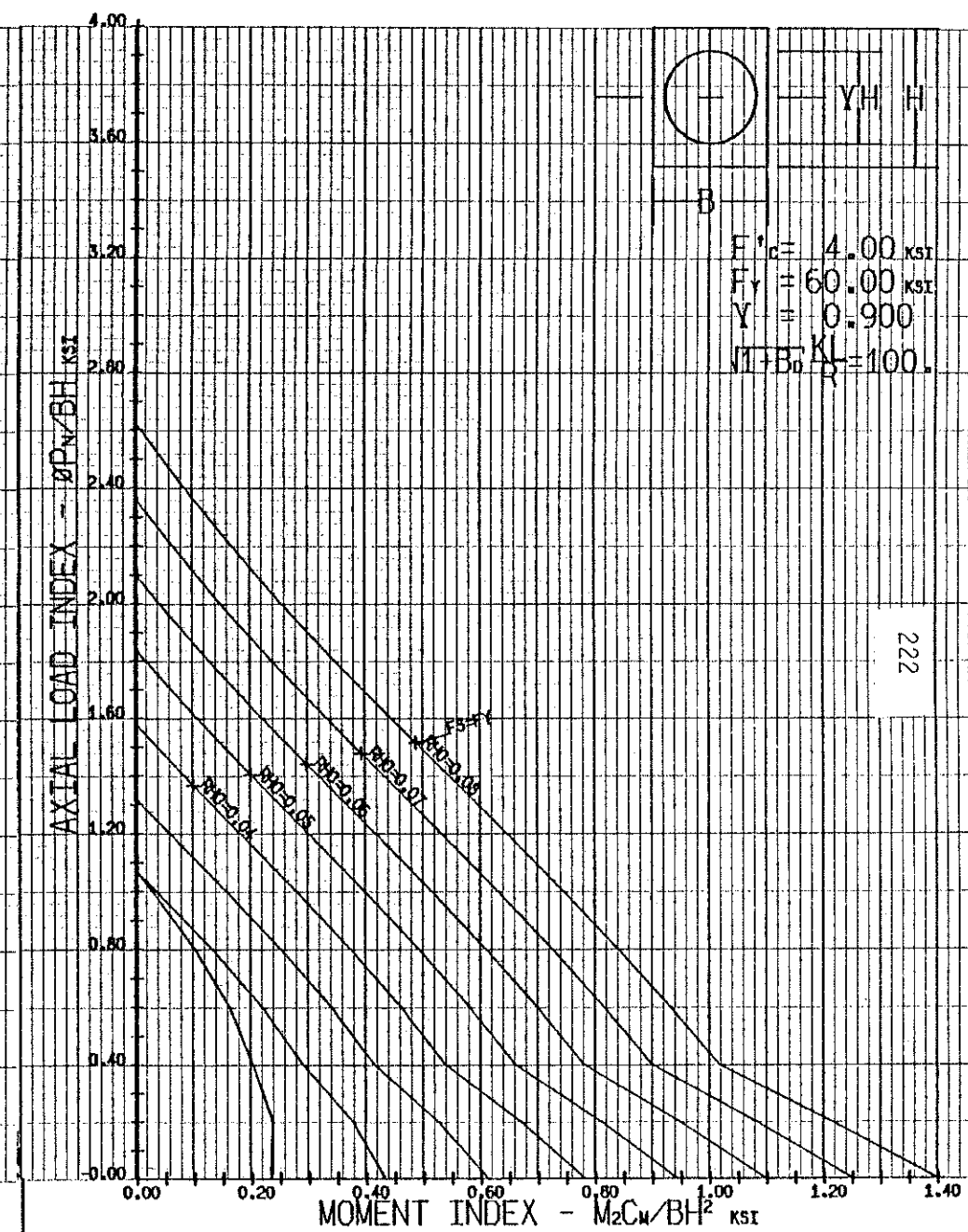


Fig. S4-60.90-100 - Interaction Diagram

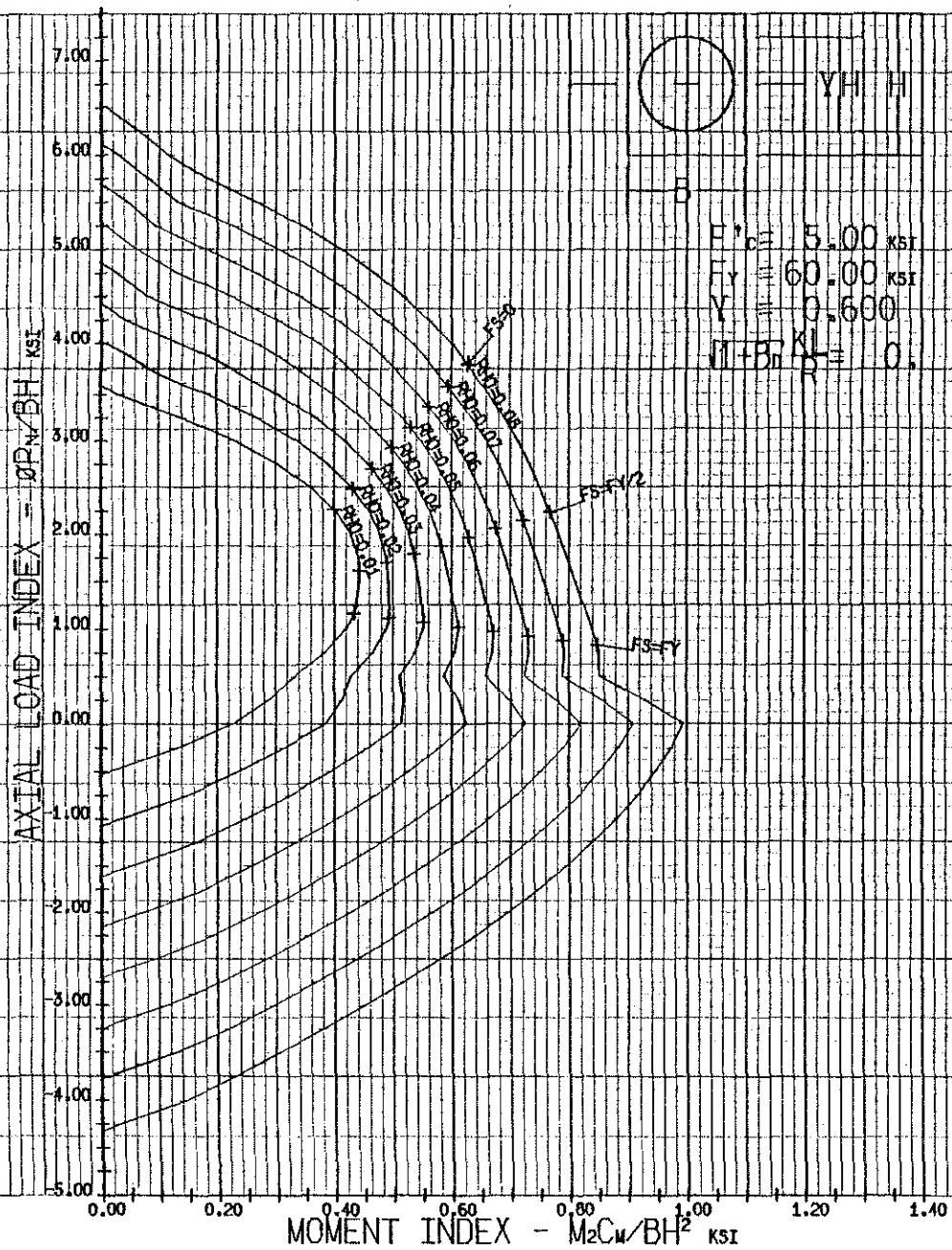


Fig. S5-60.60-0 - Interaction Diagram

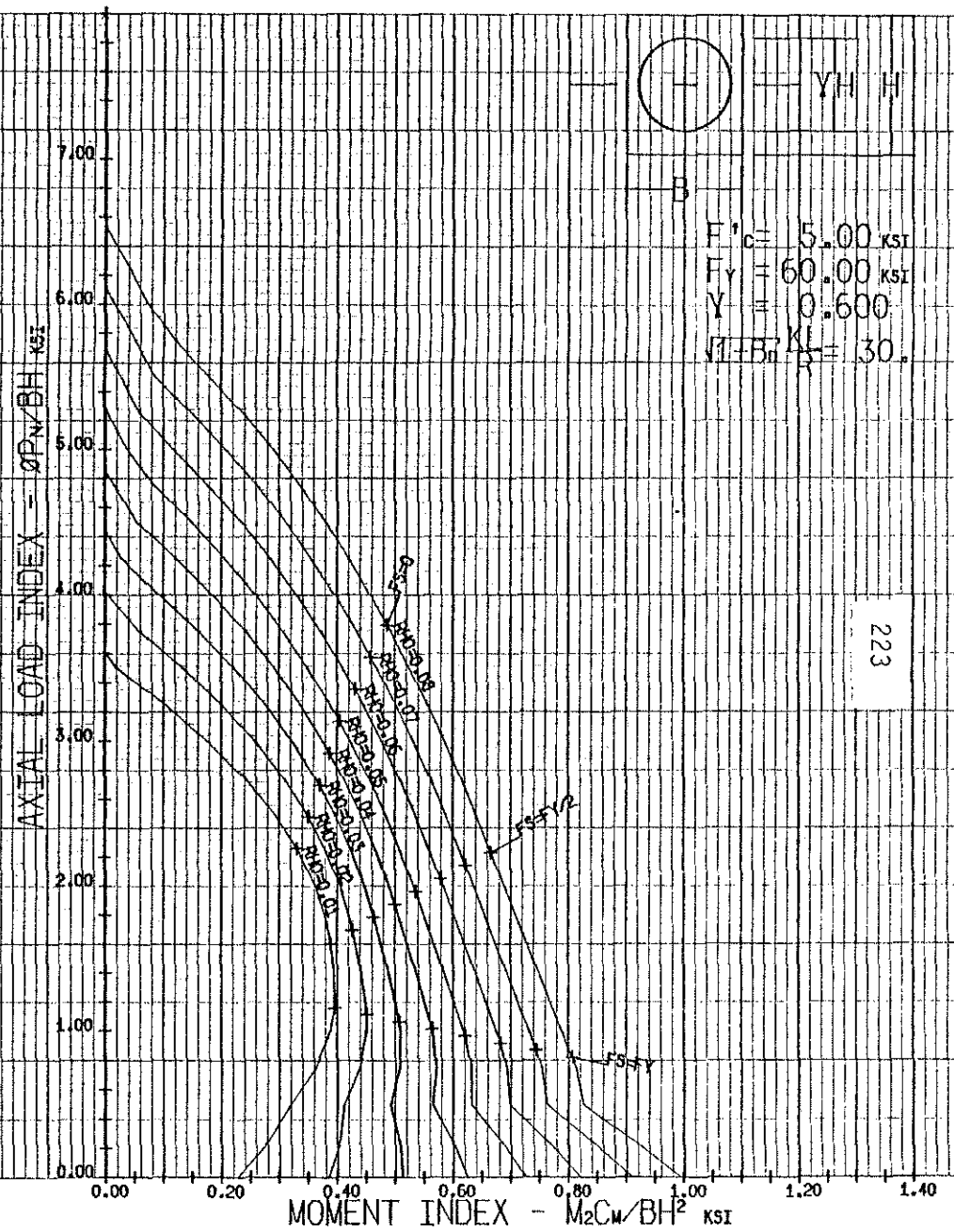


Fig. S5-60.60-30 - Interaction Diagram

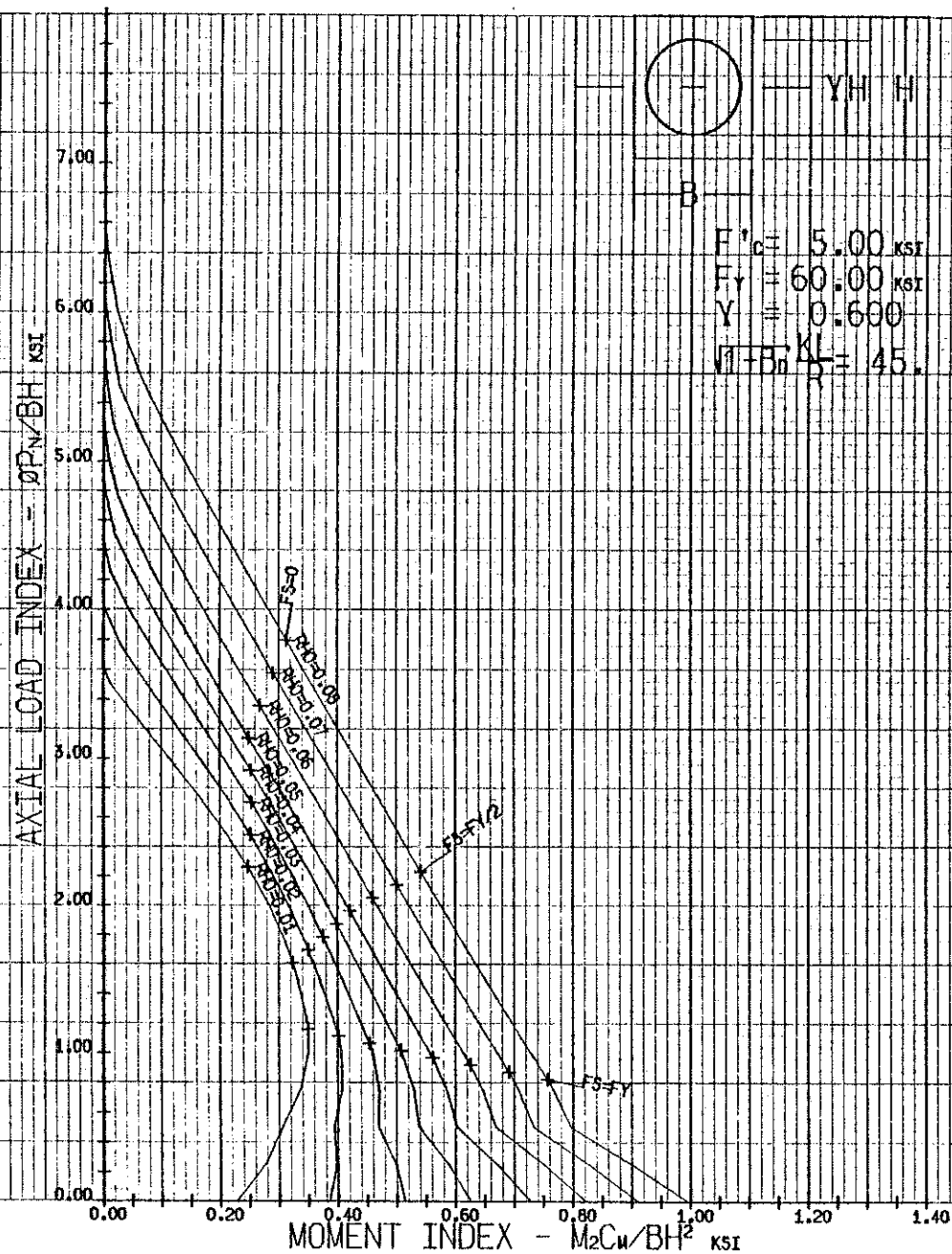


Fig. S5-60.60-45 - Interaction Diagram

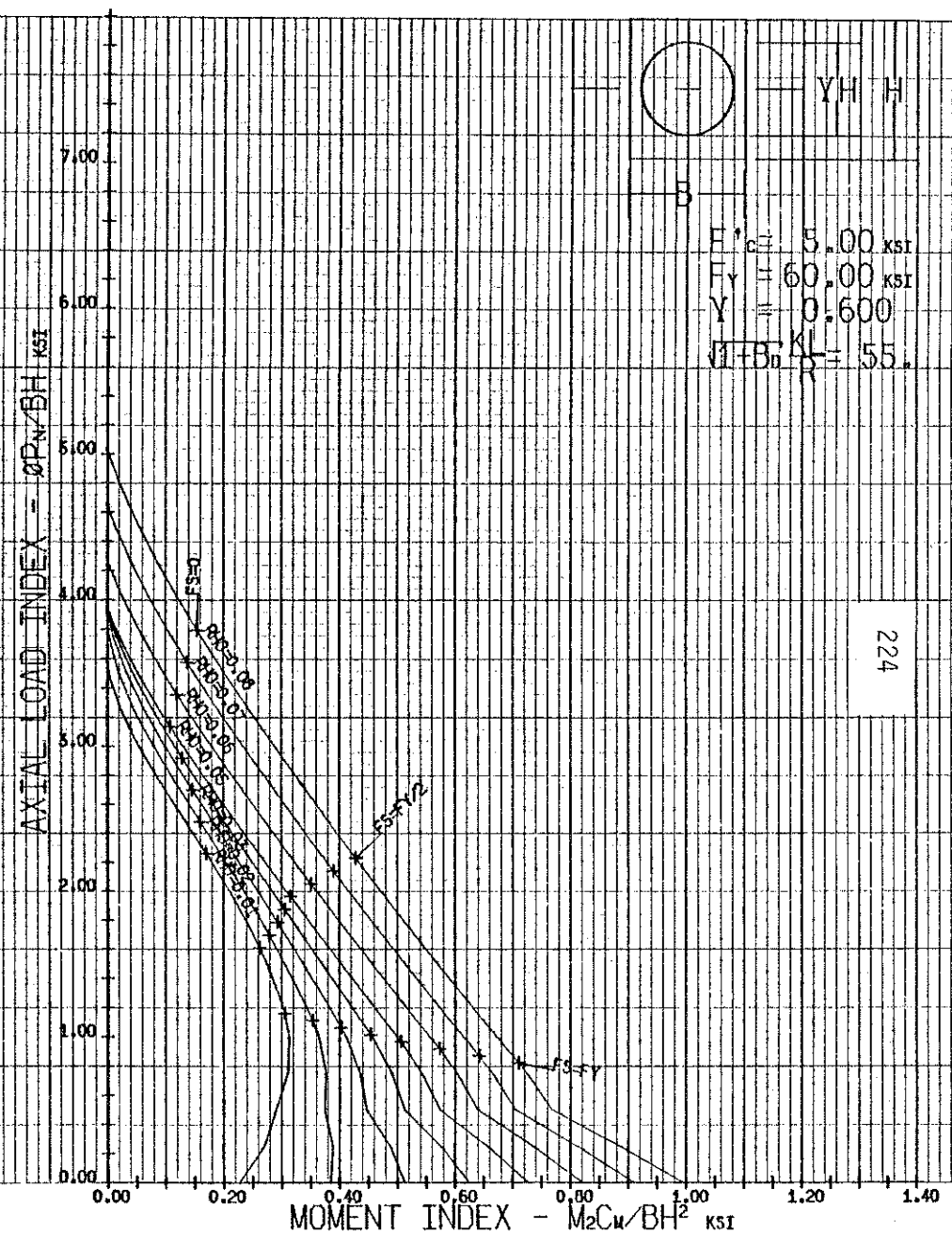


Fig. S5-60.60-55 - Interaction Diagram

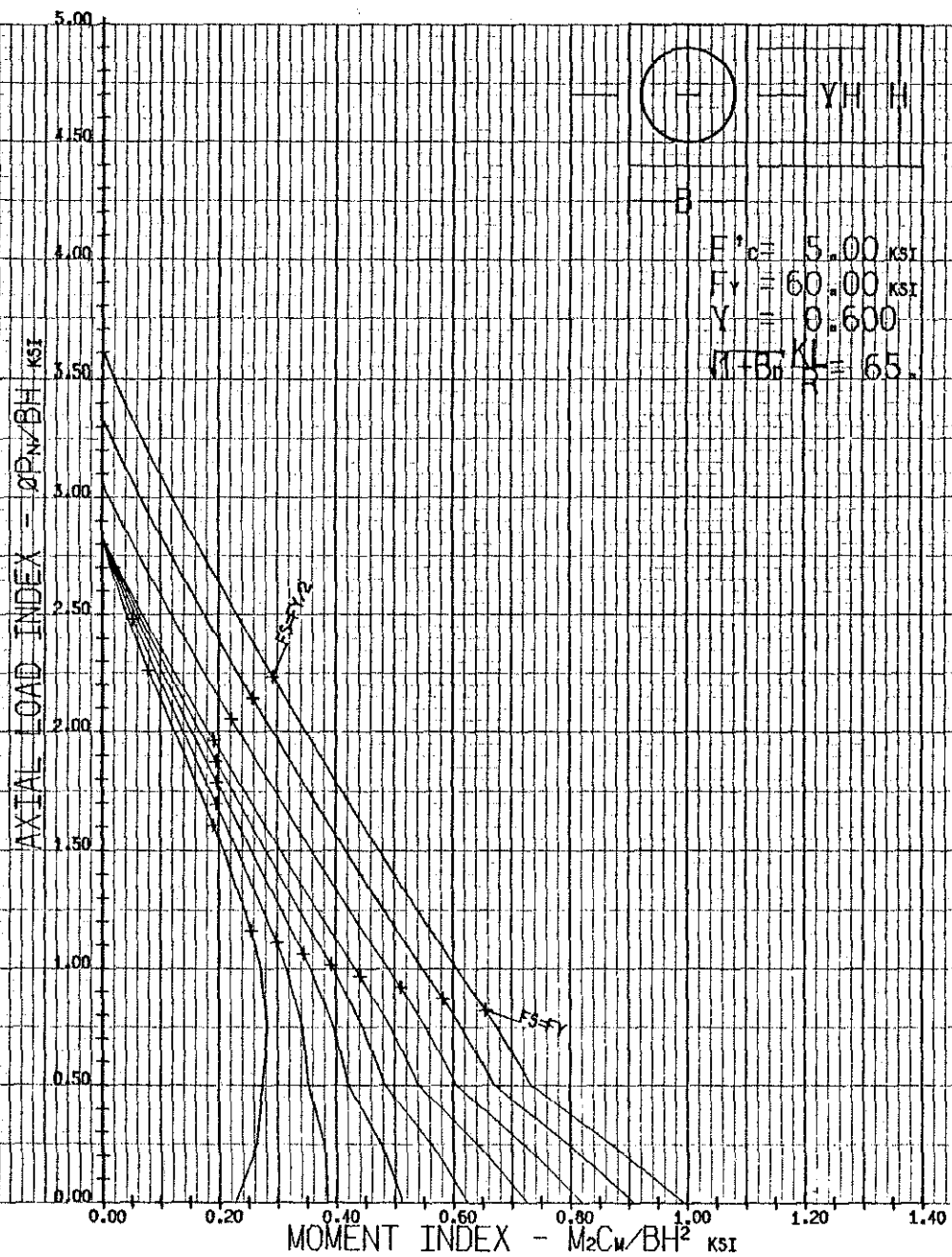


Fig. S5-60.60-65 - Interaction Diagram

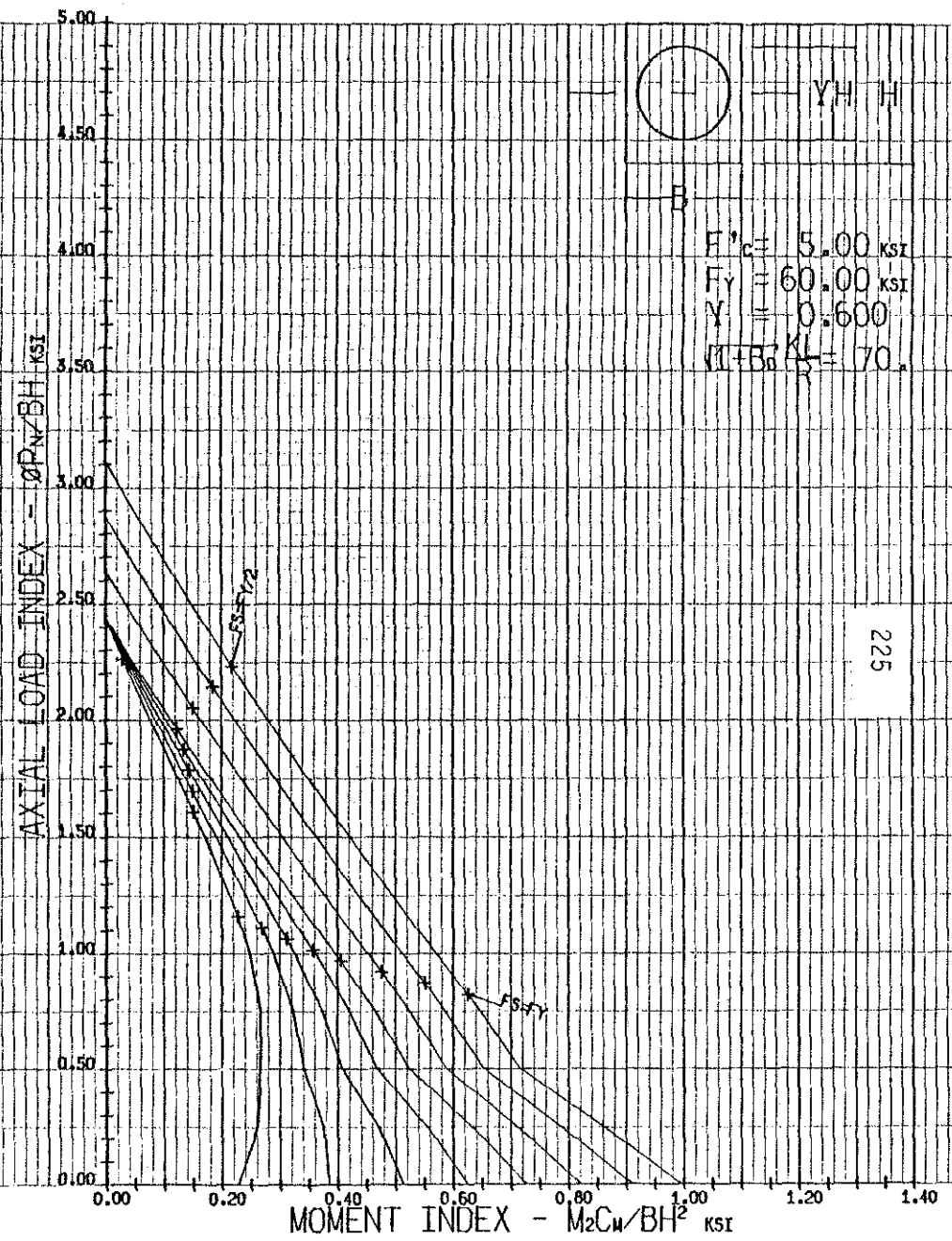


Fig. S5-60.60-70 - Interaction Diagram

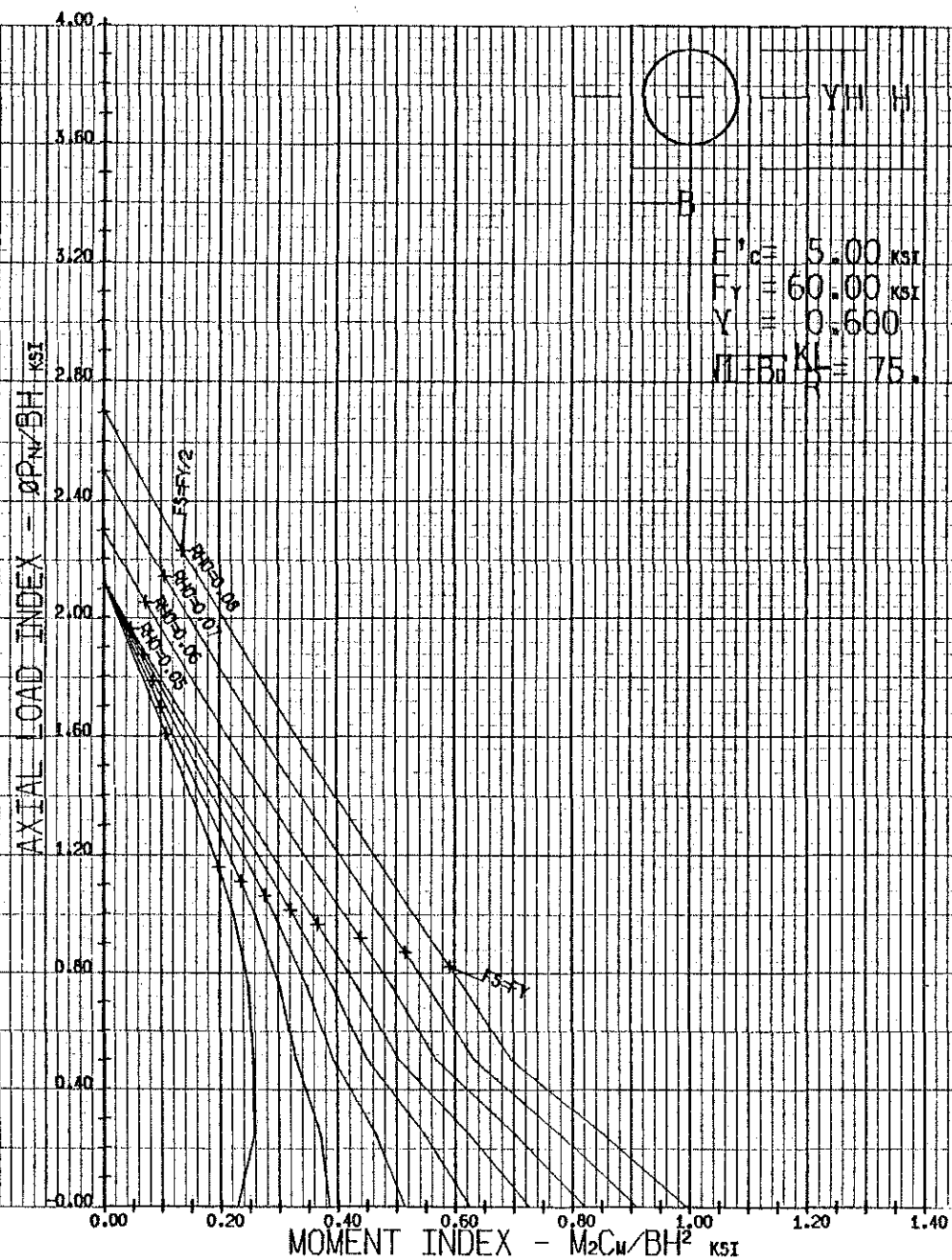


Fig. S5-60.60-75 - Interaction Diagram

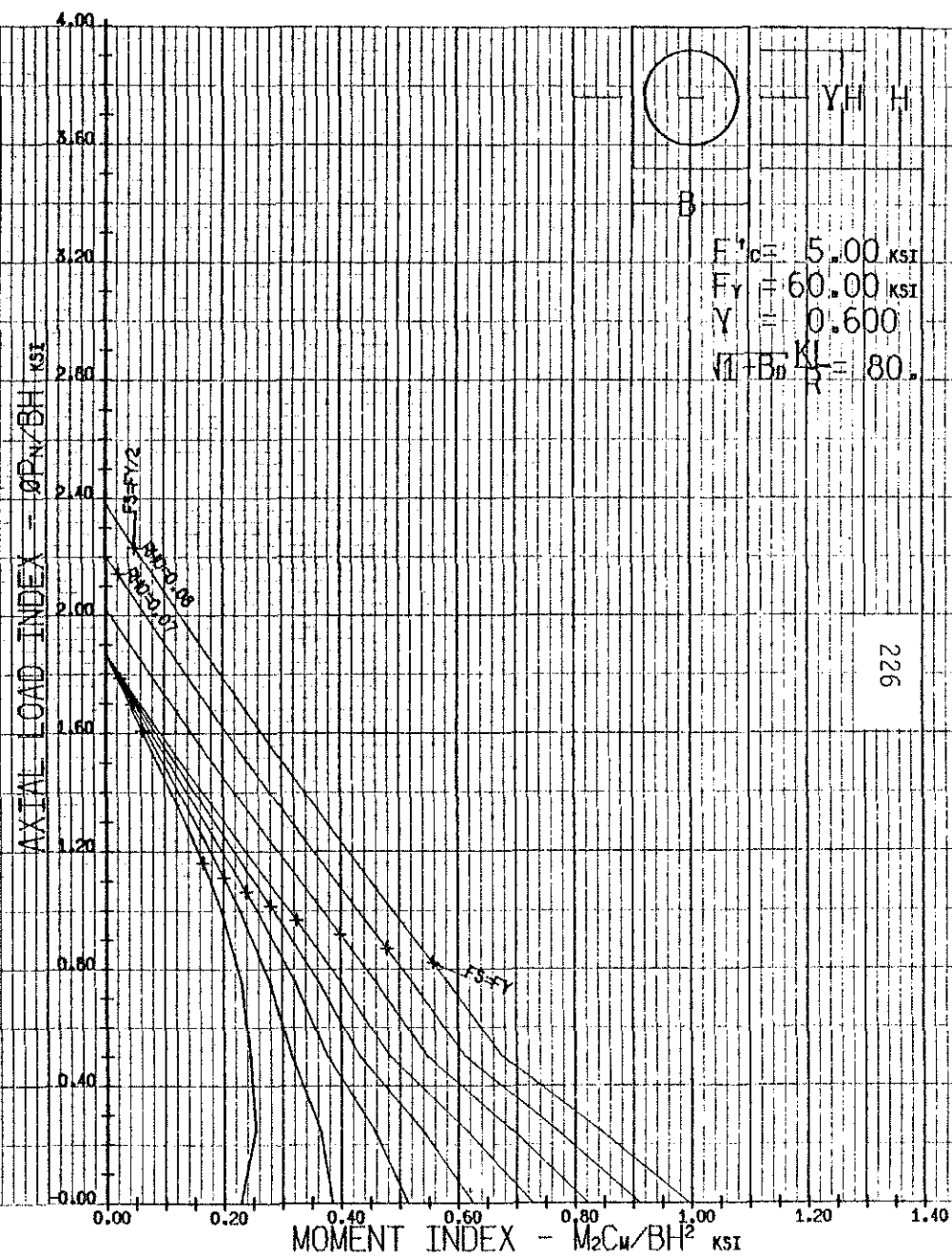


Fig. S5-60.60-80 - Interaction Diagram



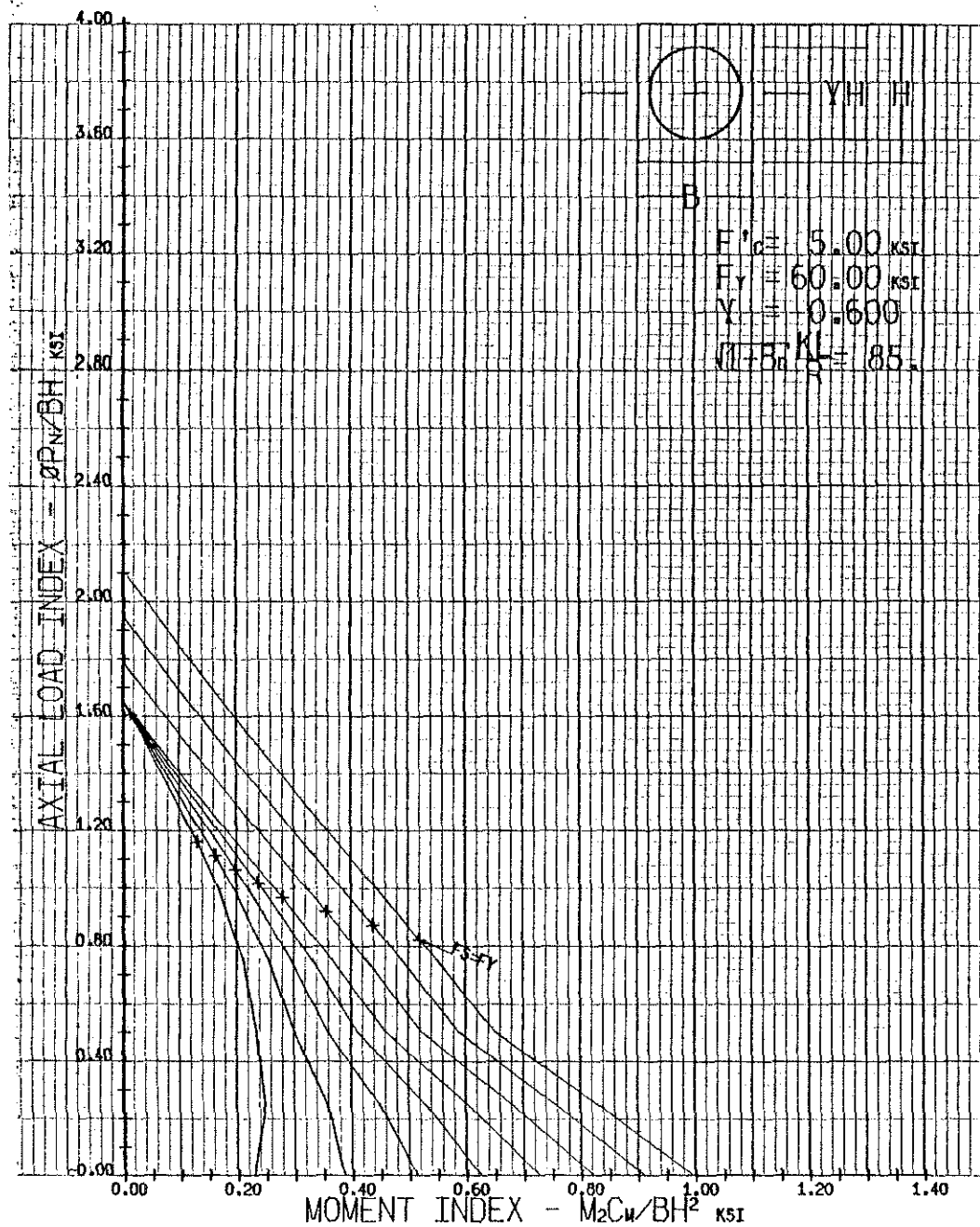


Fig. S5-60.60-85 - Interaction Diagram

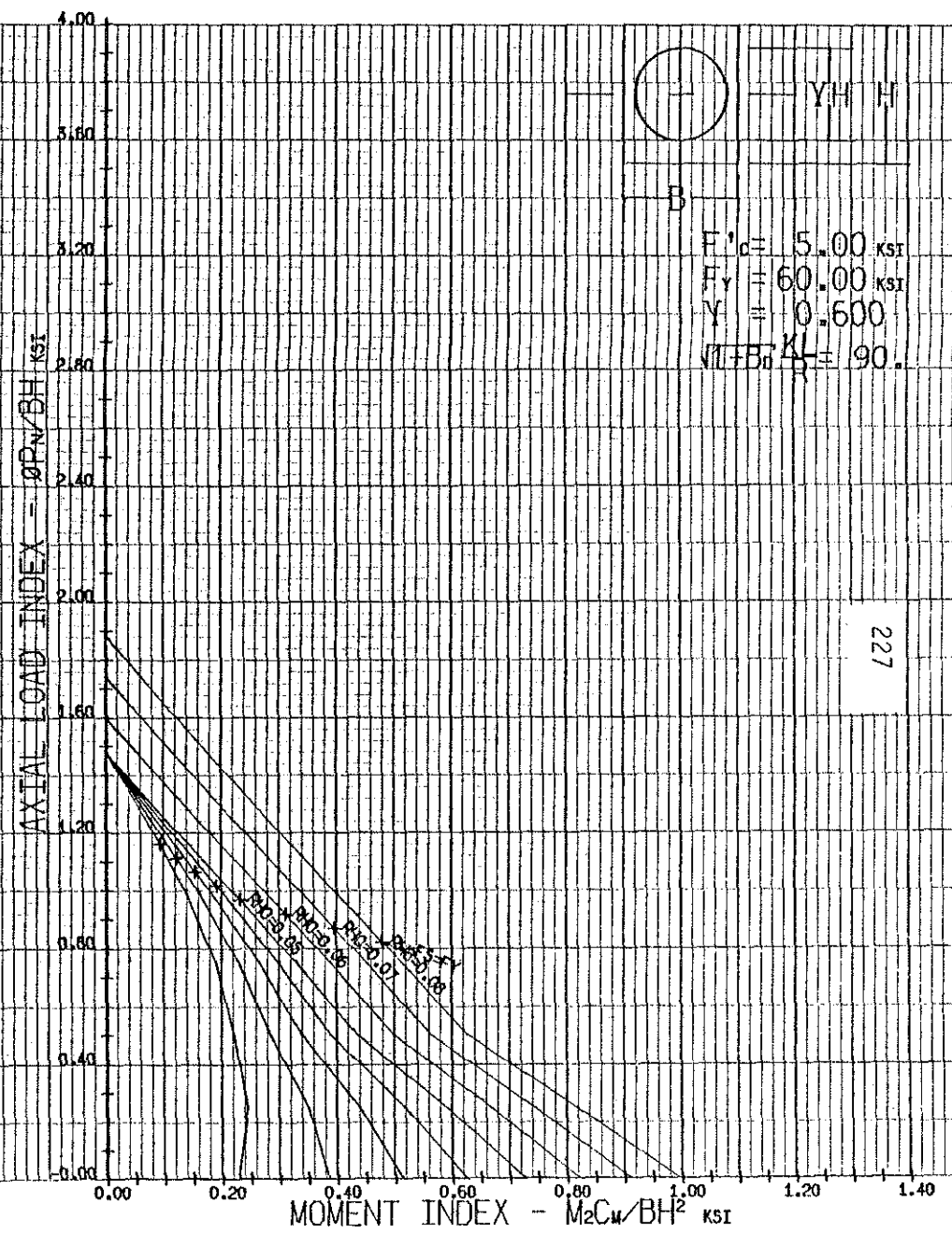


Fig. S5-60.60-90 - Interaction Diagram



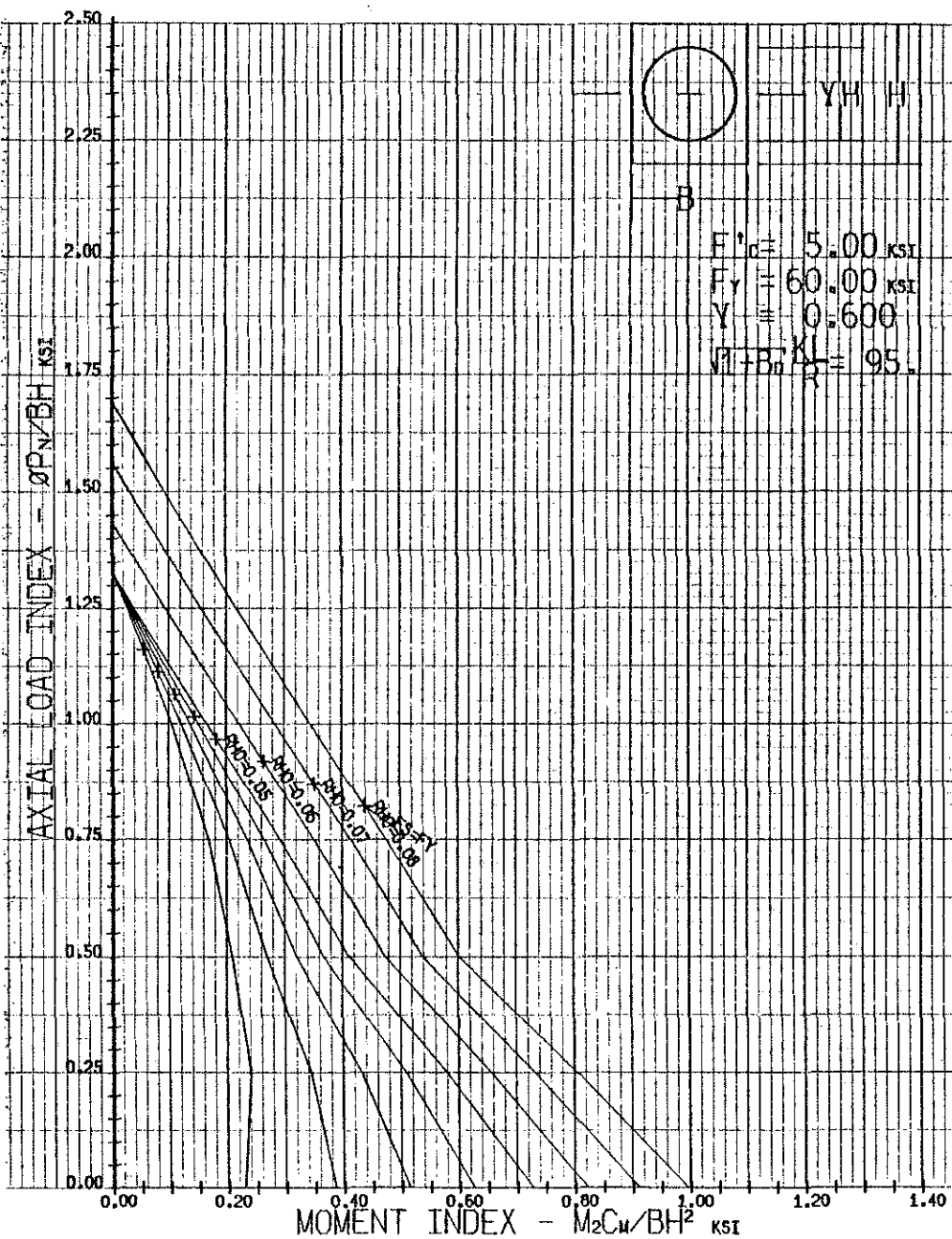


Fig. S5-60.60-95 - Interaction Diagram

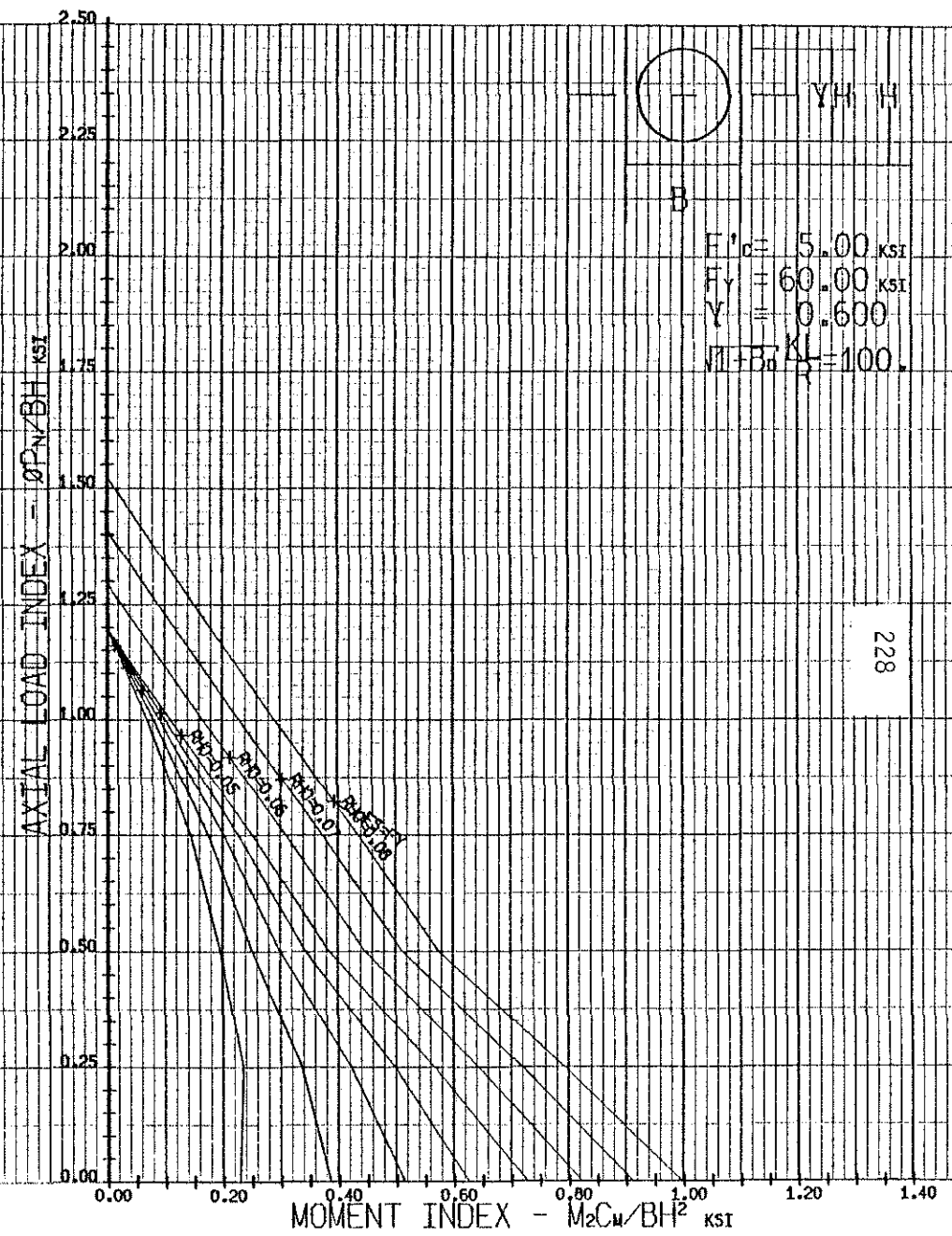


Fig. S5-60.60-100 - Interaction Diagram

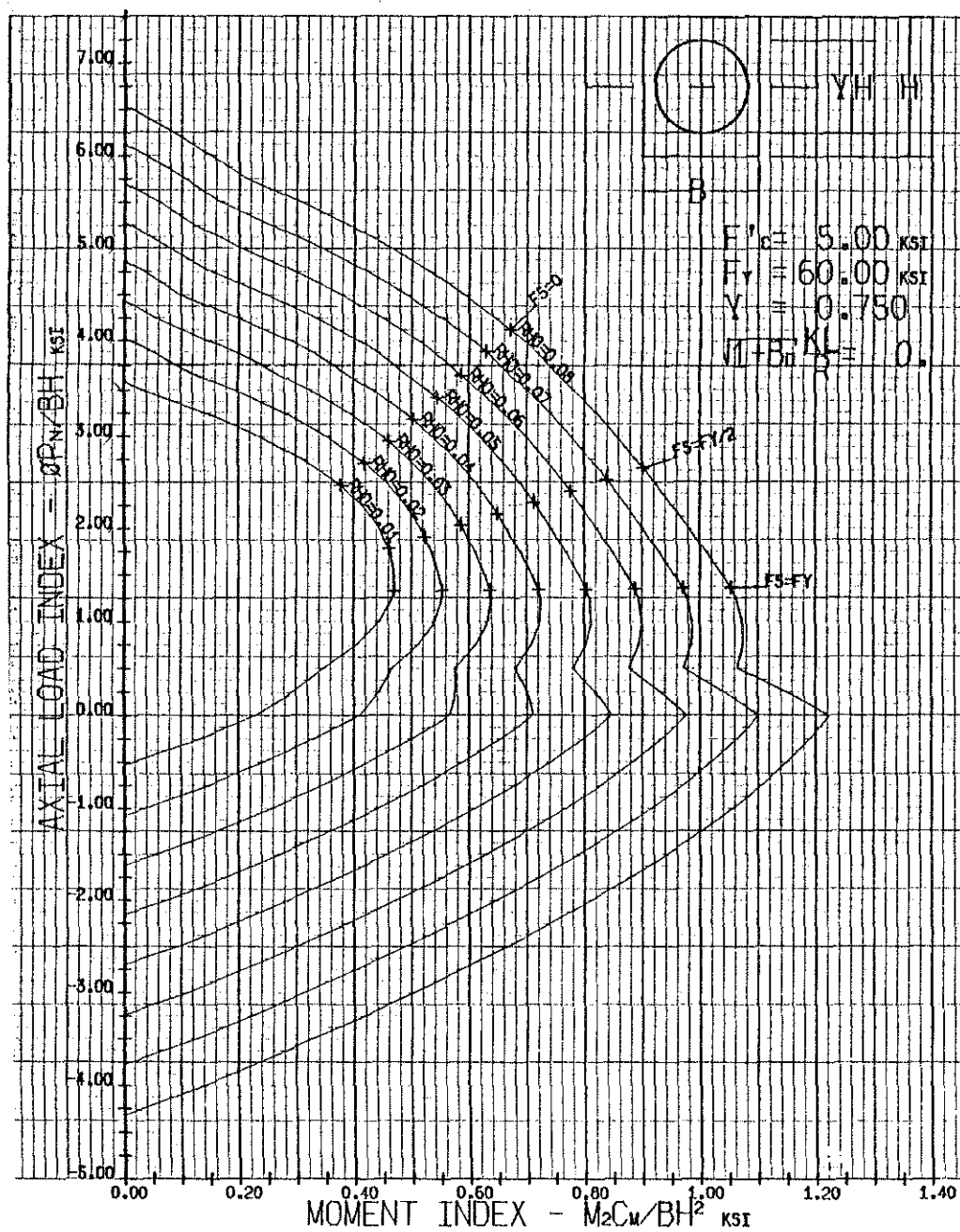


Fig. S5-60.75-0 - Interaction Diagram

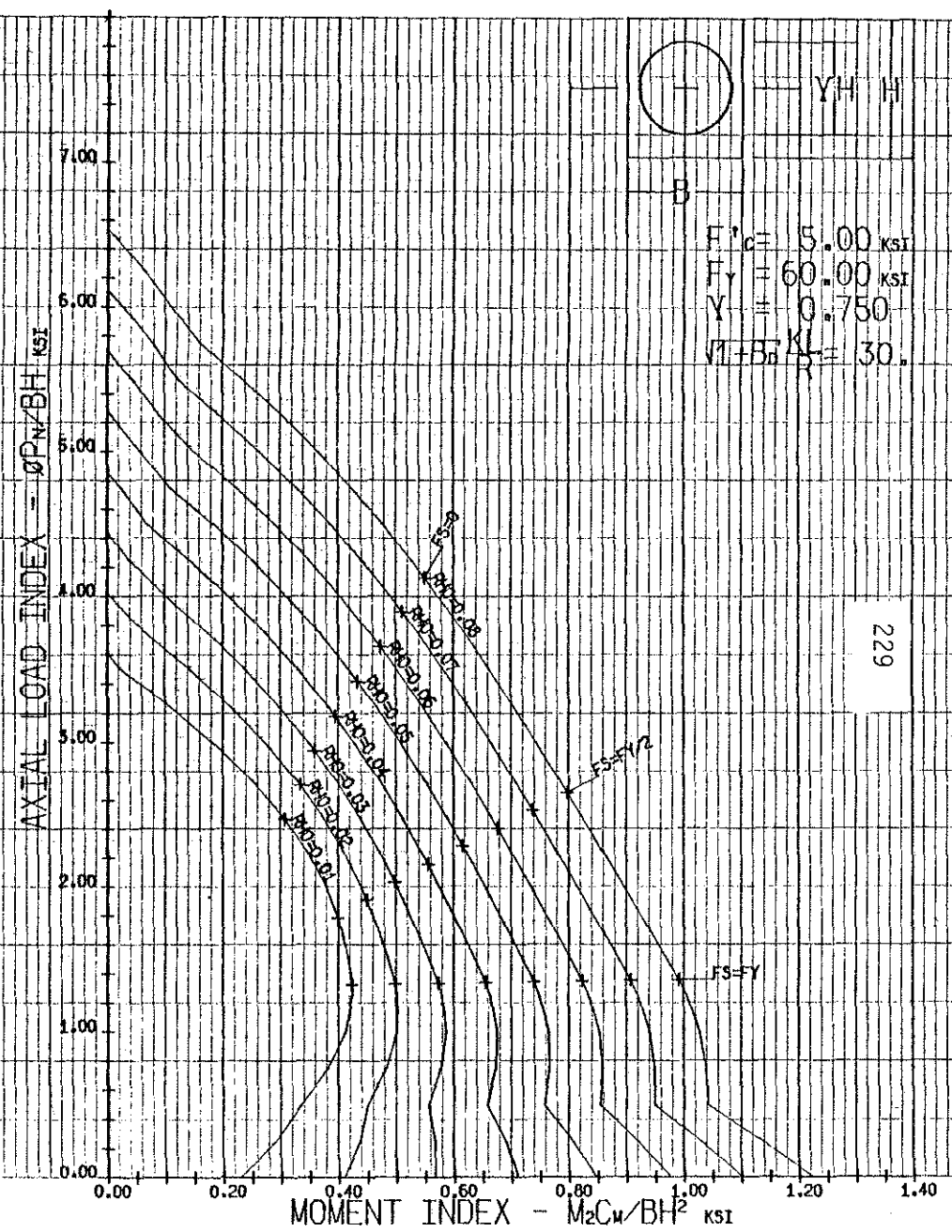


Fig. S5-60.75-30 - Interaction Diagram

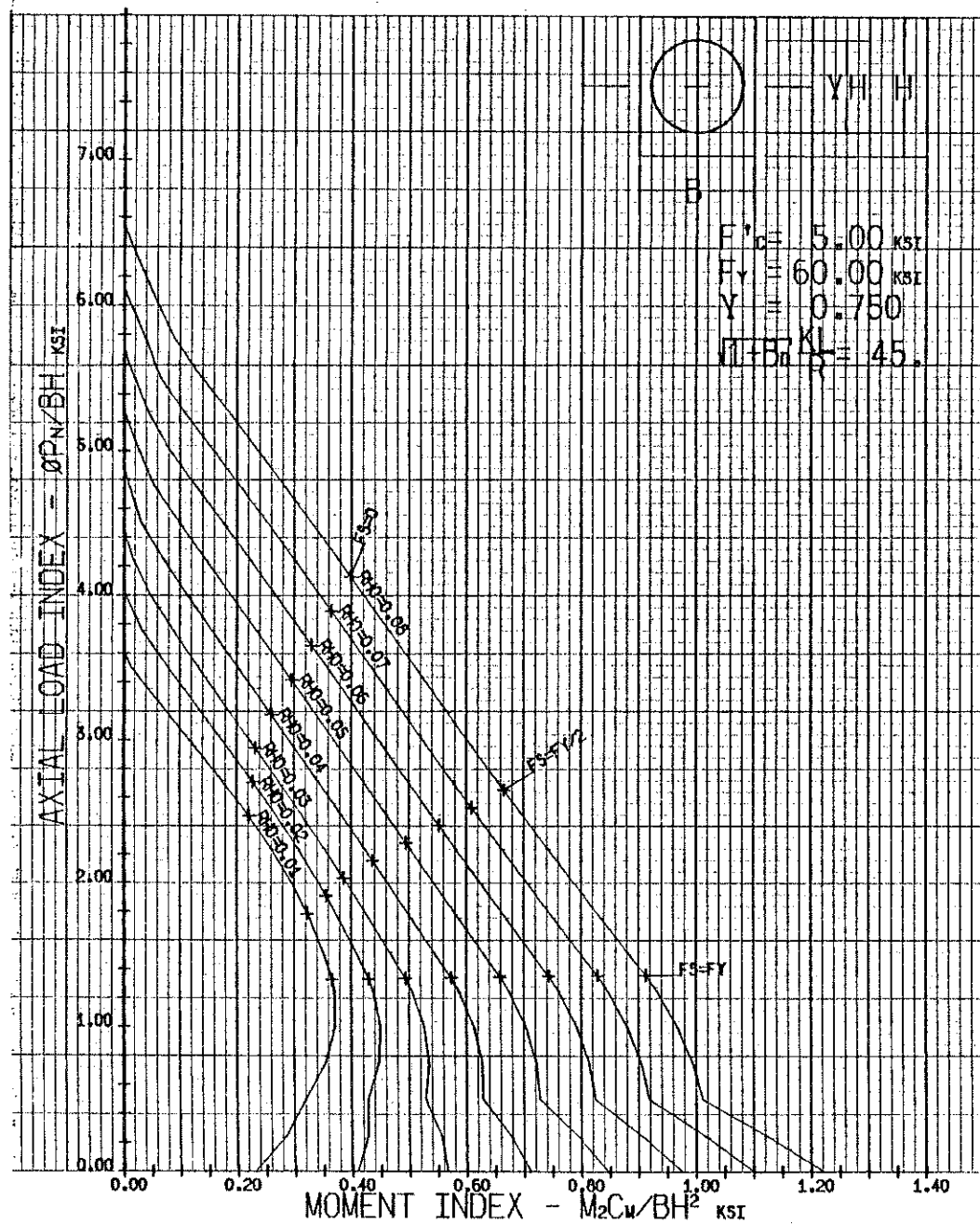


Fig. S5-60.75-45 - Interaction Diagram

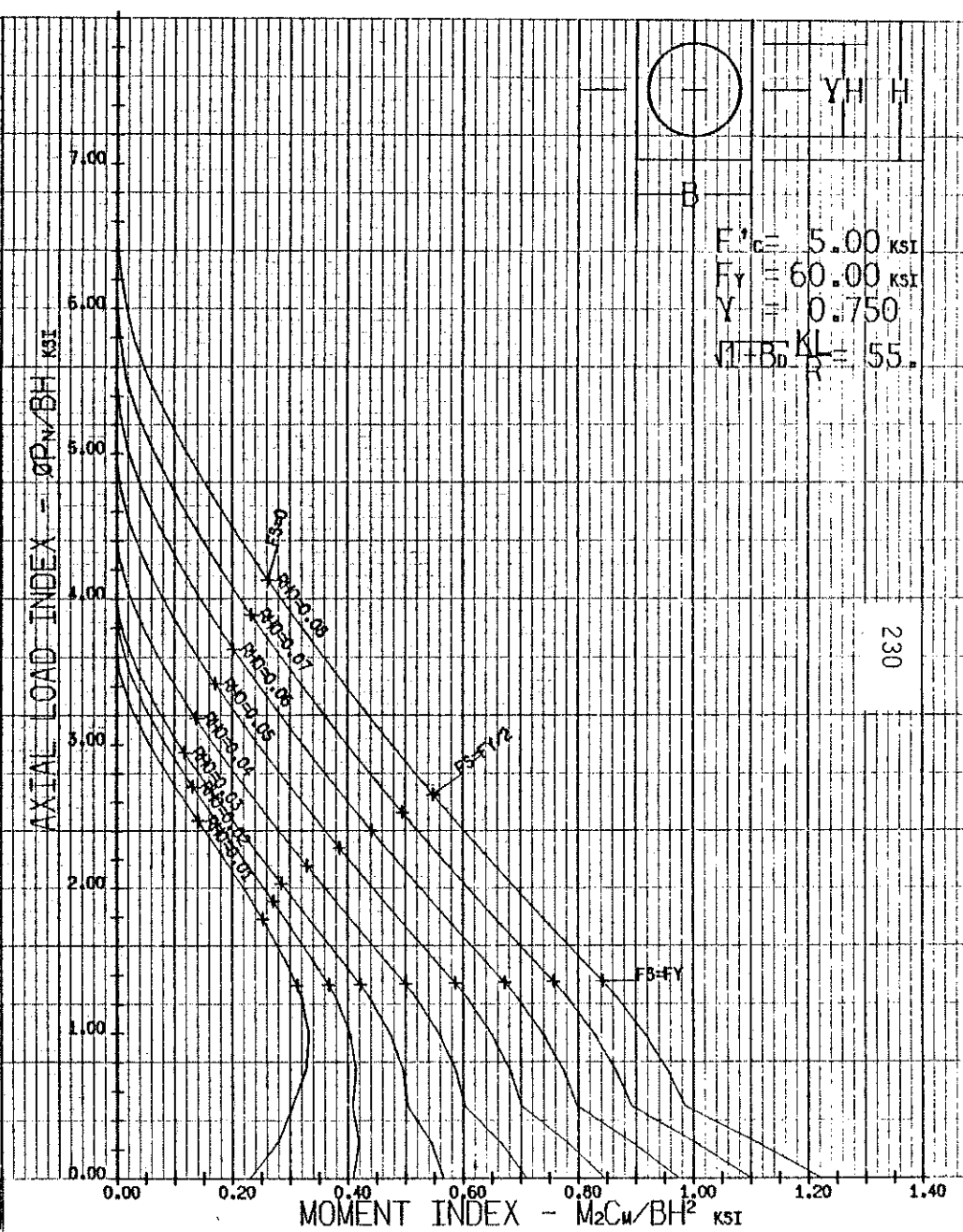


Fig. S5-60.75-55 - Interaction Diagram

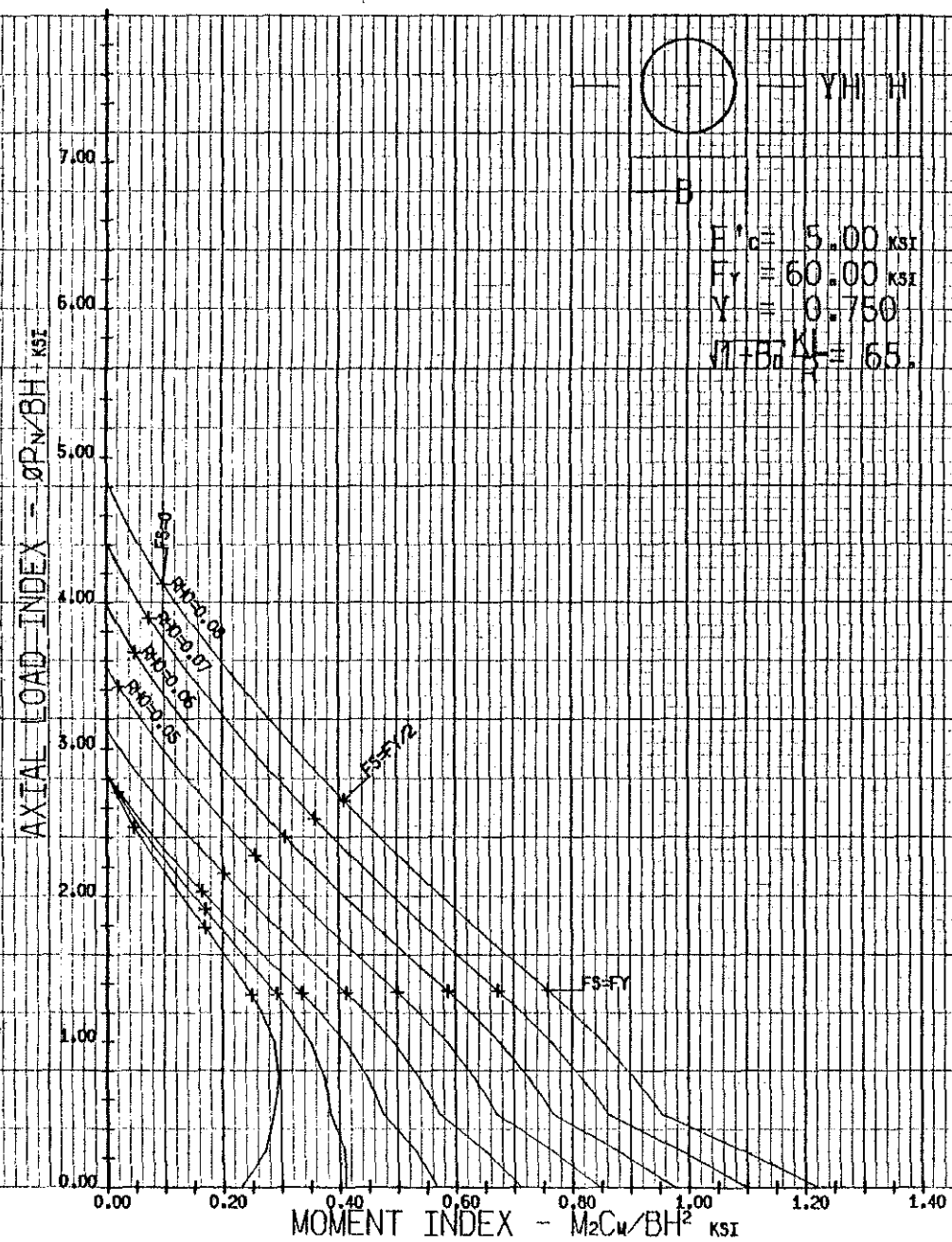


Fig. S5-60.75-65 - Interaction Diagram

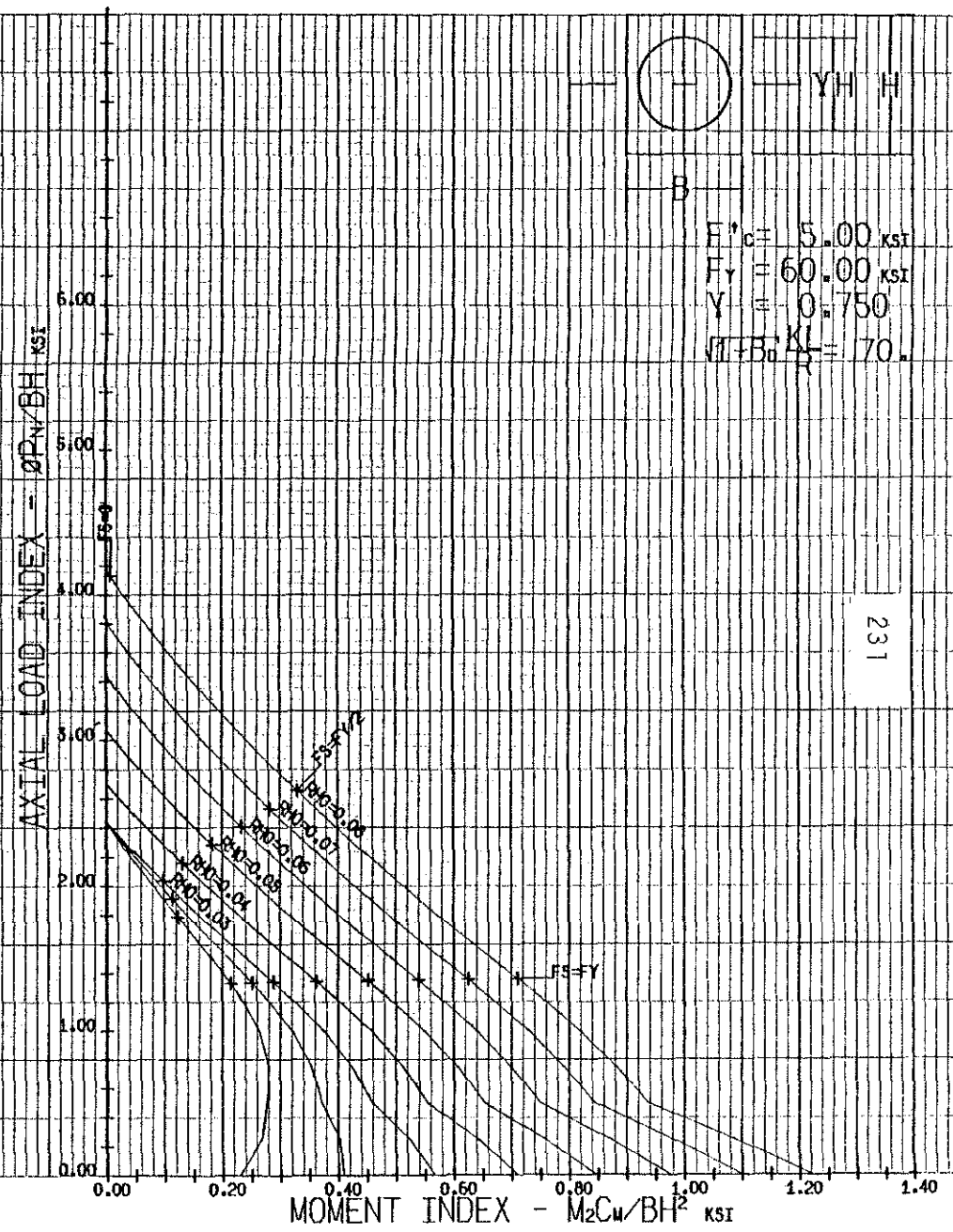


Fig. S5-60.75-70 - Interaction Diagram

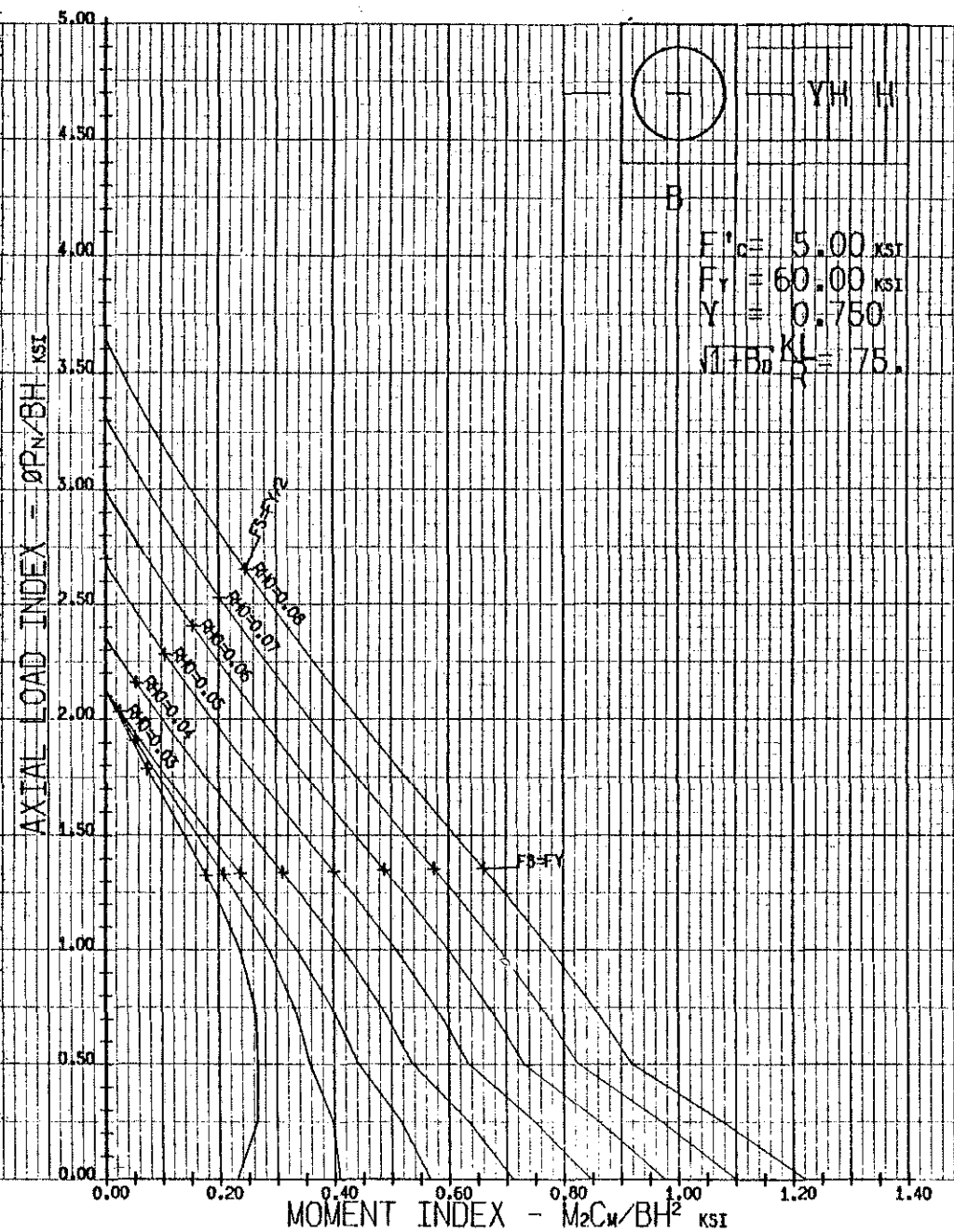


Fig. S5-60.75-75 - Interaction Diagram

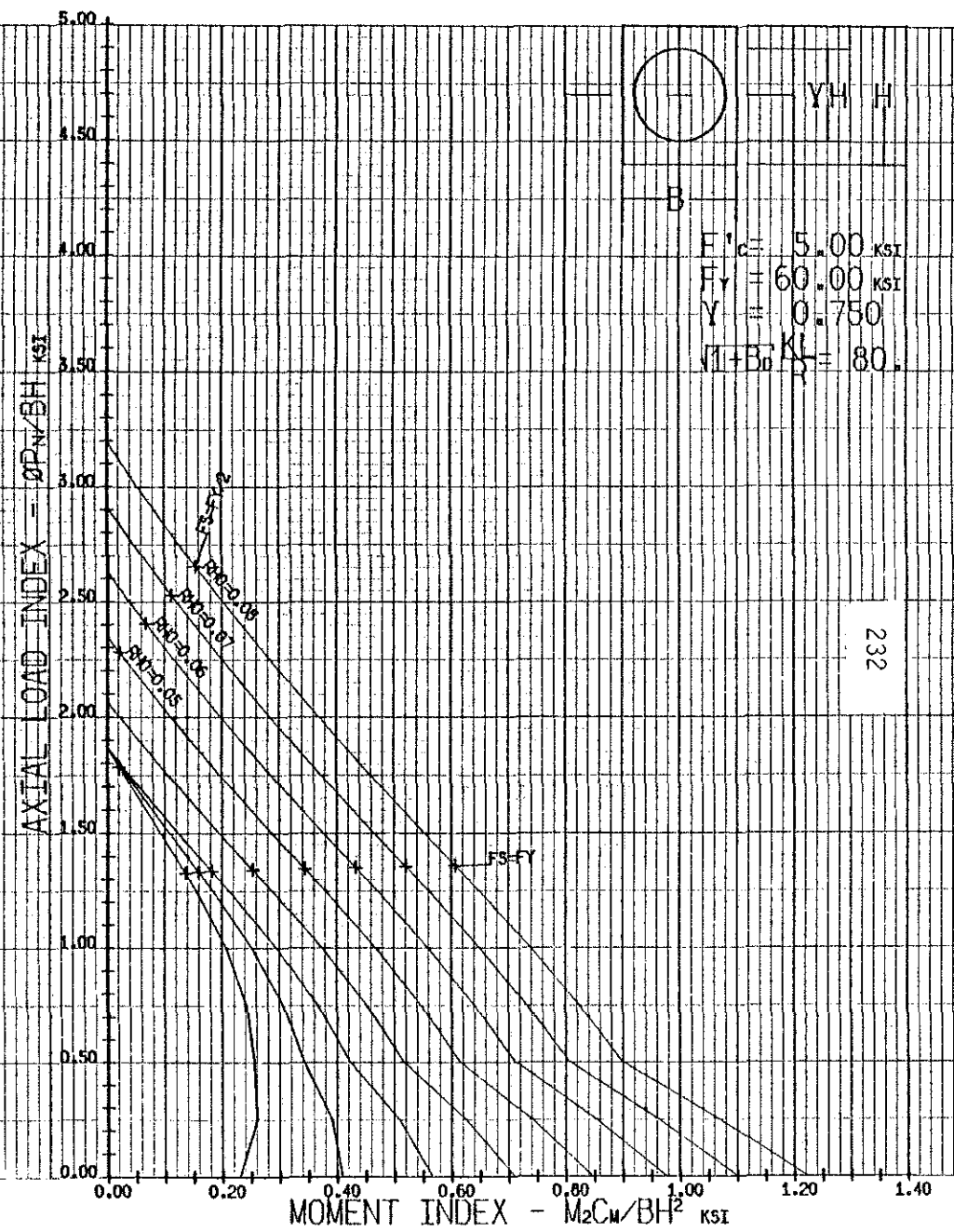


Fig. S5-60.75-80 - Interaction Diagram



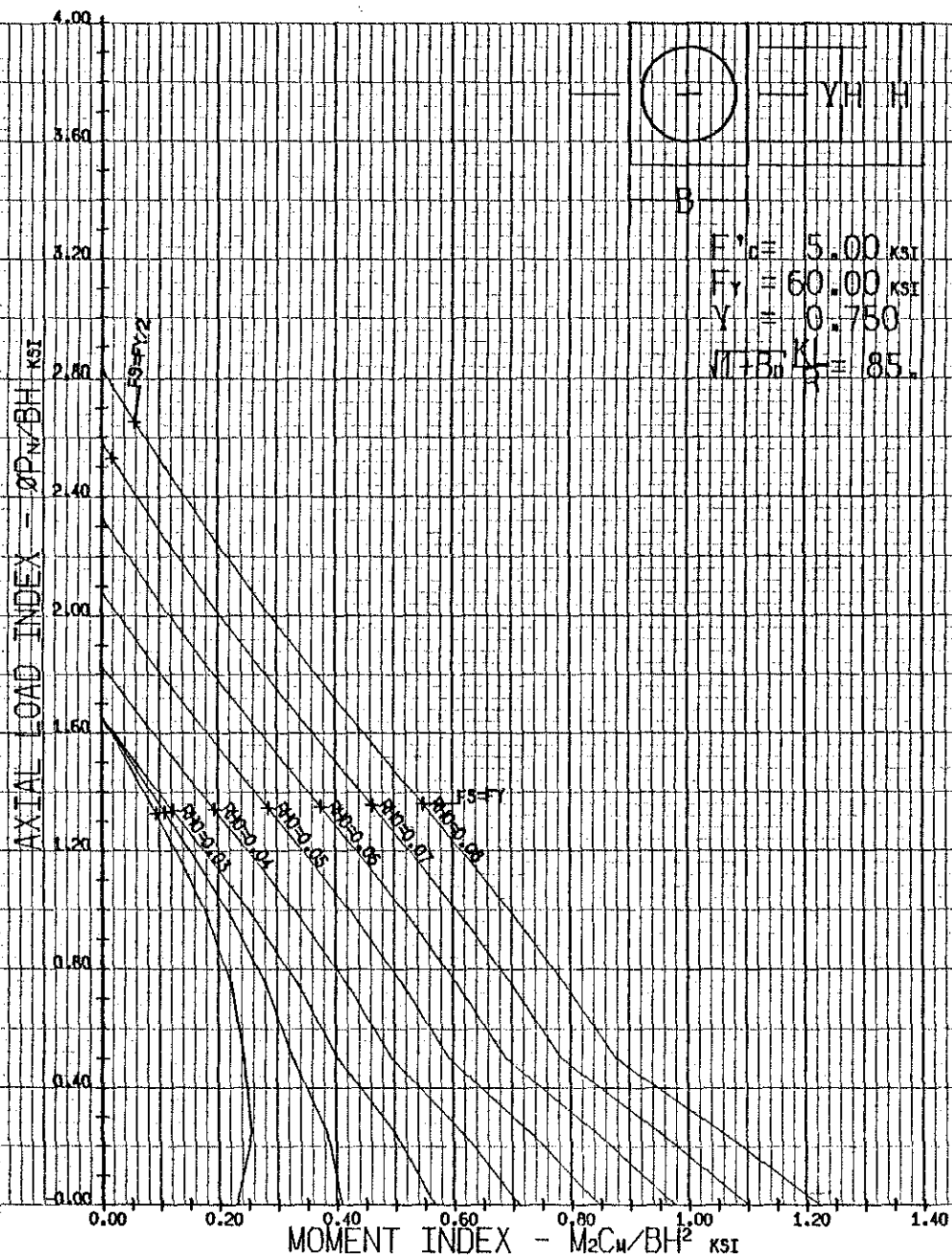


Fig. S5-60.75-85 - Interaction Diagram

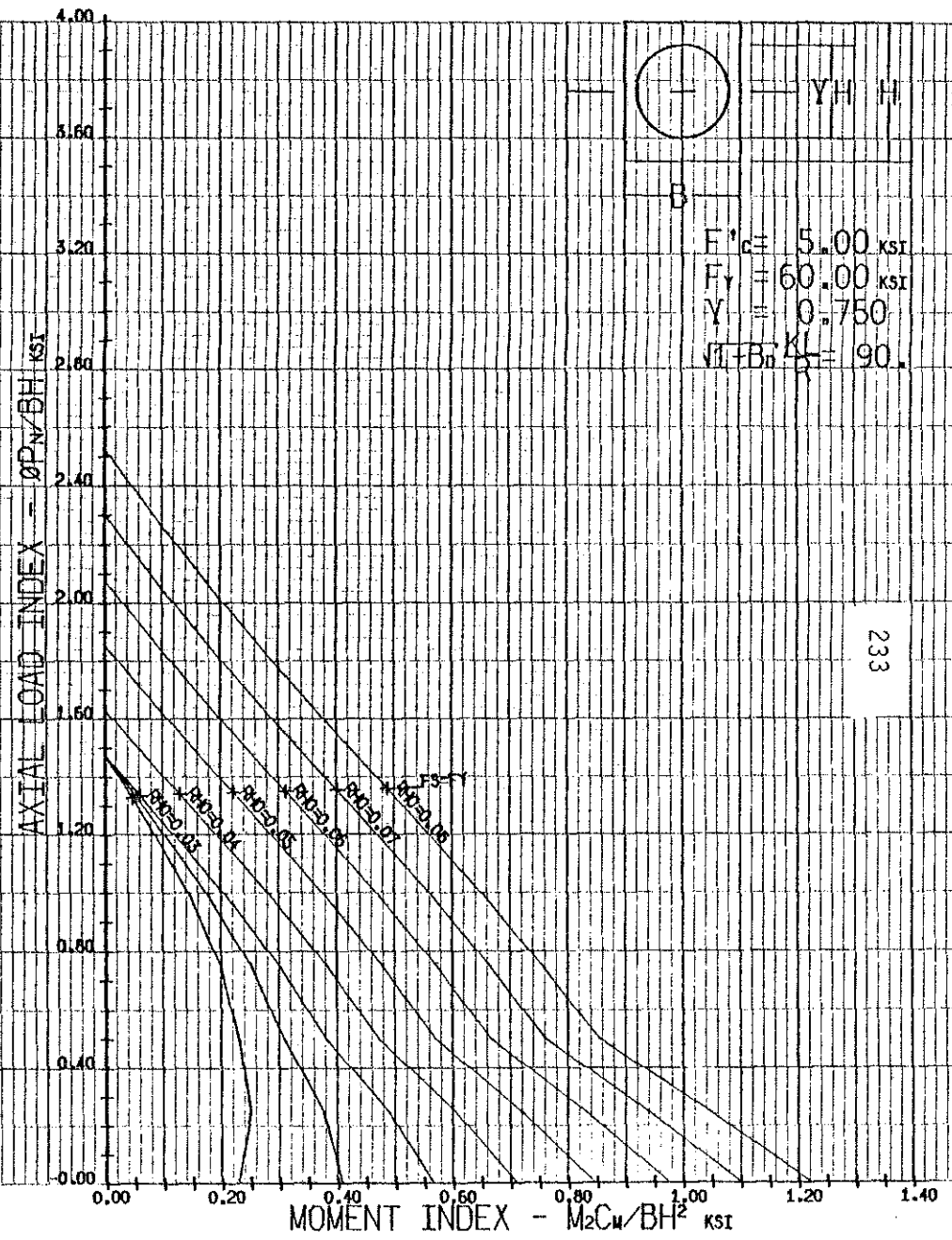


Fig. S5-60.75-90 - Interaction Diagram



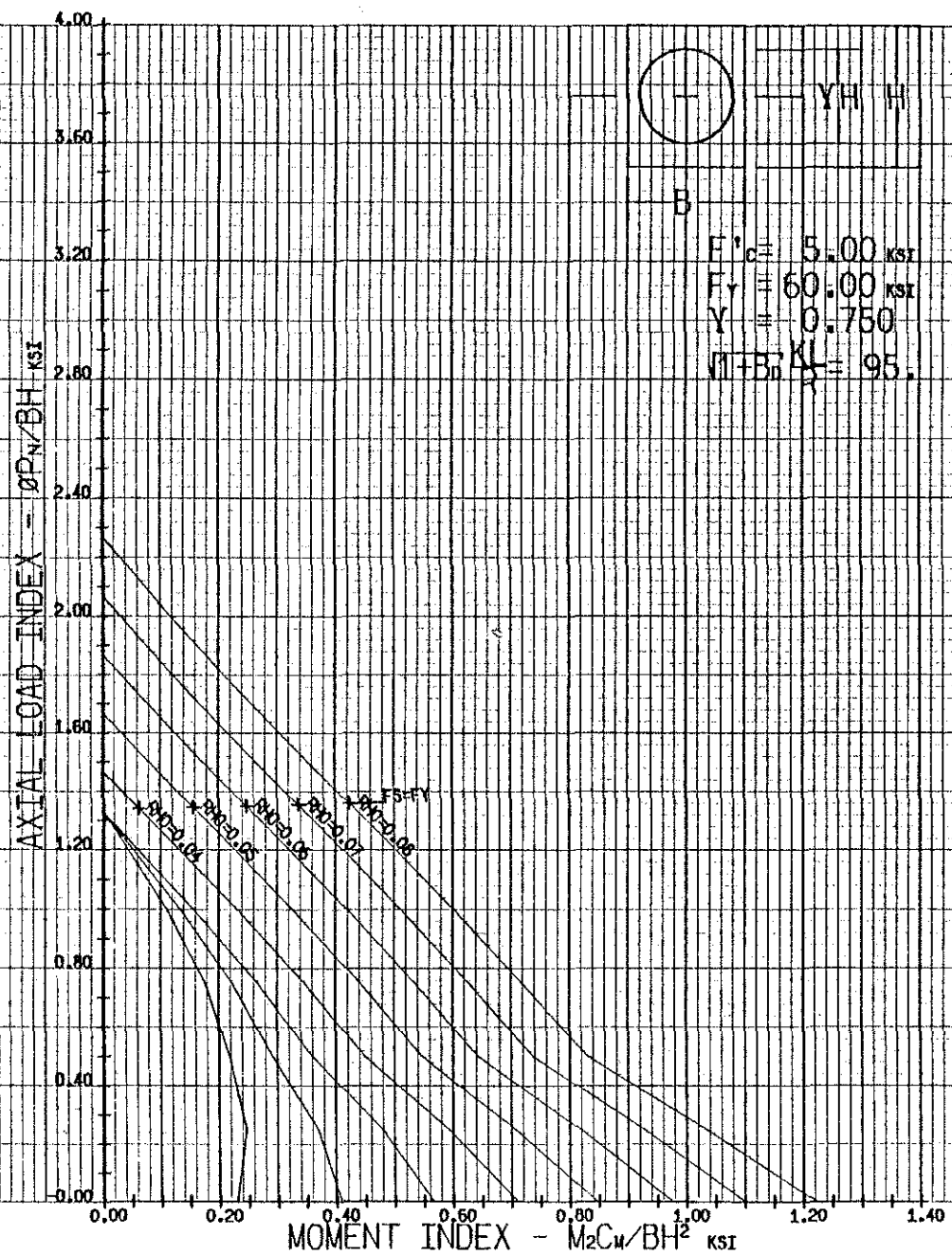


Fig. S5-60.75-95 - Interaction Diagram

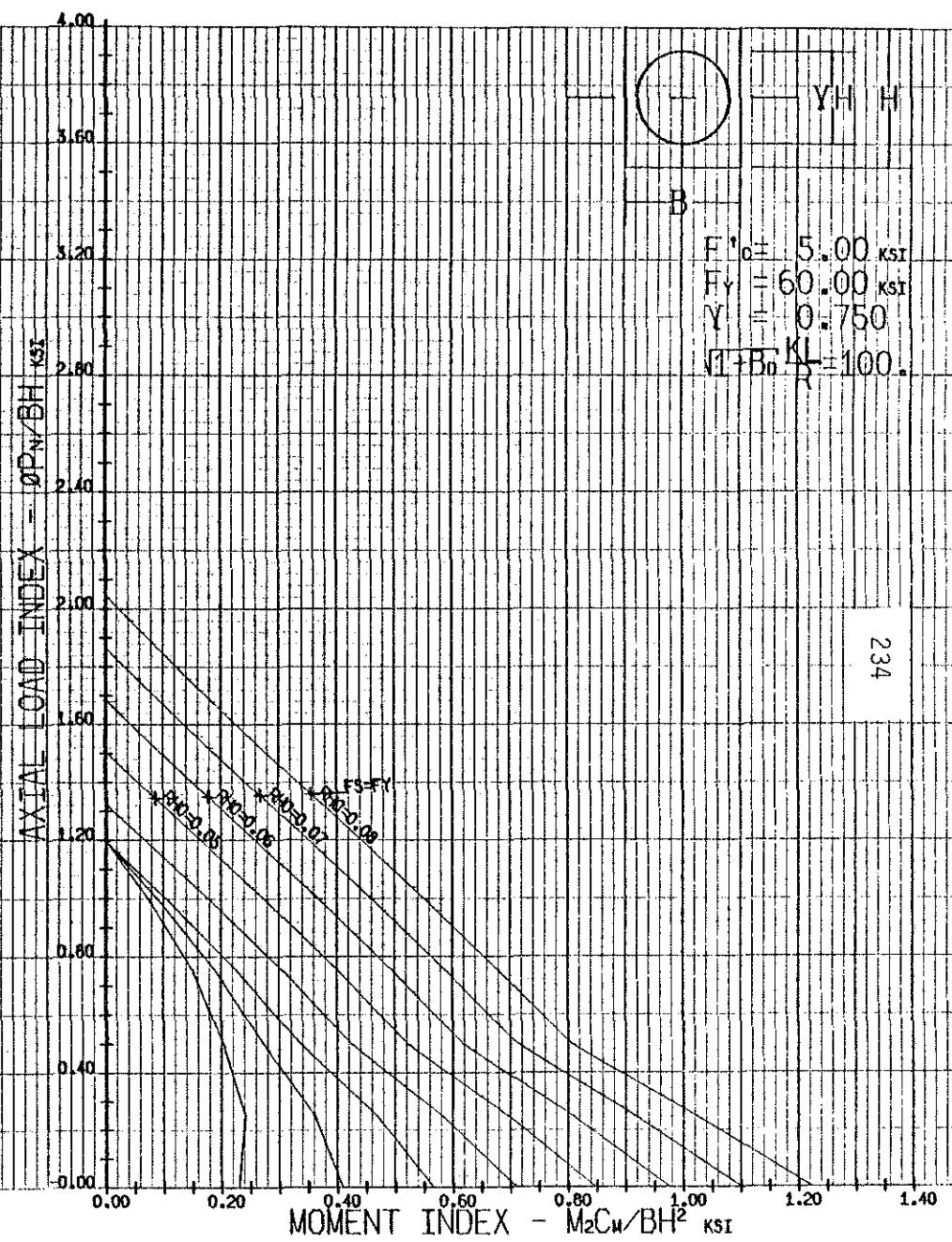


Fig. S5-60.75-100 - Interaction Diagram

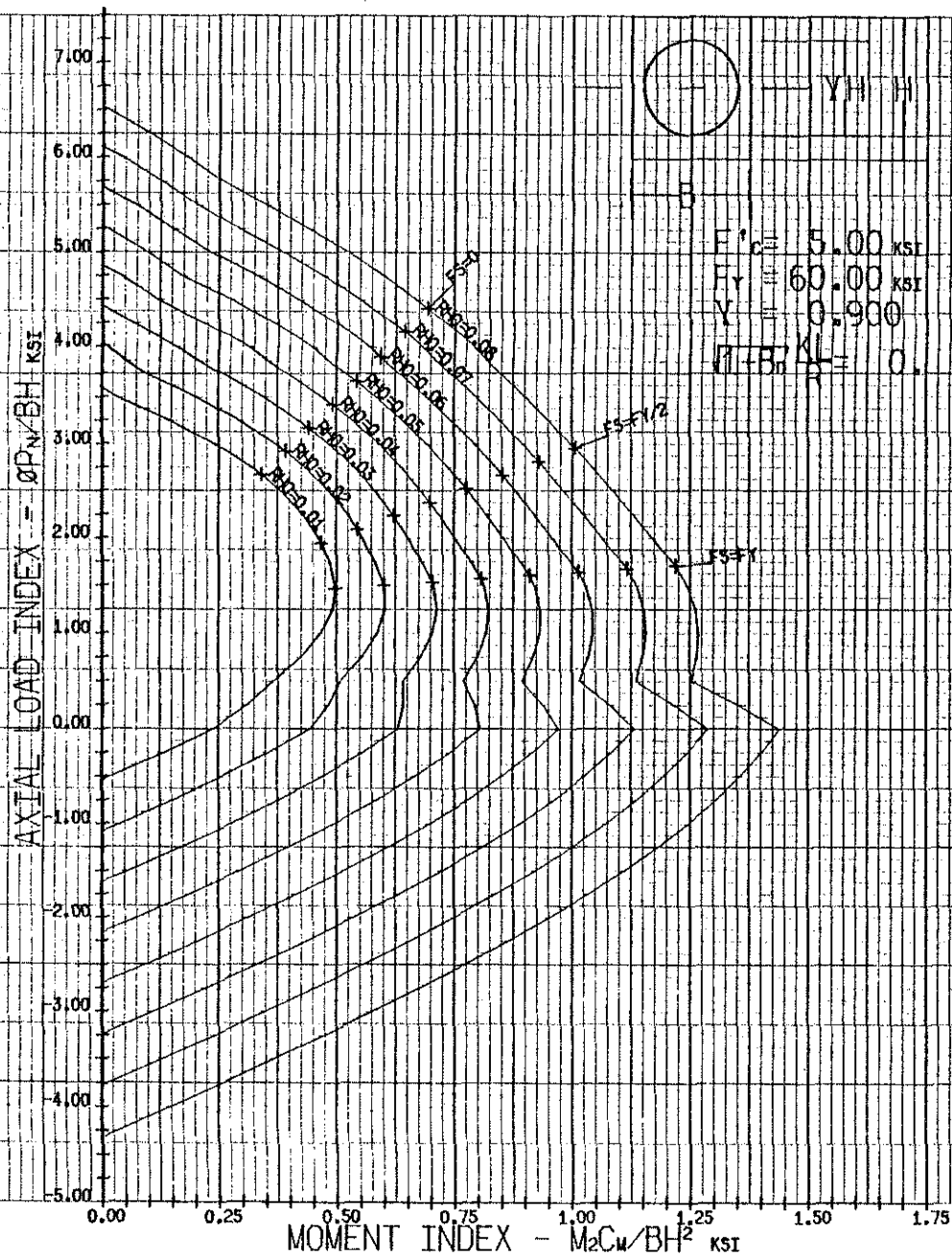


Fig. S5-60.90-0 - Interaction Diagram

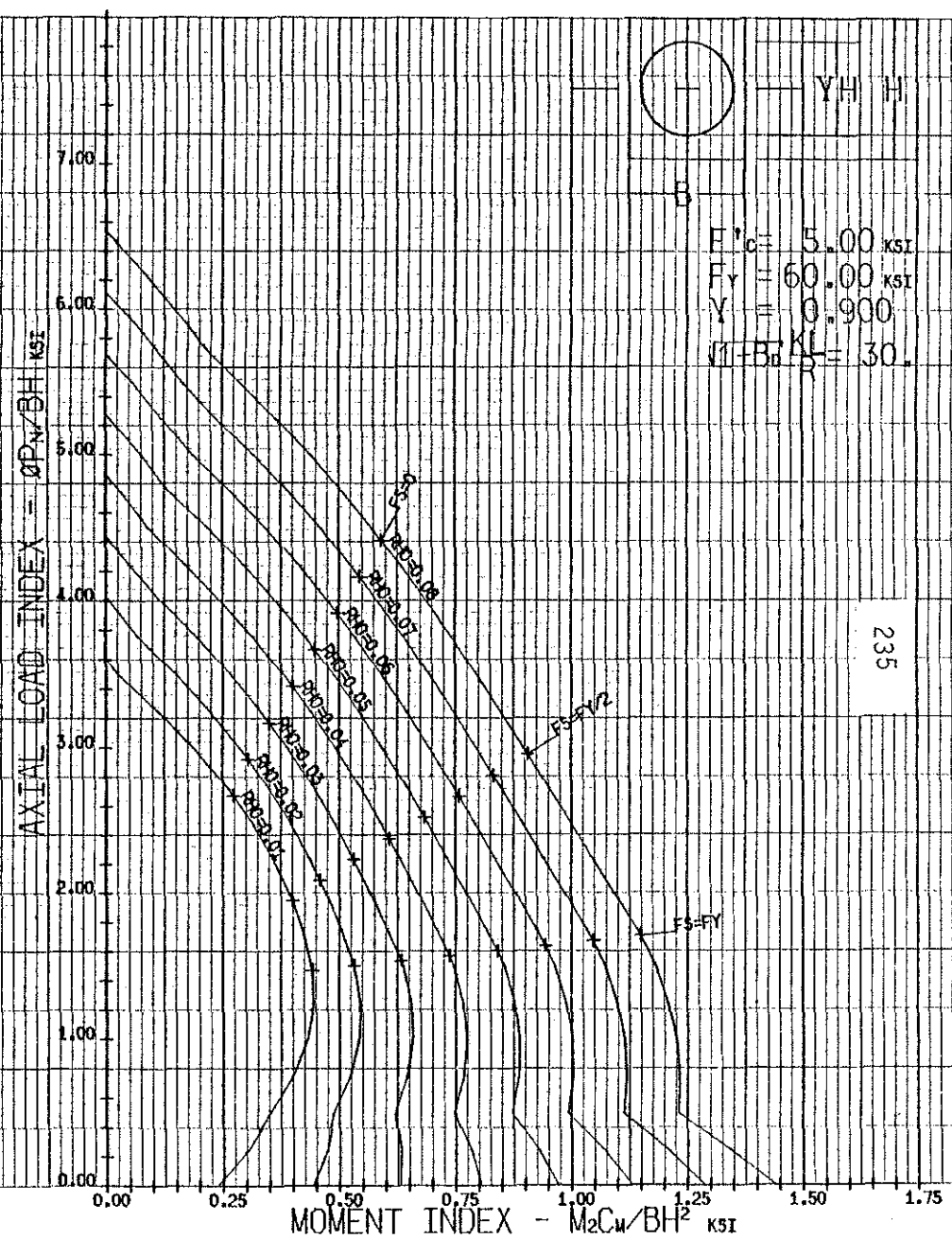


Fig. S5-60.90-30 - Interaction Diagram

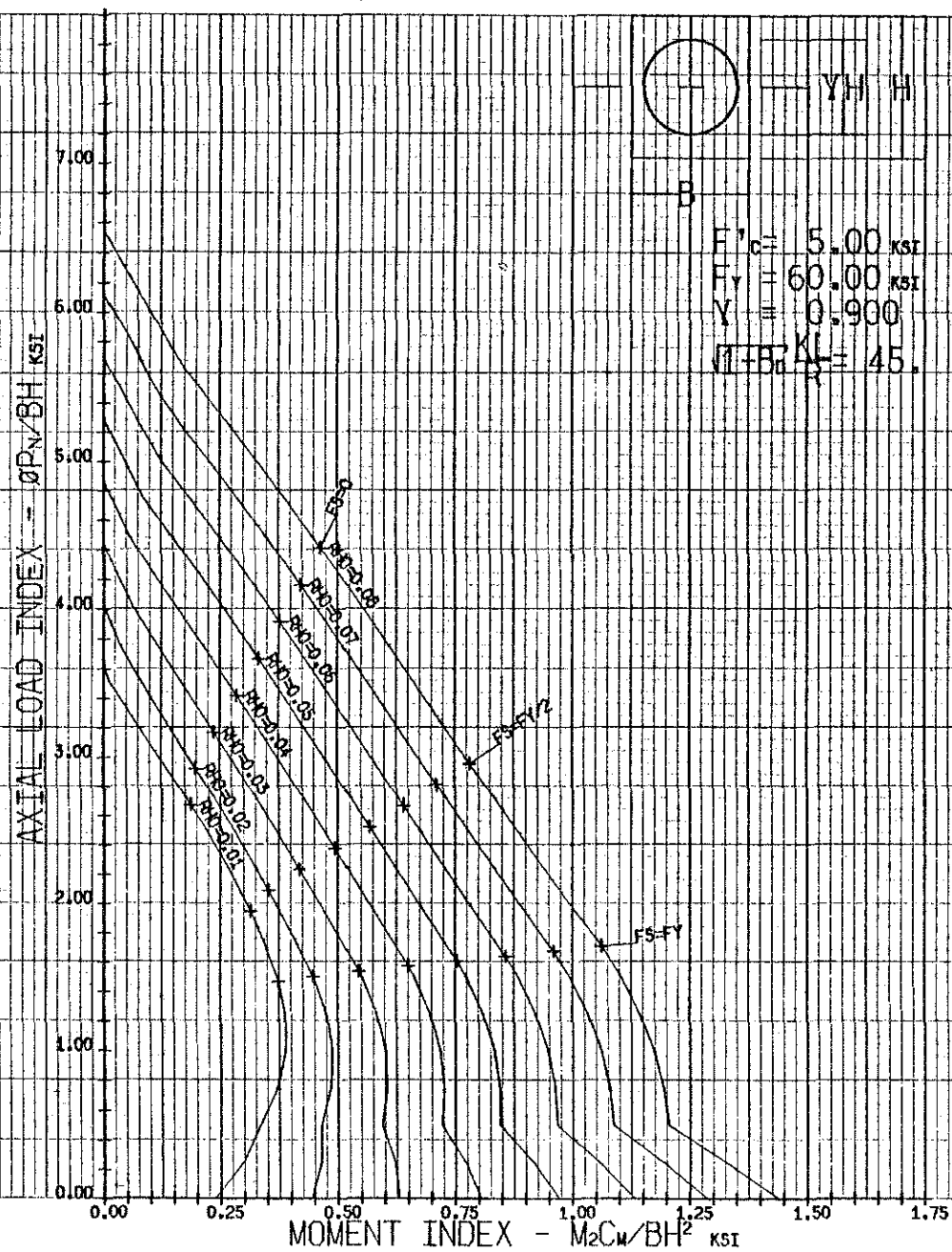


Fig. S5-60.90-45 - Interaction Diagram

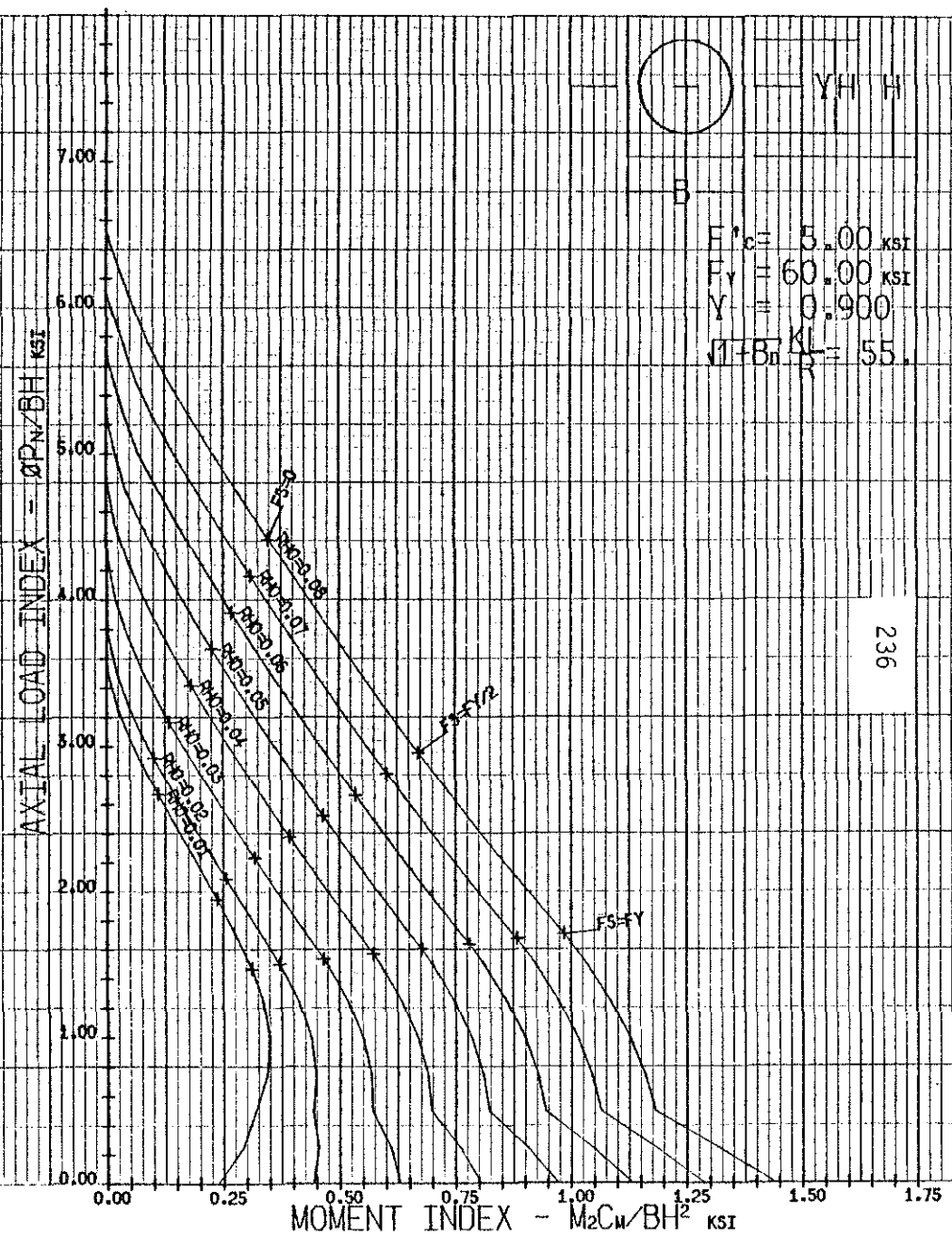


Fig. S5-60.90-55 - Interaction Diagram

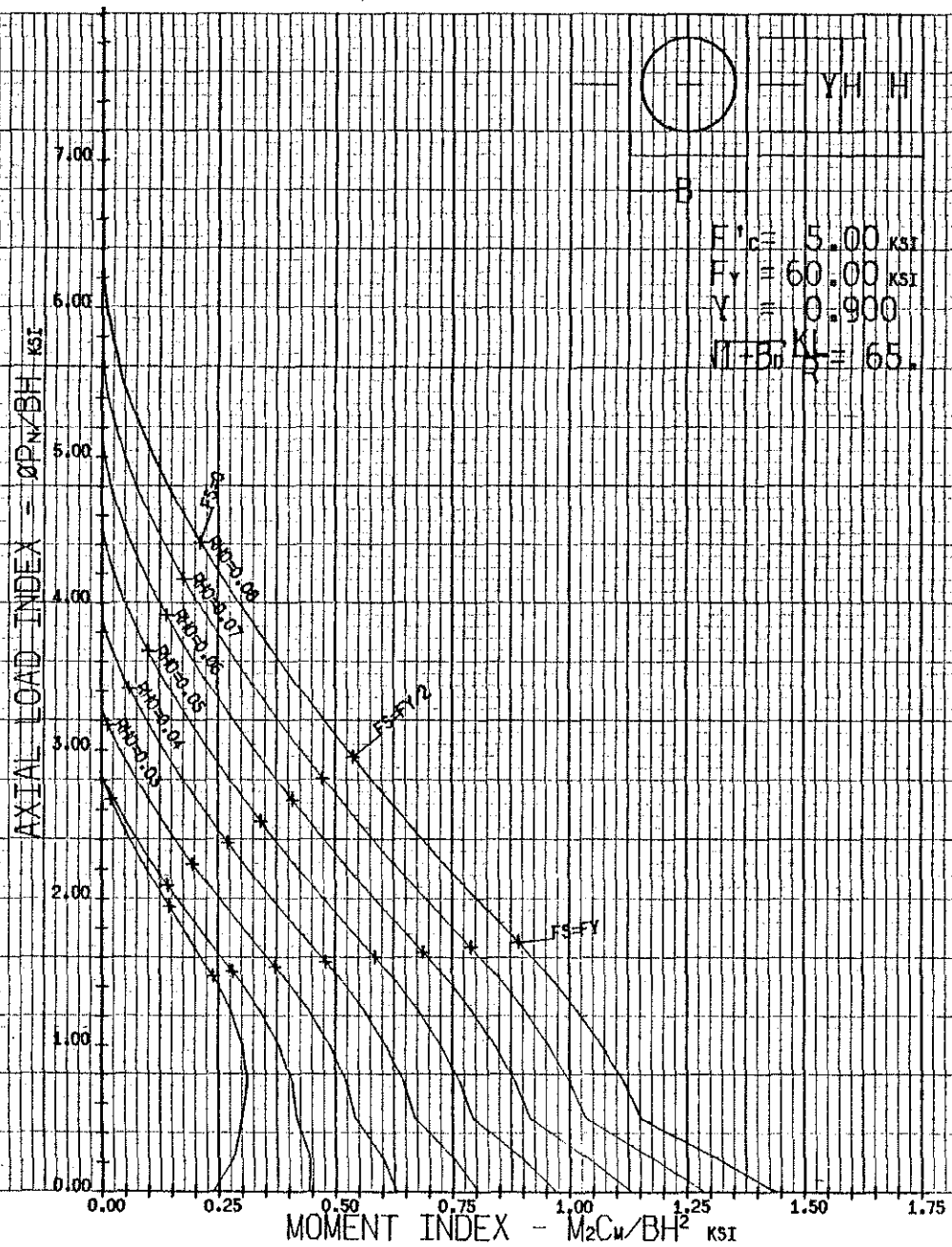


Fig. S5-60.90-65 - Interaction Diagram

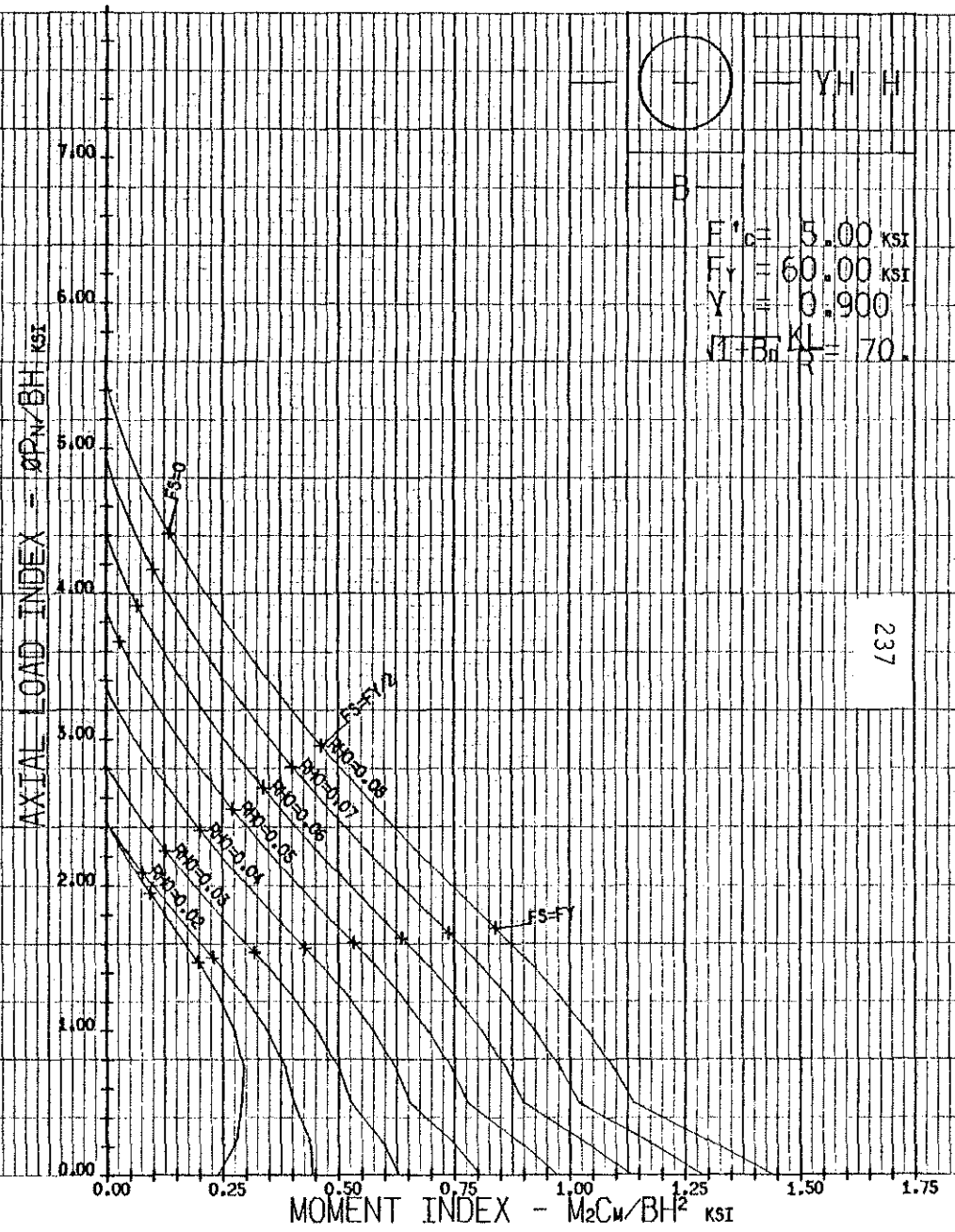


Fig. S5-60.90-70 - Interaction Diagram

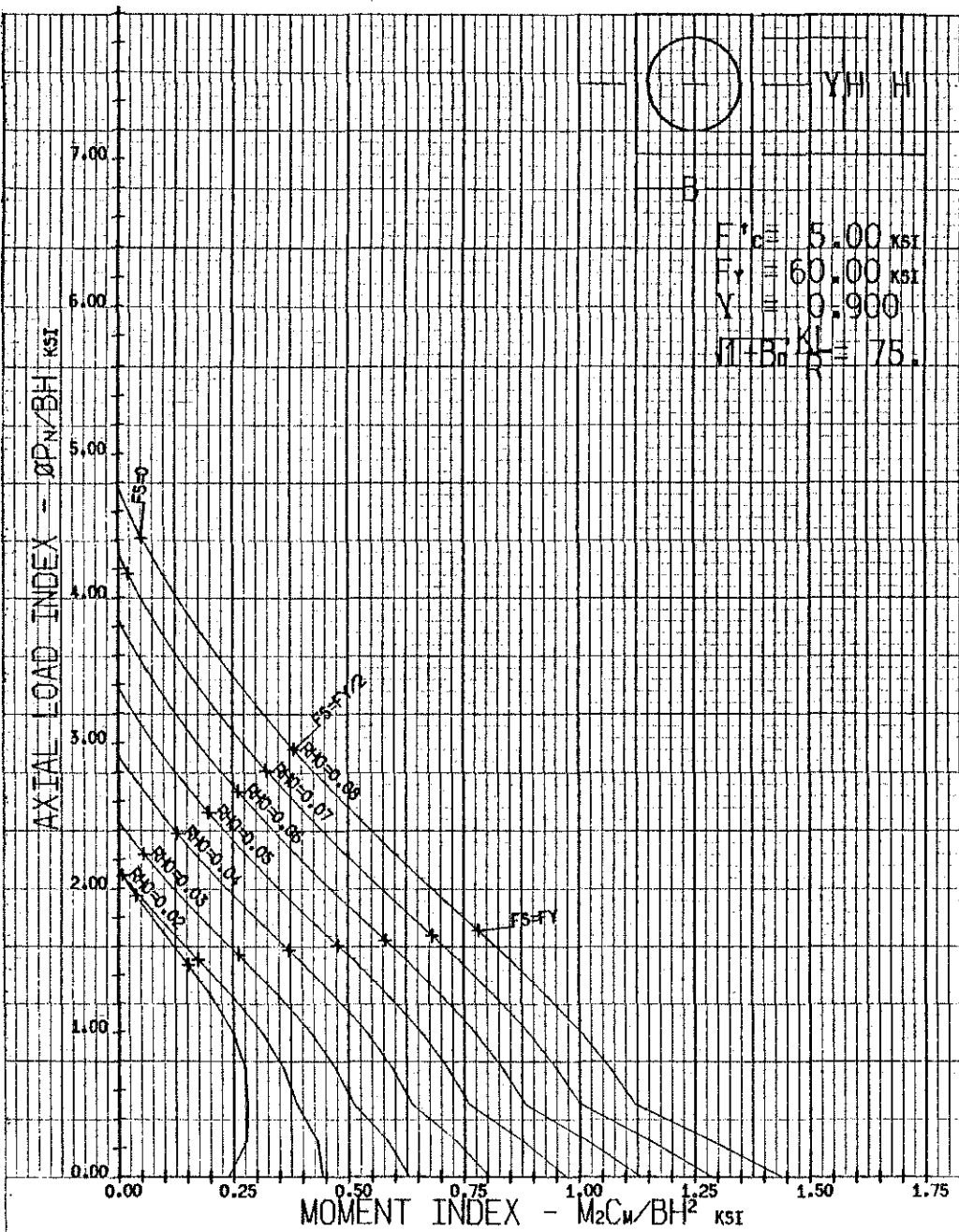


Fig. S5-60.90-75 - Interaction Diagram

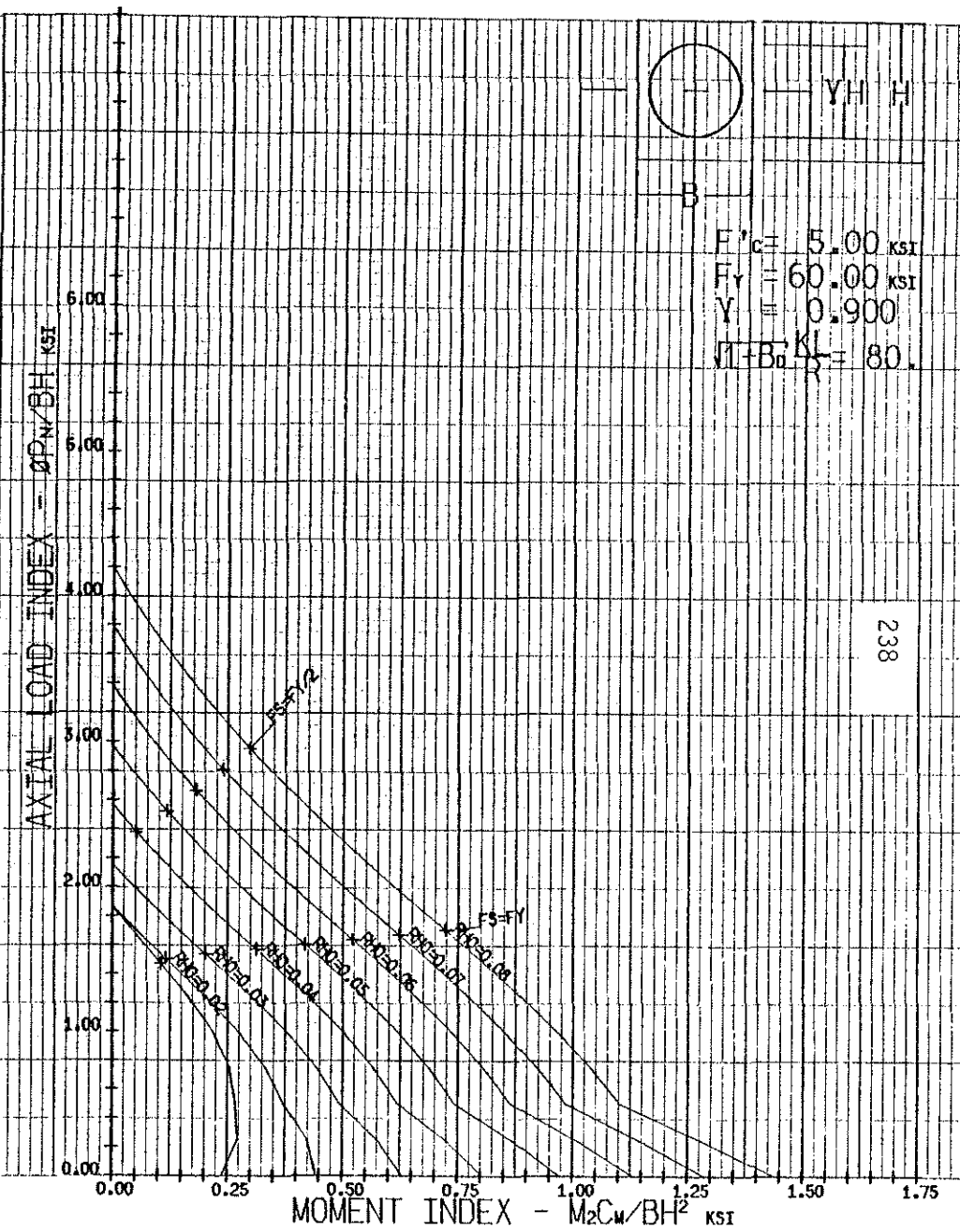


Fig. S5-60.90-80 - Interaction Diagram



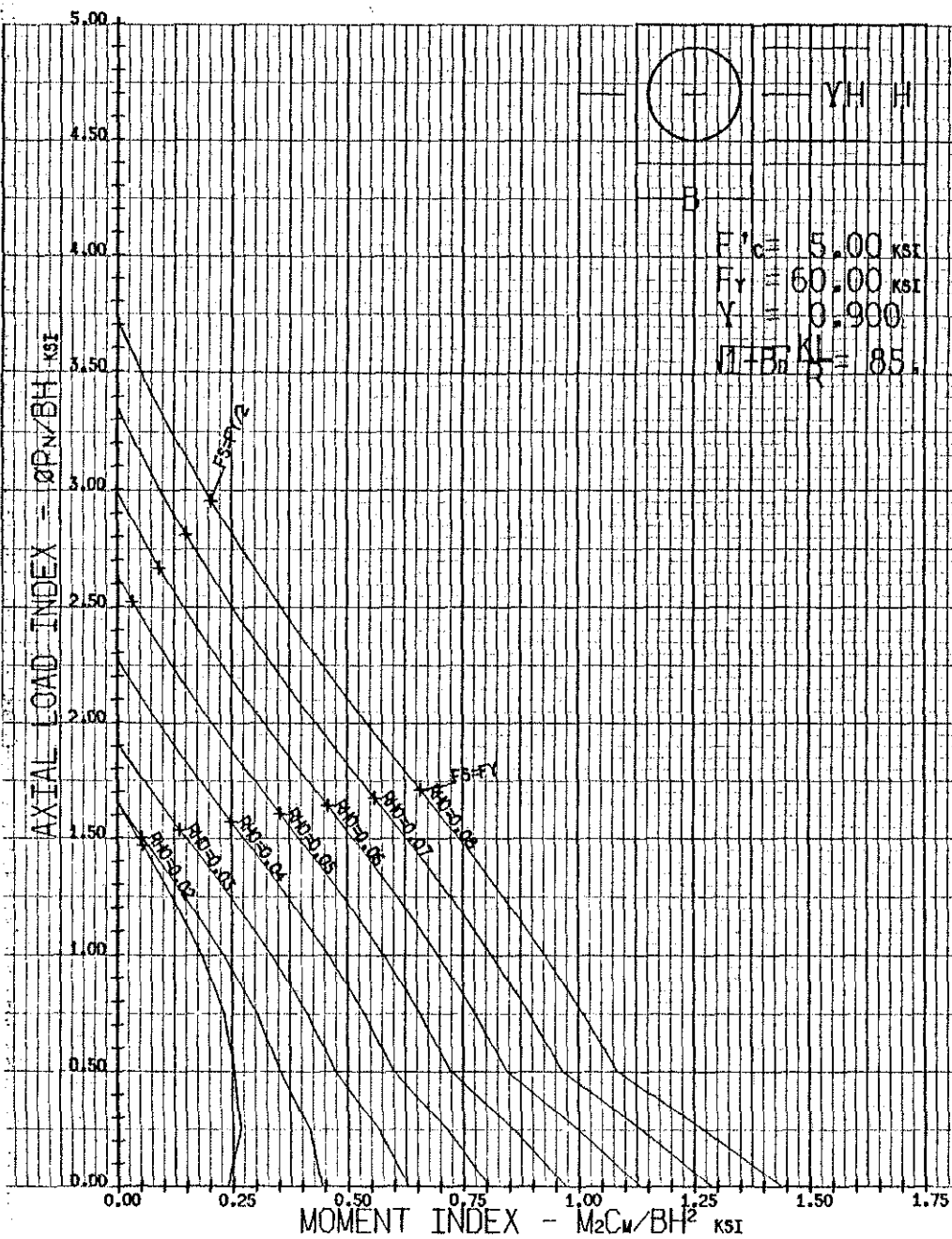


Fig. S5-60.90-85 - Interaction Diagram

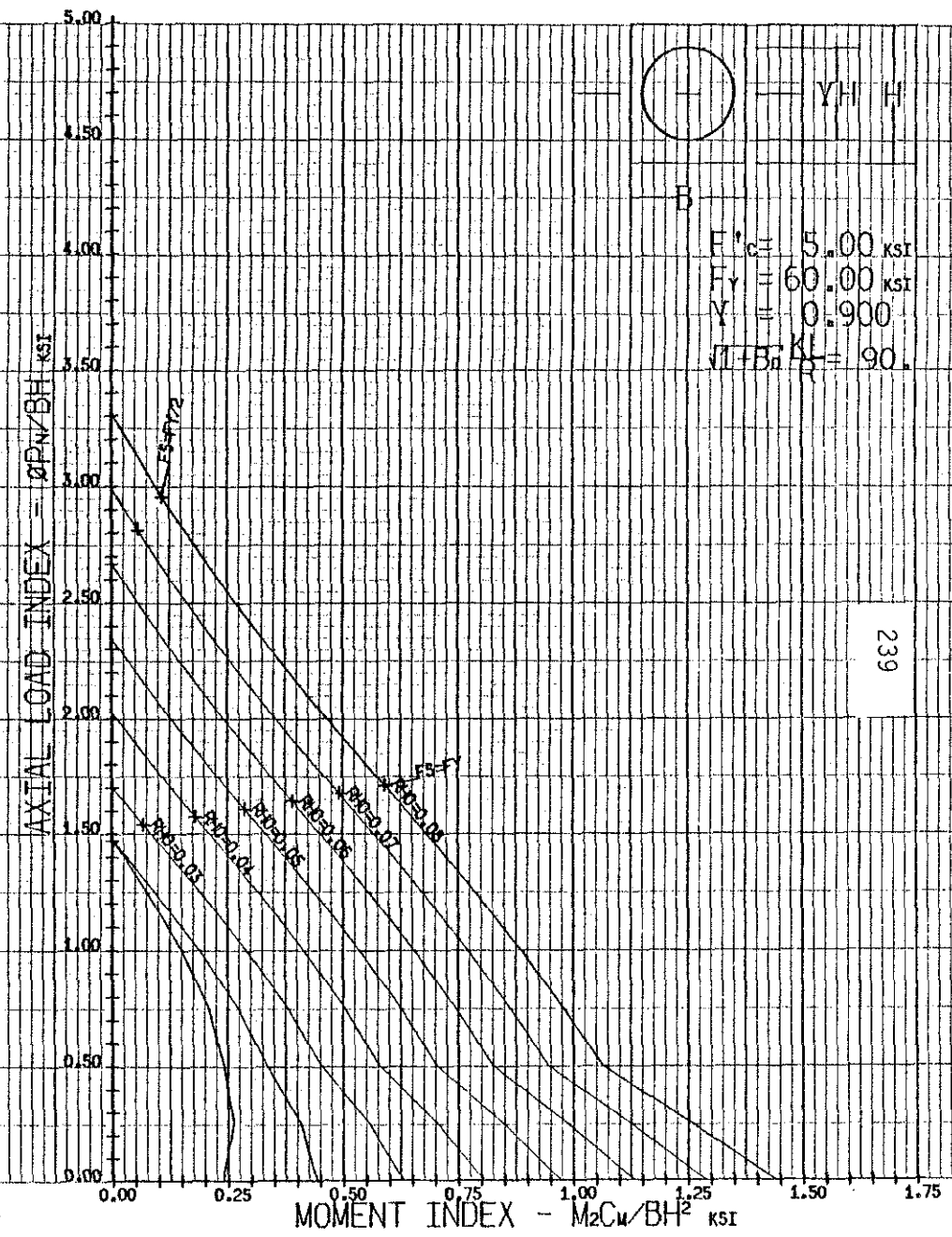


Fig. S5-60.90-90 - Interaction Diagram



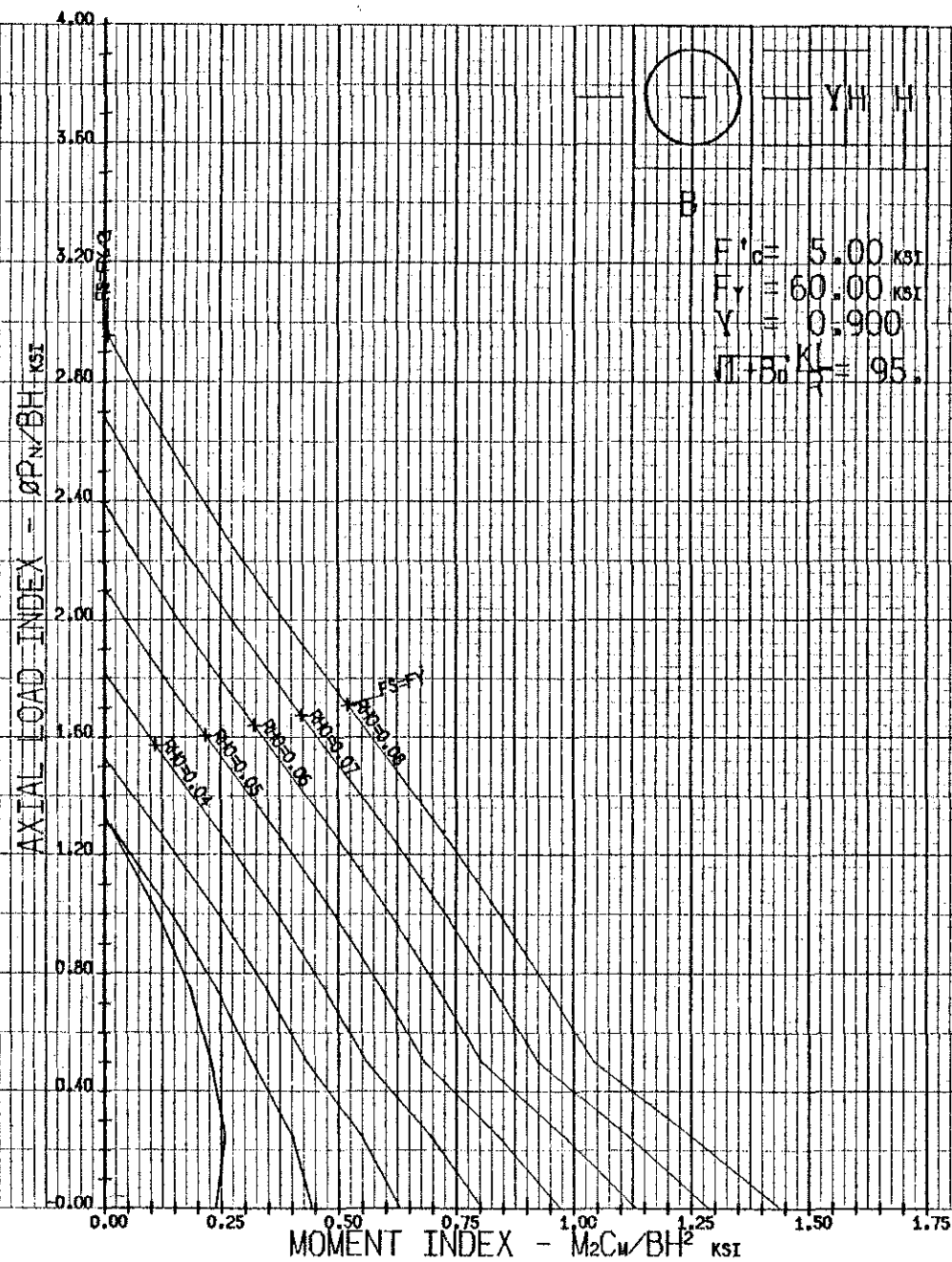


Fig. S5-60.90-95 - Interaction Diagram

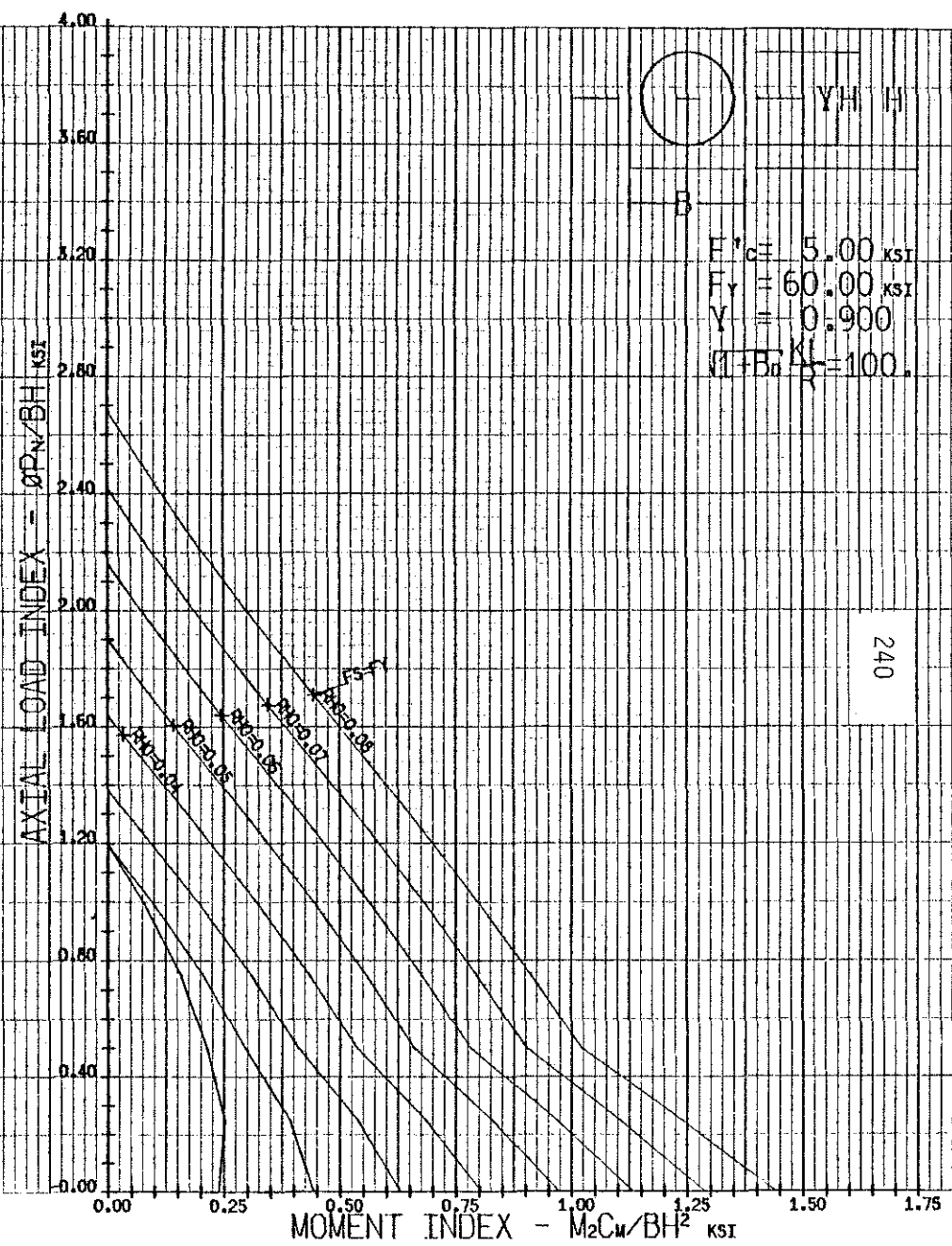


Fig. S5-60.90-100 - Interaction Diagram

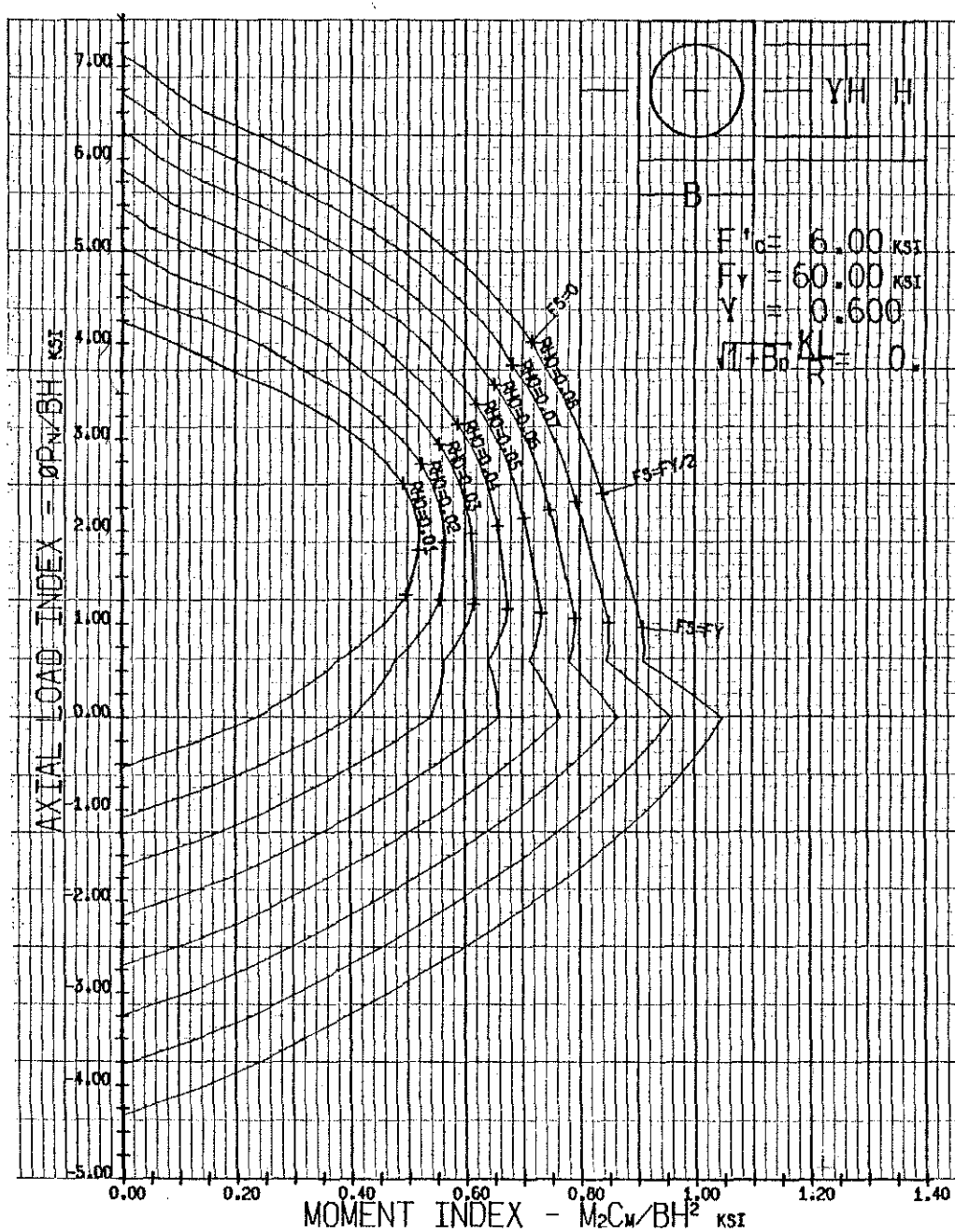


Fig. S6-60.60-0 - Interaction Diagram

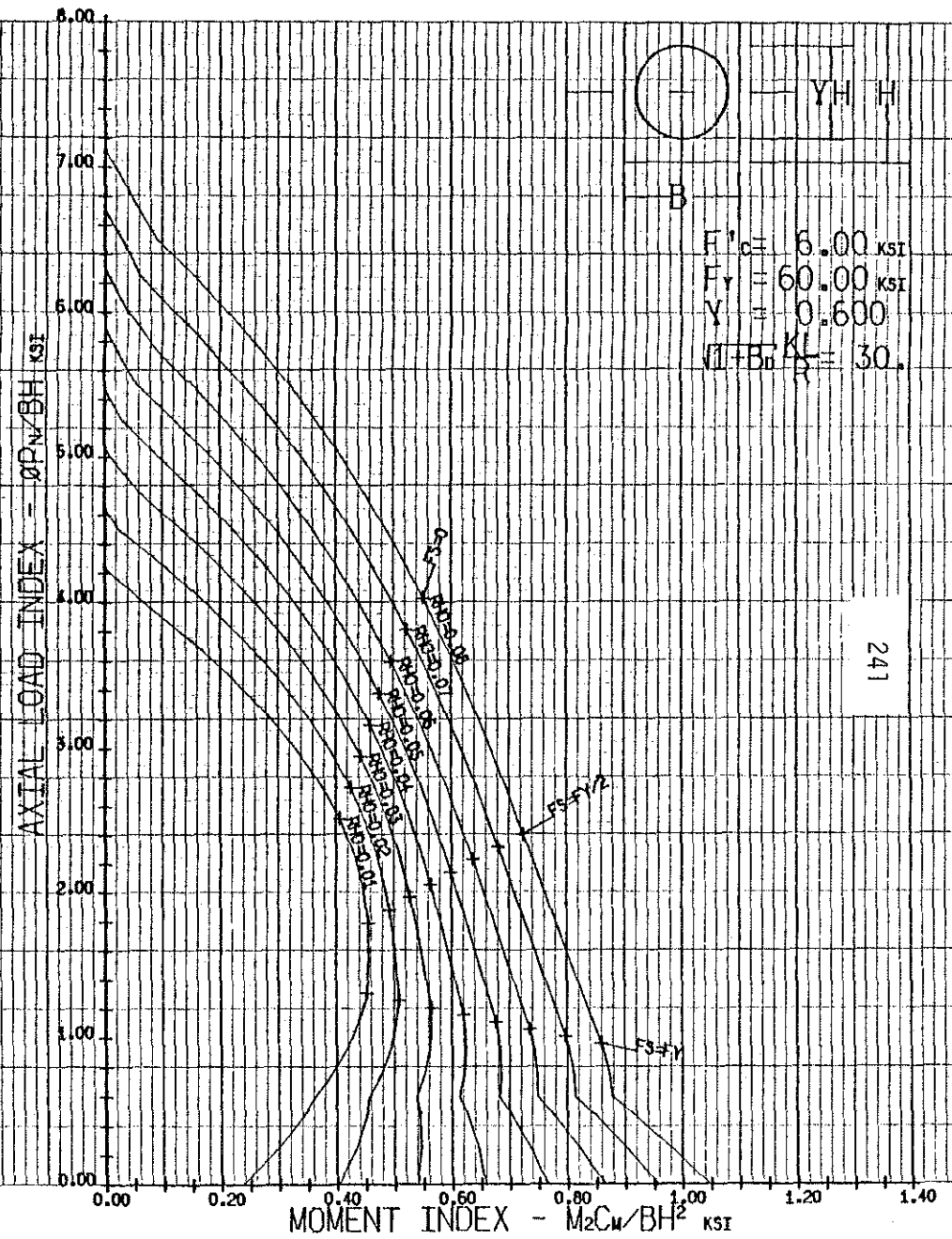


Fig. S6-60.60-30 - Interaction Diagram

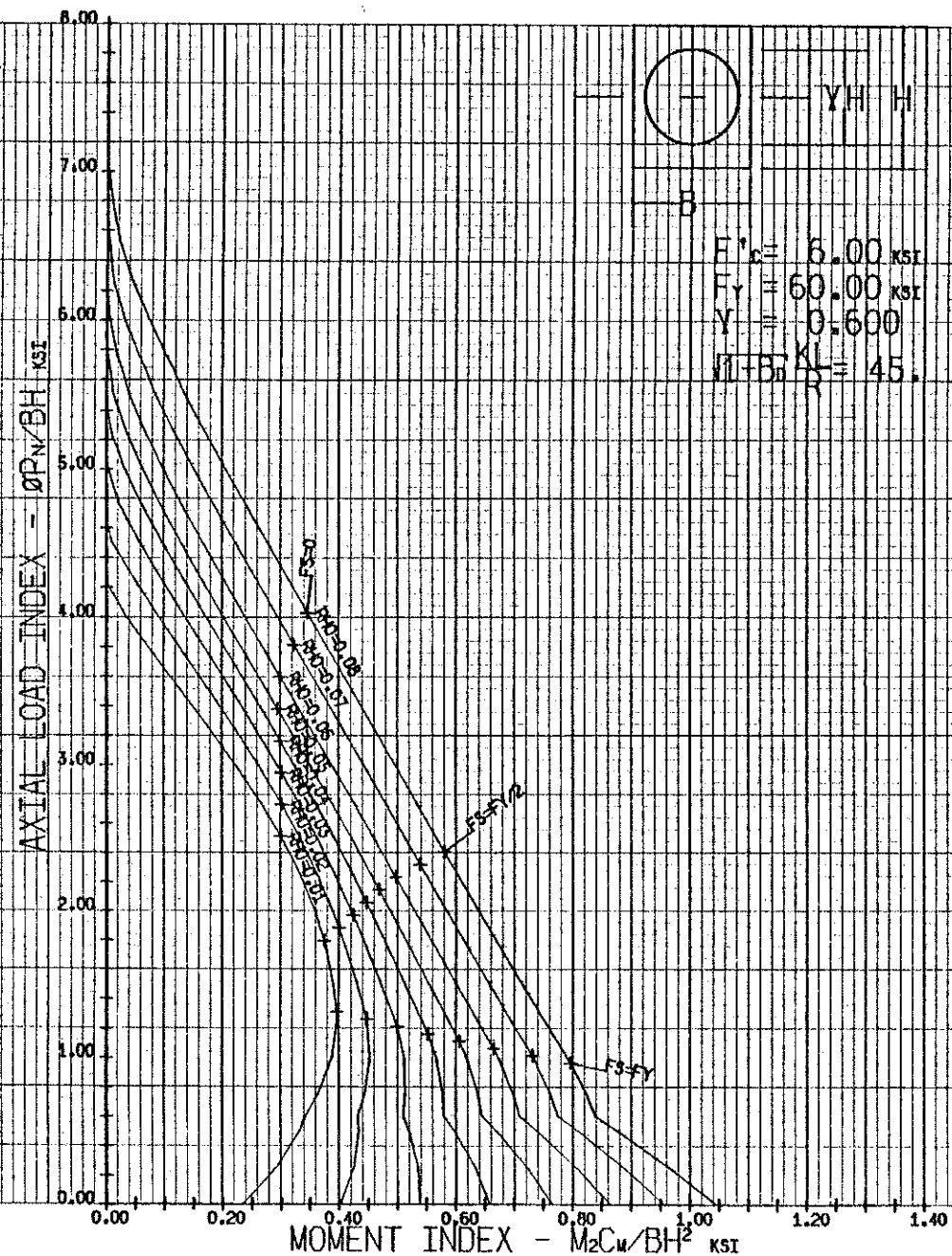


Fig. S6-60.60-45 - Interaction Diagram

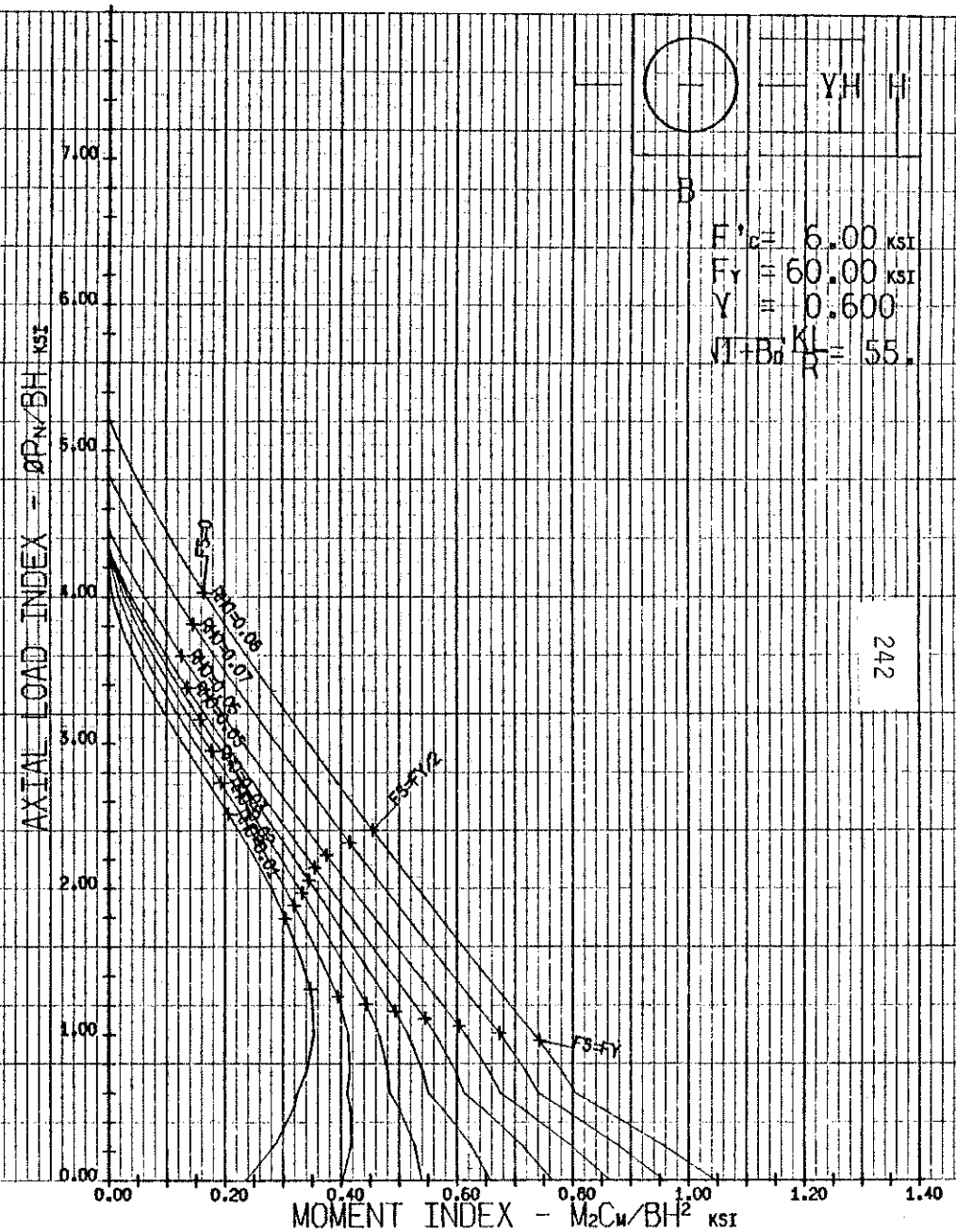


Fig. S6-60.60-55 - Interaction Diagram

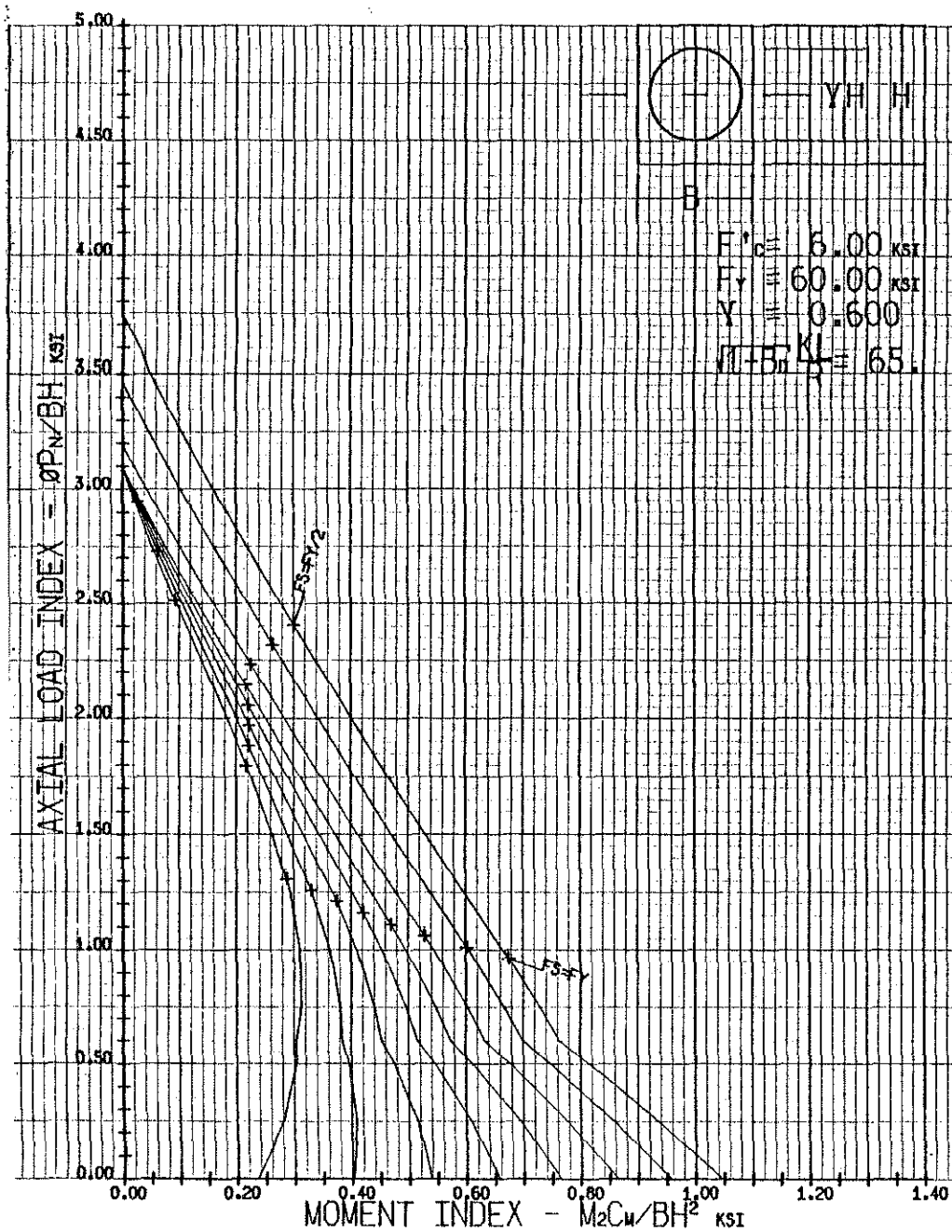


Fig. S6-60.60-65 - Interaction Diagram

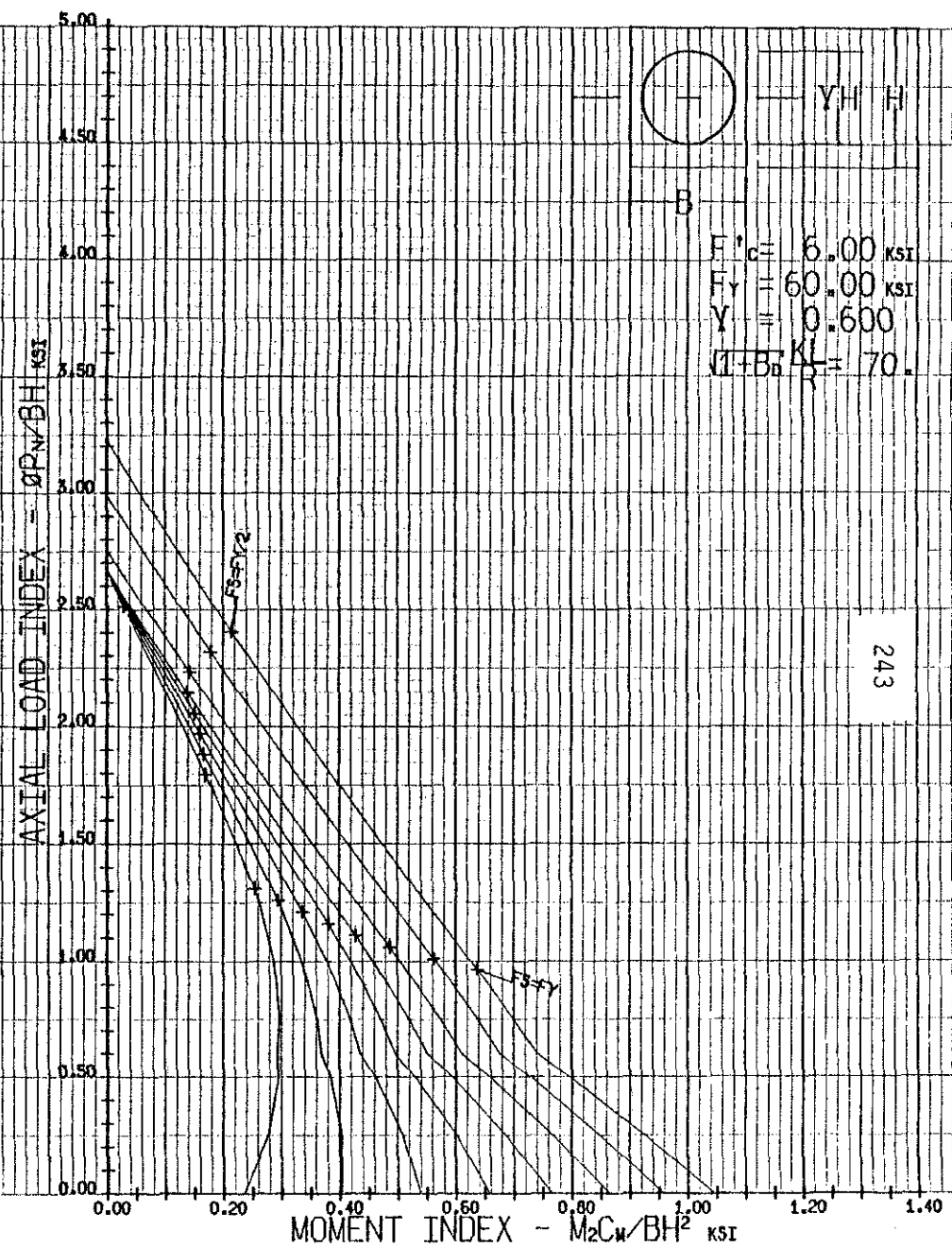


Fig. S6-60.60-70 - Interaction Diagram

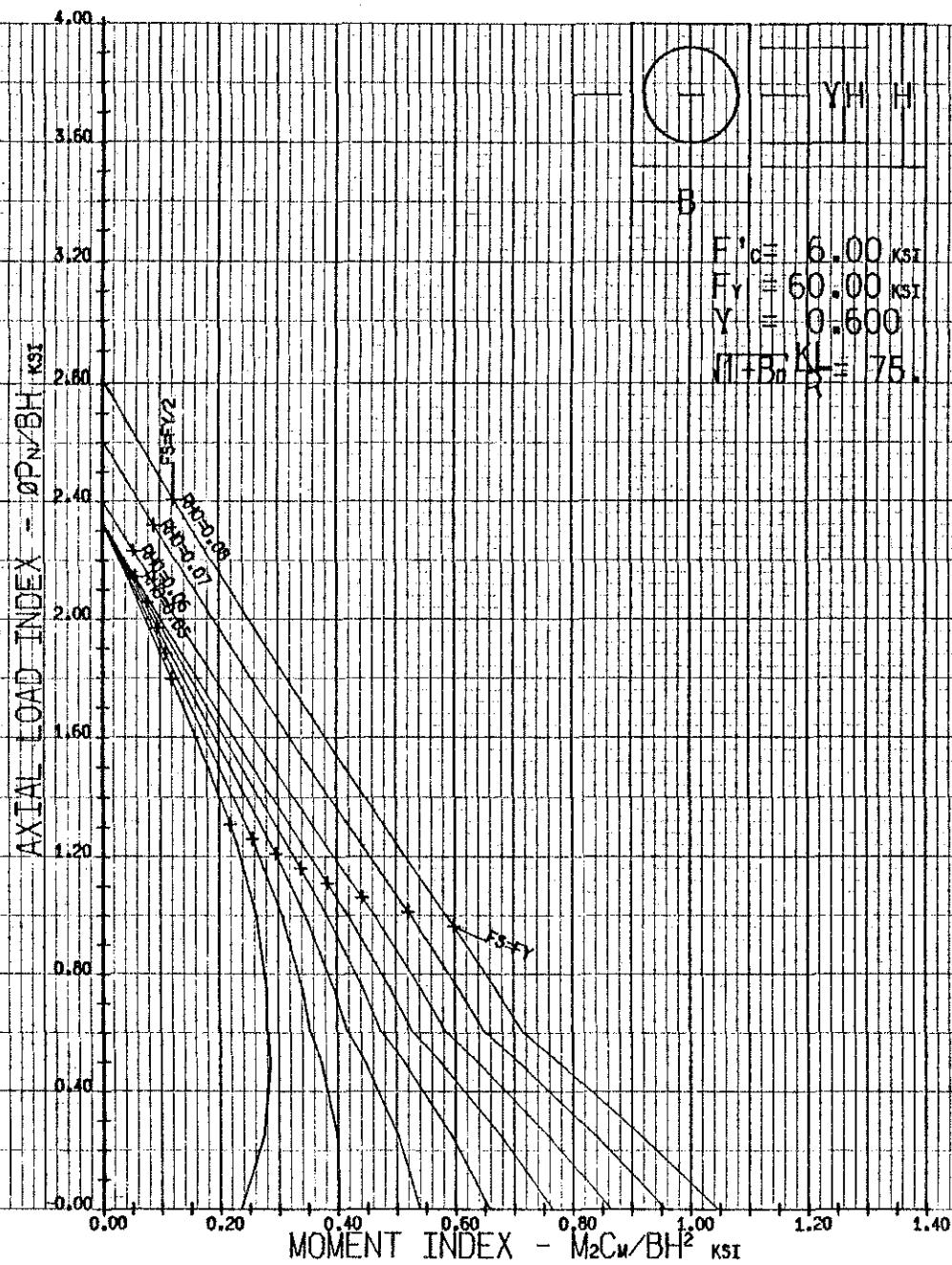


Fig. S6-60.60-75 - Interaction Diagram

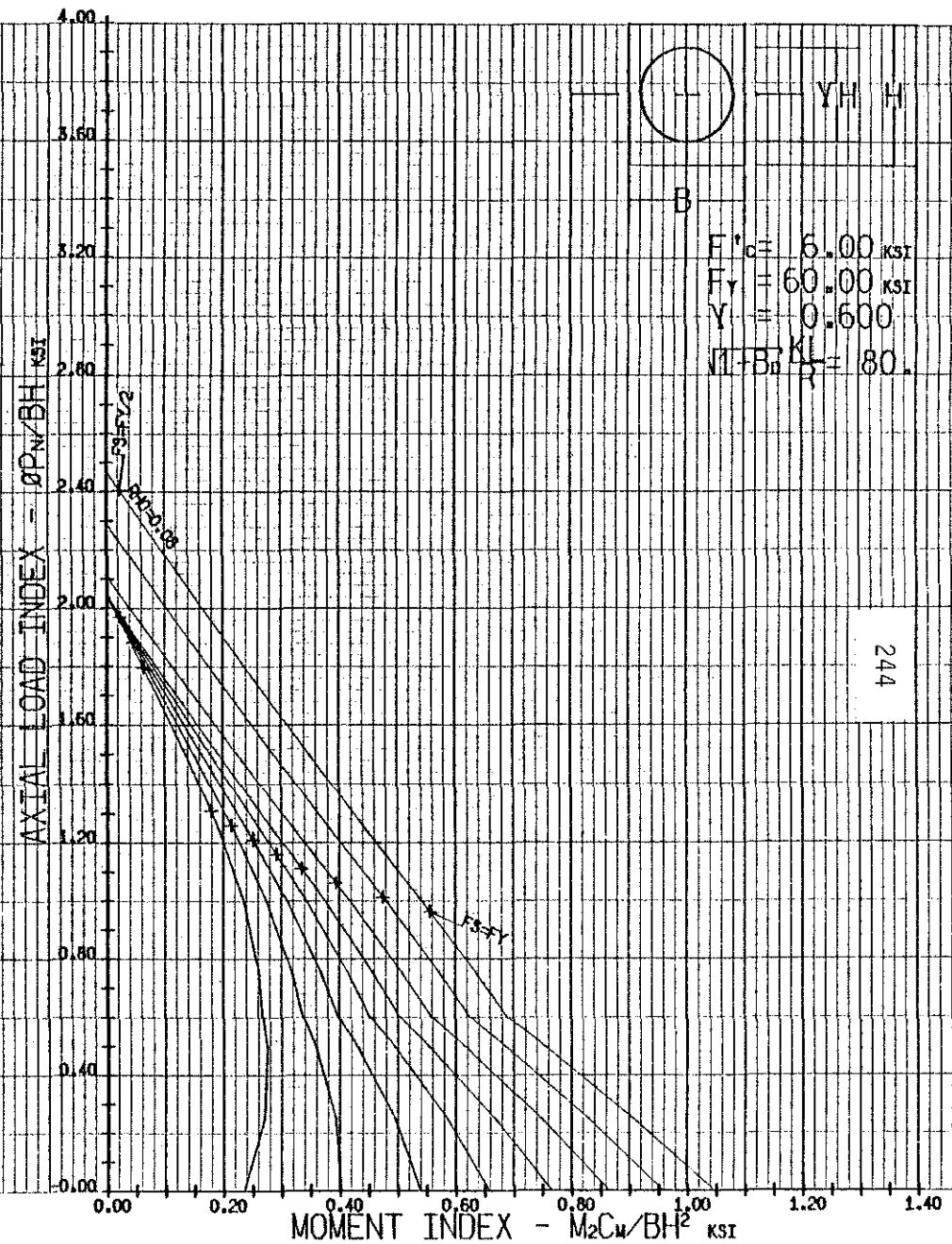


Fig. S6-60.60-80 - Interaction Diagram



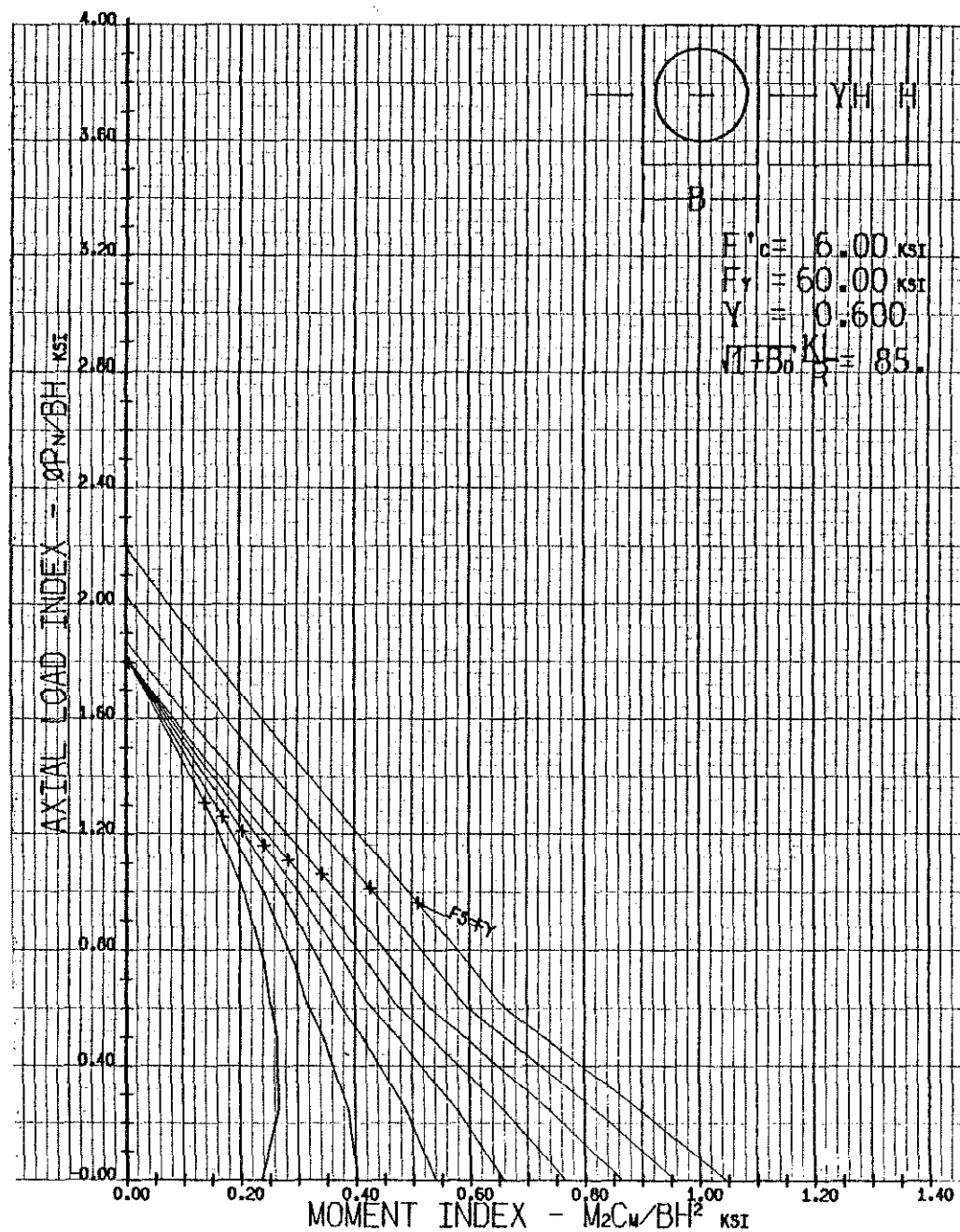


Fig. S6-60.60-85 - Interaction Diagram

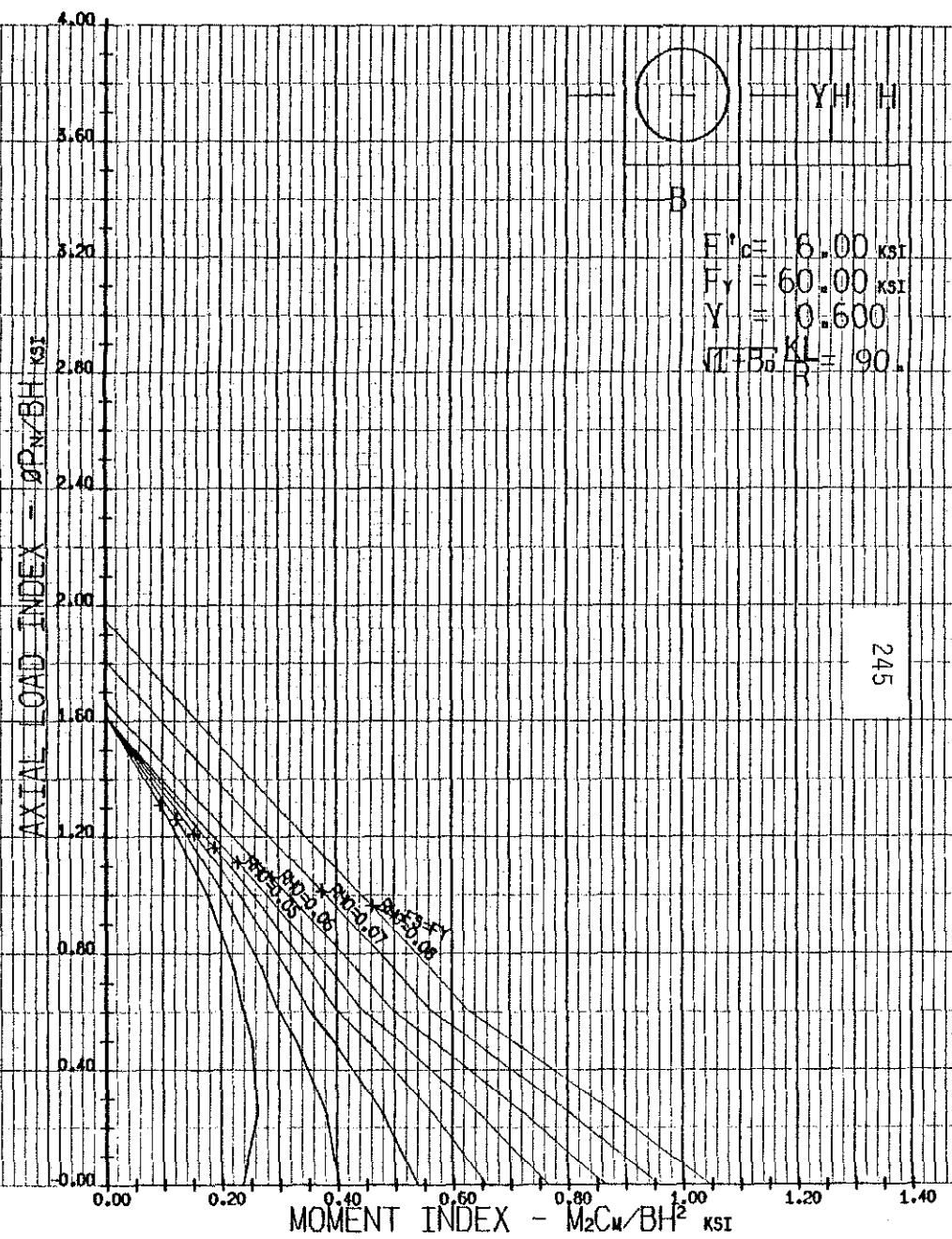


Fig. S6-60.60-90 - Interaction Diagram



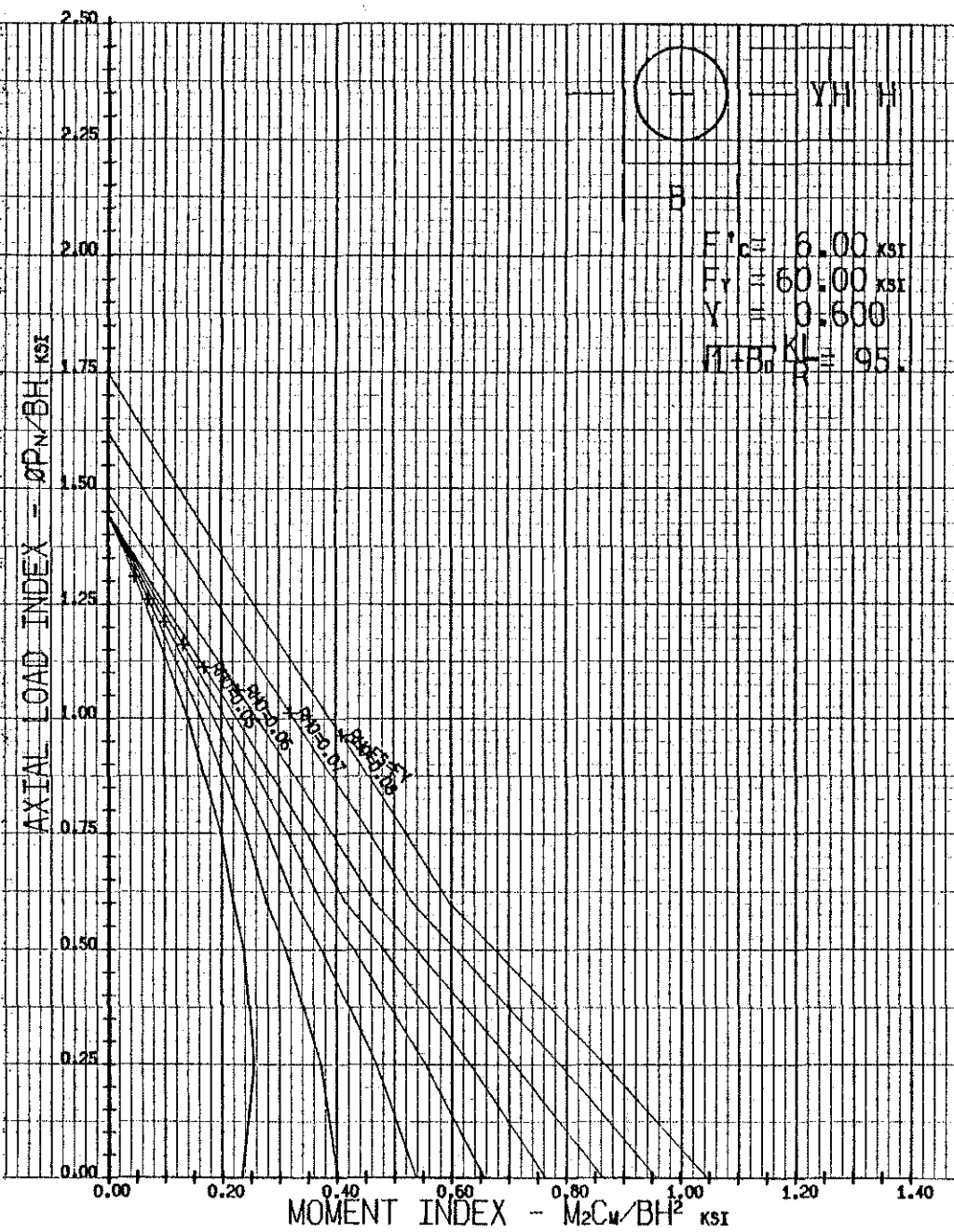


Fig. S6-60.60-95 - Interaction Diagram

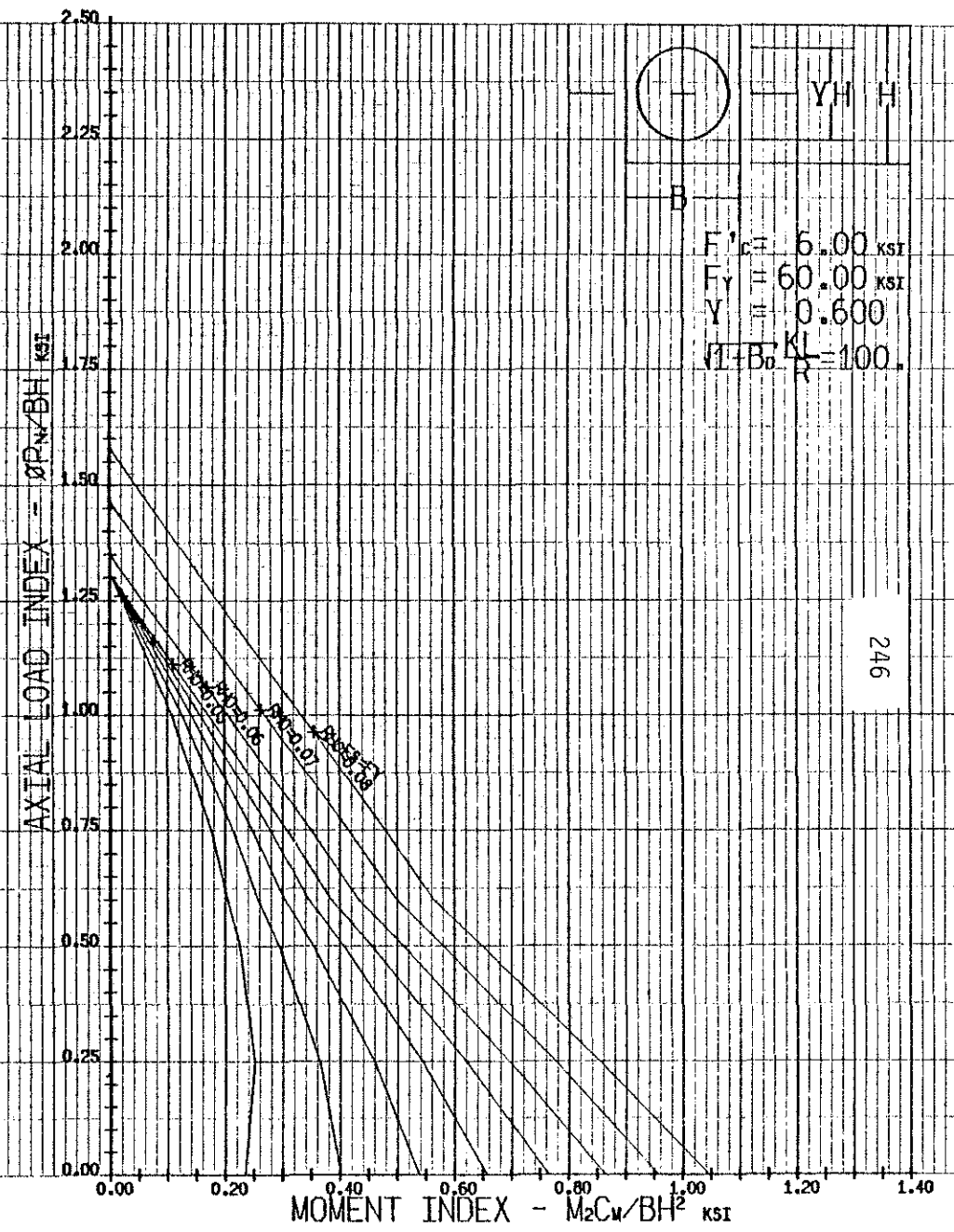


Fig. S6-60.60-100 - Interaction Diagram

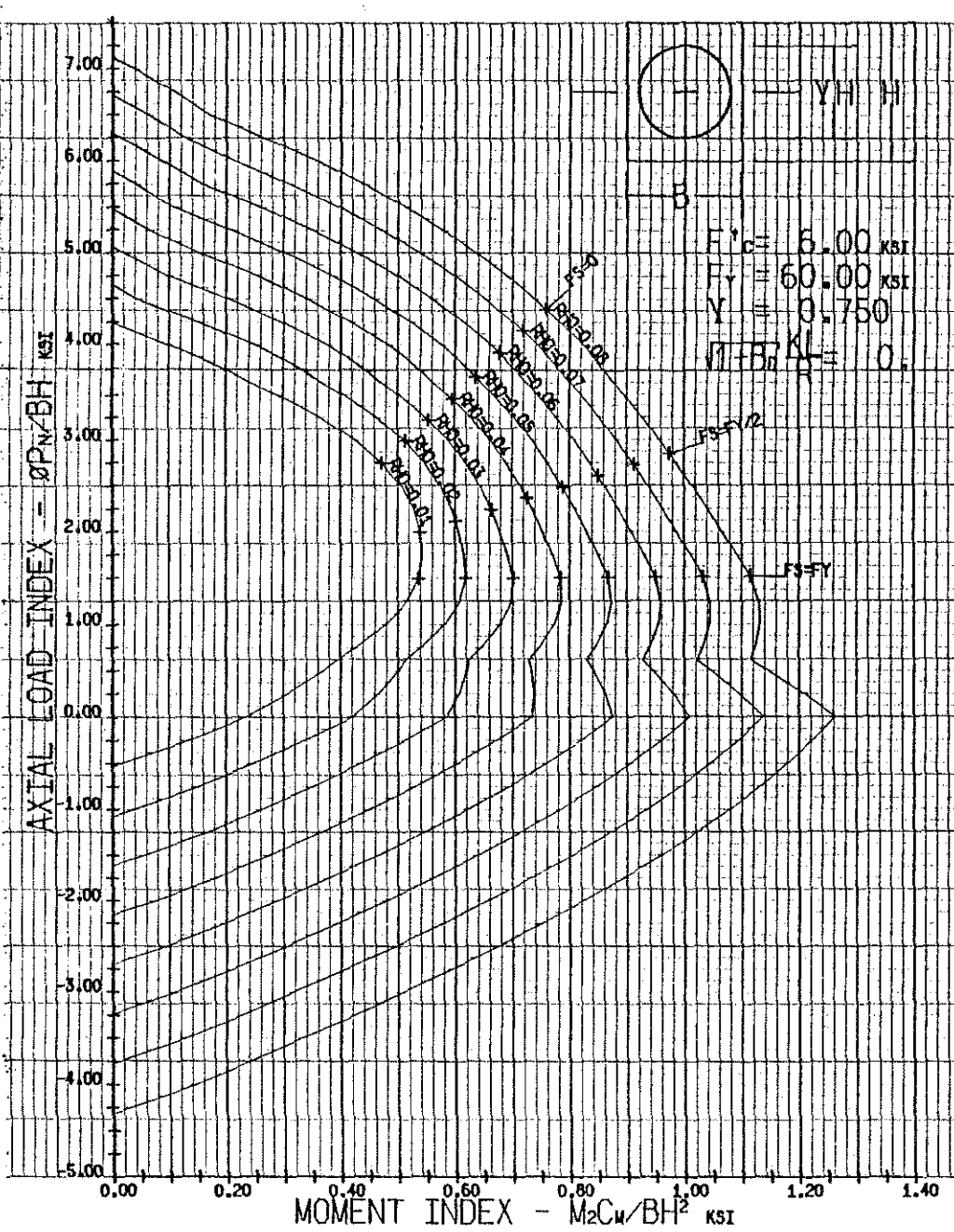


Fig. S6-60.75-0 - Interaction Diagram

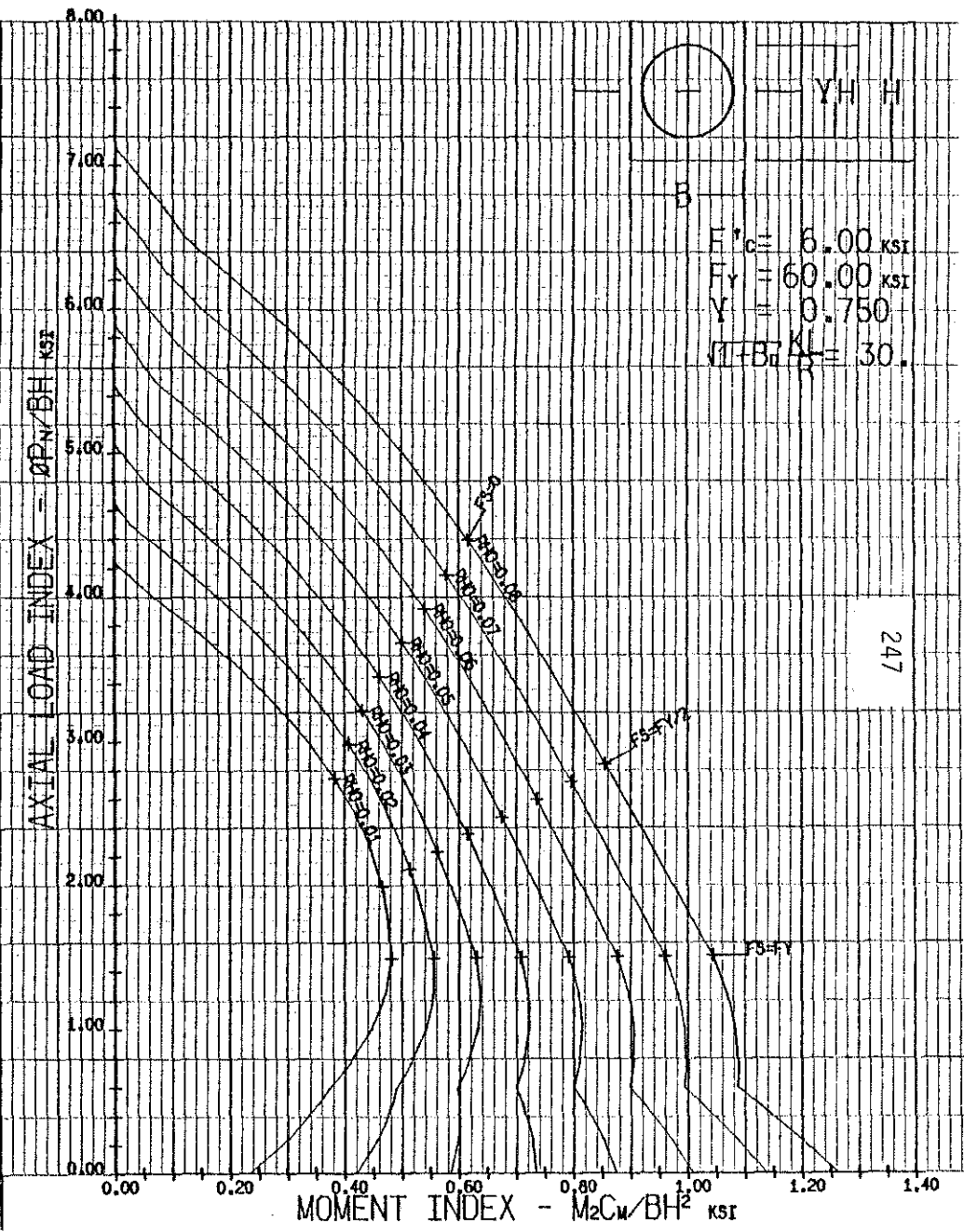


Fig. S6-60.75-30 - Interaction Diagram

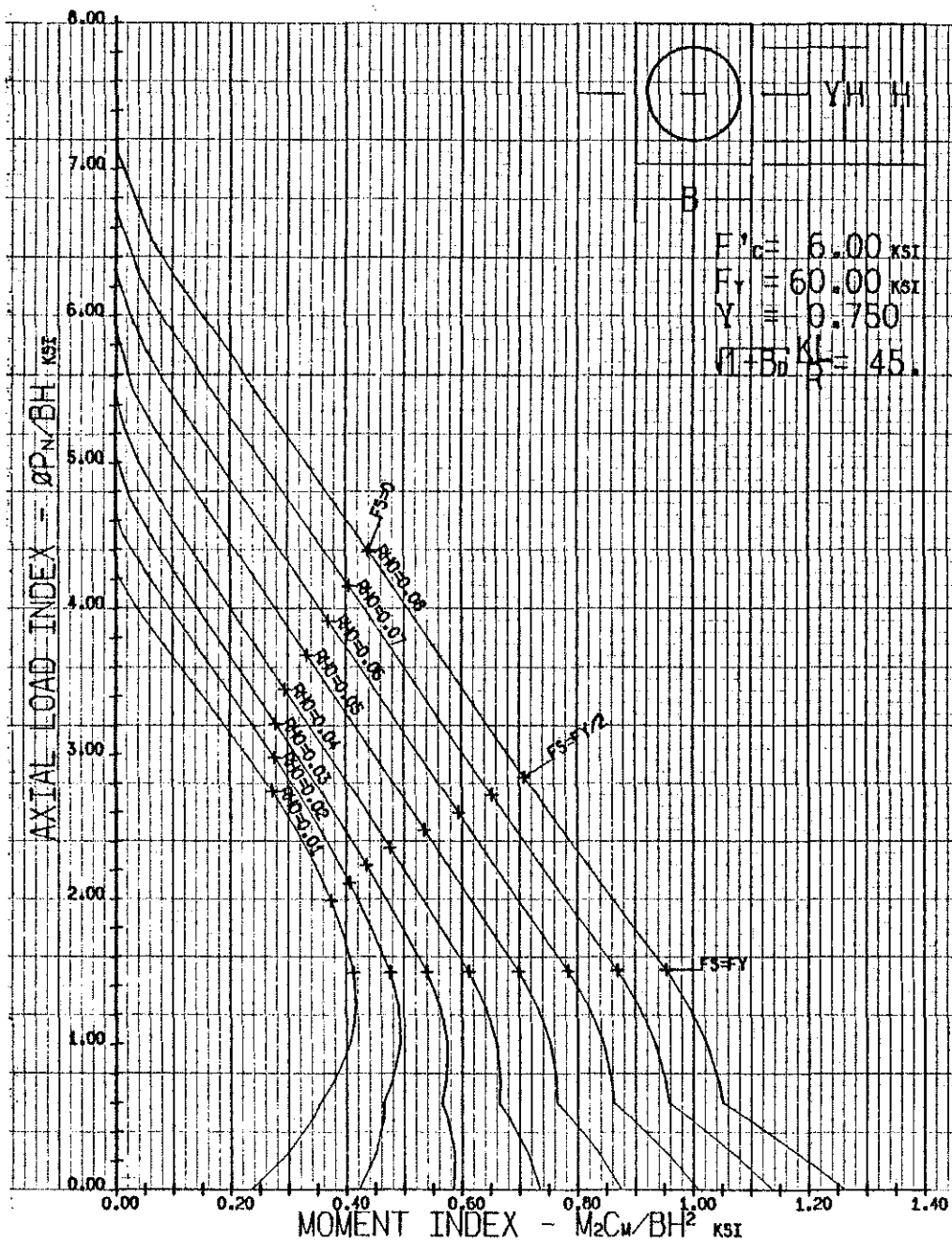


Fig. S6-60.75-45 - Interaction Diagram

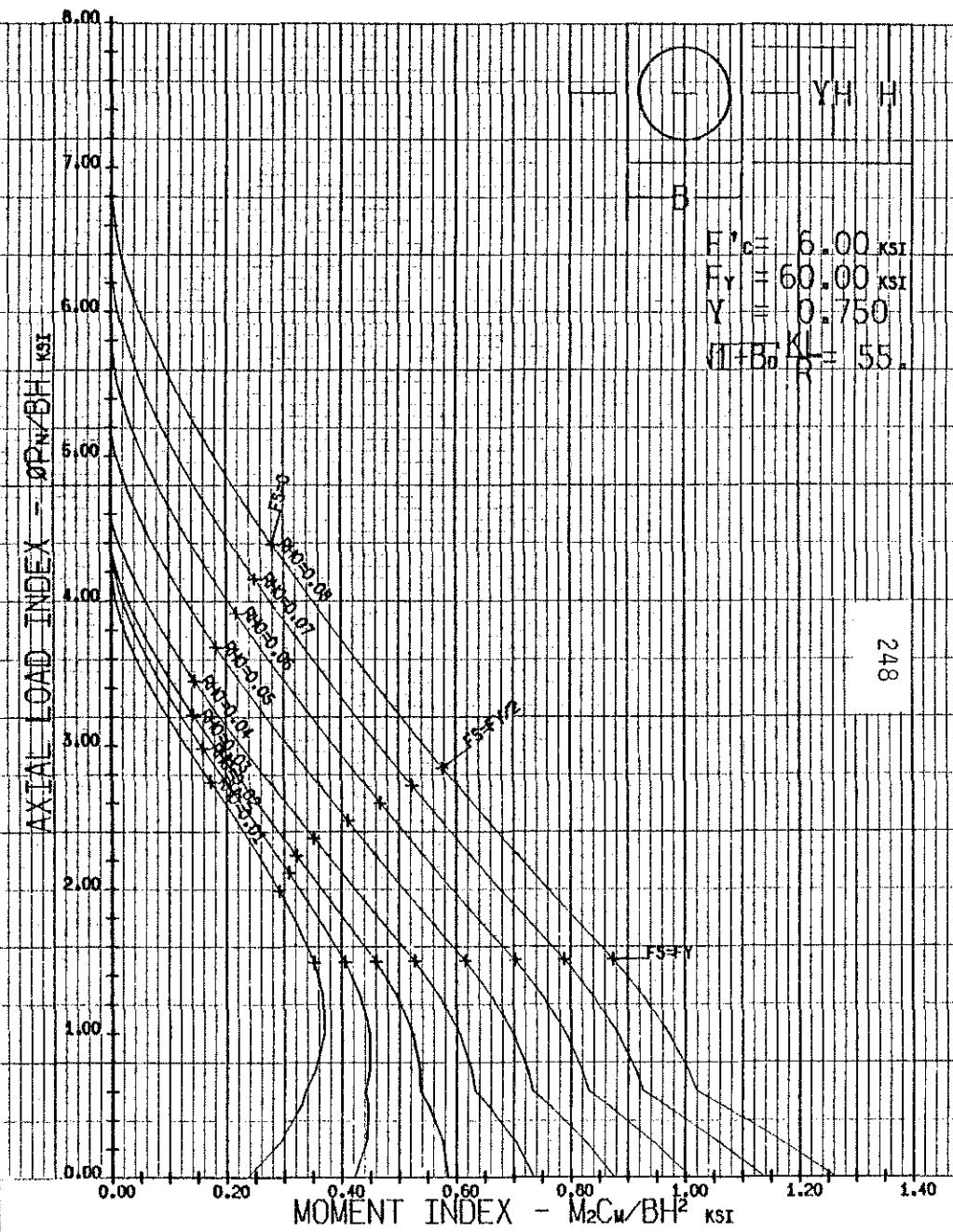


Fig. S6-60.75-55 - Interaction Diagram

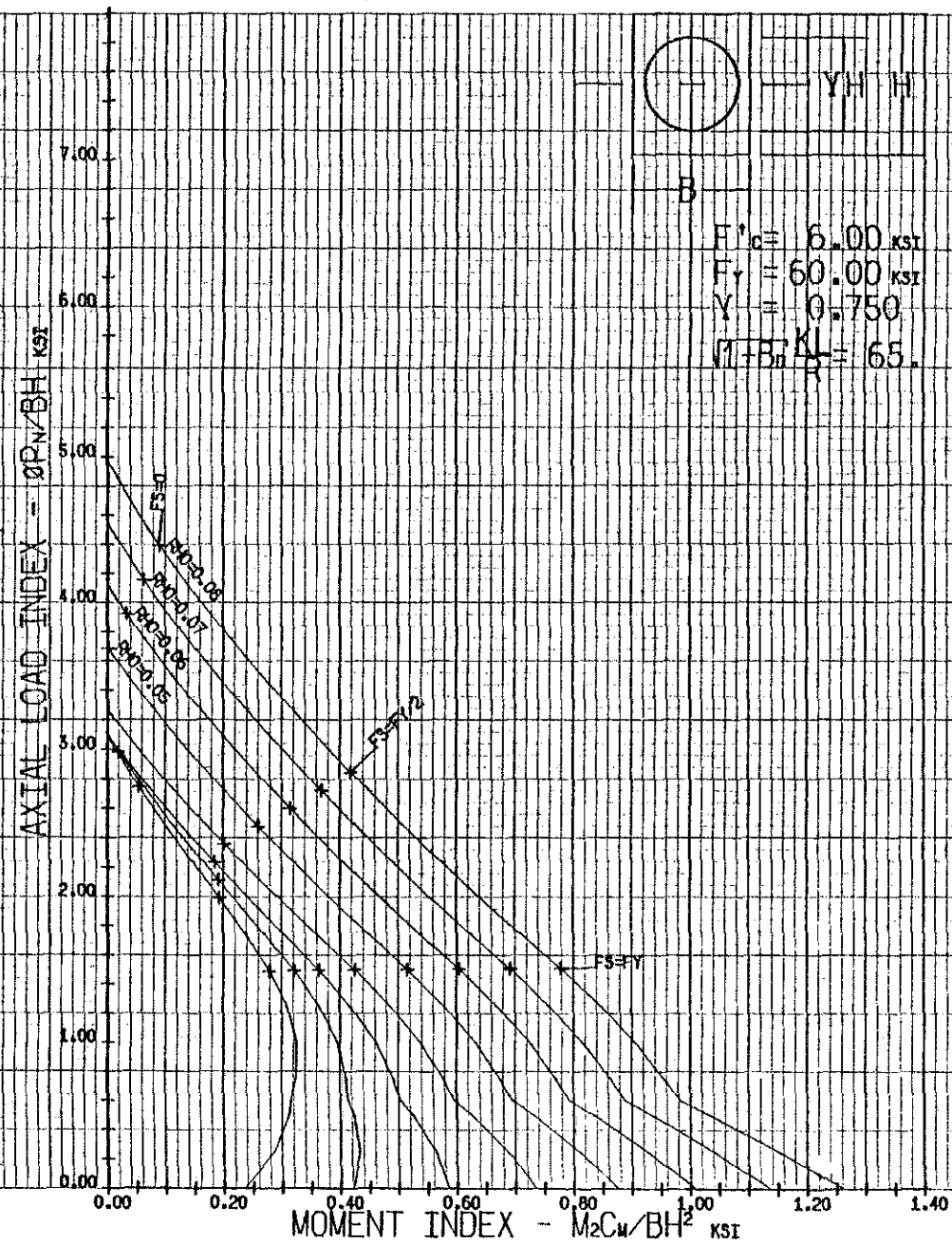


Fig. S6-60.75-65 - Interaction Diagram

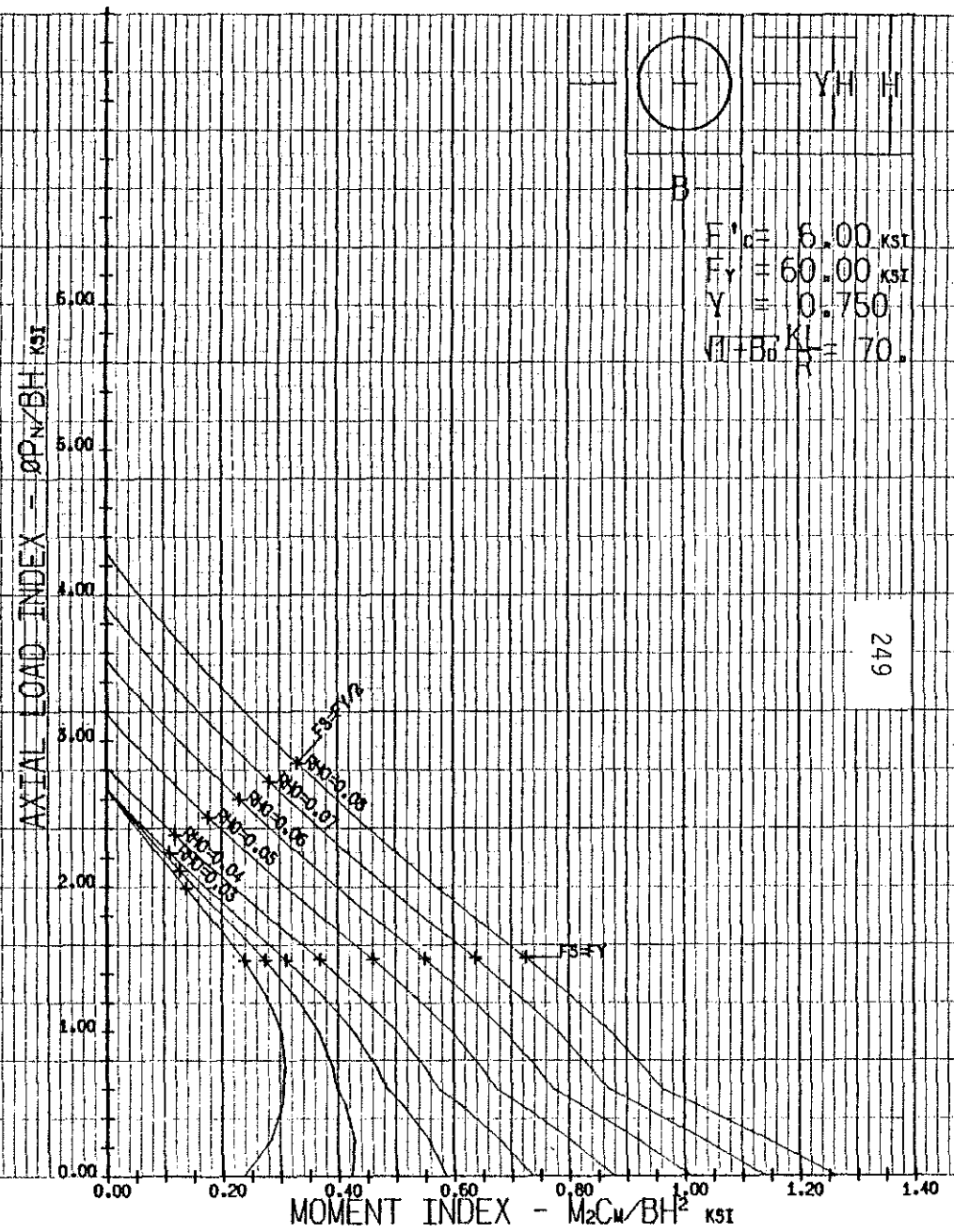


Fig. S6-60.75-70 - Interaction Diagram

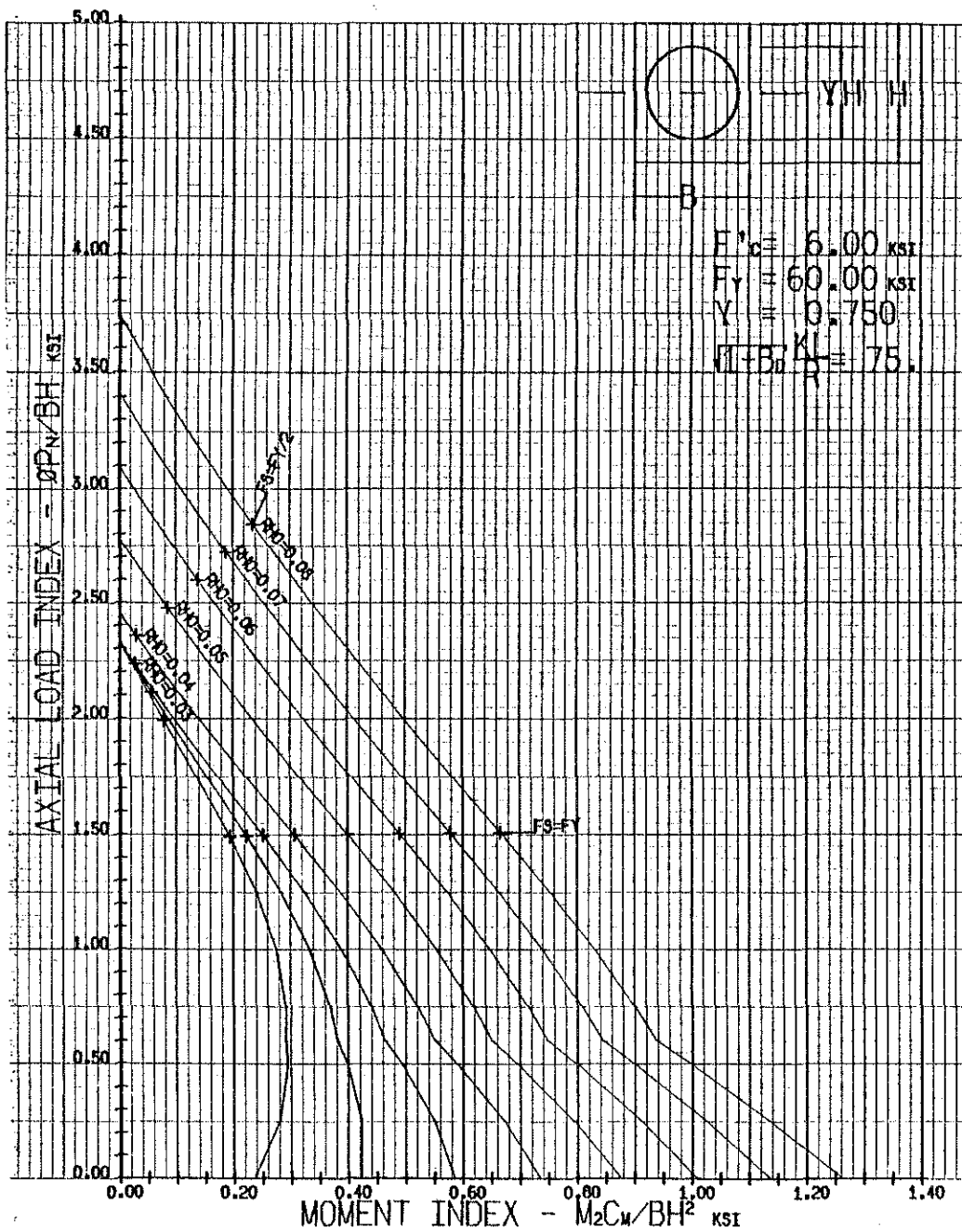


Fig. S6-60.75-75 - Interaction Diagram

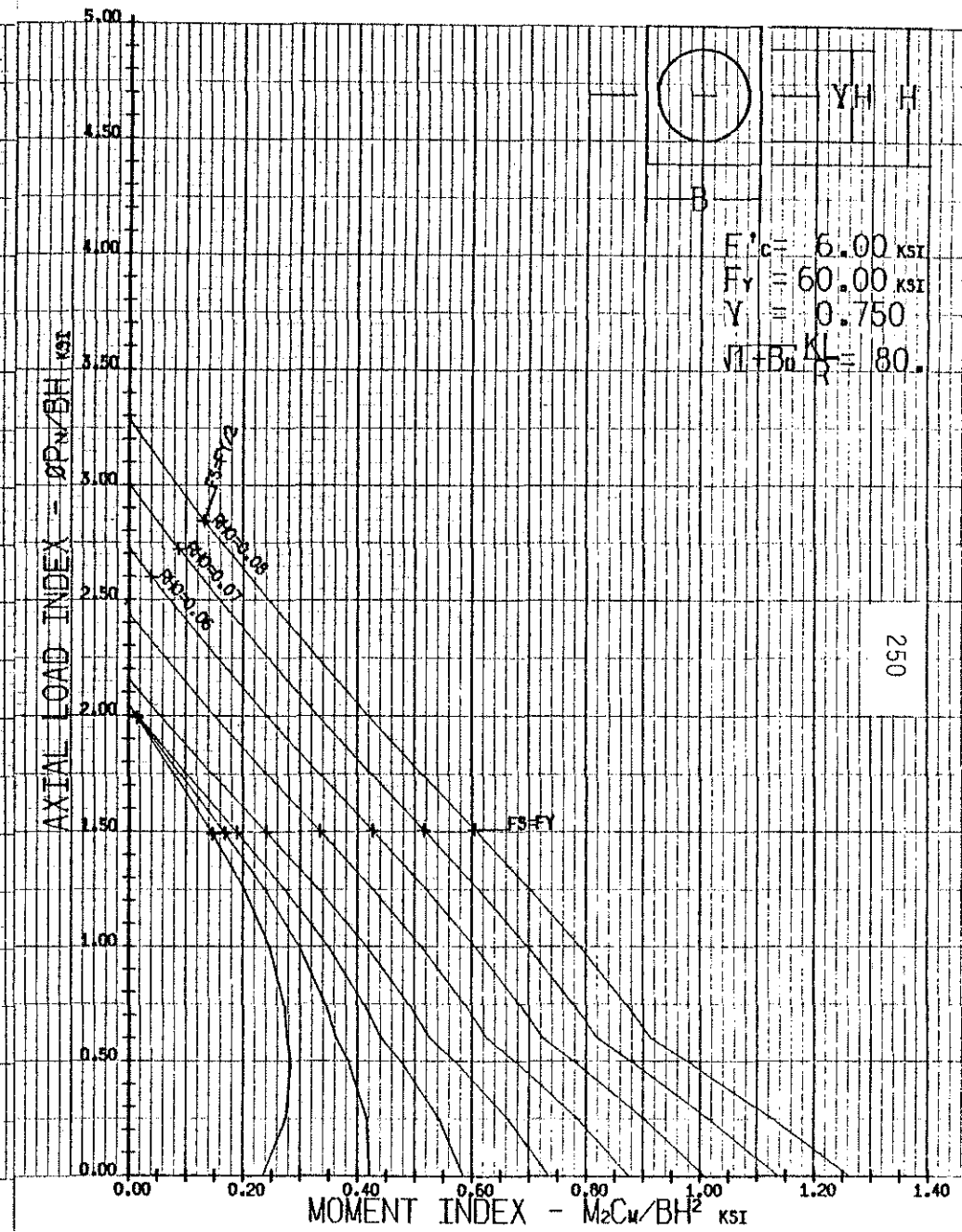


Fig. S6-60.75-80 - Interaction Diagram



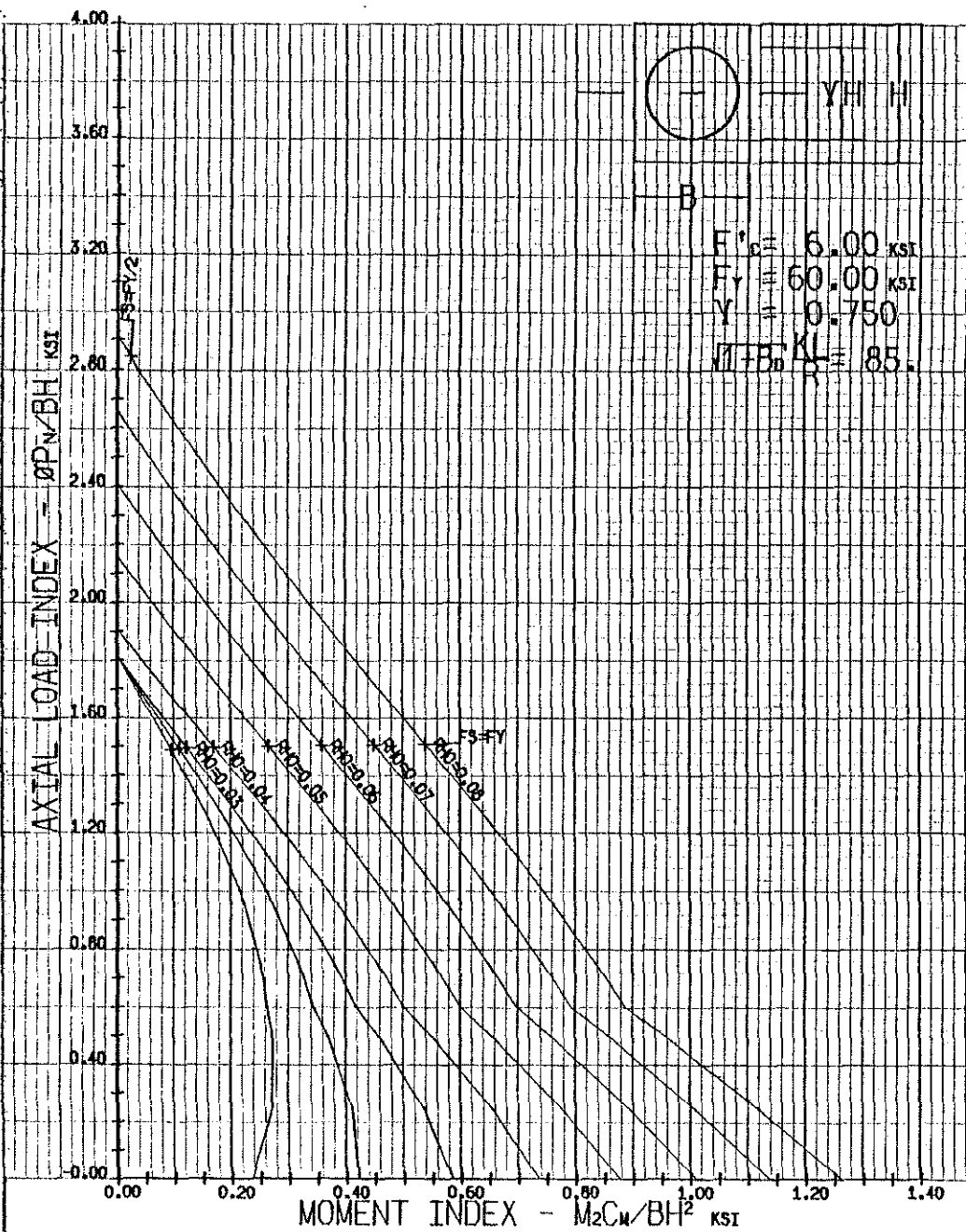


Fig. S6-60.75-85 - Interaction Diagram

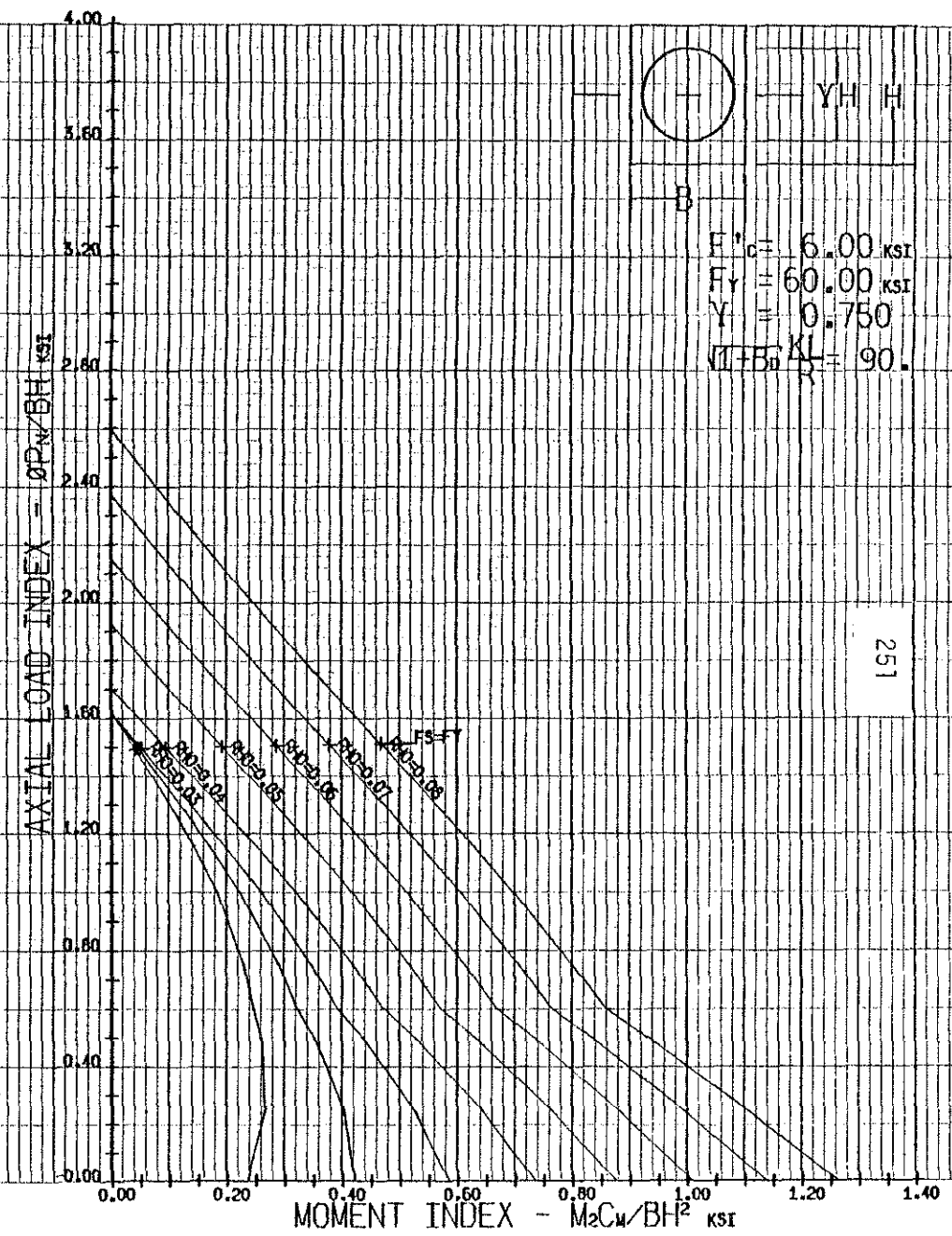


Fig. S6-60.75-90 - Interaction Diagram



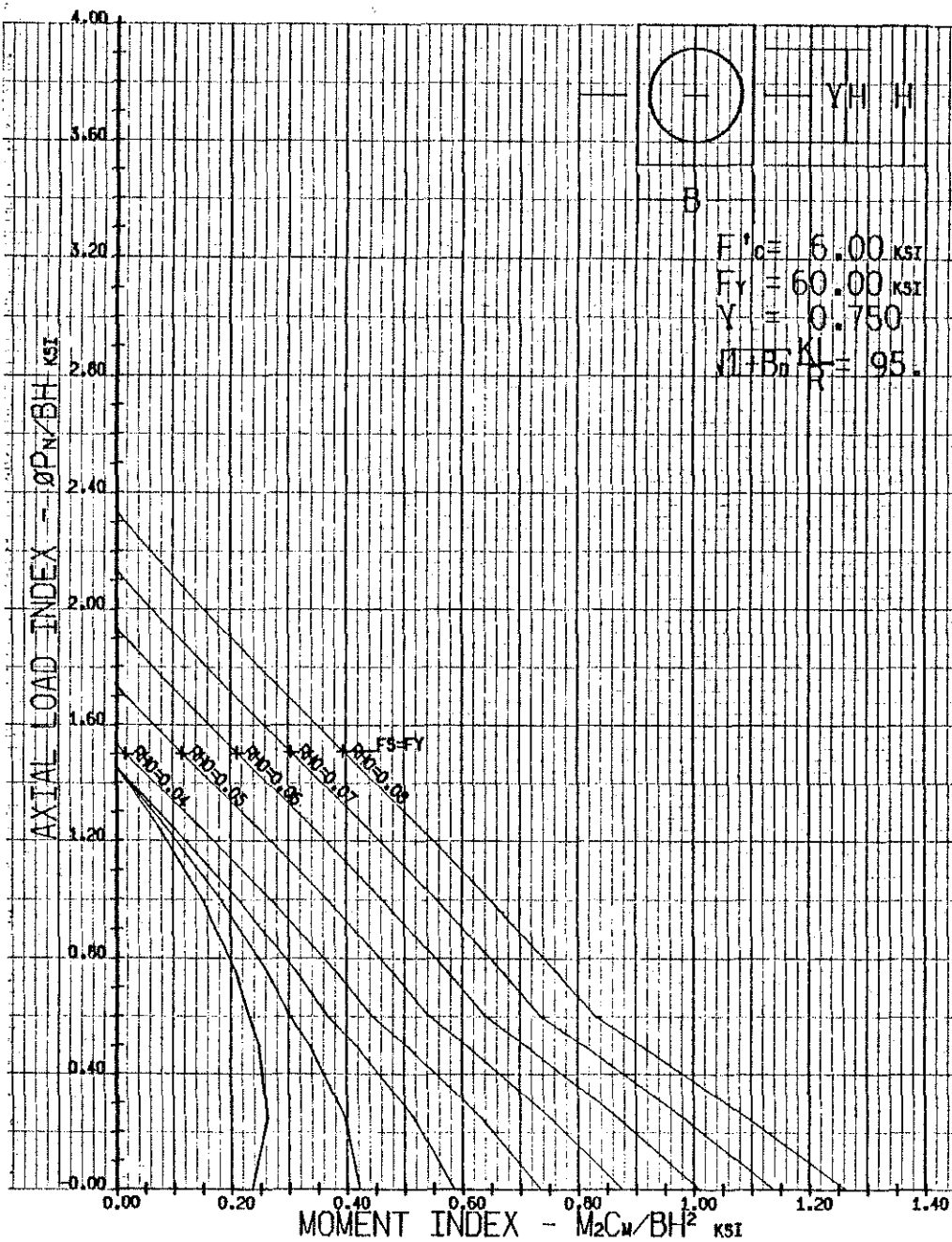


Fig. S6-60.75-95 - Interaction Diagram

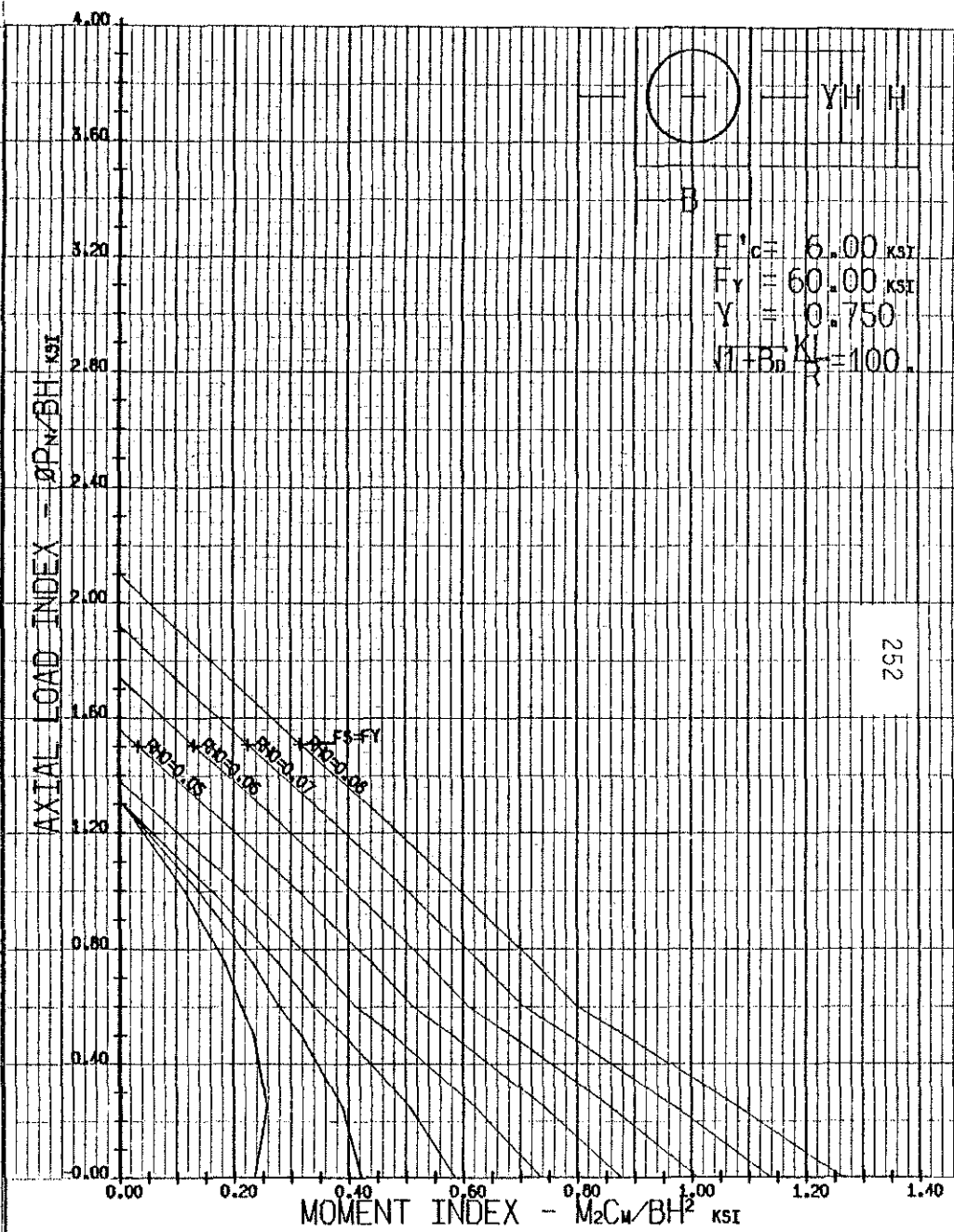


Fig. S6-60.75-100 - Interaction Diagram

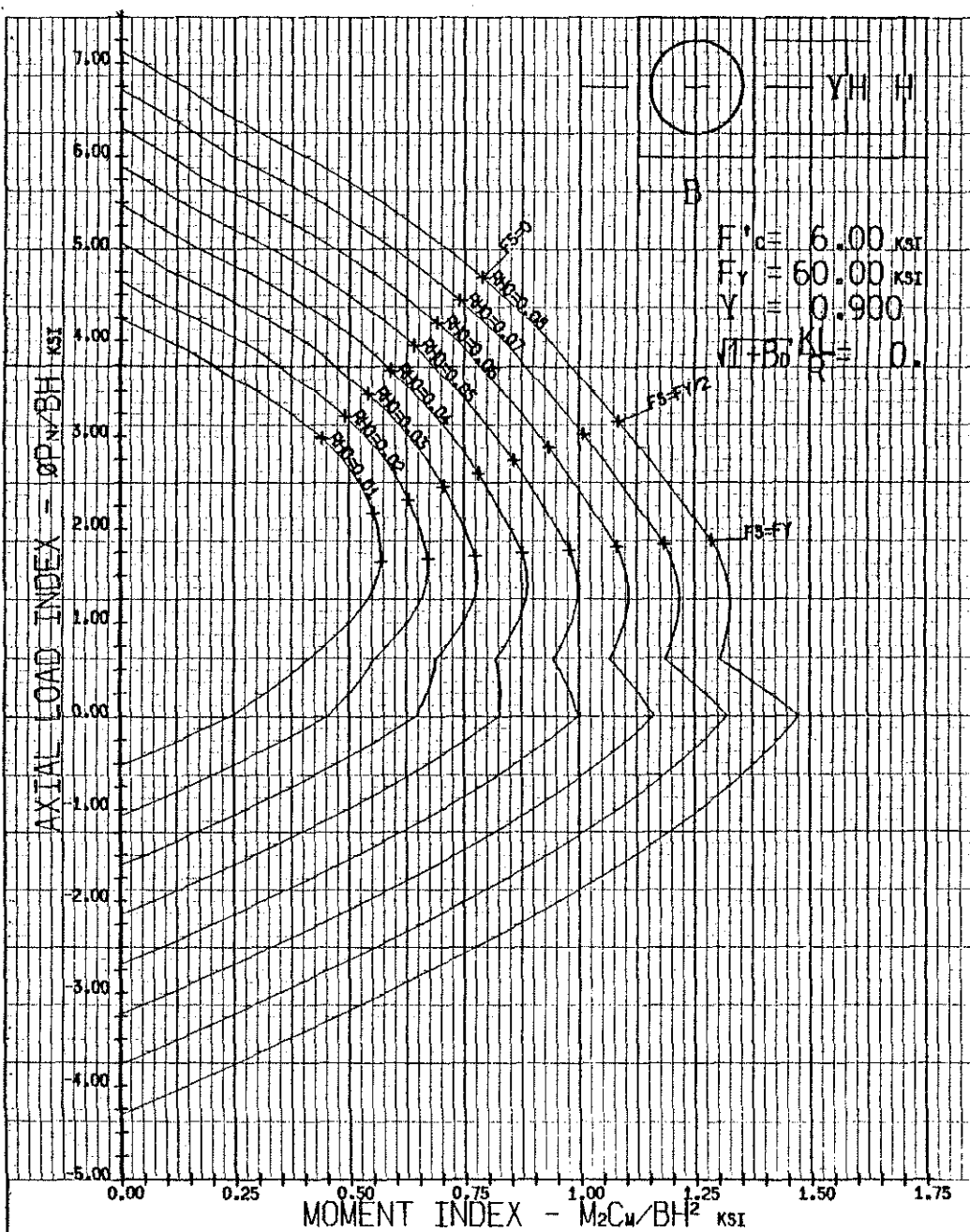


Fig. S6-60.90-0 - Interaction Diagram

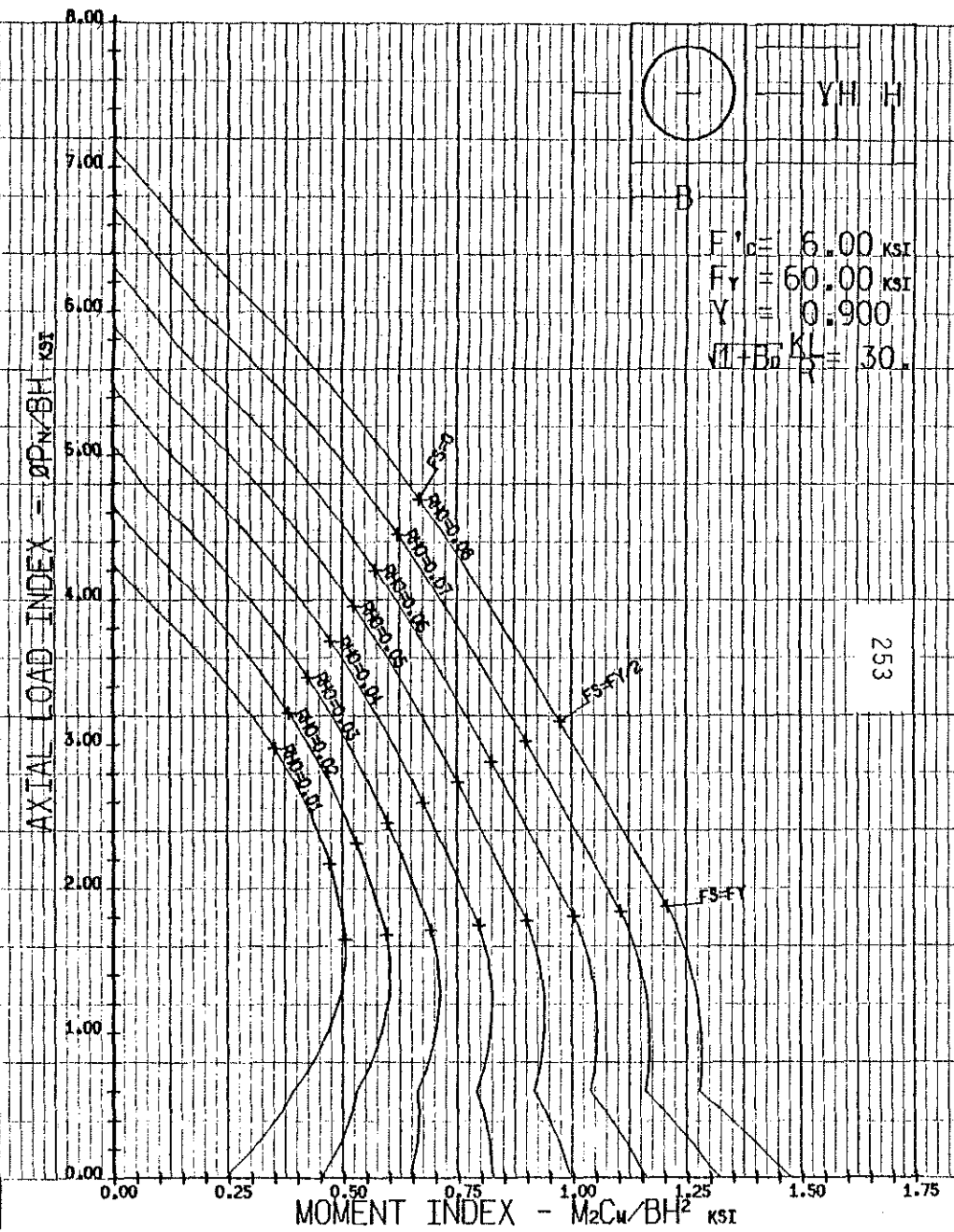


Fig. S6-60.90-30 - Interaction Diagram

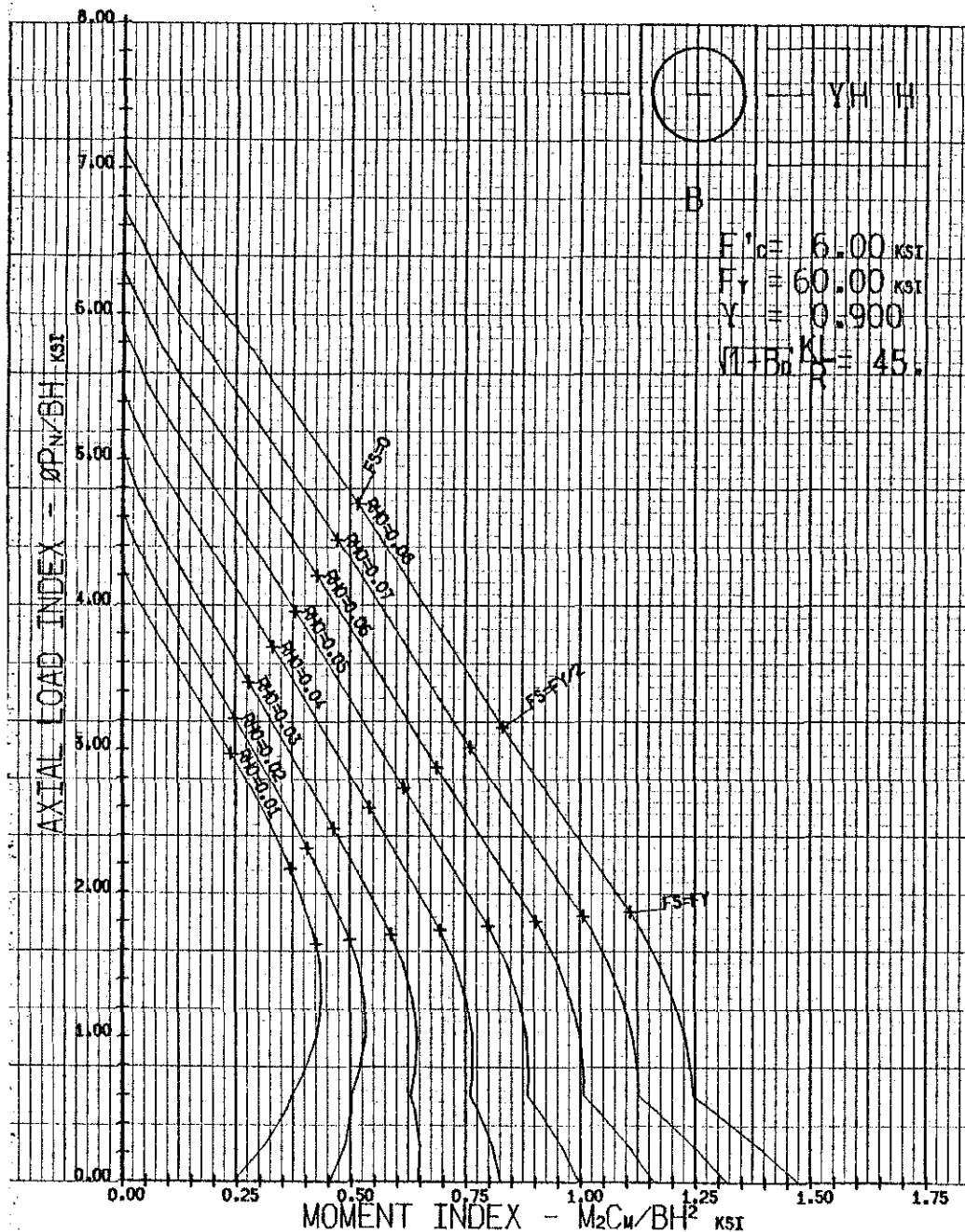


Fig. S6-60.90-45 - Interaction Diagram

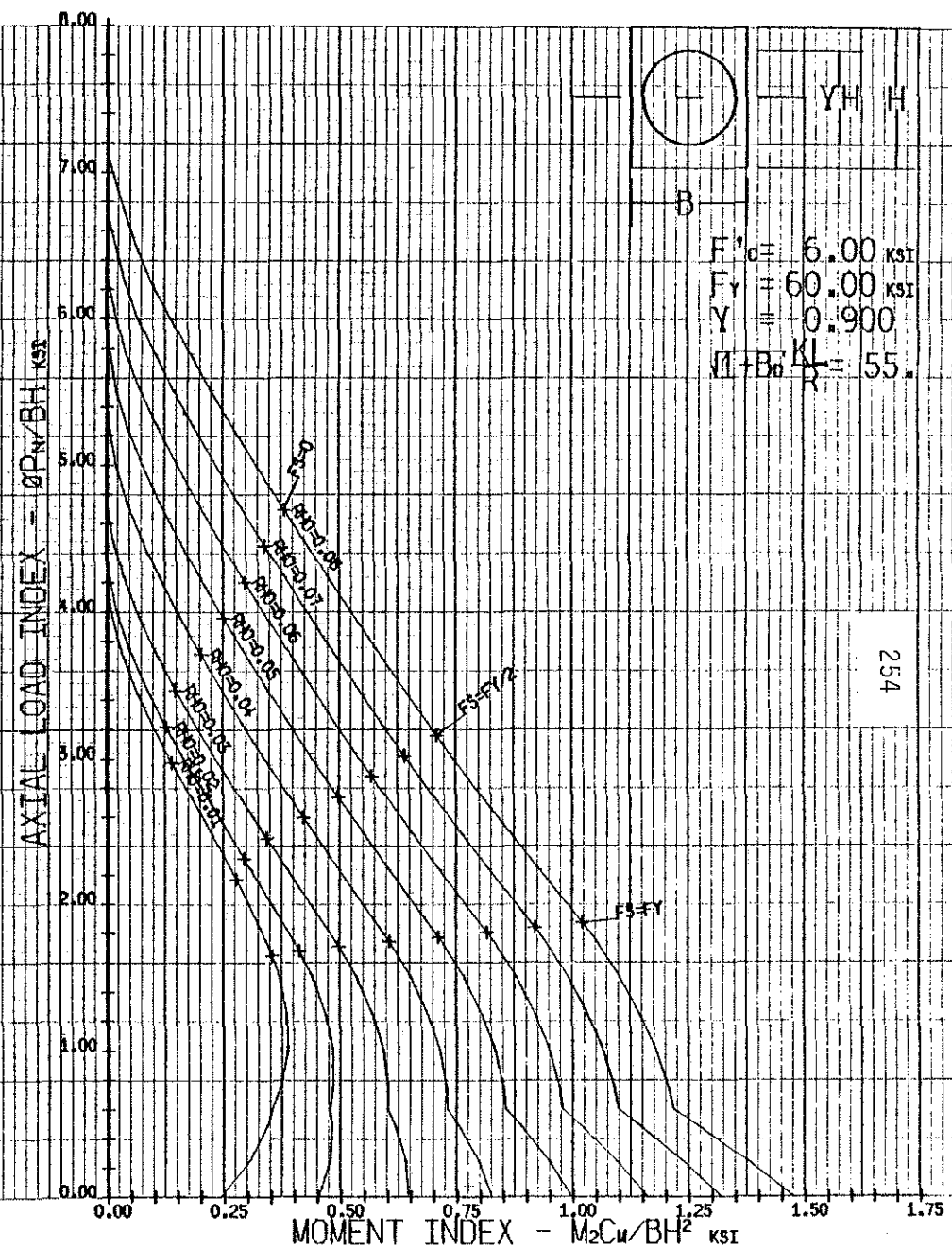


Fig. S6-60.90-55 - Interaction Diagram

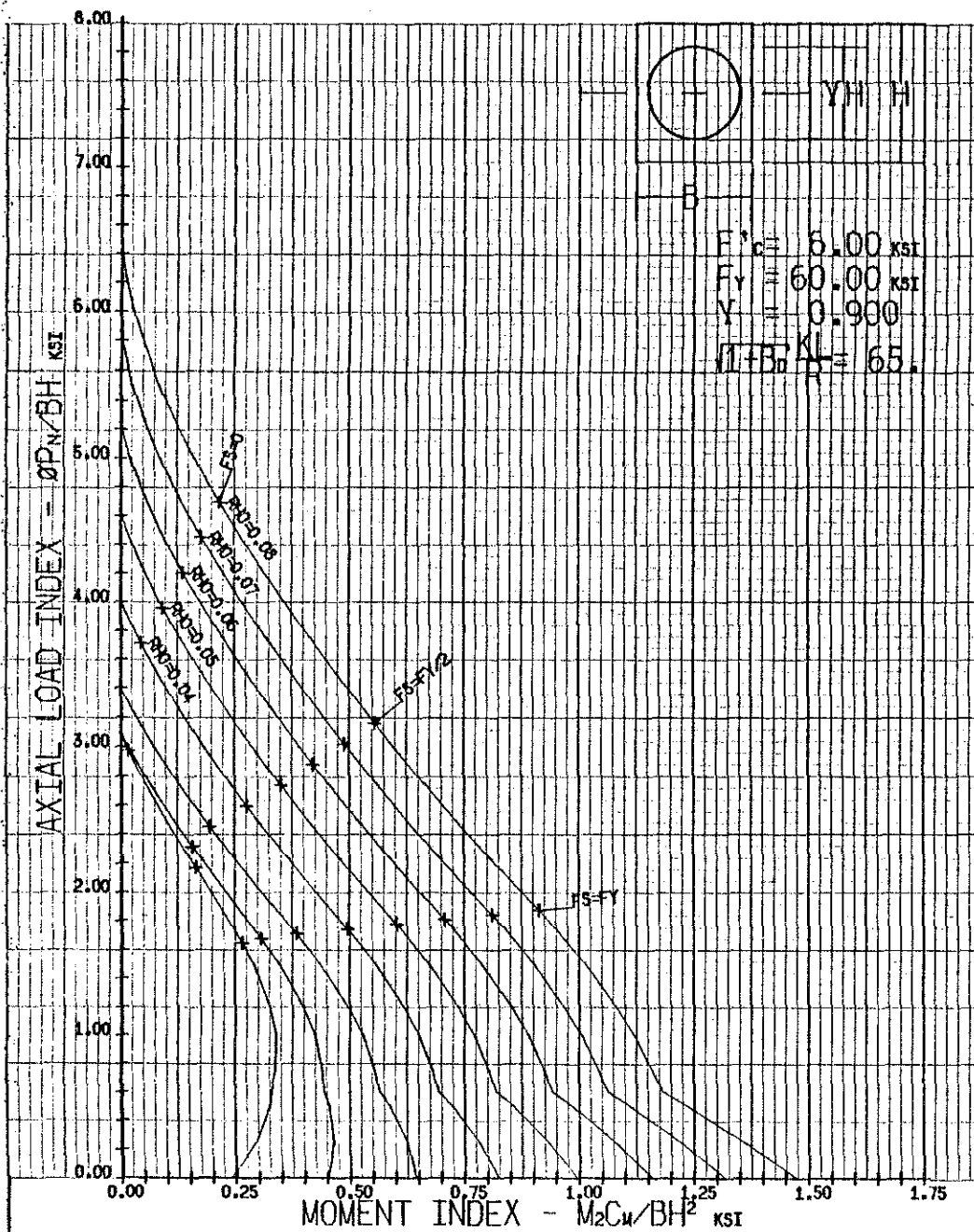


Fig. S6-60.90-65 - Interaction Diagram

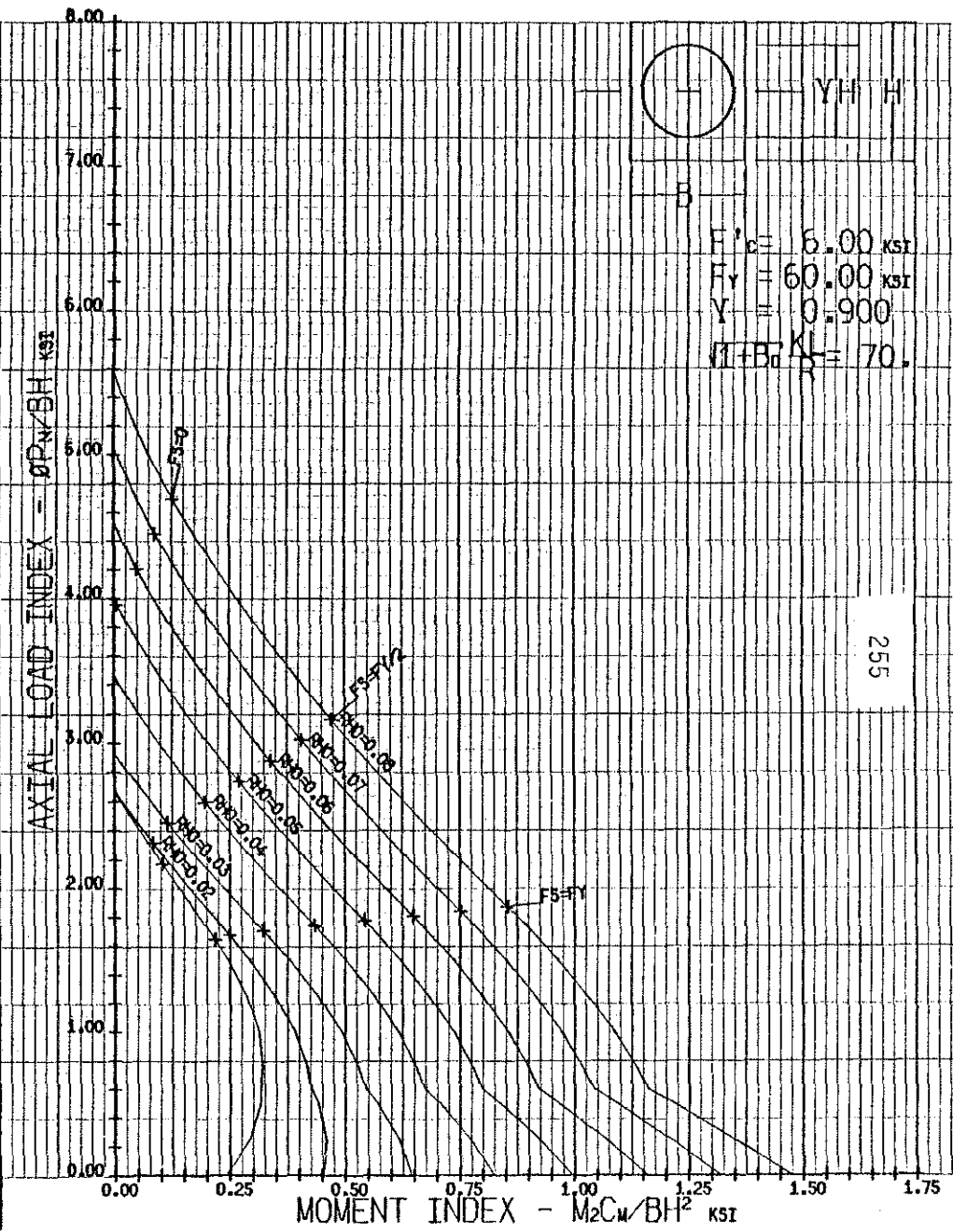


Fig. S6-60.90-70 - Interaction Diagram

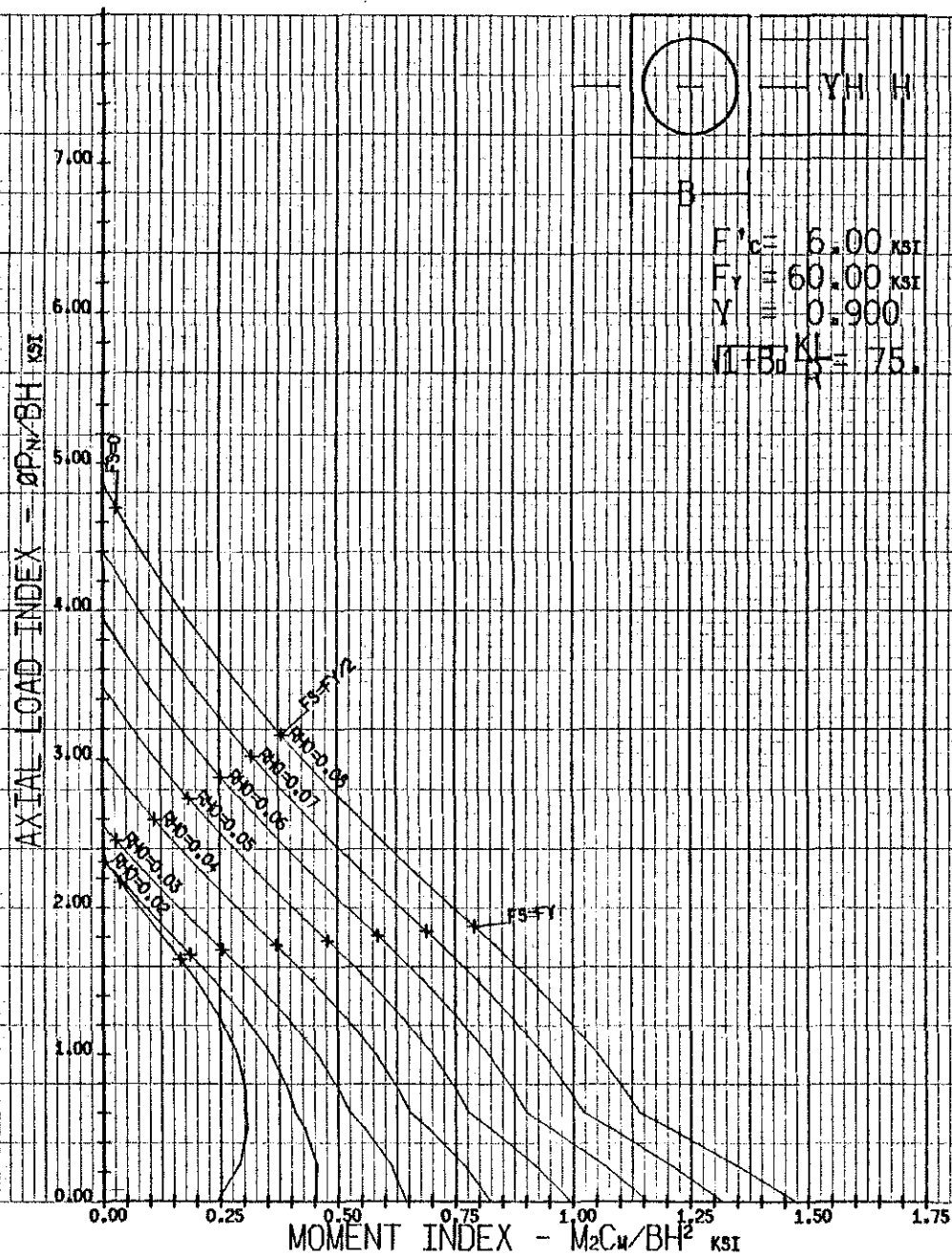


Fig. S6-60.90-75 - Interaction Diagram

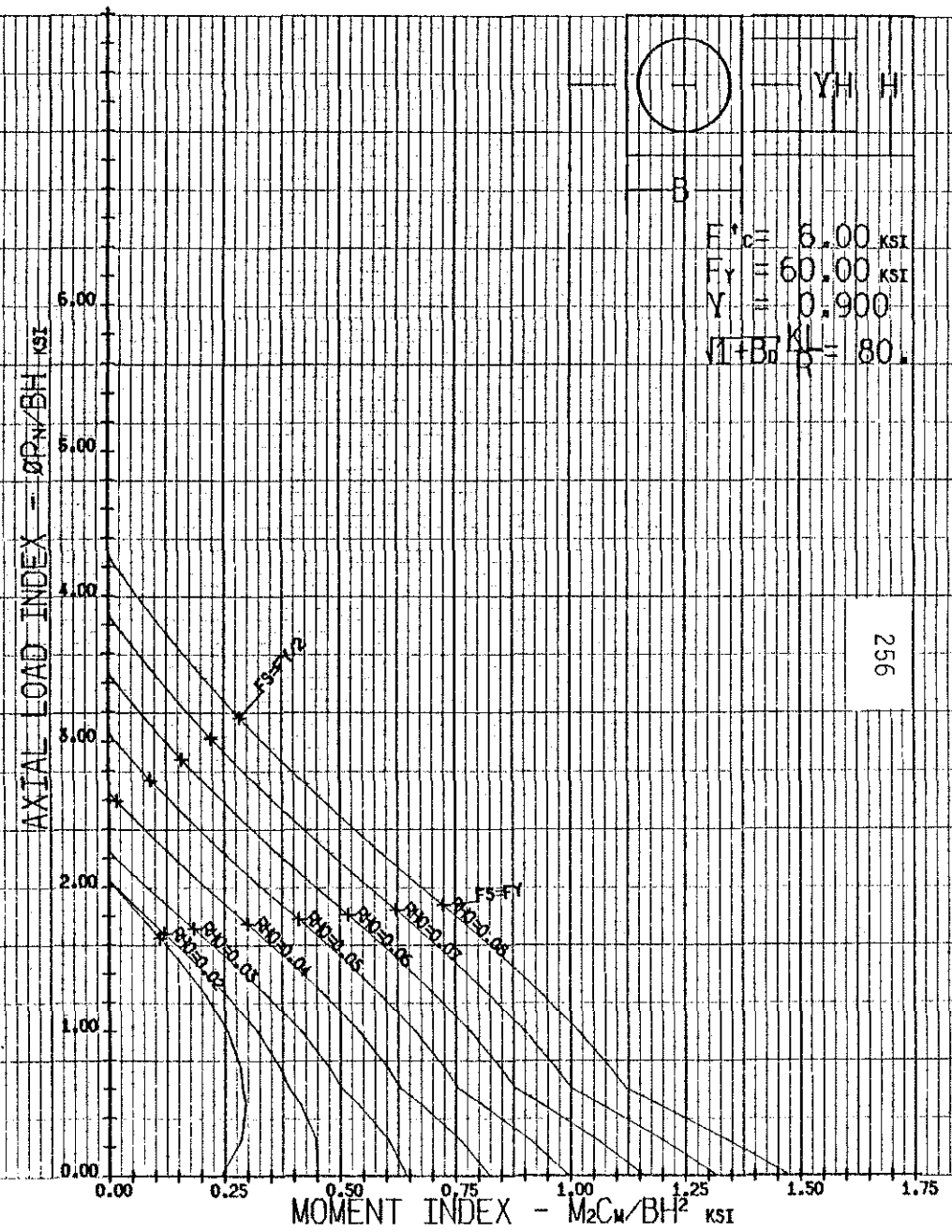


Fig. S6-60.90-80 - Interaction Diagram



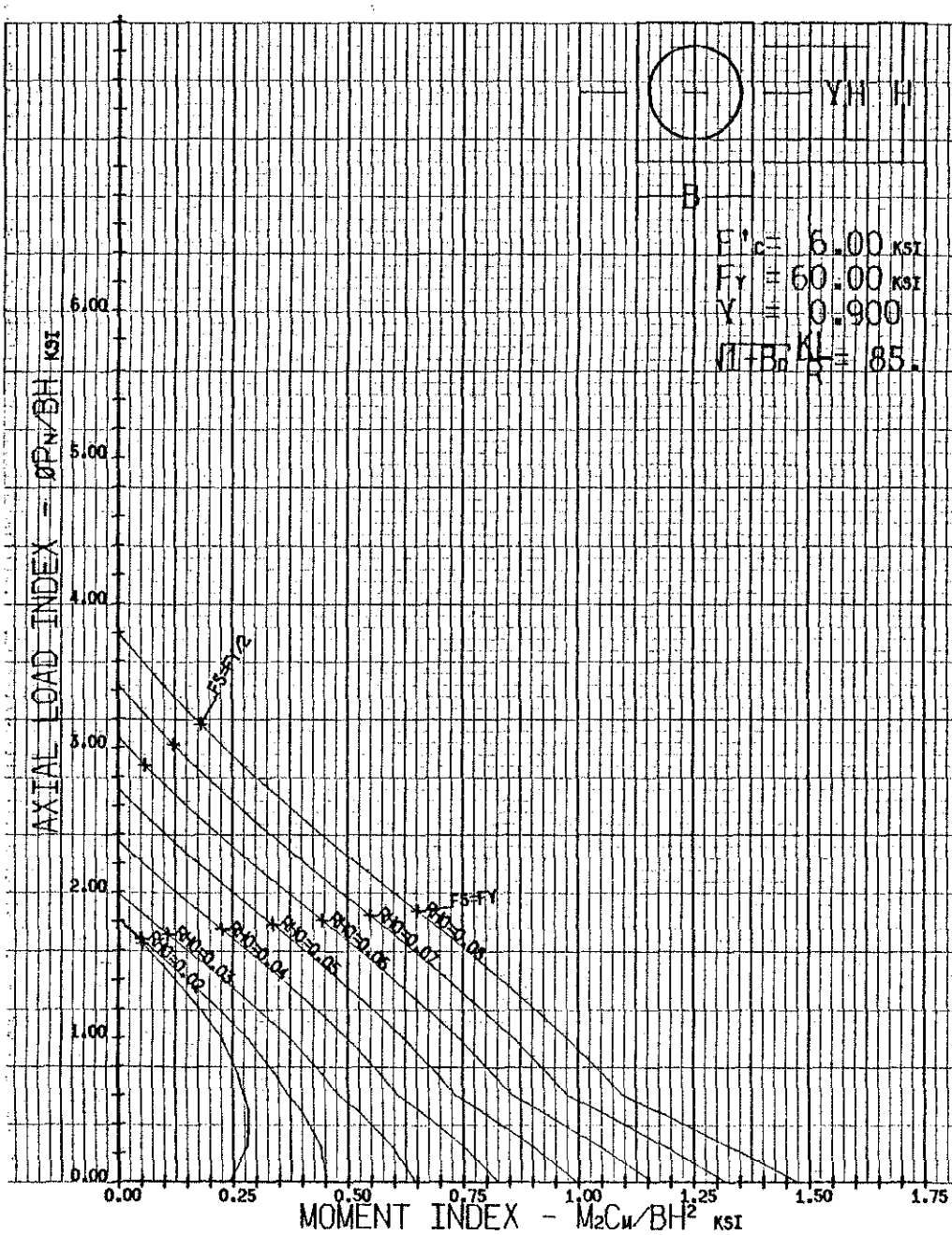


Fig. S6-60.90-85 - Interaction Diagram

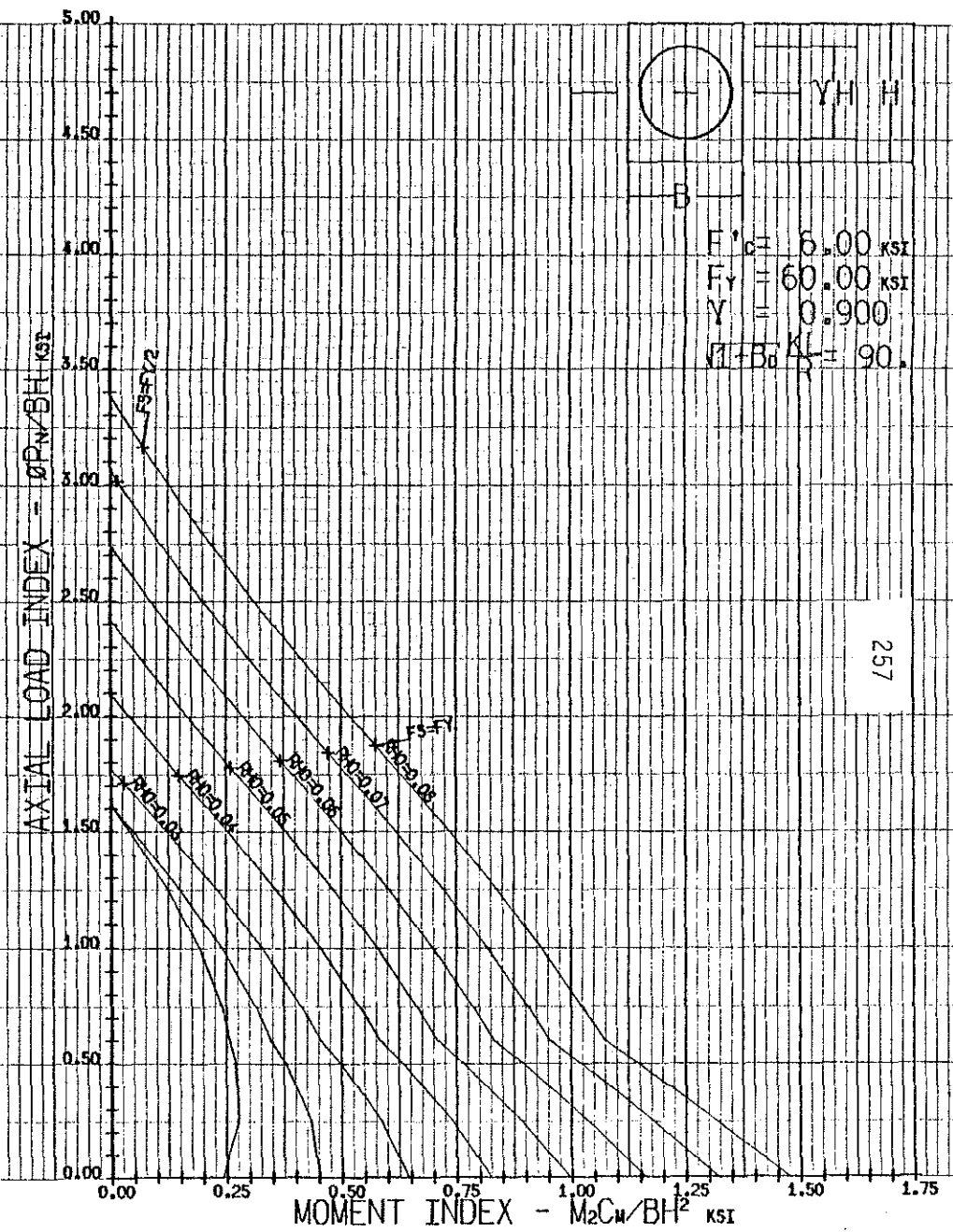


Fig. S6-60.90-90 - Interaction Diagram



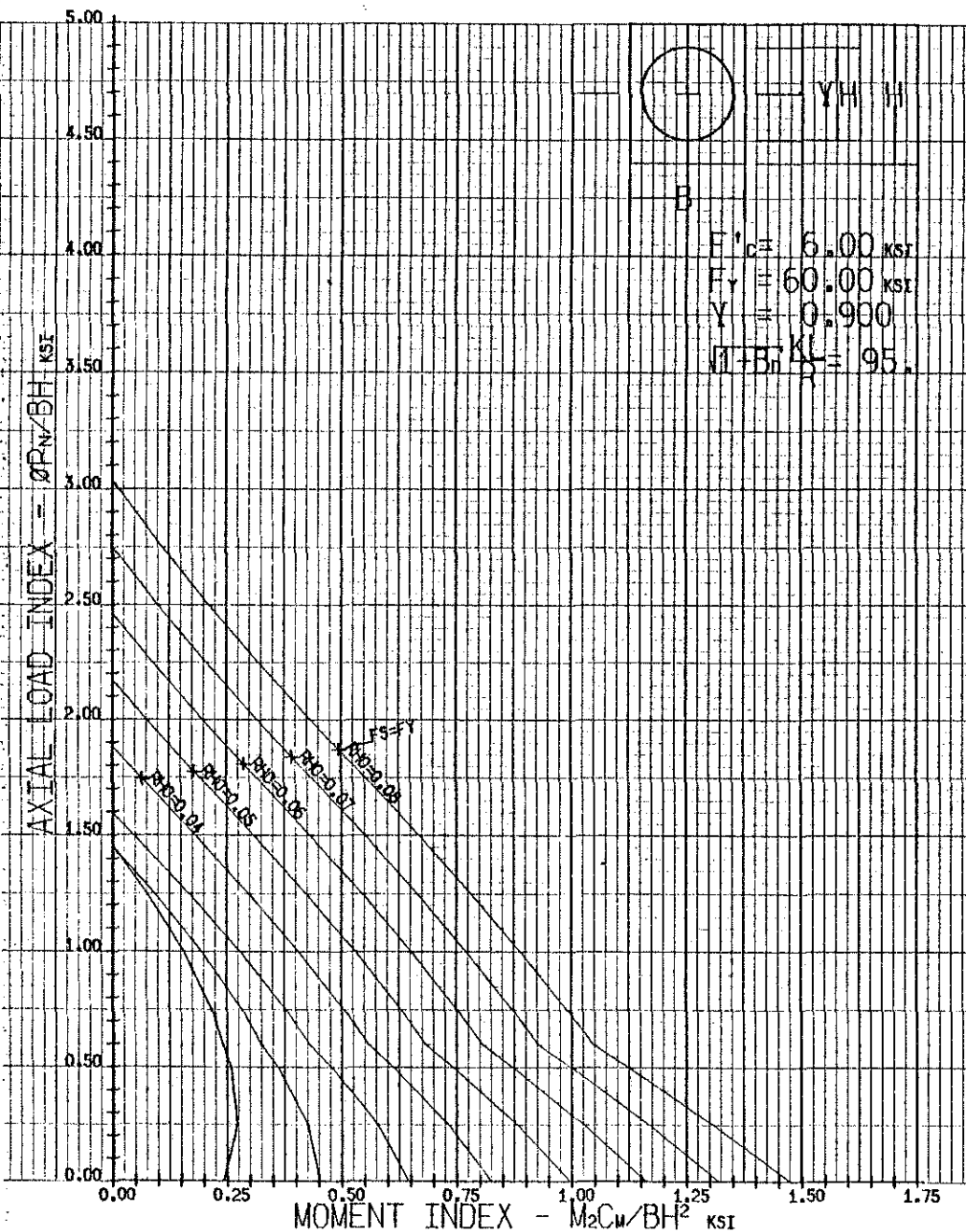


Fig. S6-60.90-95 - Interaction Diagram

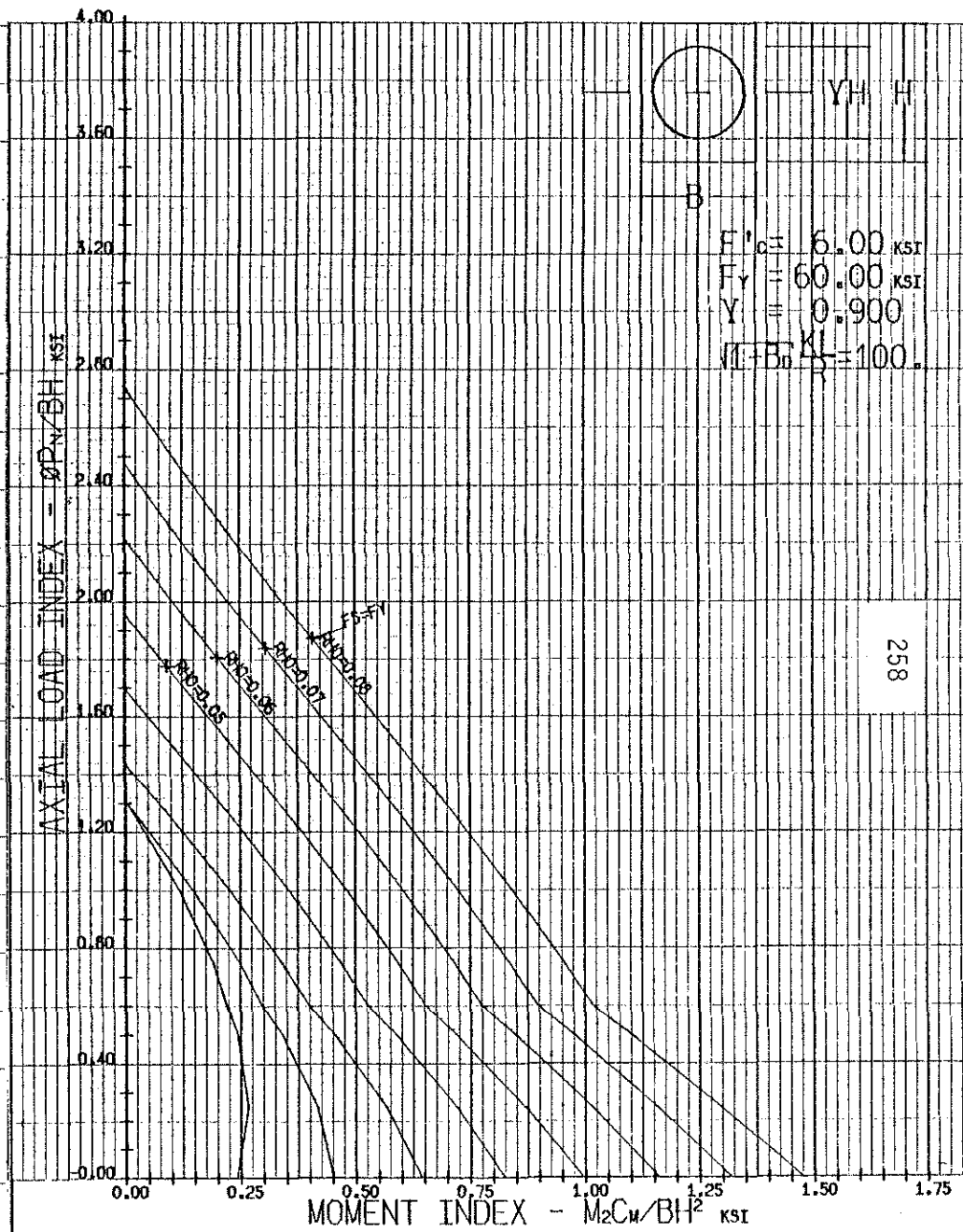


Fig. S6-60.90-100 - Interaction Diagram

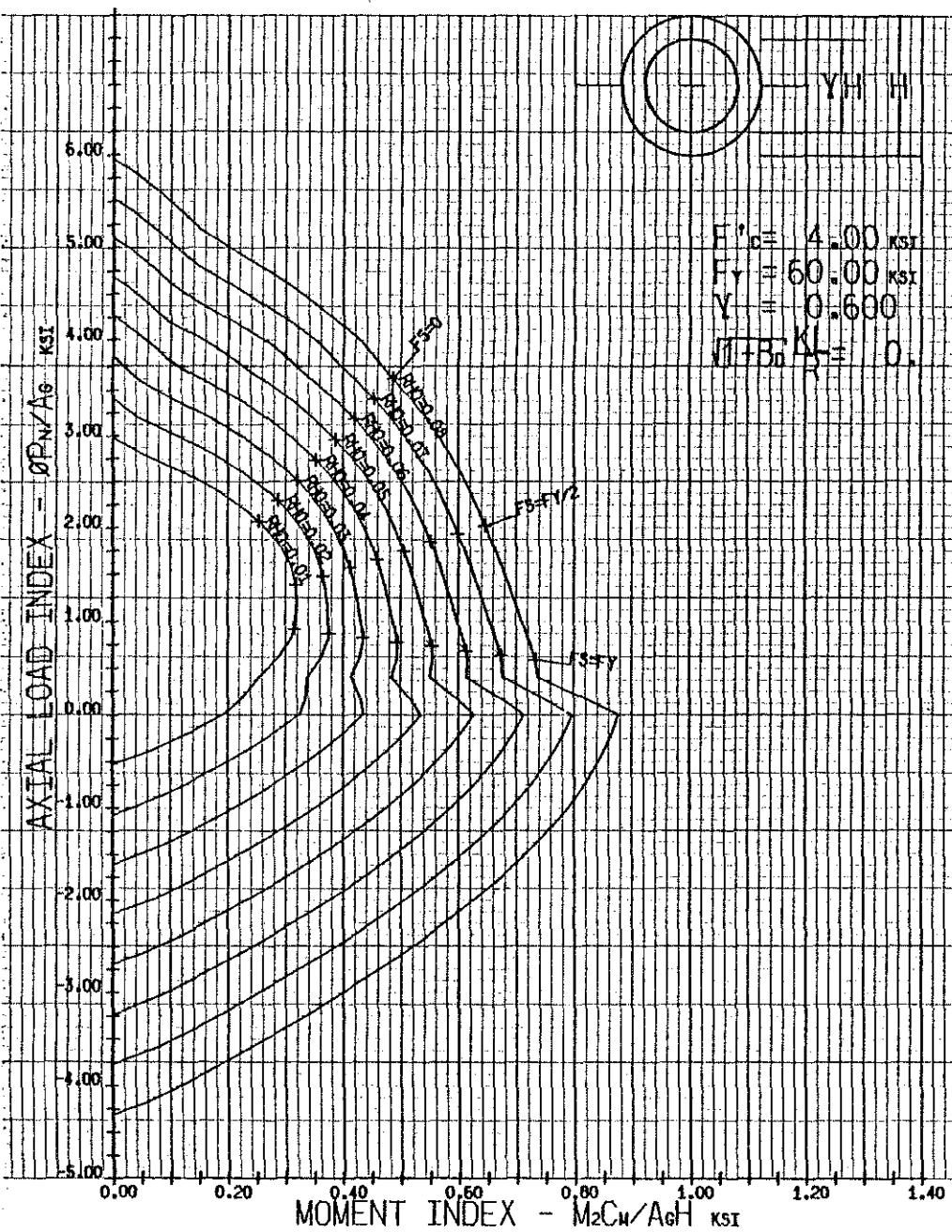


Fig. C4-60.60-0 - Interaction Diagram

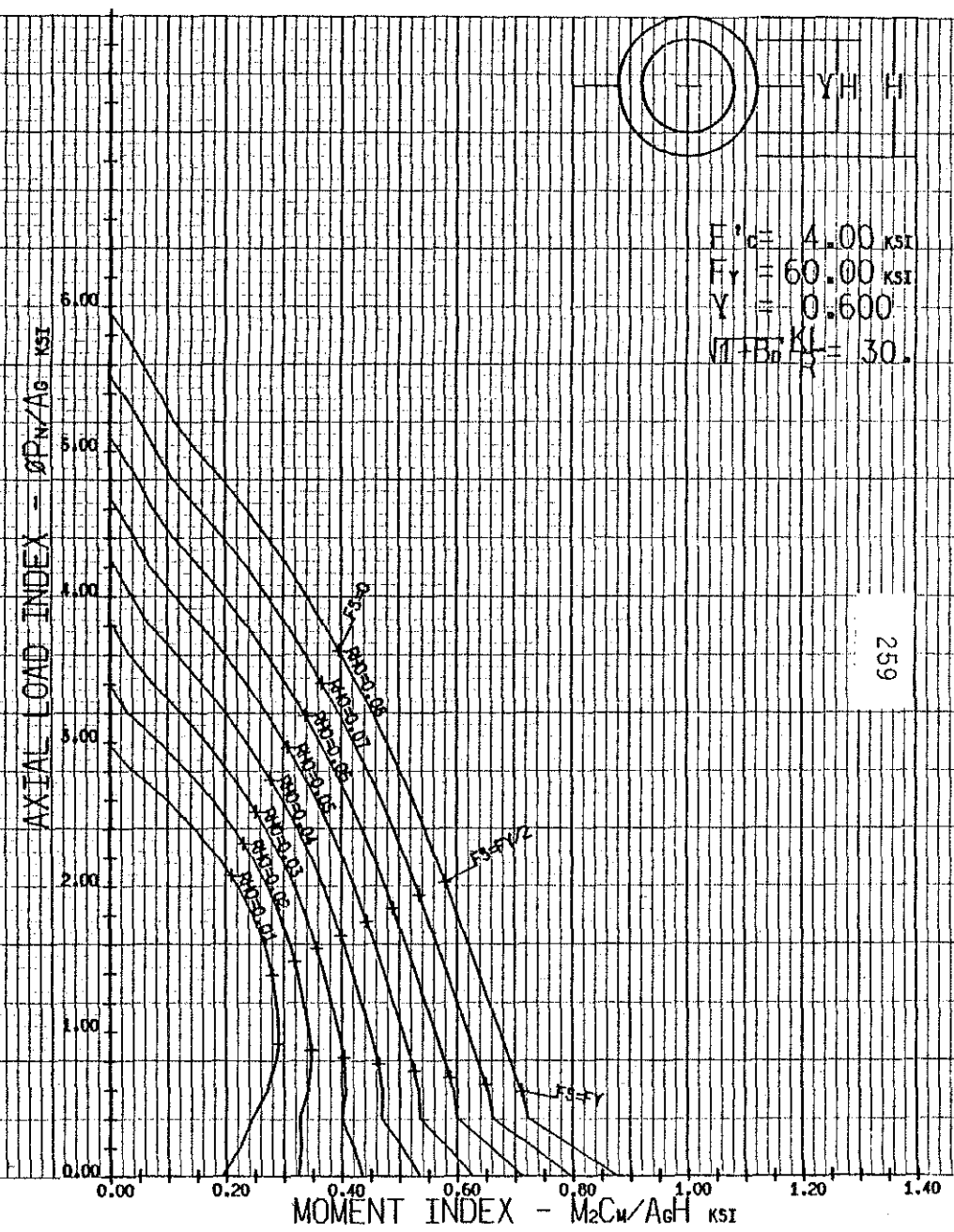


Fig. C4-60.60-30 - Interaction Diagram

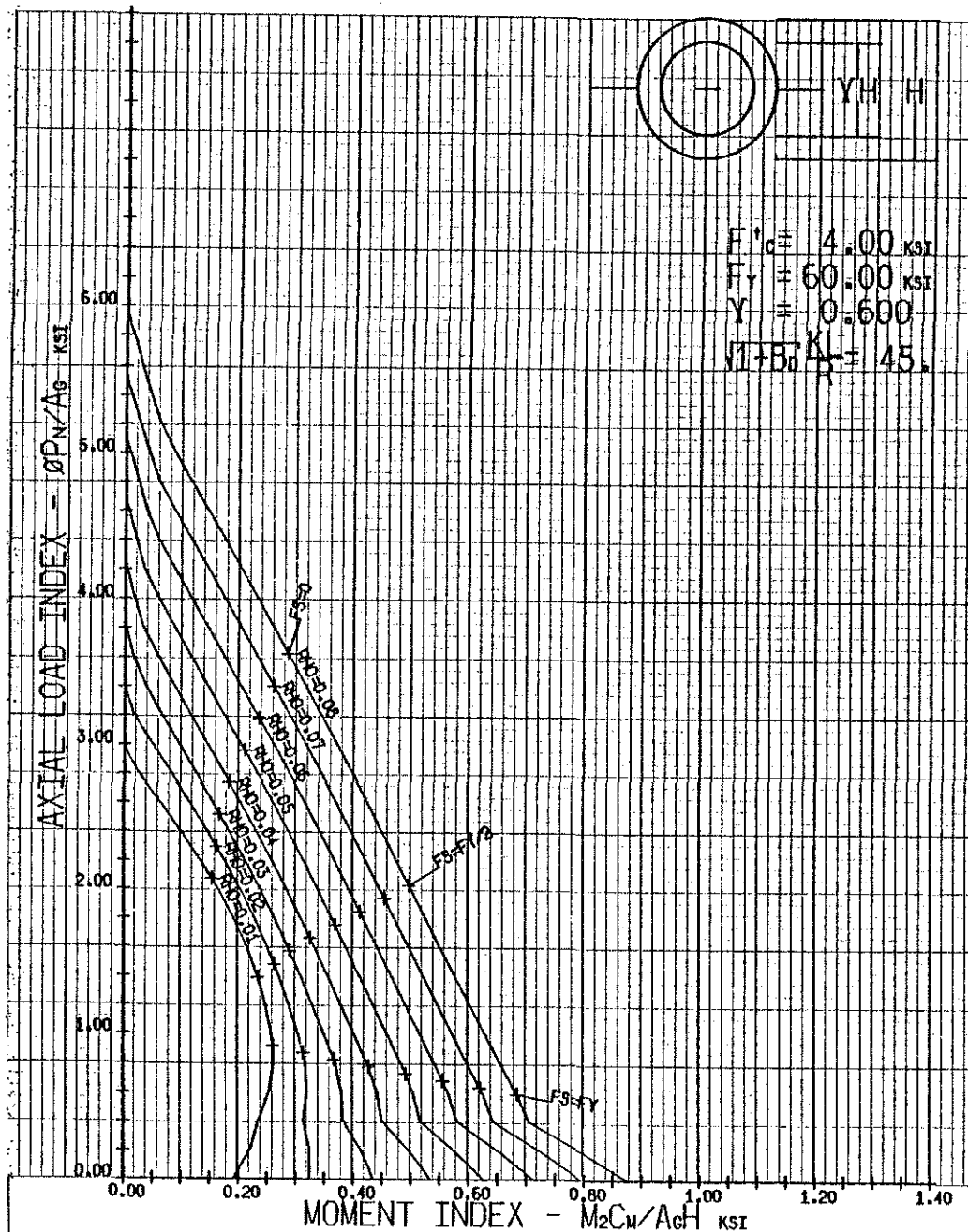


Fig. C4-60.60-45 - Interaction Diagram

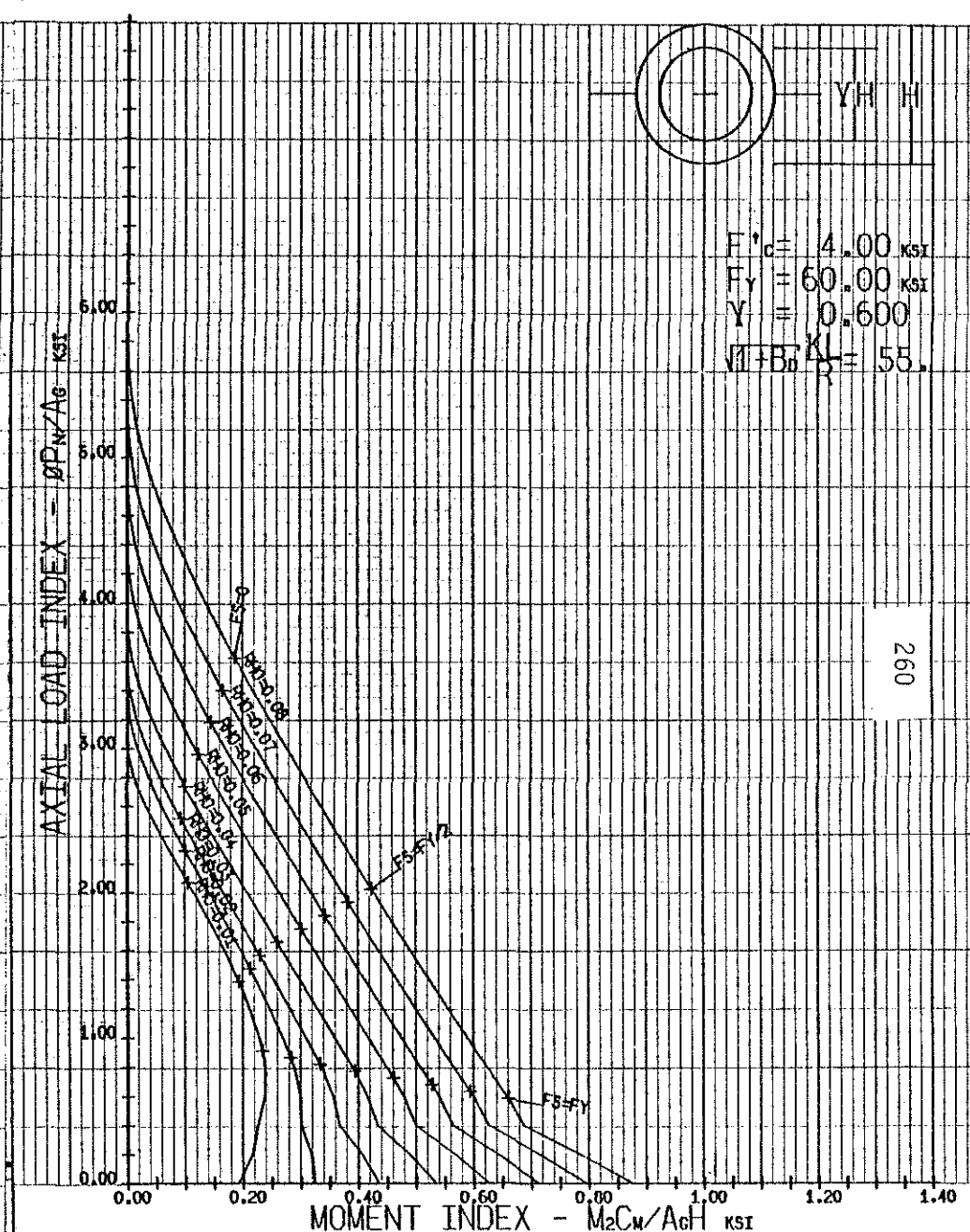


Fig. C4-60.60-55 - Interaction Diagram

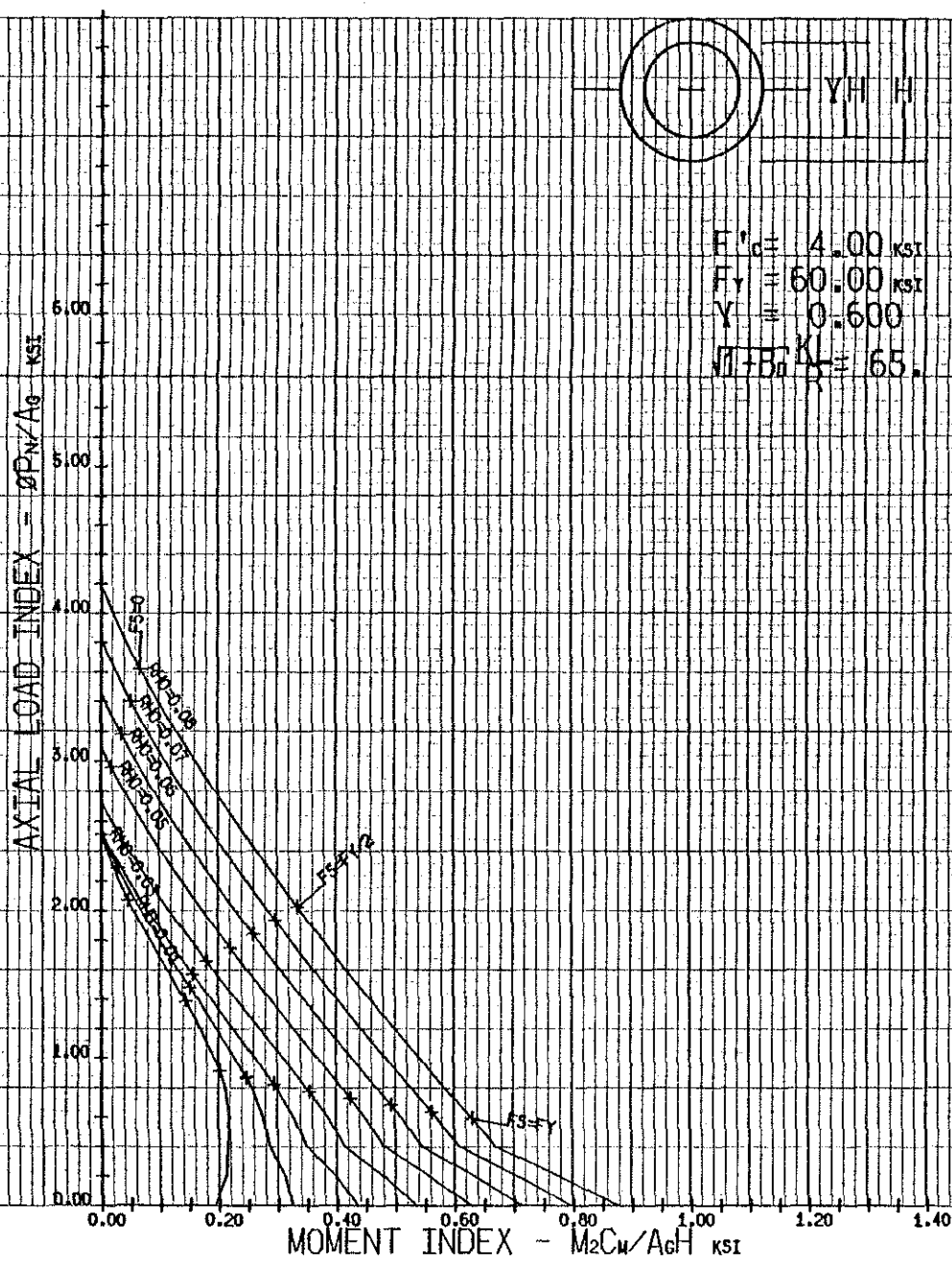


Fig. C4-60.60-65 - Interaction Diagram

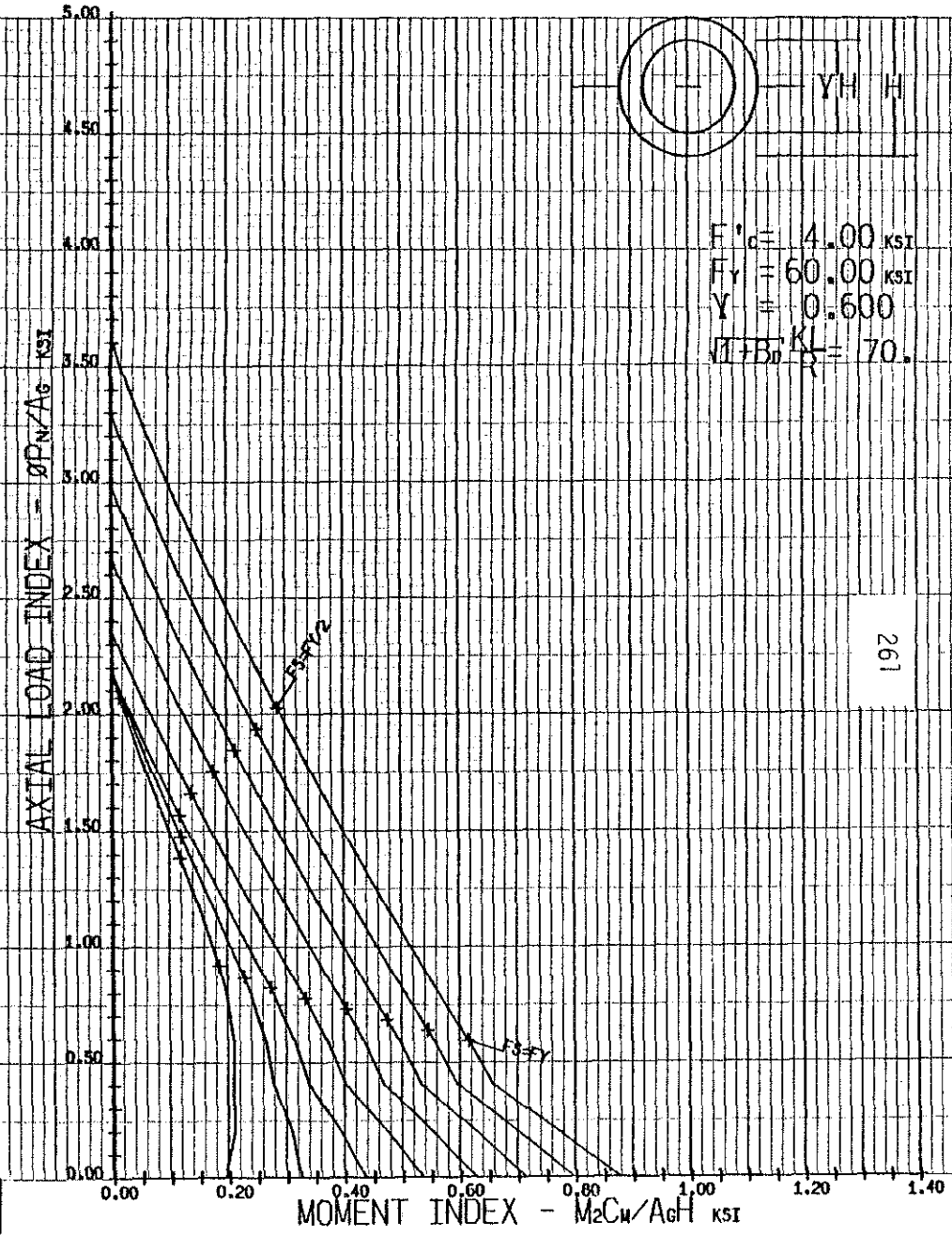


Fig. C4-60.60-70 - Interaction Diagram

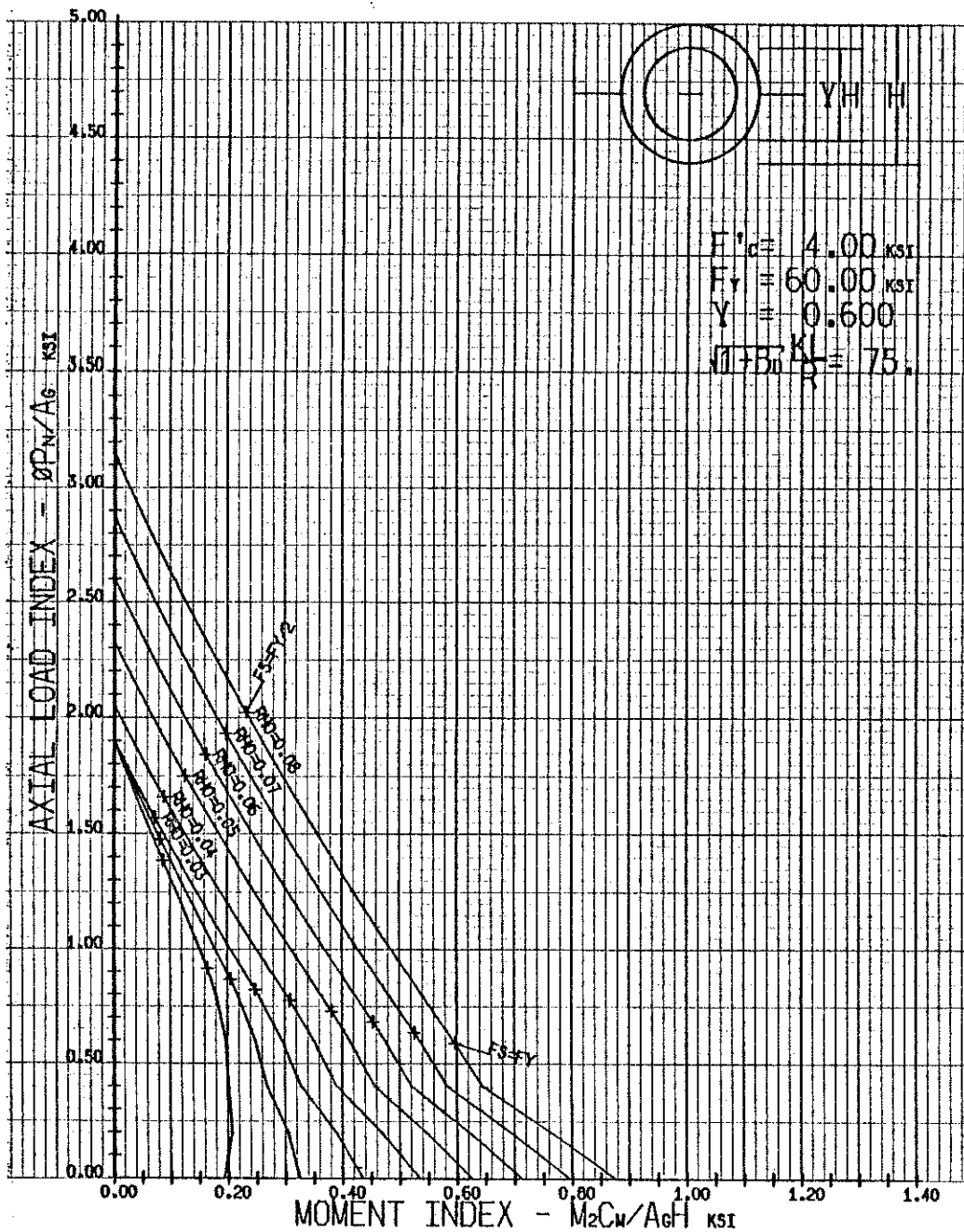


Fig. C4-60.60-75 - Interaction Diagram

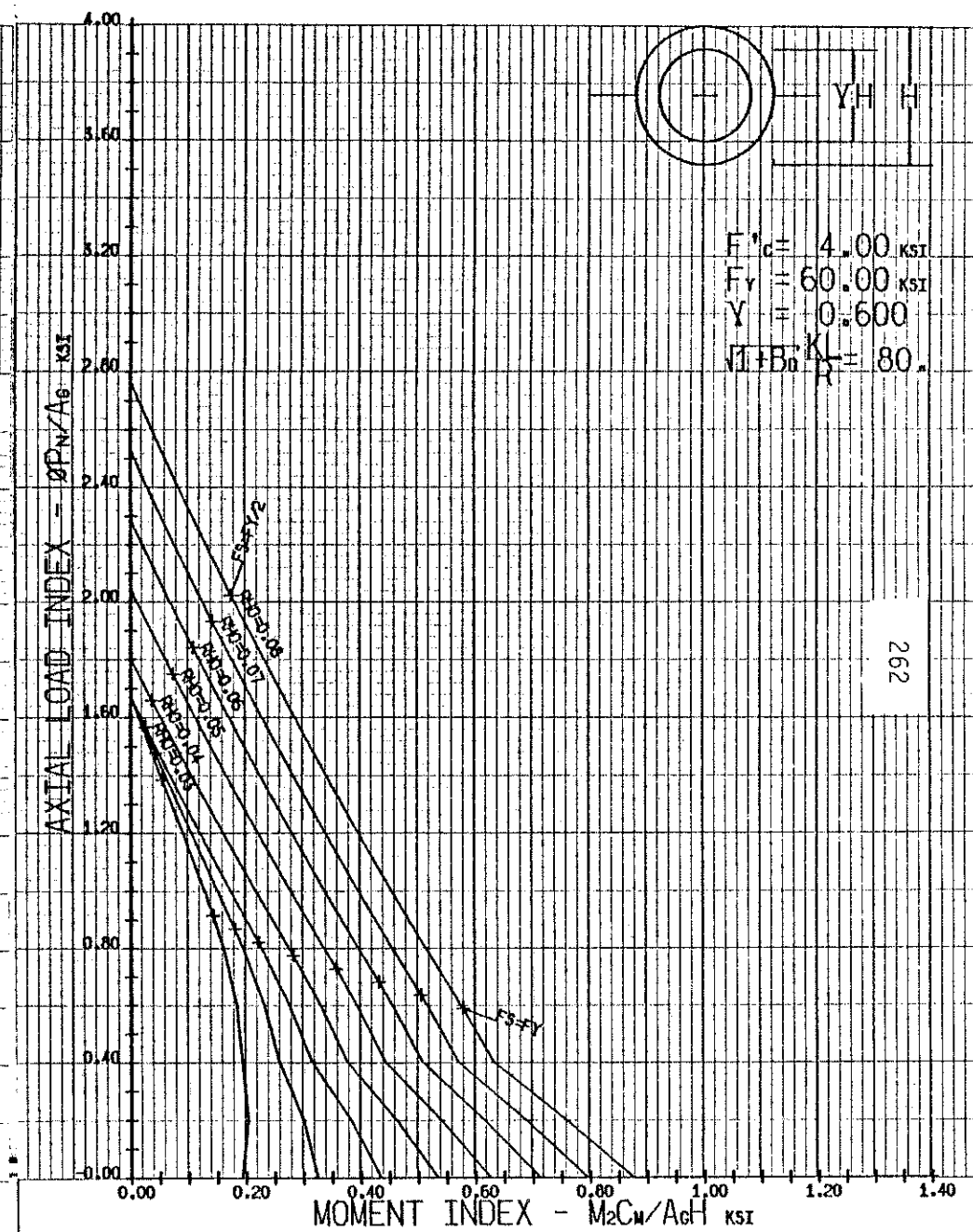


Fig. C4-60.60-80 - Interaction Diagram



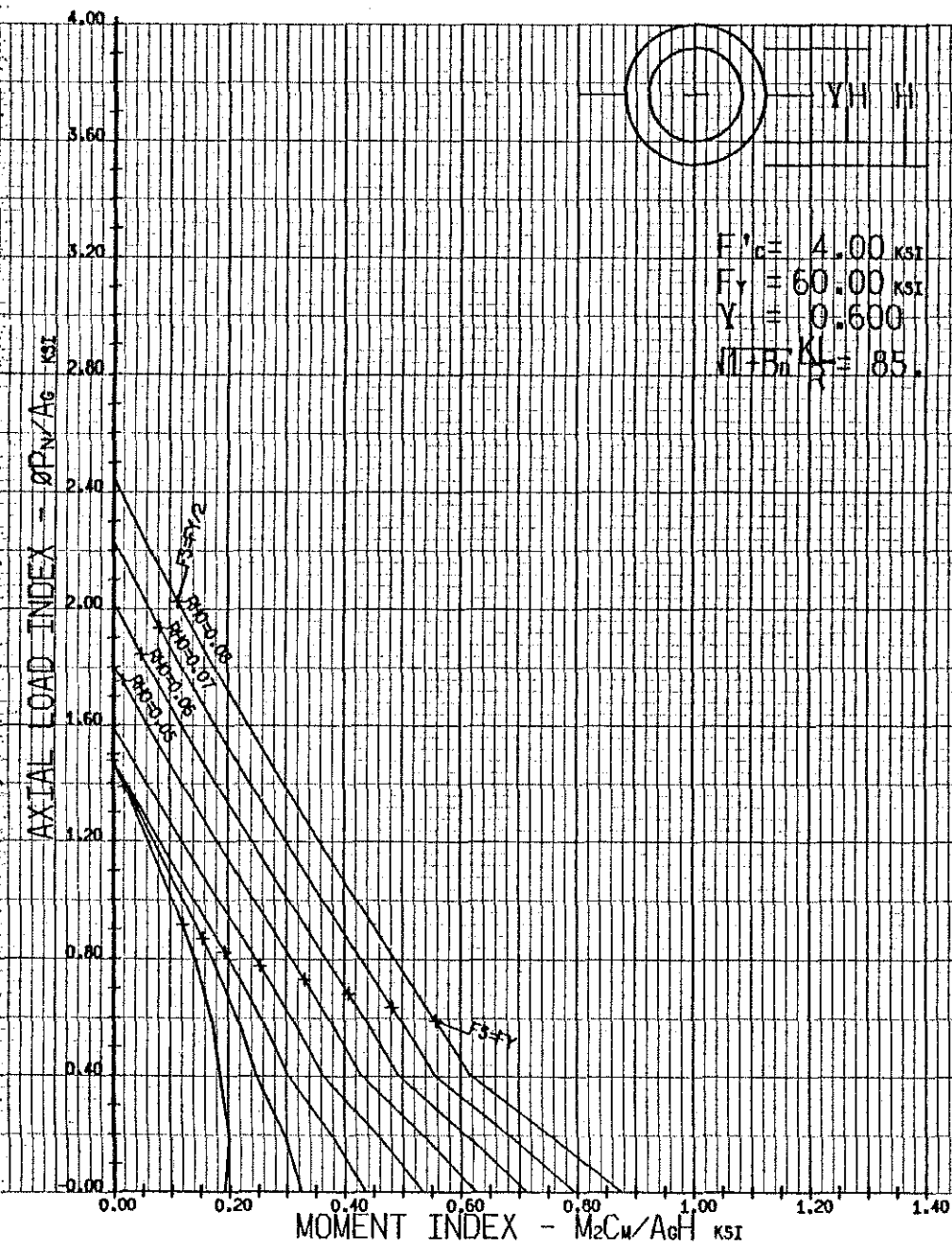


Fig. C4-60.60-85 - Interaction Diagram

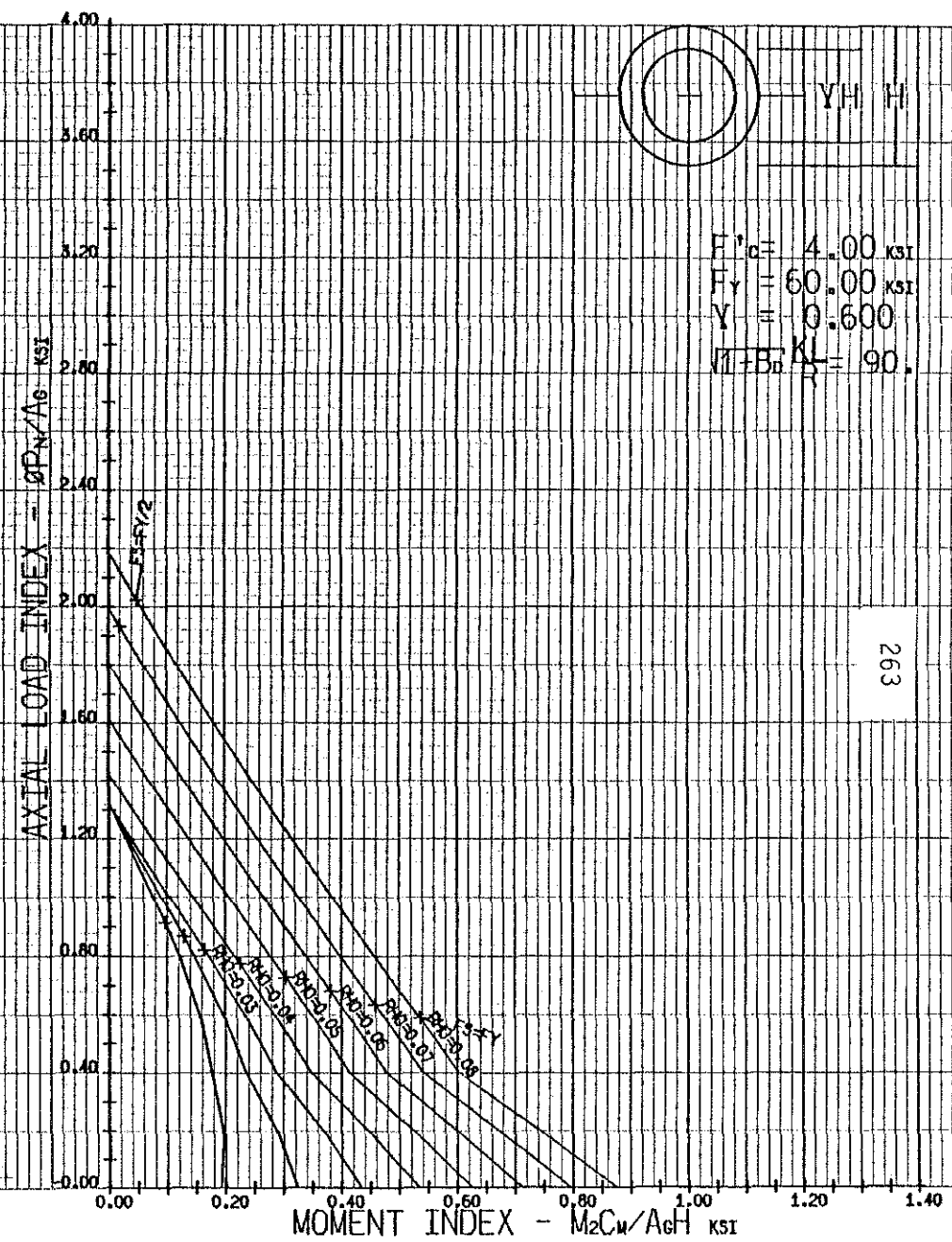


Fig. C4-60.60-90 - Interaction Diagram



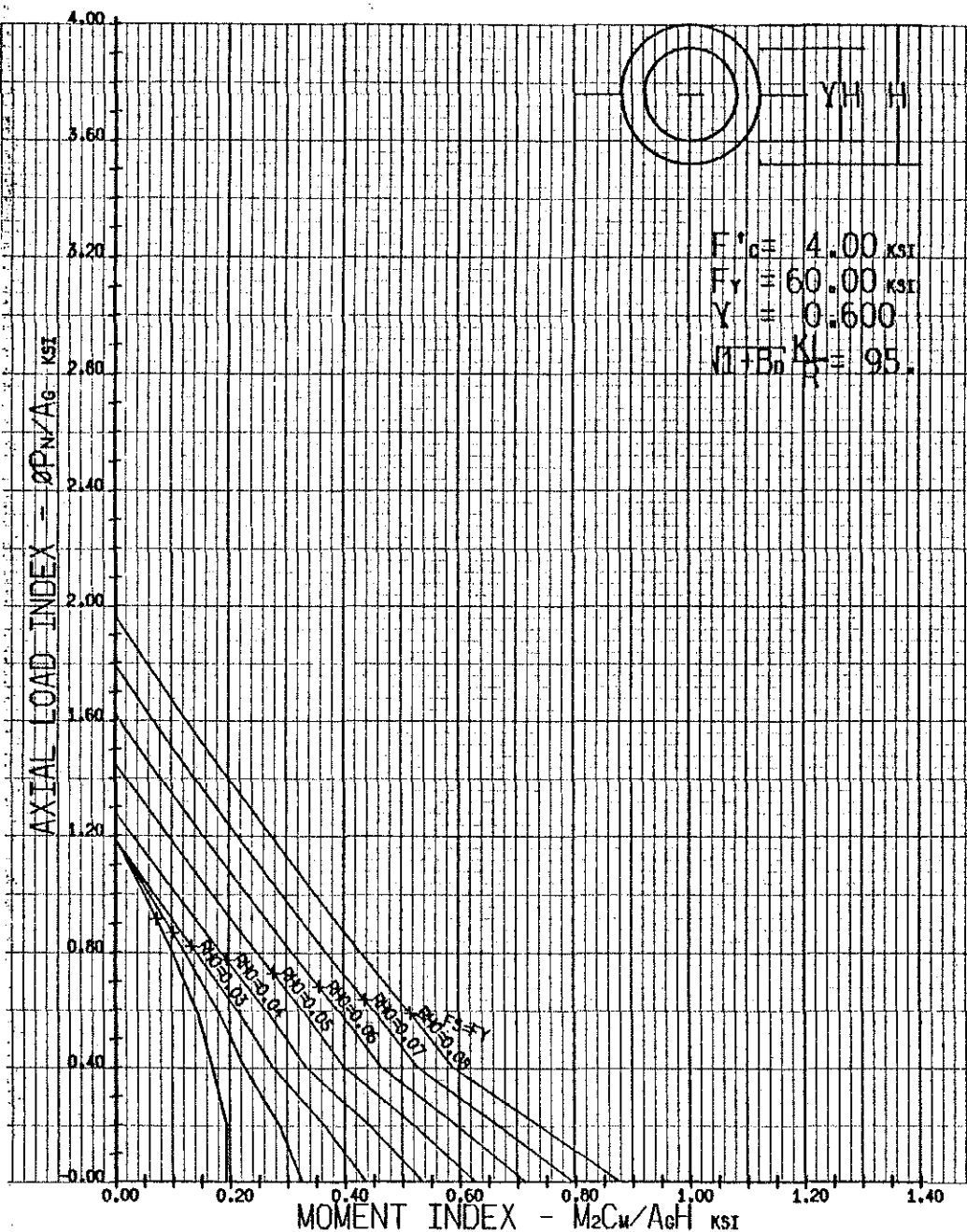


Fig. C4-60.60-95 - Interaction Diagram

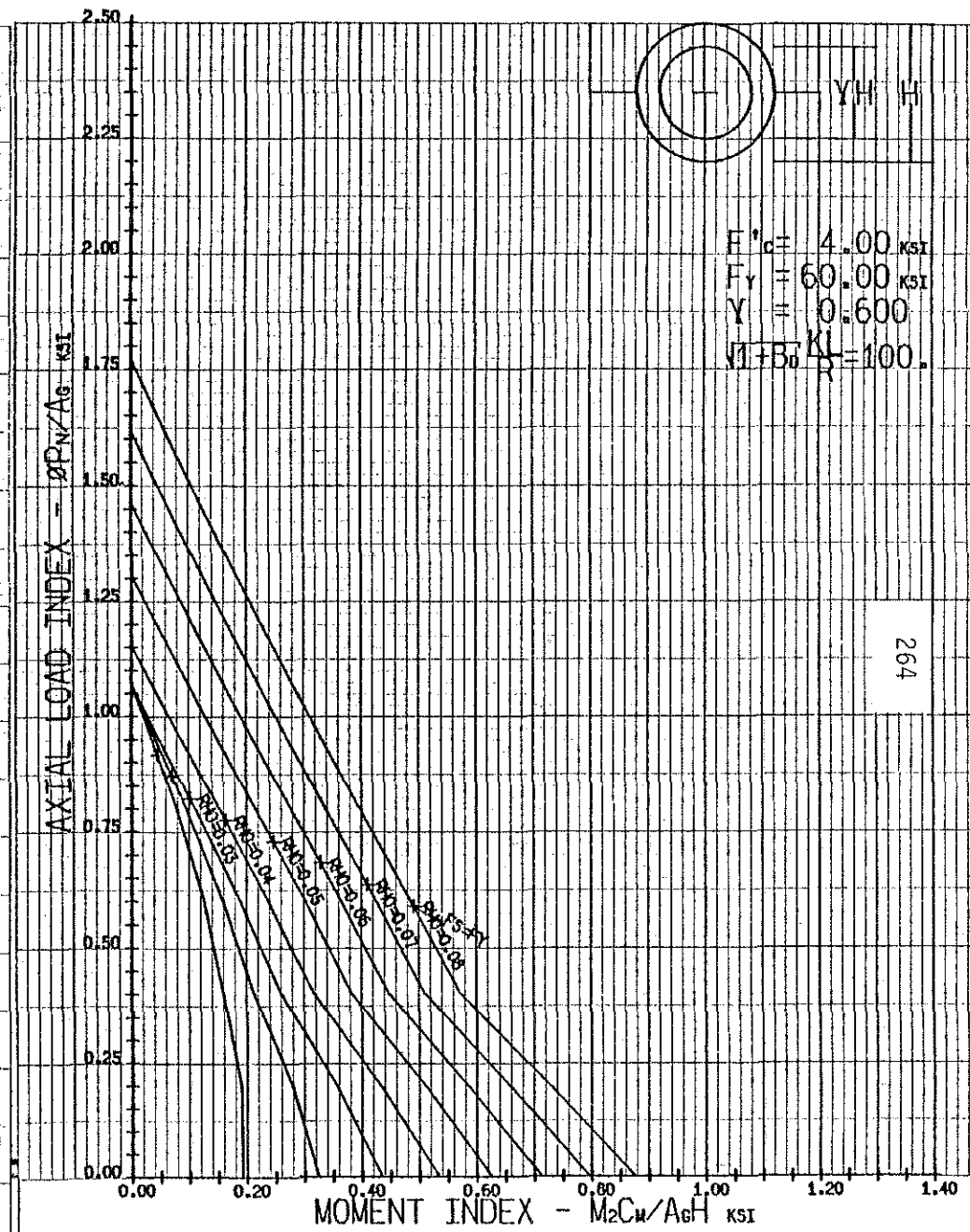


Fig. C4-60.60-100 - Interaction Diagram

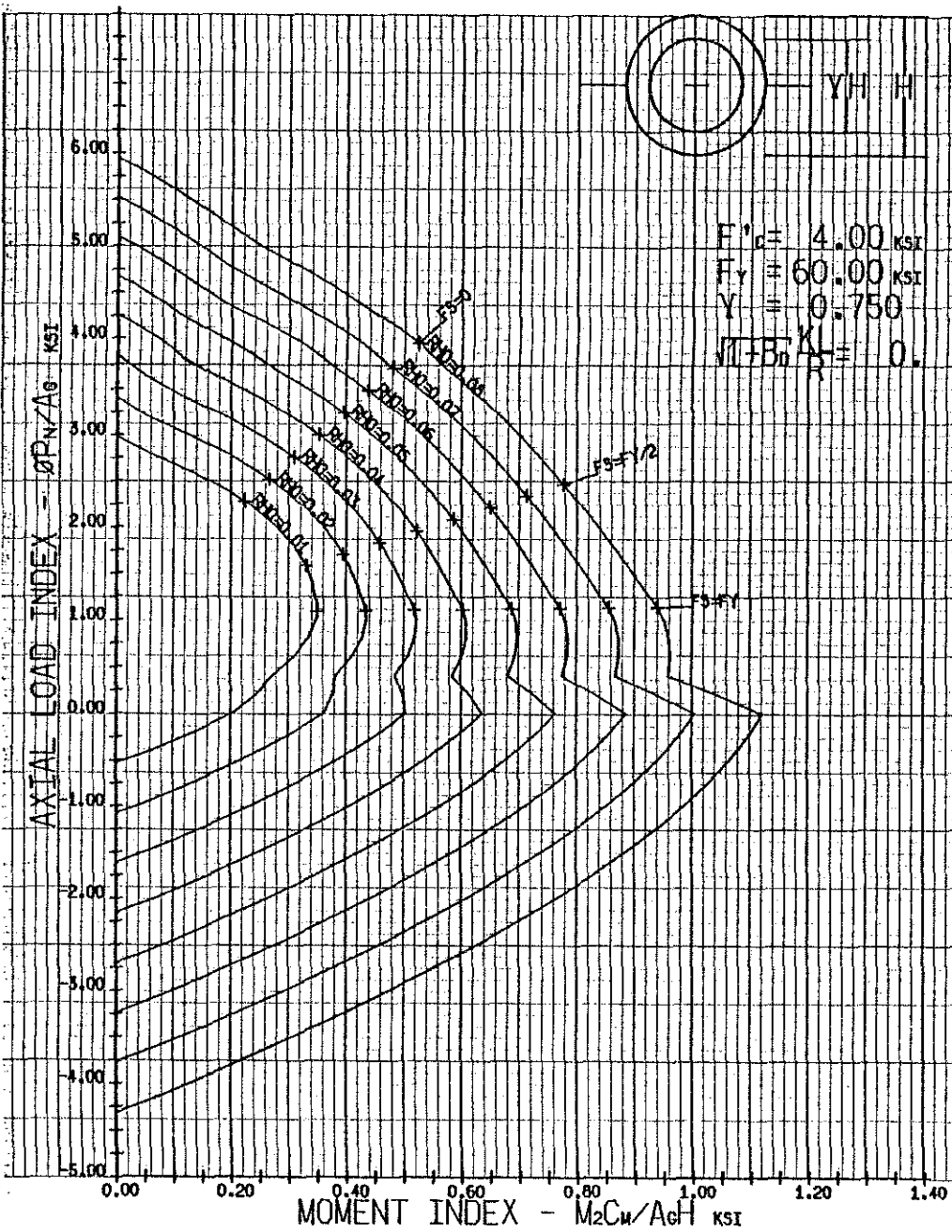


Fig. C4-60.75-0 - Interaction Diagram

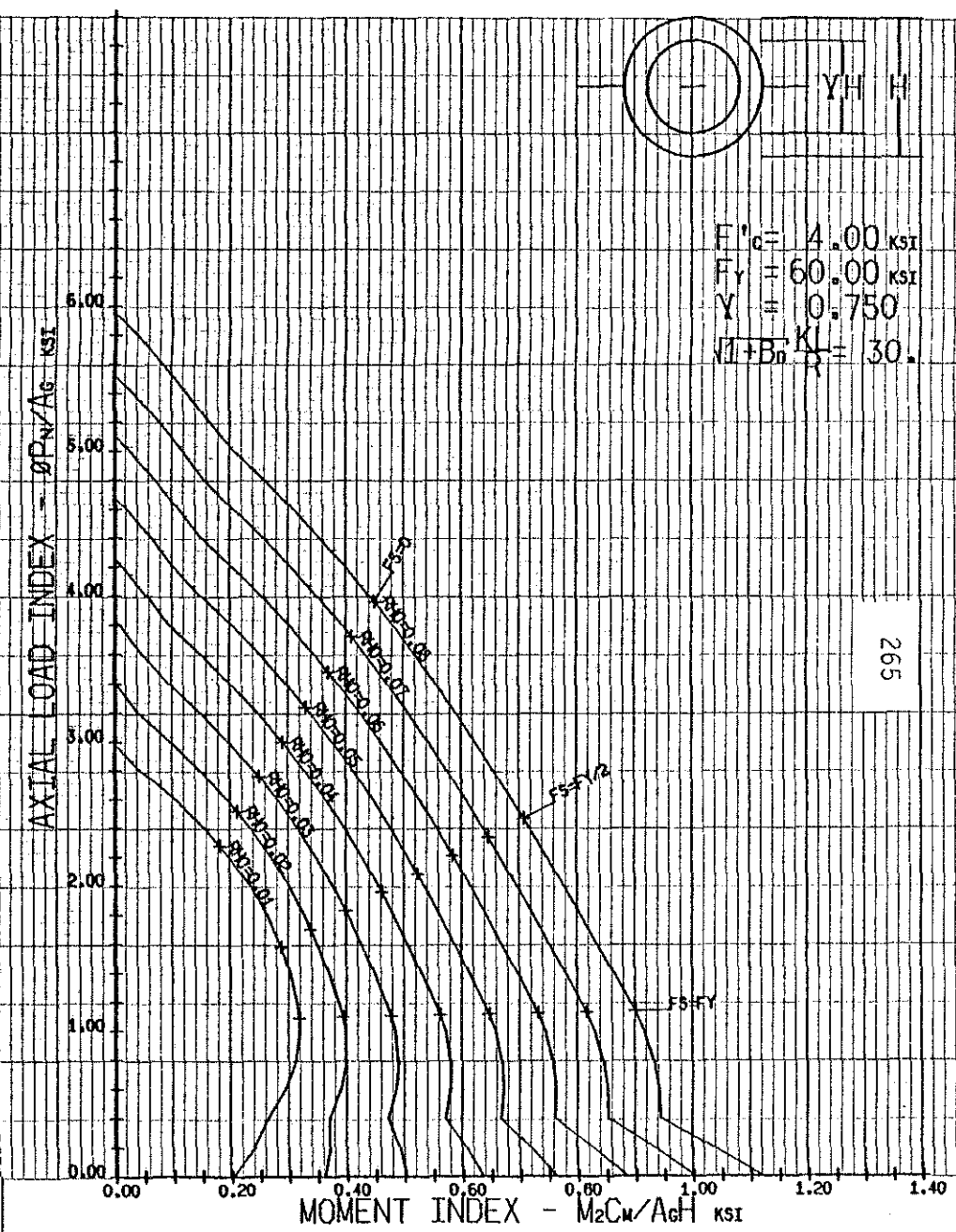


Fig. C4-60.75-30 - Interaction Diagram

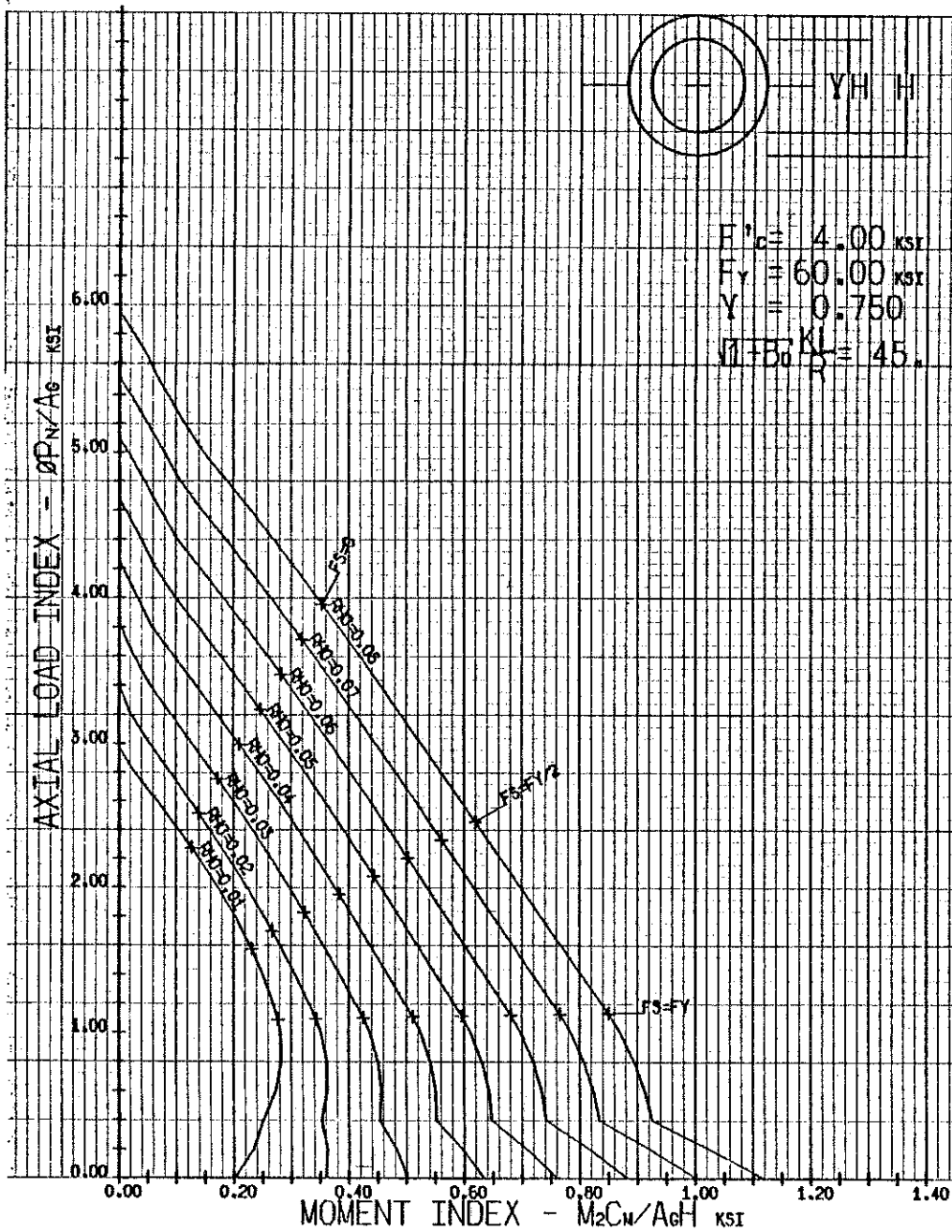


Fig. C4-60.75-45 - Interaction Diagram

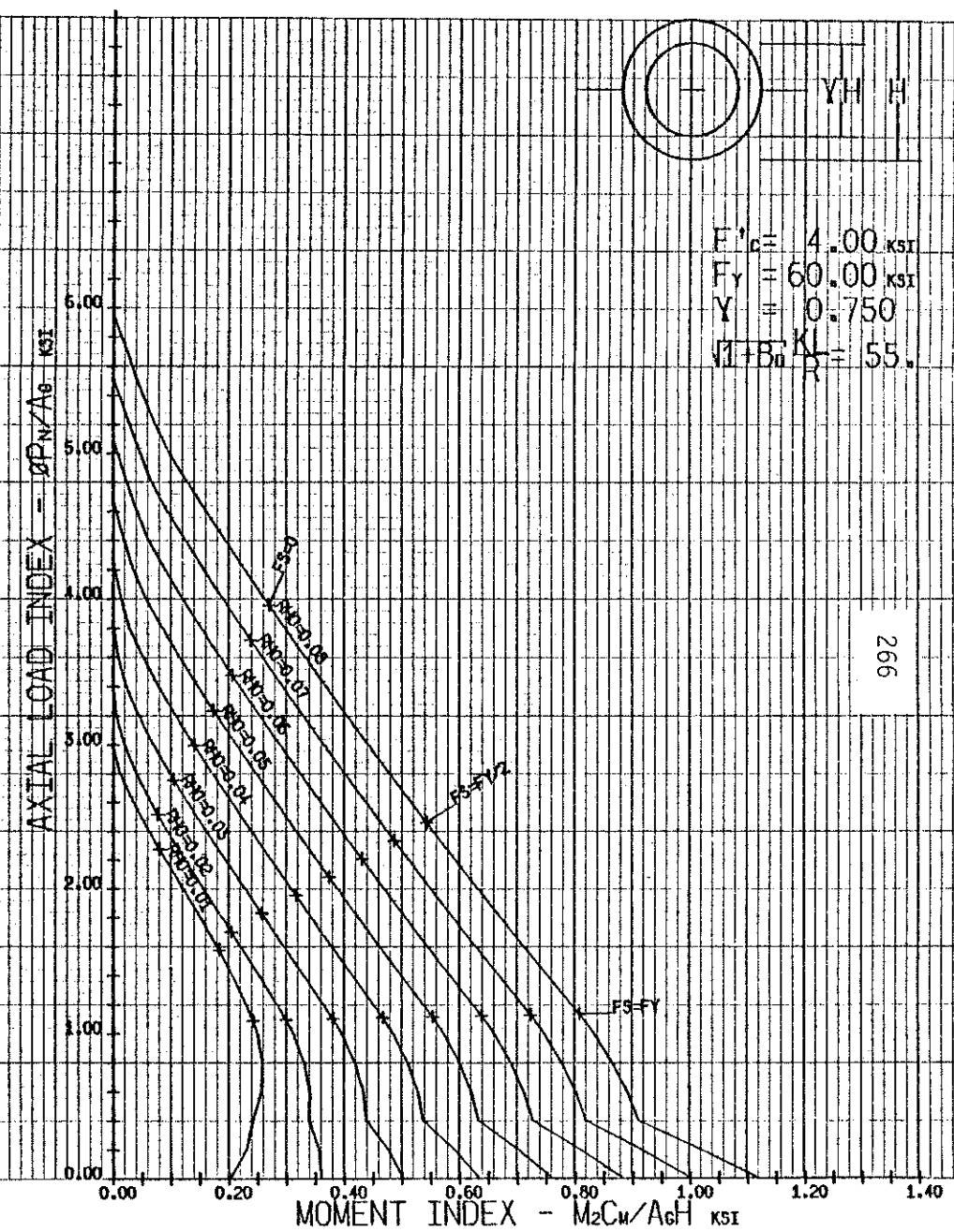


Fig. C4-60.75-55 - Interaction Diagram

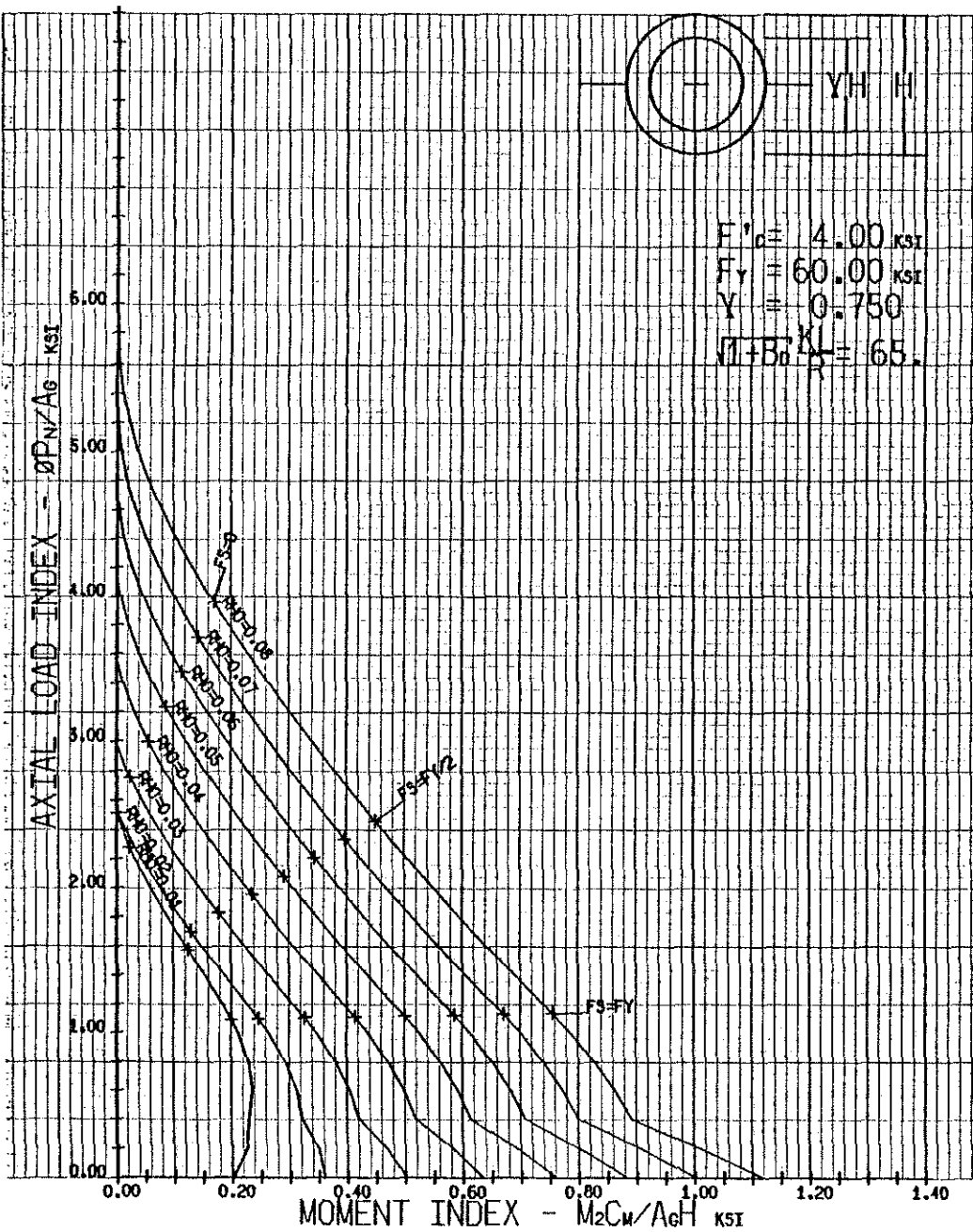


Fig. C4-60.75-65 - Interaction Diagram

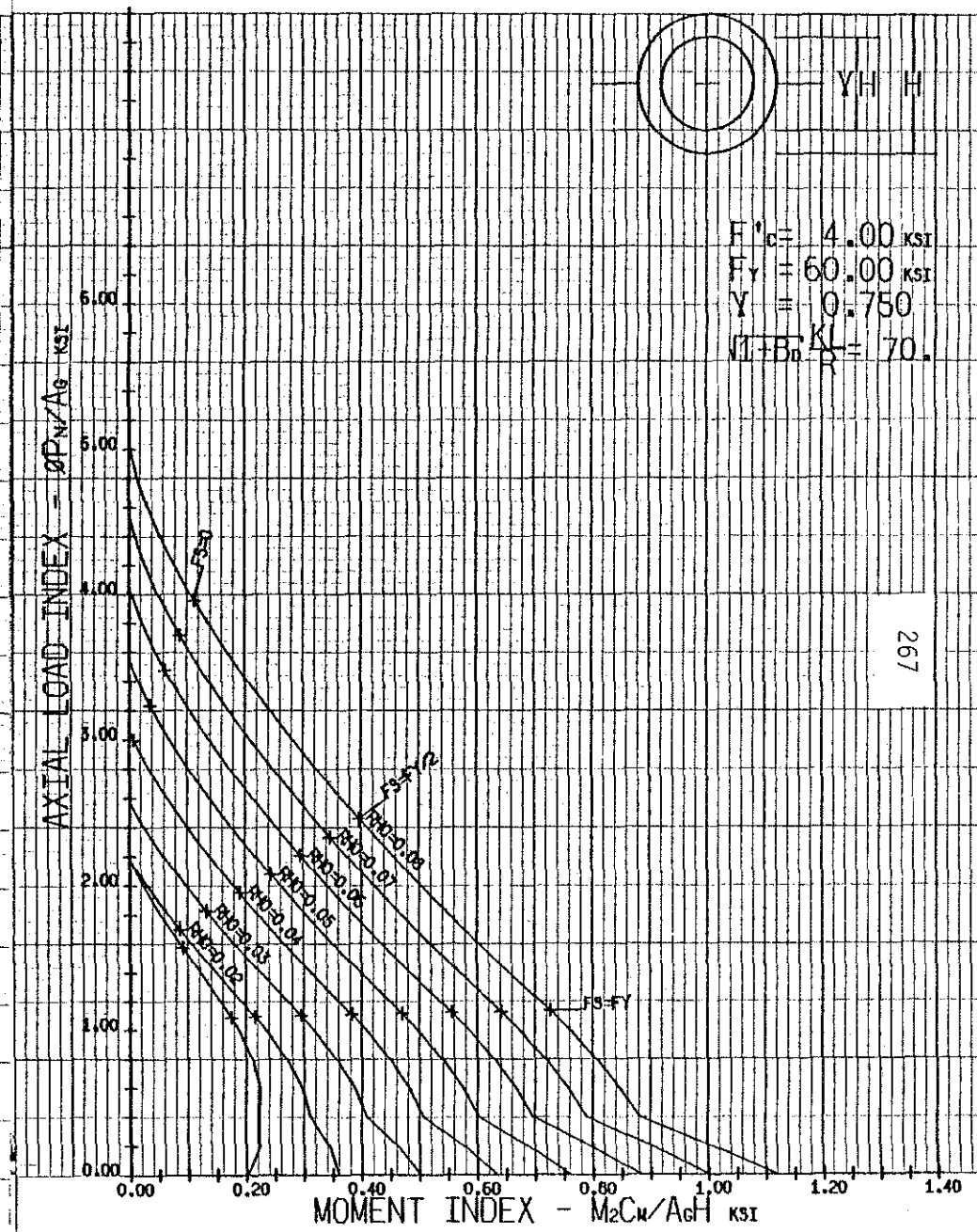


Fig. C4-60.75-70 - Interaction Diagram

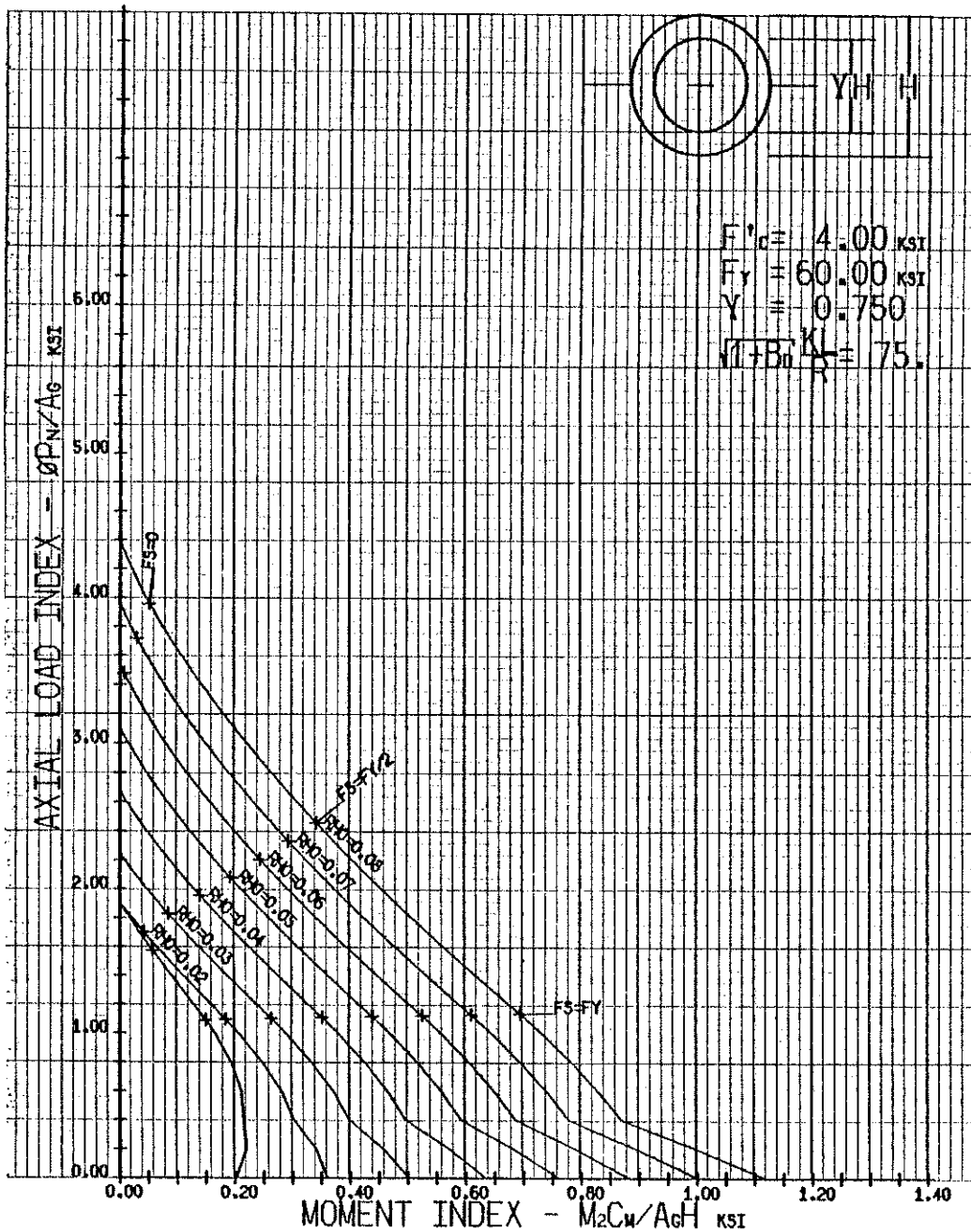


Fig. C4-60.75-75 - Interaction Diagram

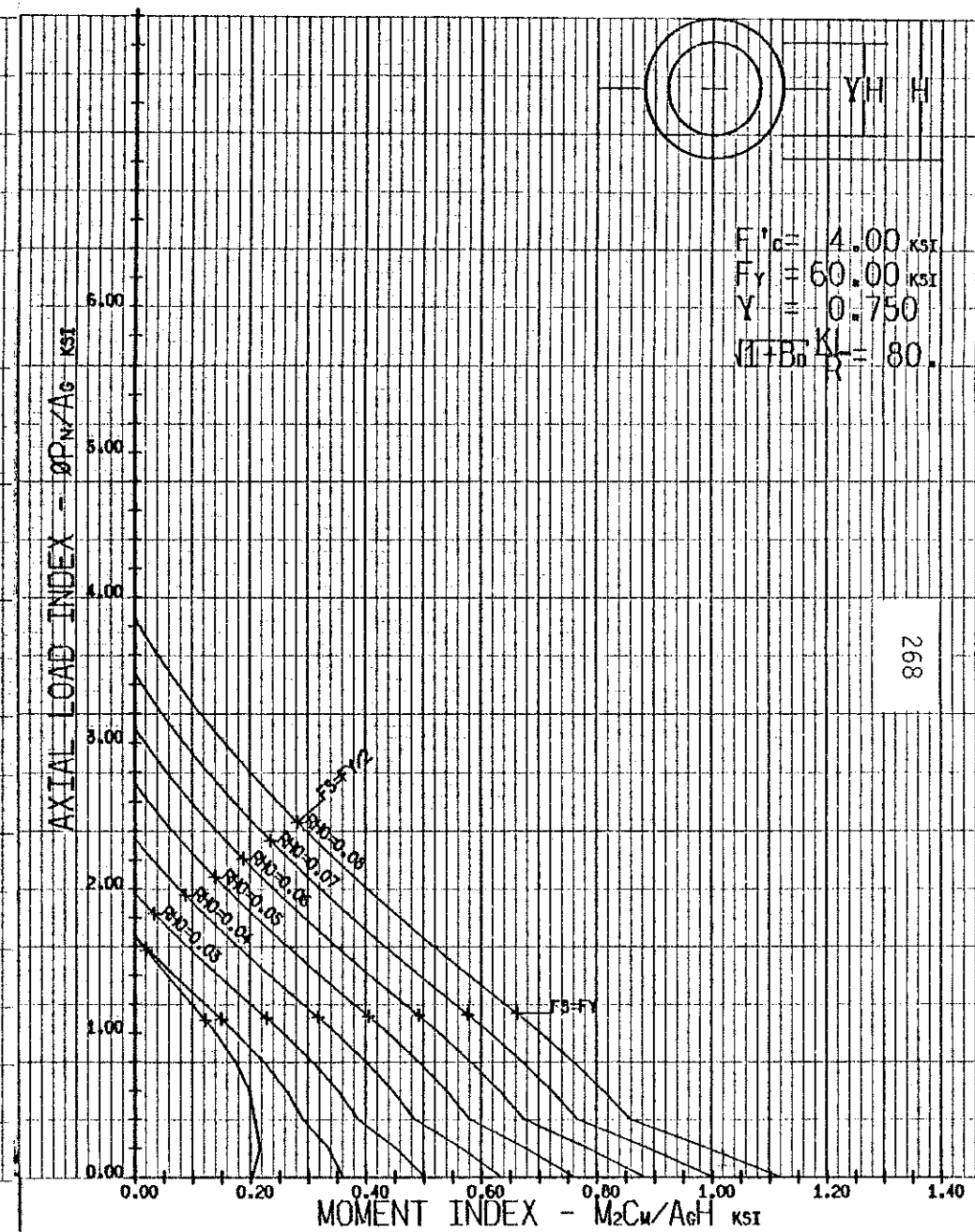


Fig. C4-60.75-80 - Interaction Diagram



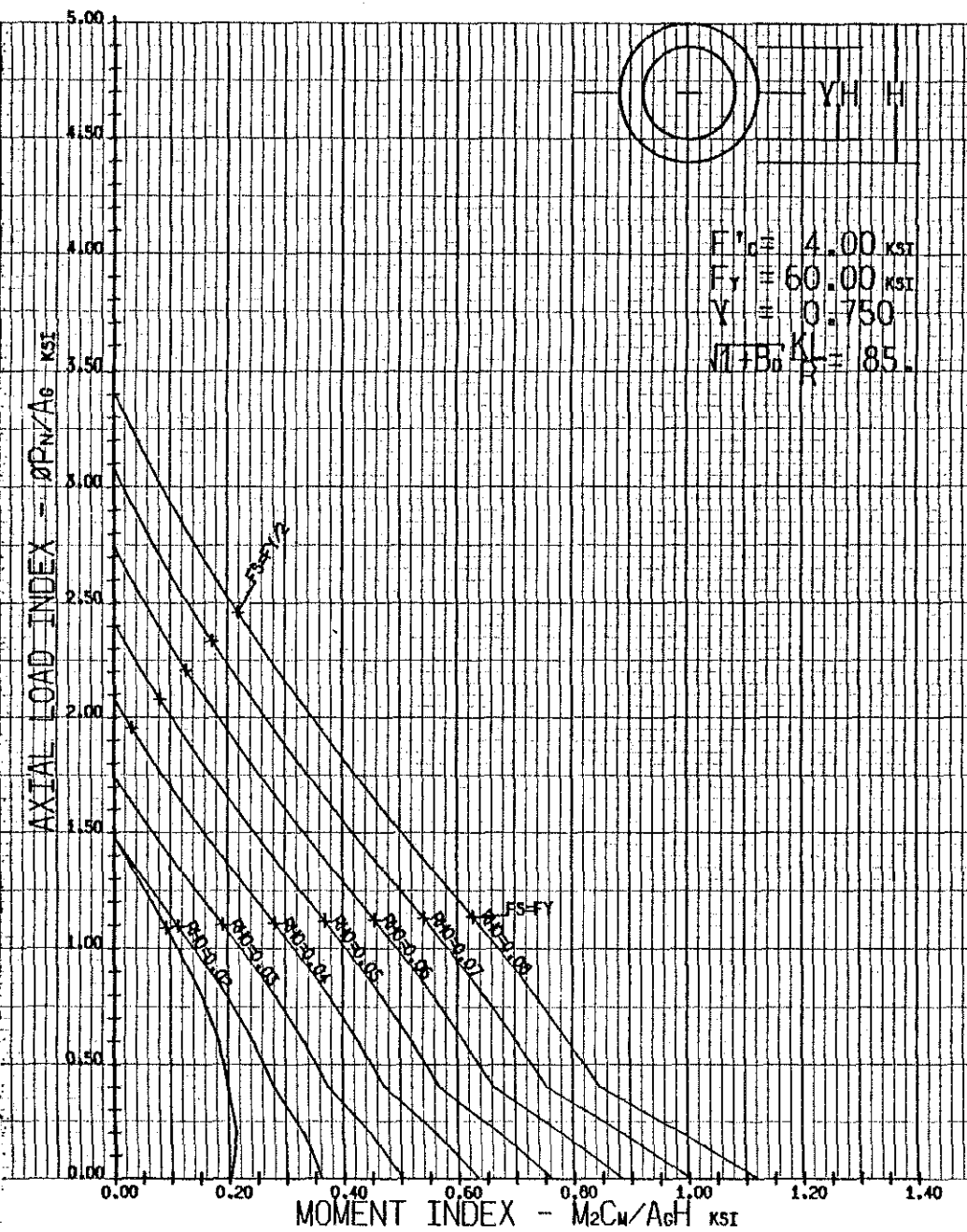


Fig. C4-60.75-85 - Interaction Diagram

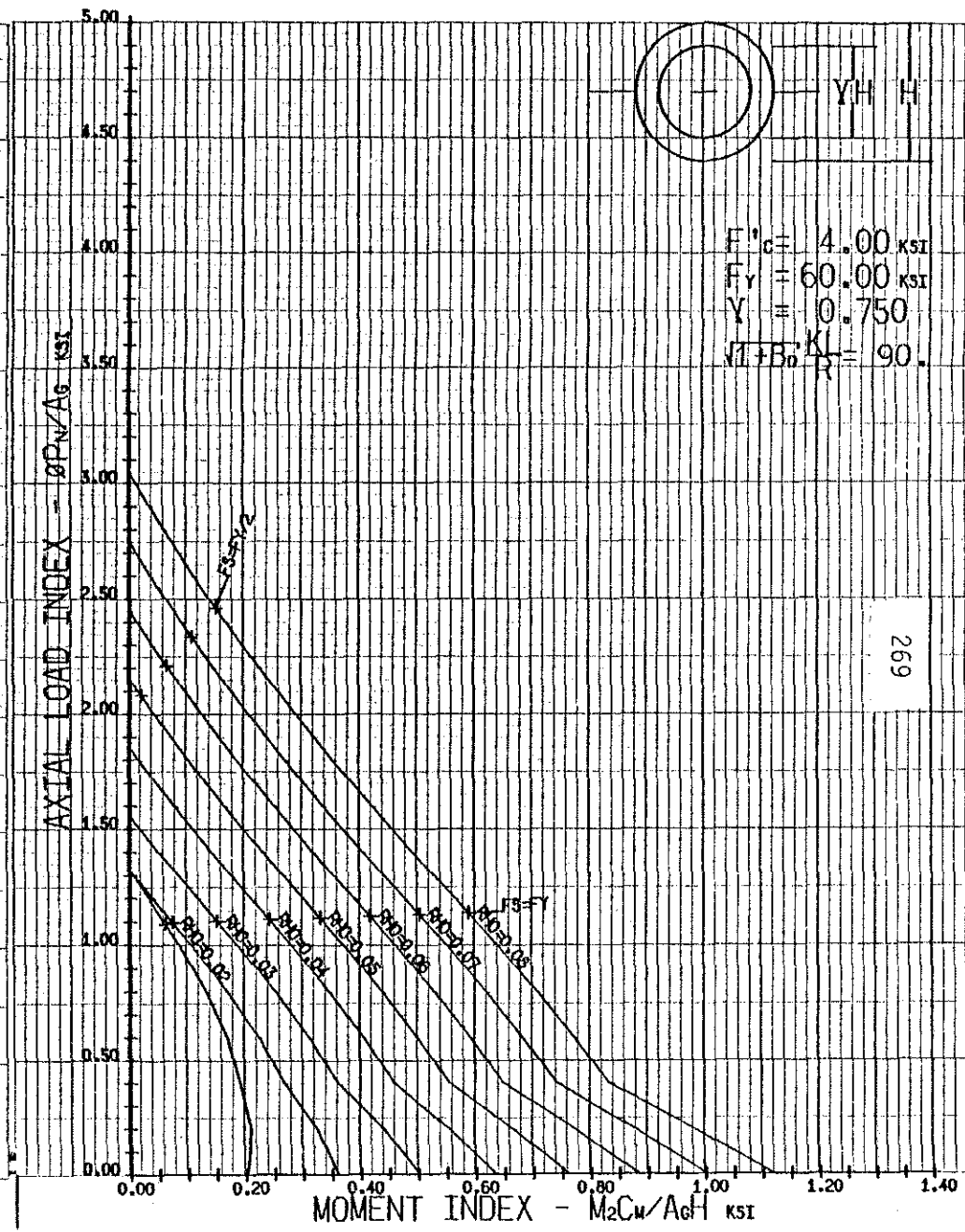


Fig. C4-60.75-90 - Interaction Diagram



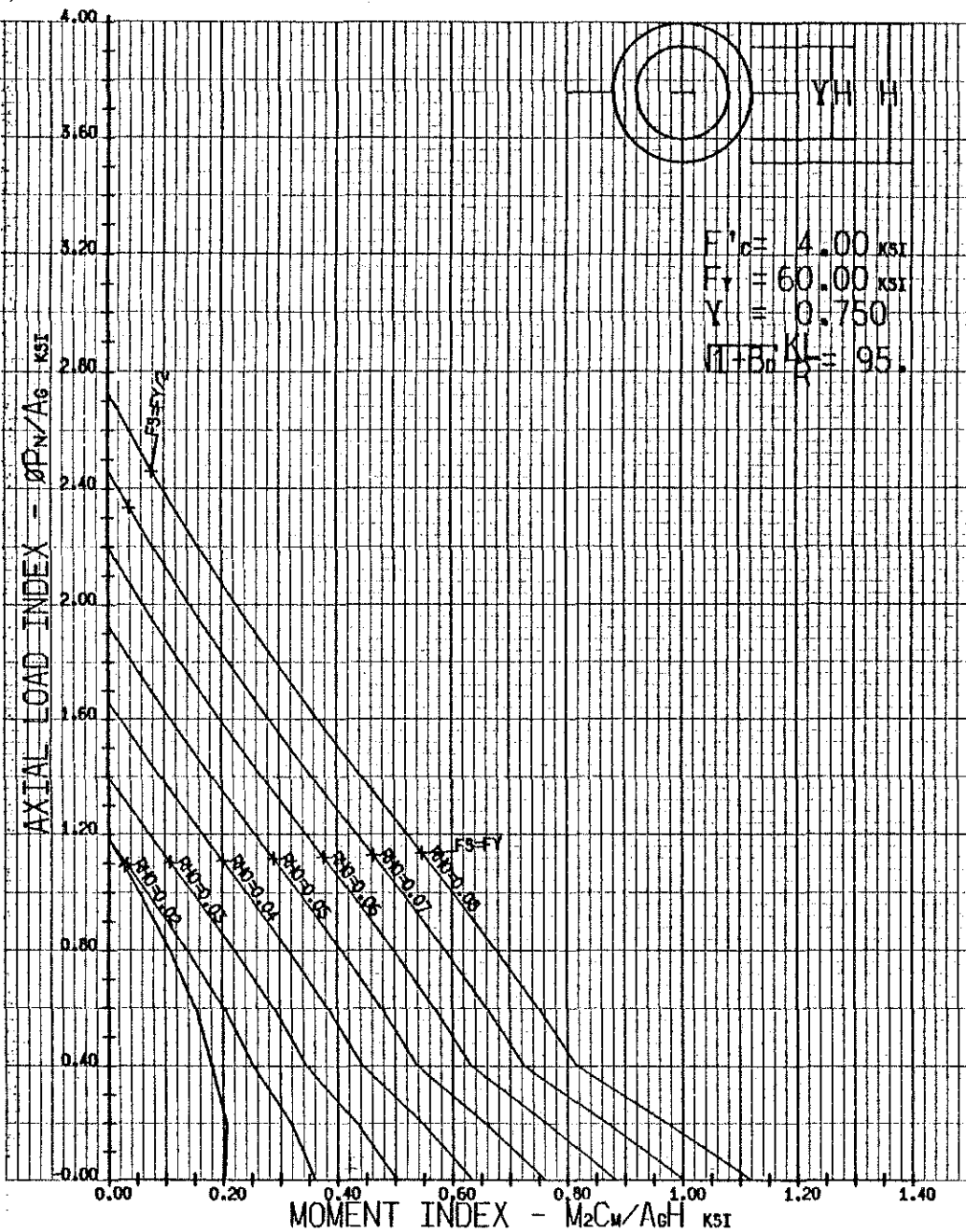


Fig. C4-60.75-95 - Interaction Diagram

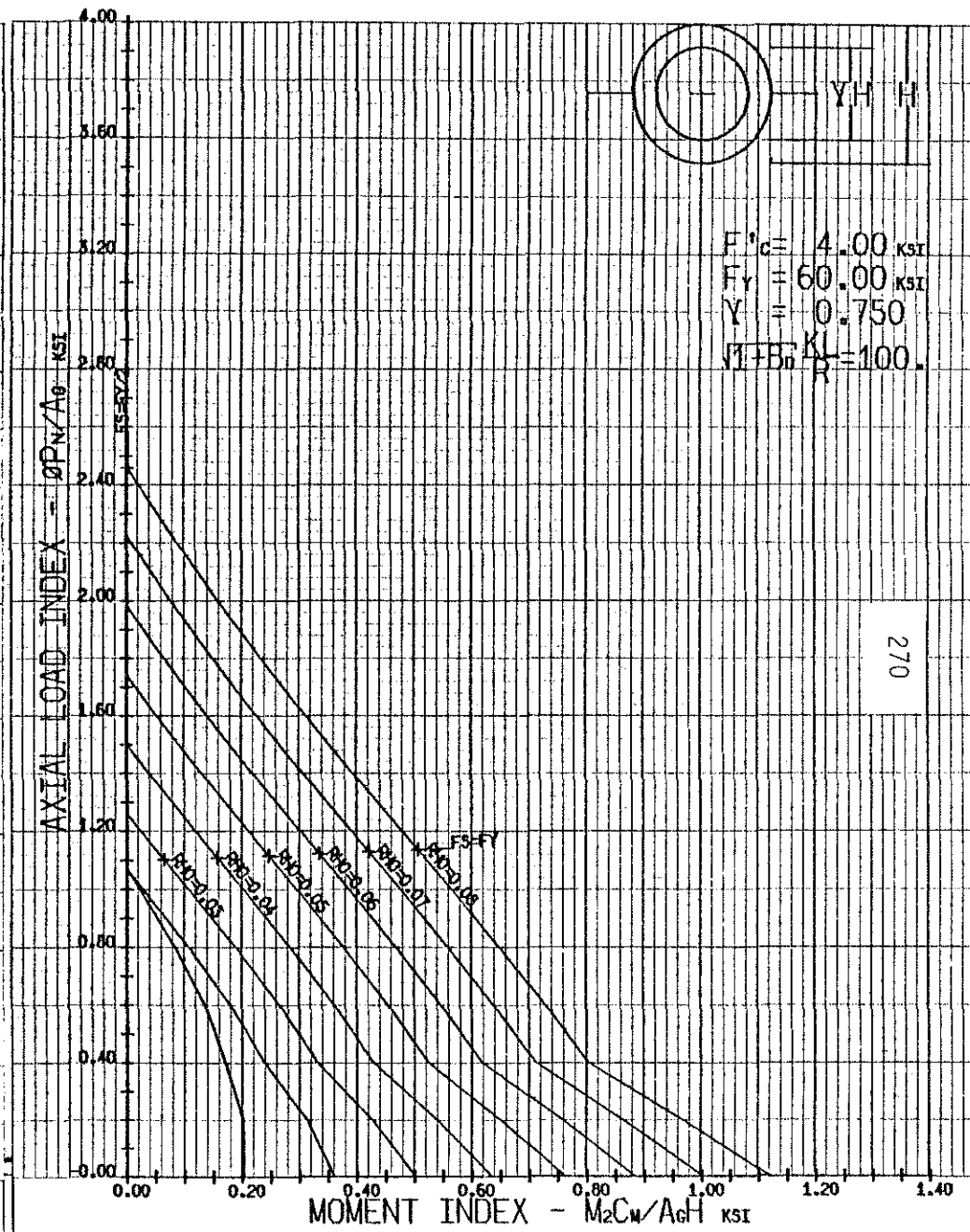


Fig. C4-60.75-100 - Interaction Diagram

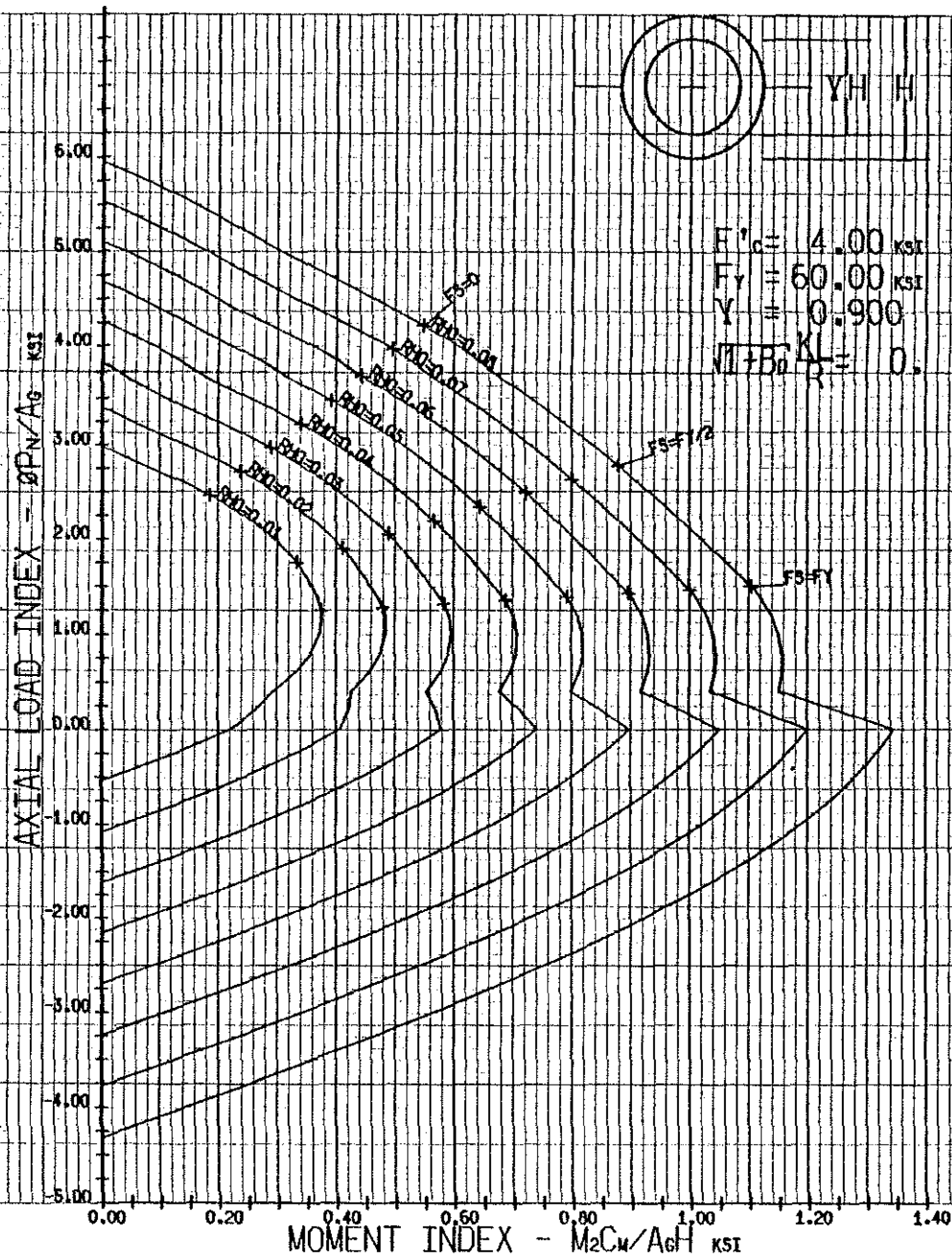


Fig. C4-60.90-0 - Interaction Diagram

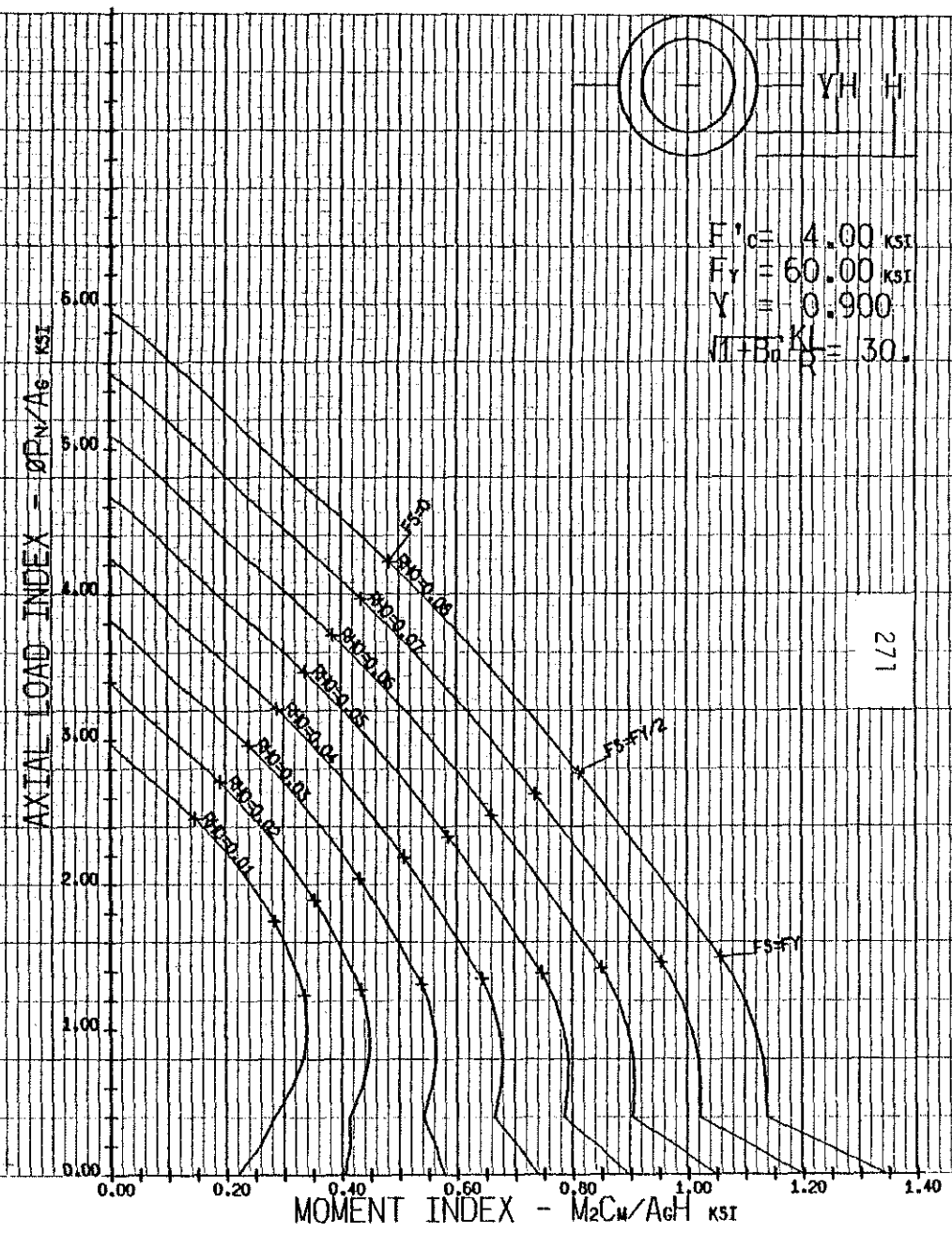


Fig. C4-60.90-30 - Interaction Diagram

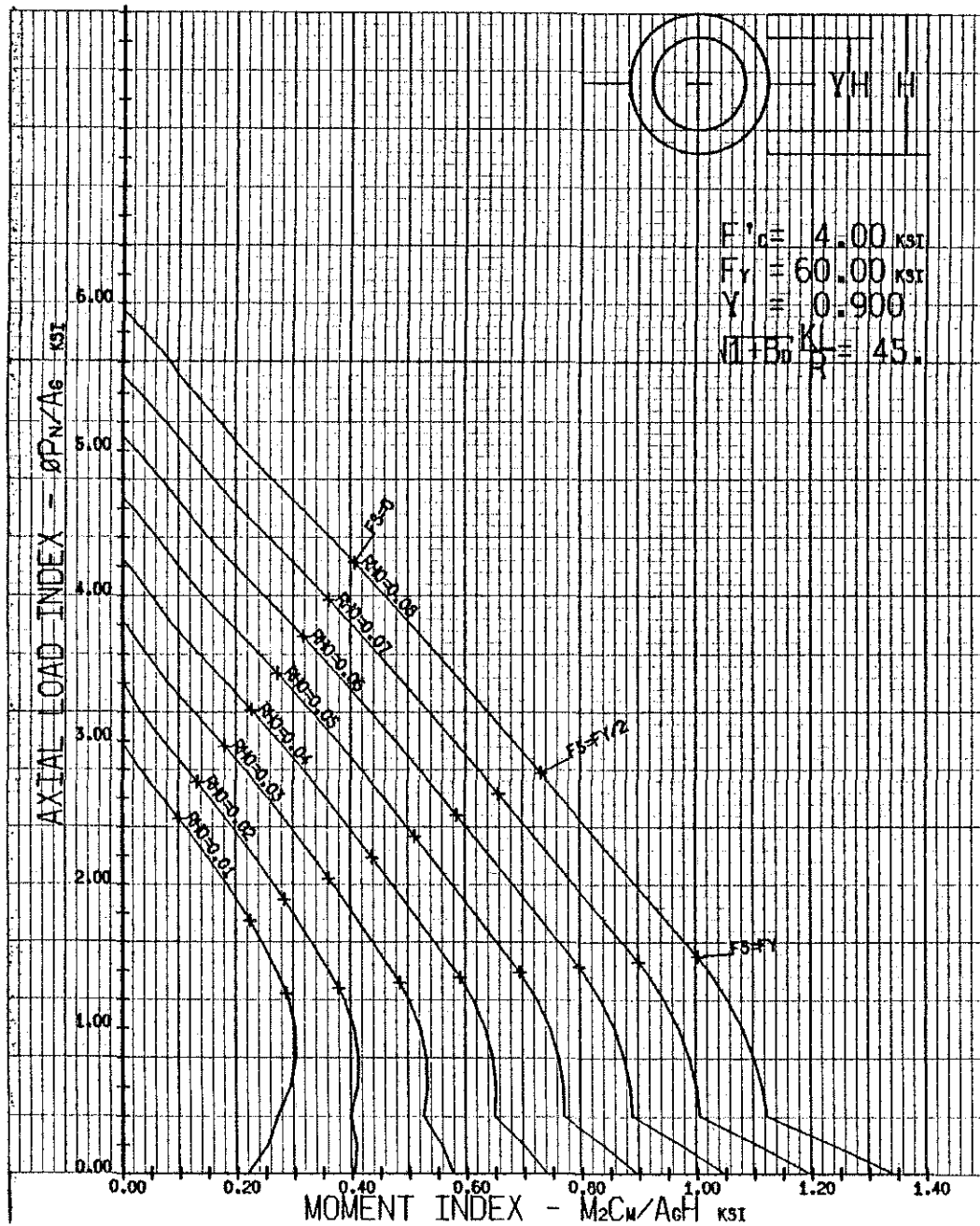


Fig. C4-60.90-45 - Interaction Diagram

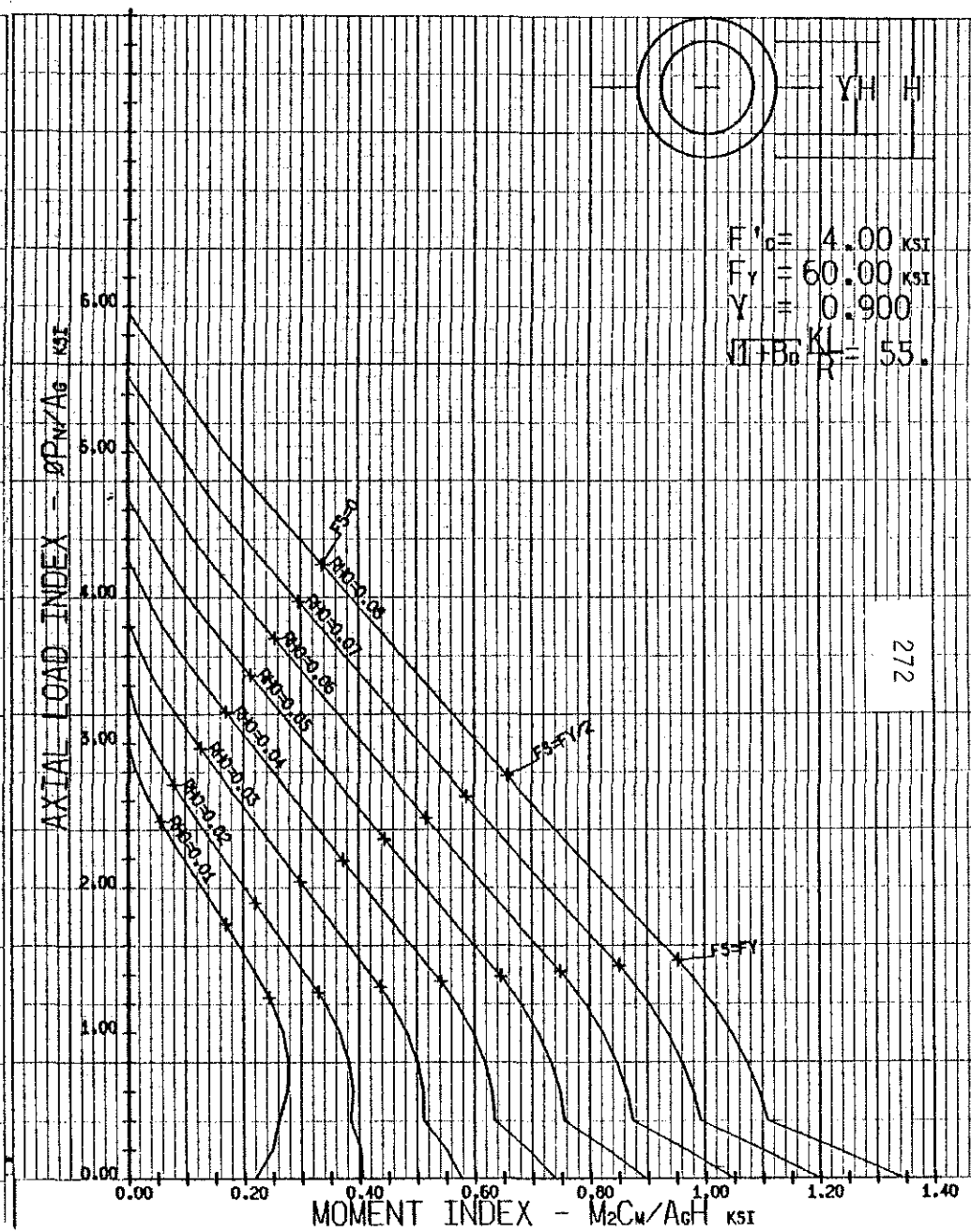


Fig. C4-60.90-55 - Interaction Diagram

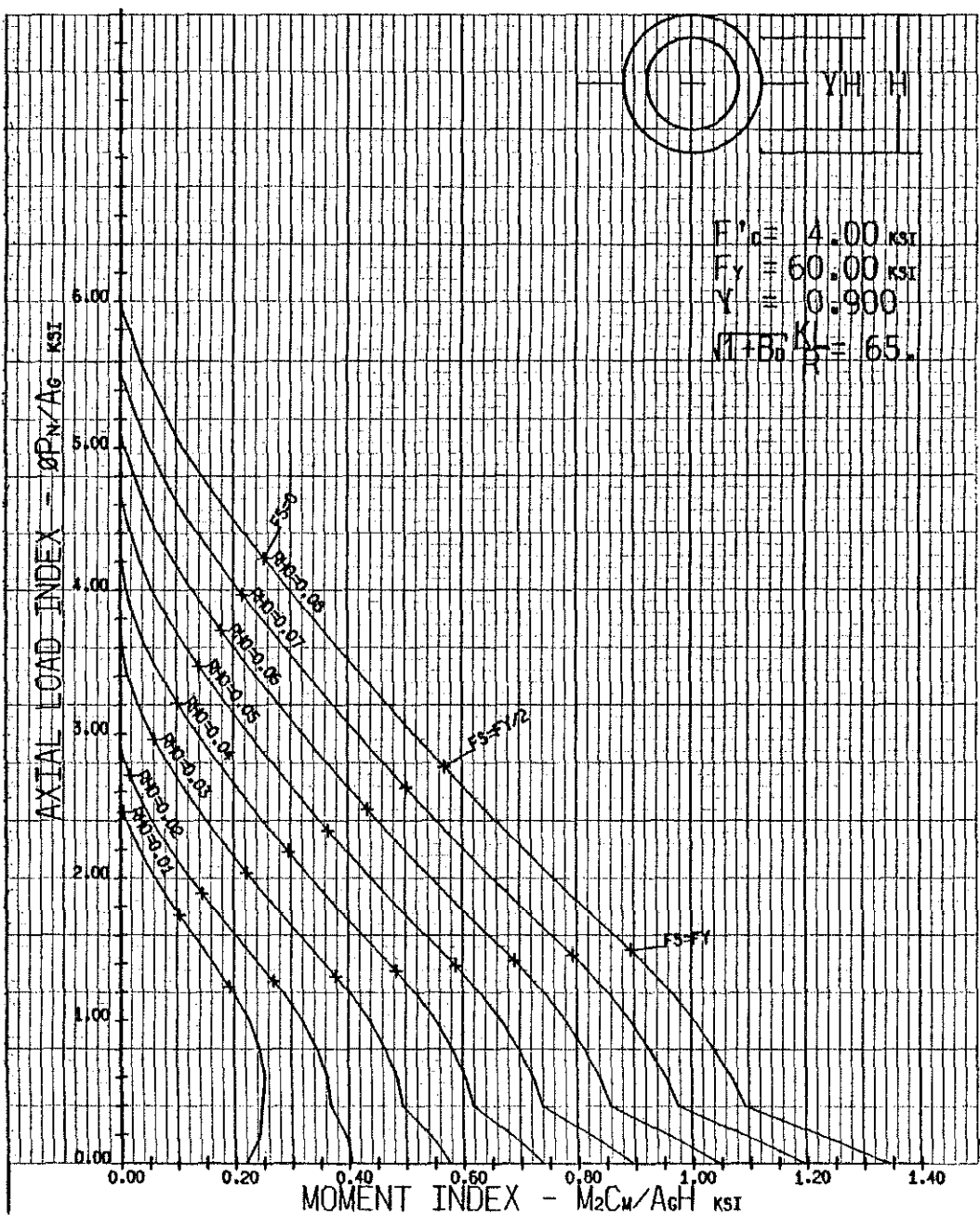


Fig. C4-60.90-65 - Interaction Diagram

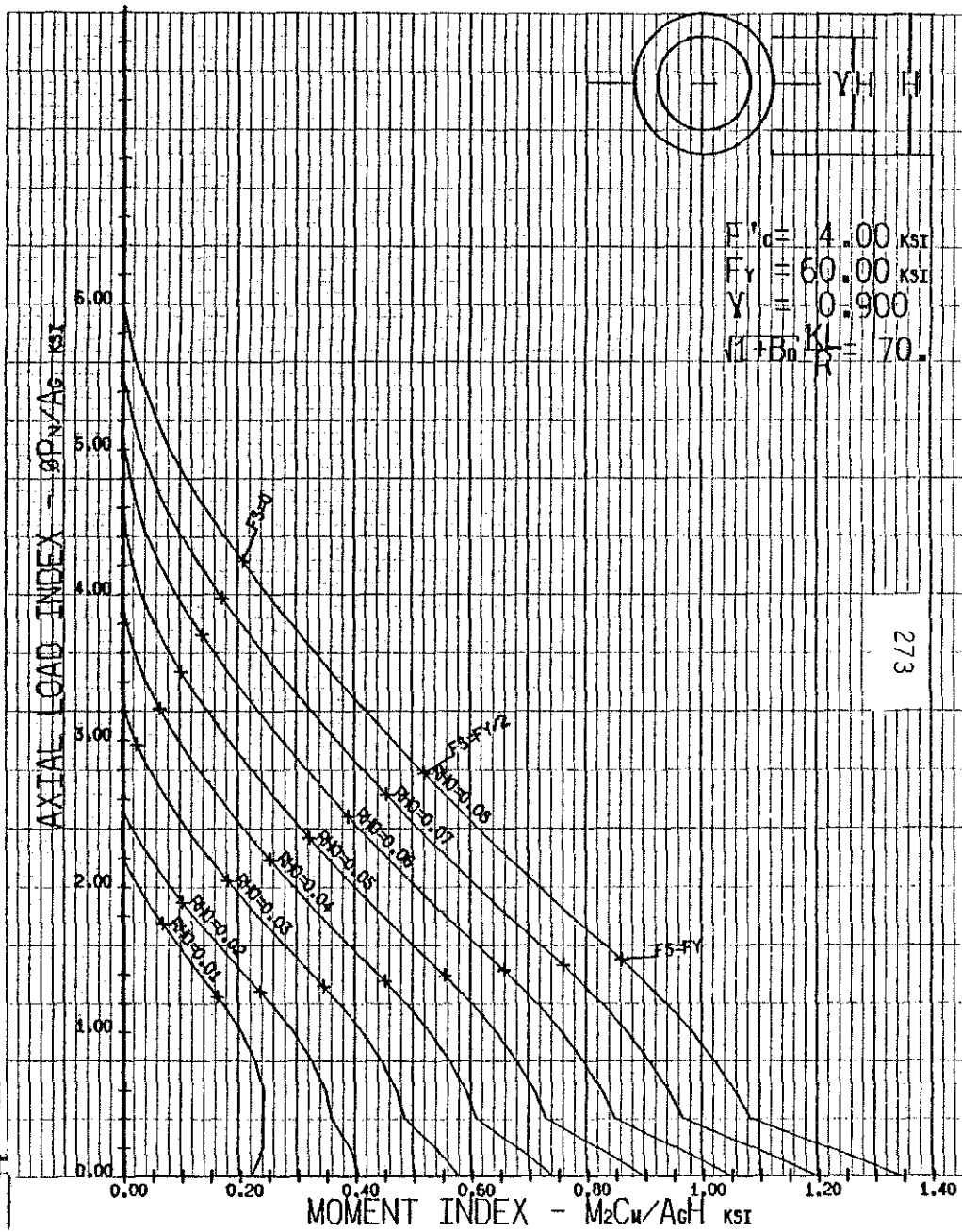


Fig. C4-60.90-70 - Interaction Diagram

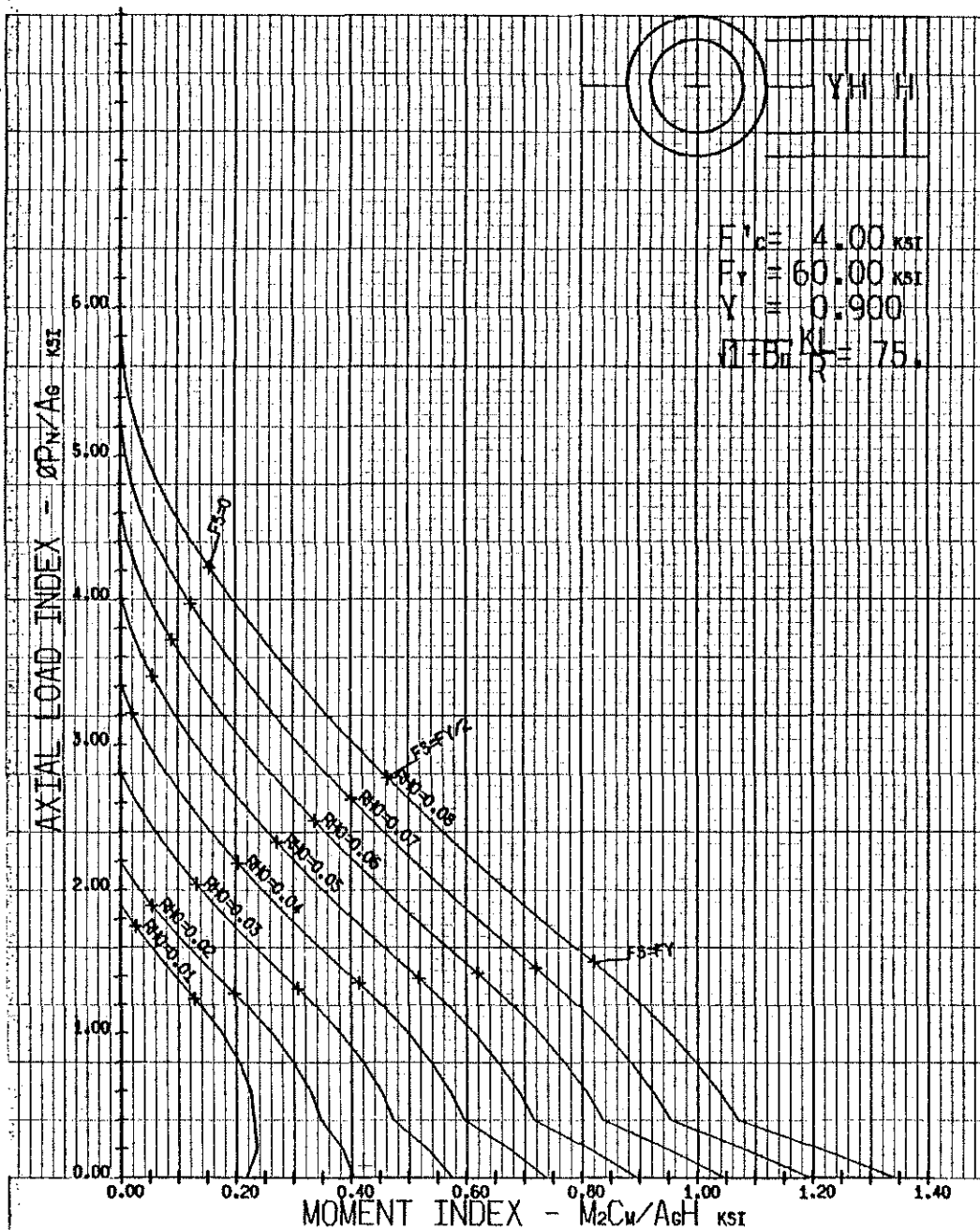


Fig. C4-60.90-75 - Interaction Diagram

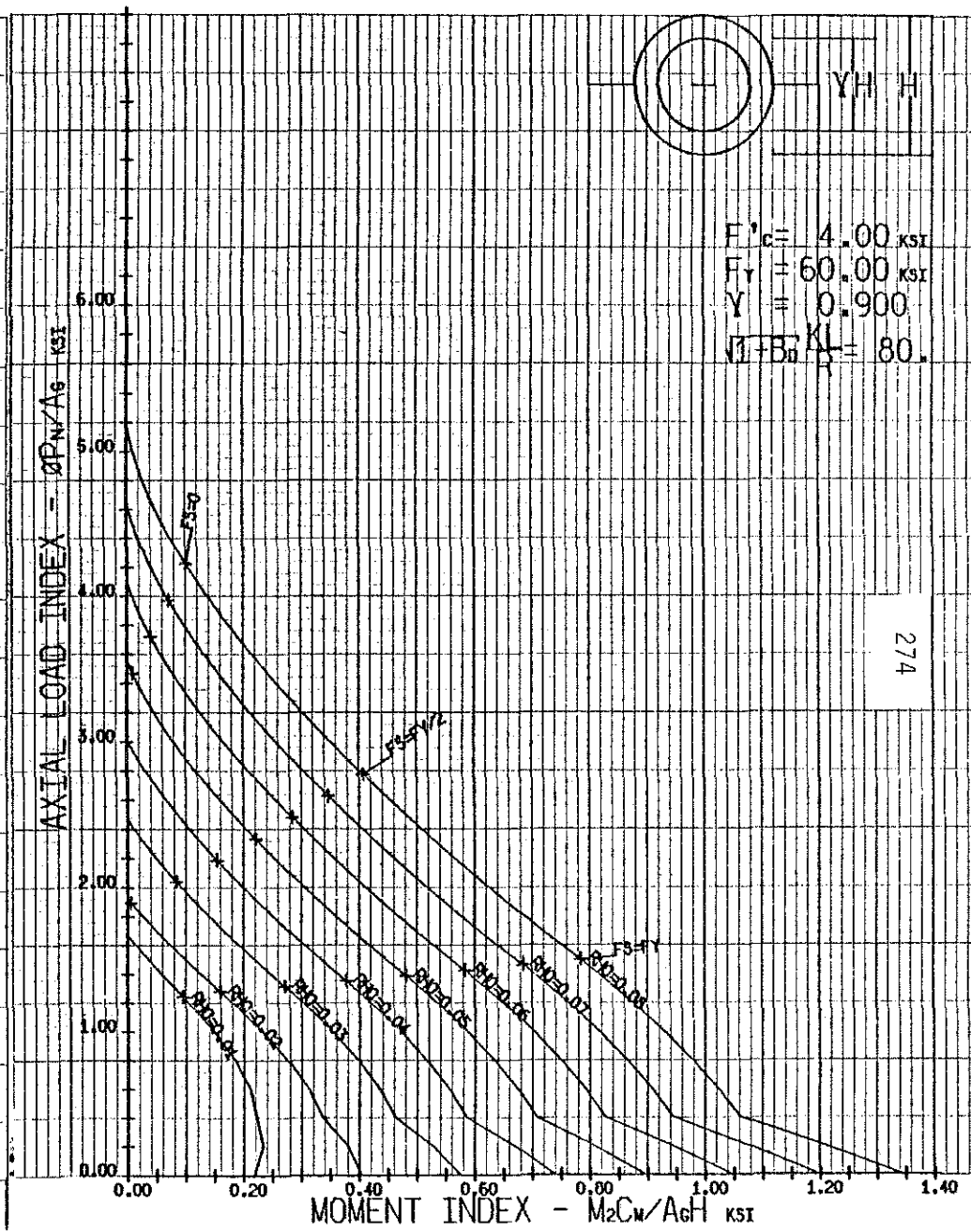


Fig. C4-60.90-80 - Interaction Diagram



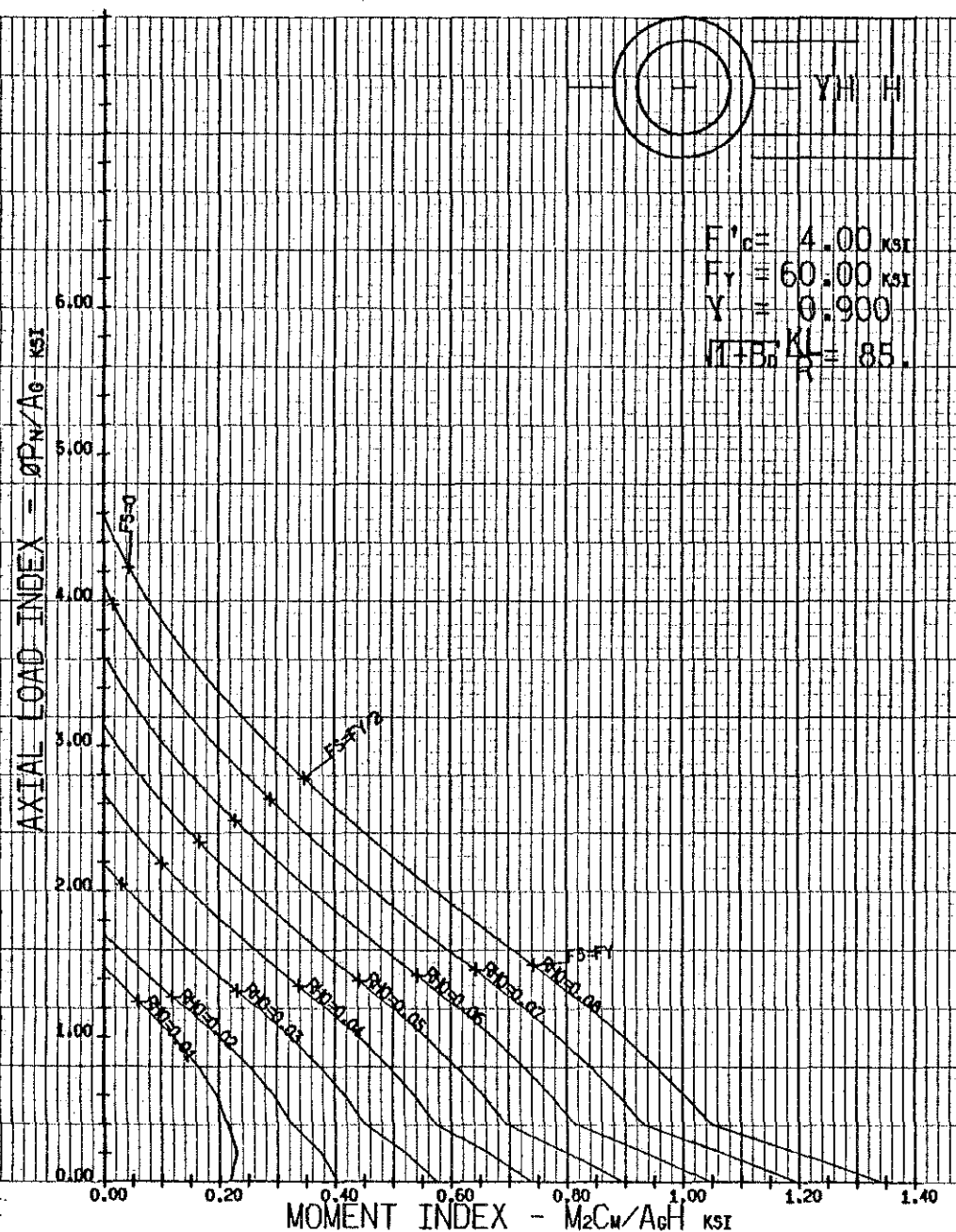


Fig. C4-60.90-85 - Interaction Diagram

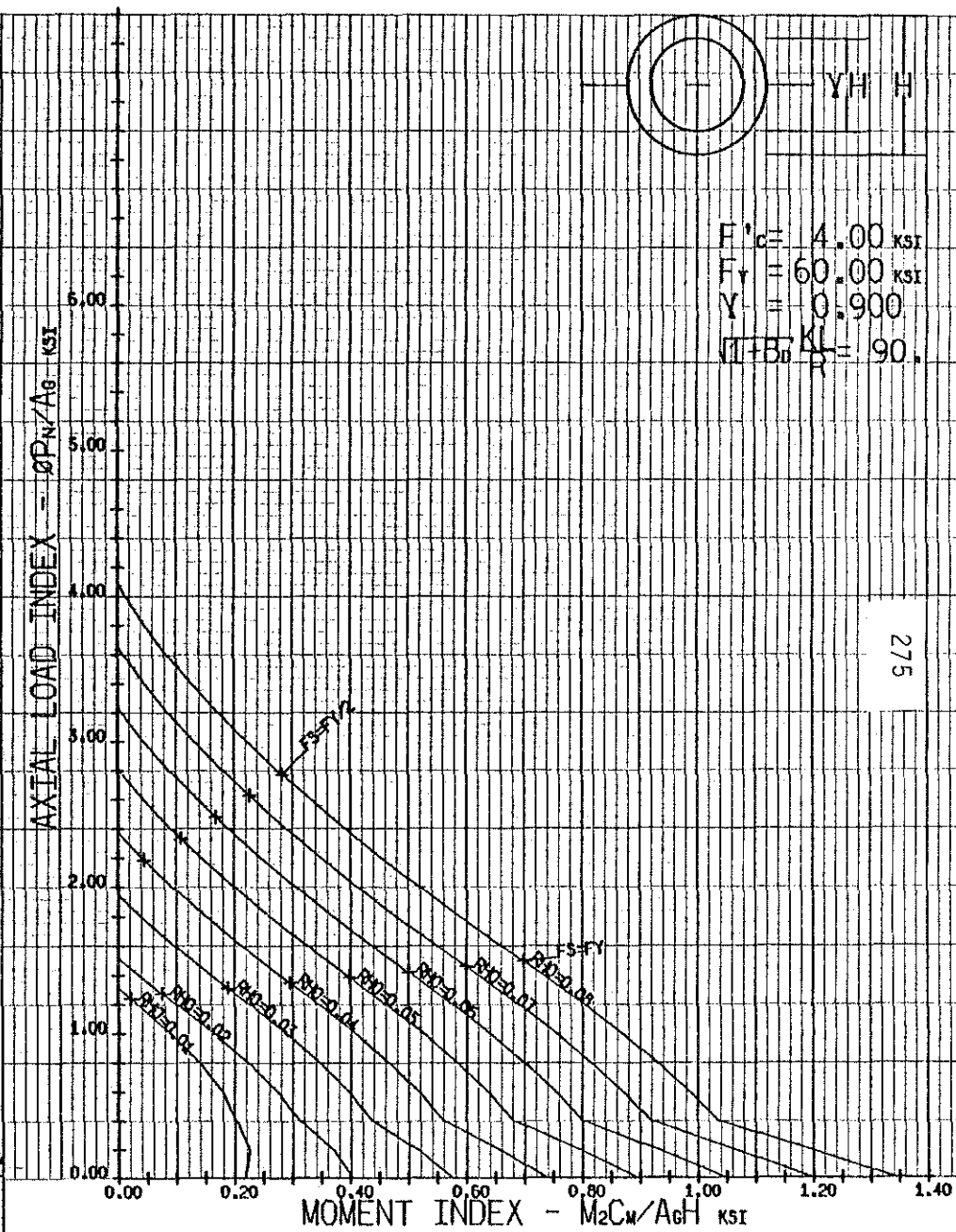


Fig. C4-60.90-90 - Interaction Diagram



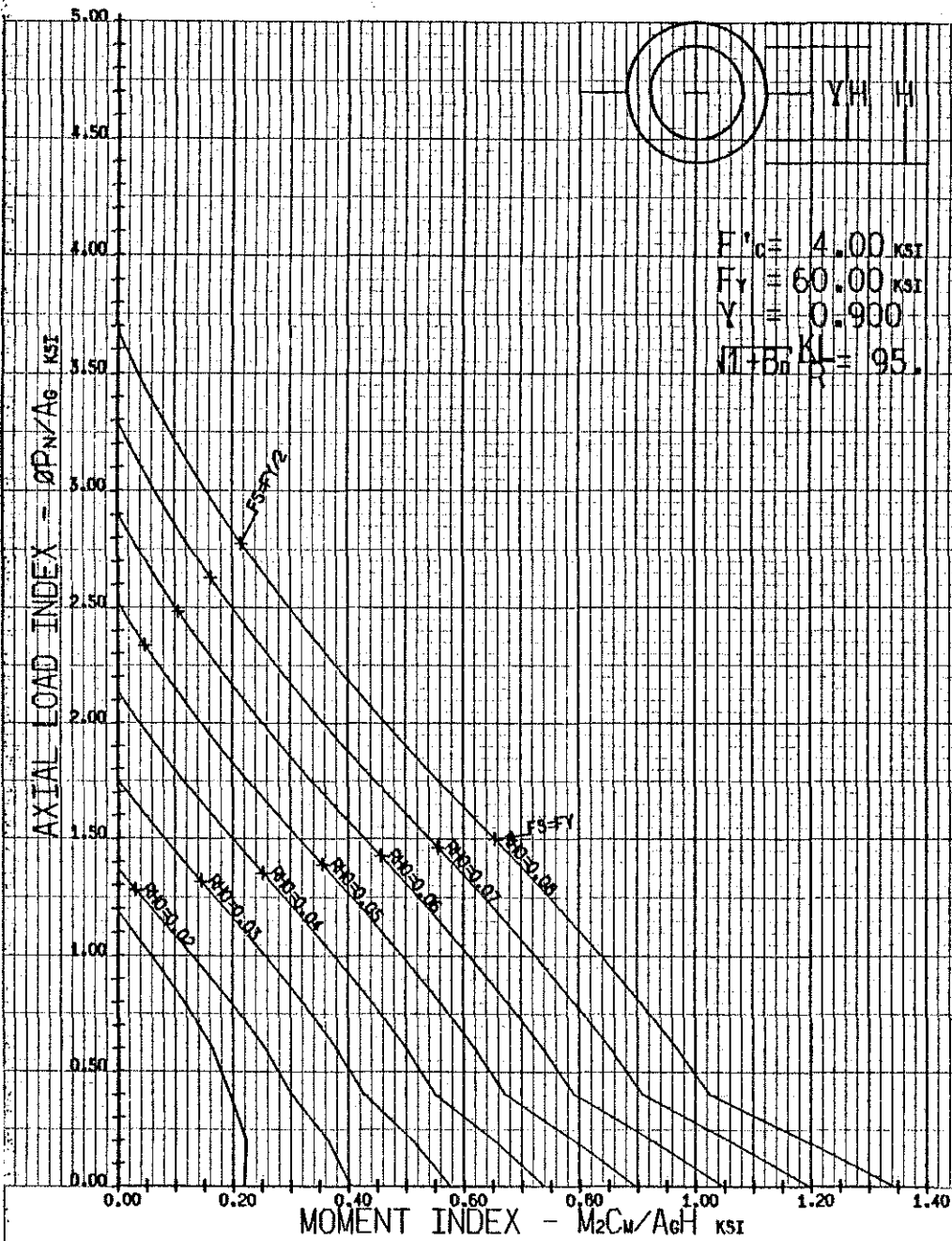


Fig. C4-60.90-95 - Interaction Diagram

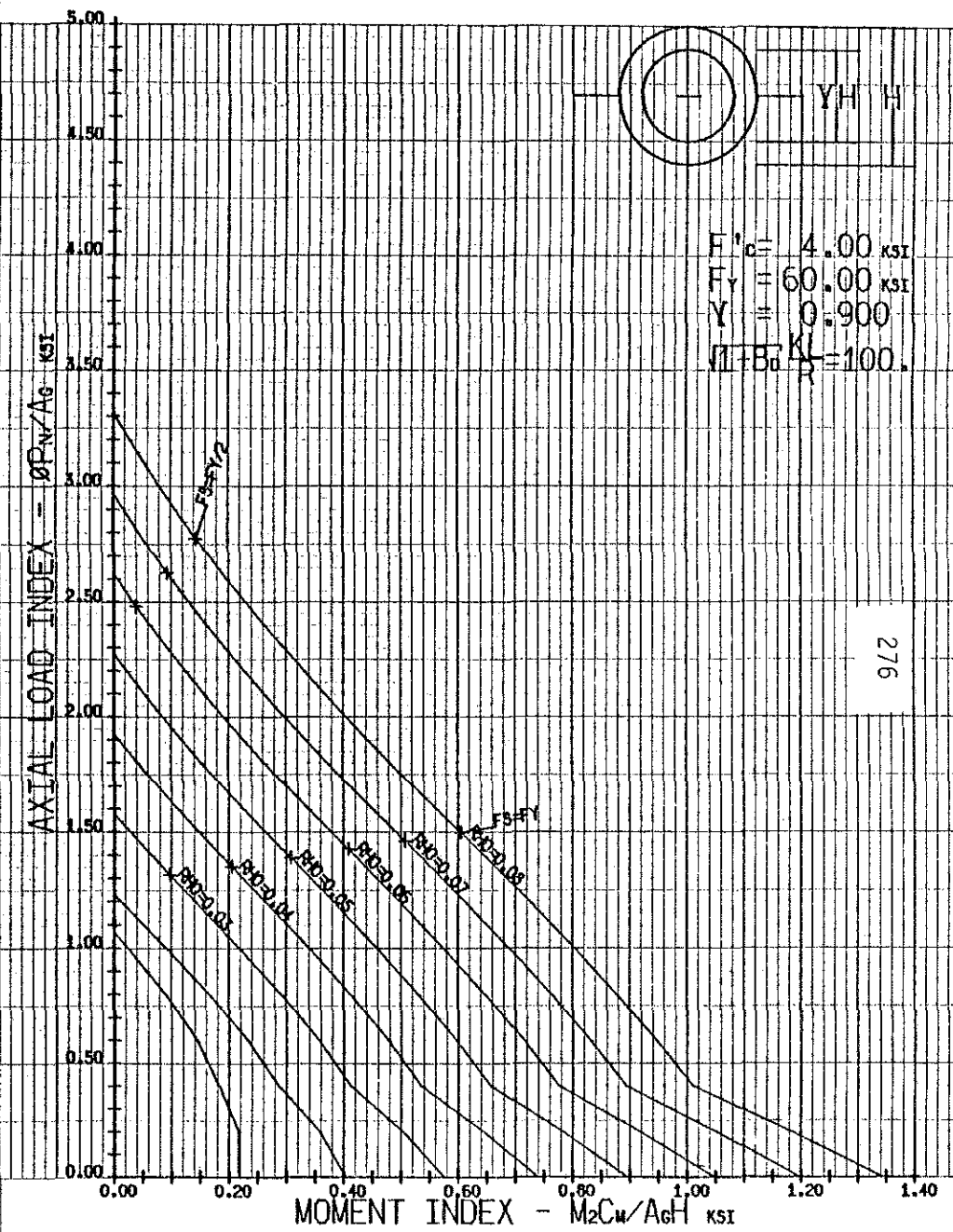


Fig. C4-60.90-100 - Interaction Diagram

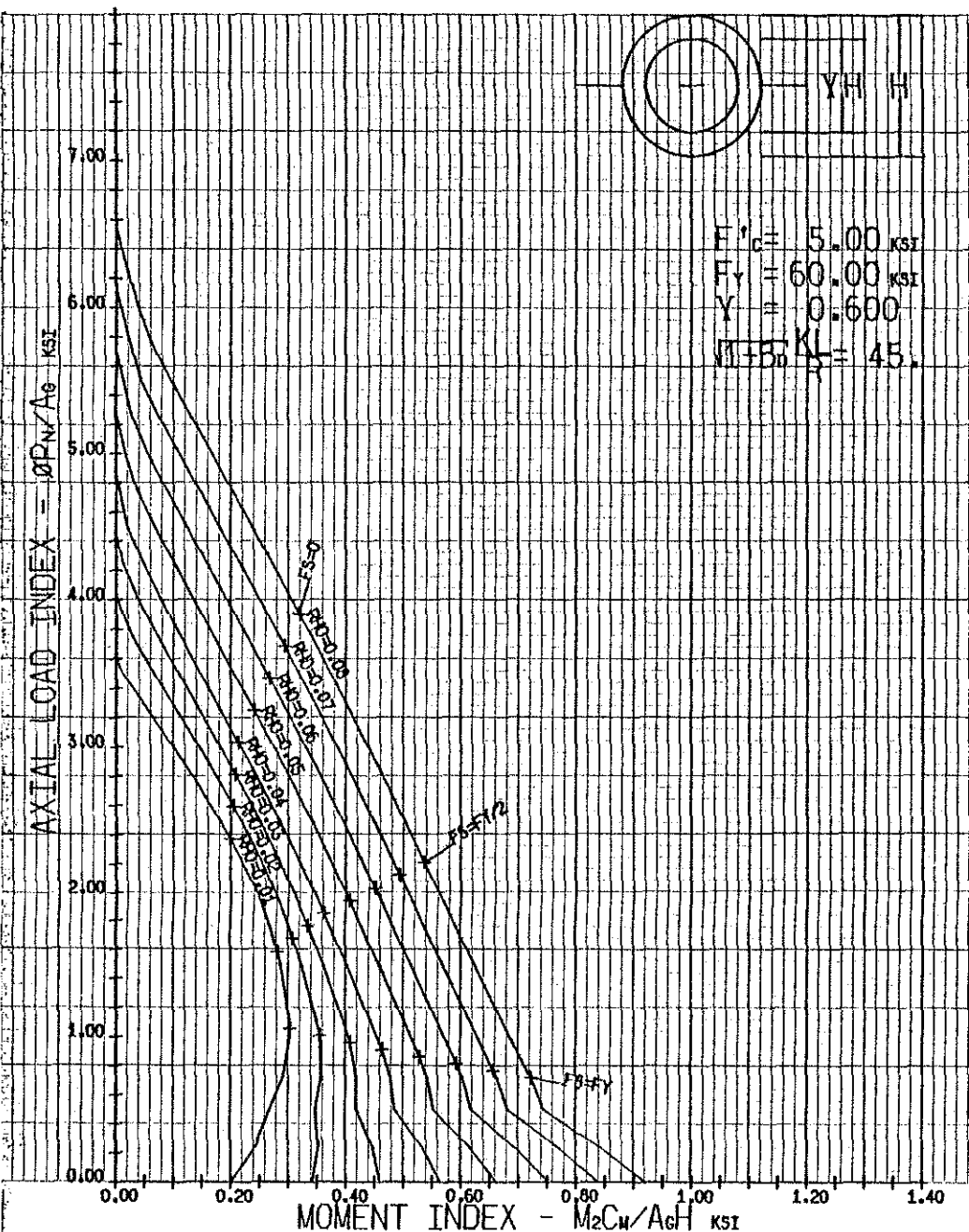


Fig. C5-60.60-45 - Interaction Diagram

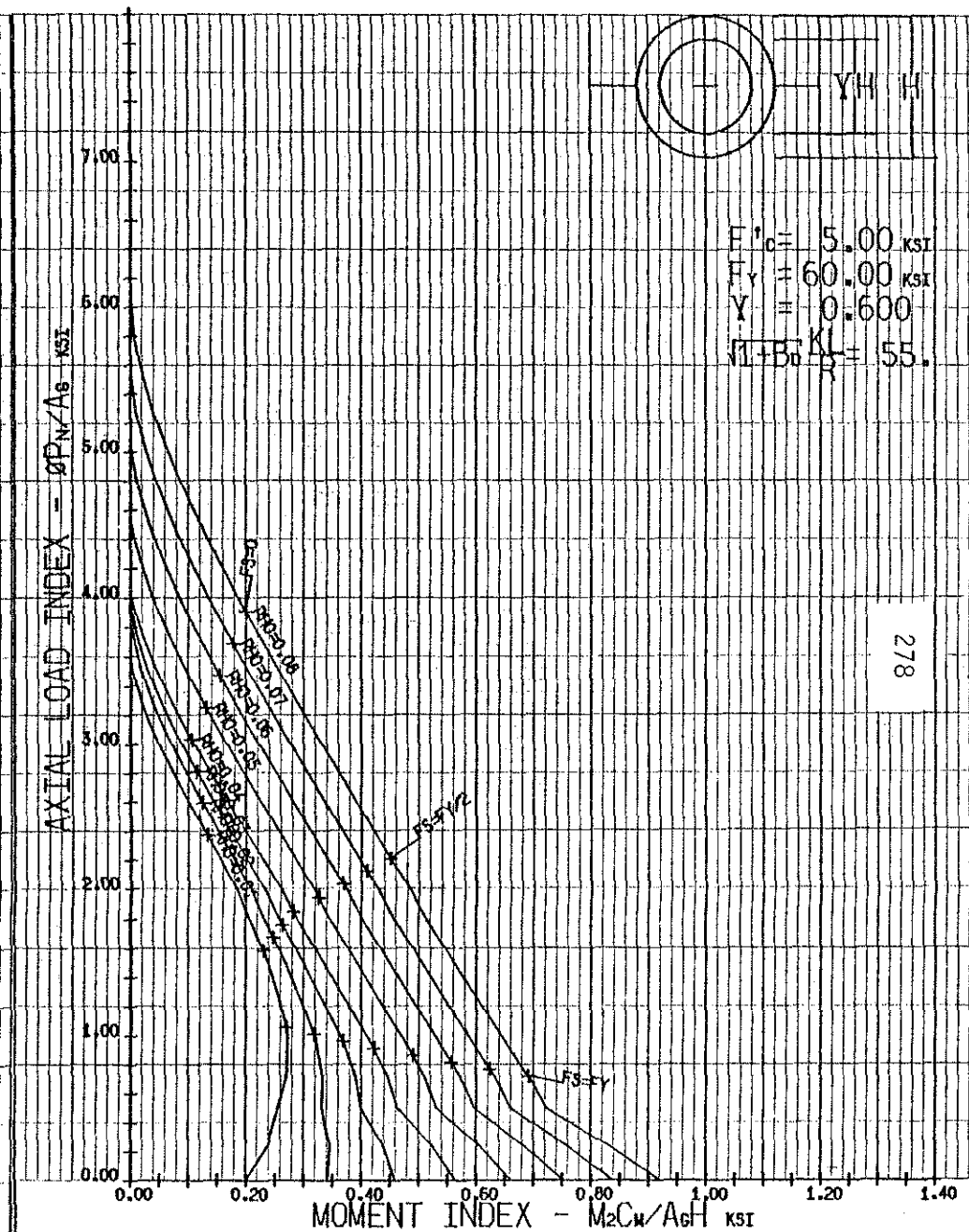
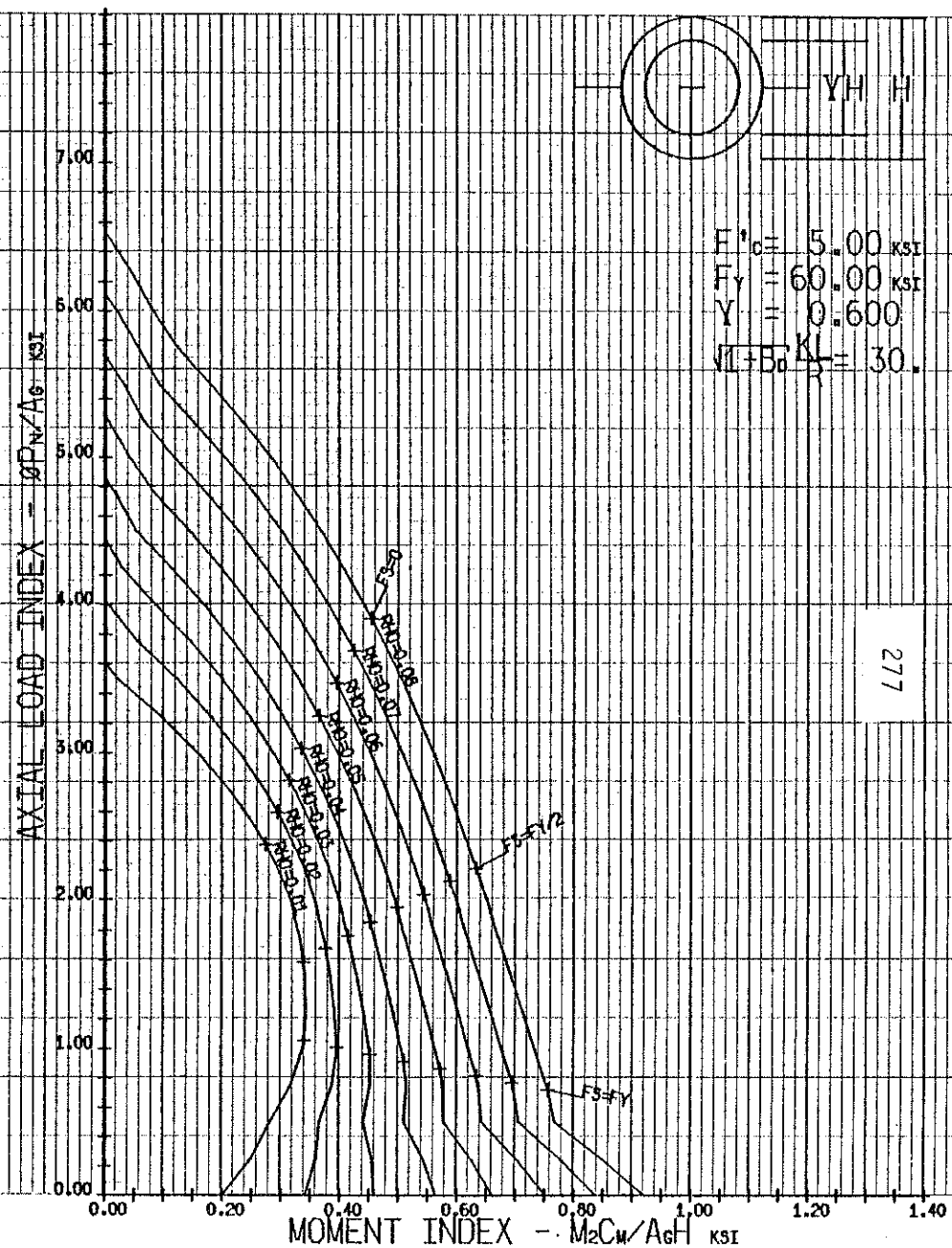
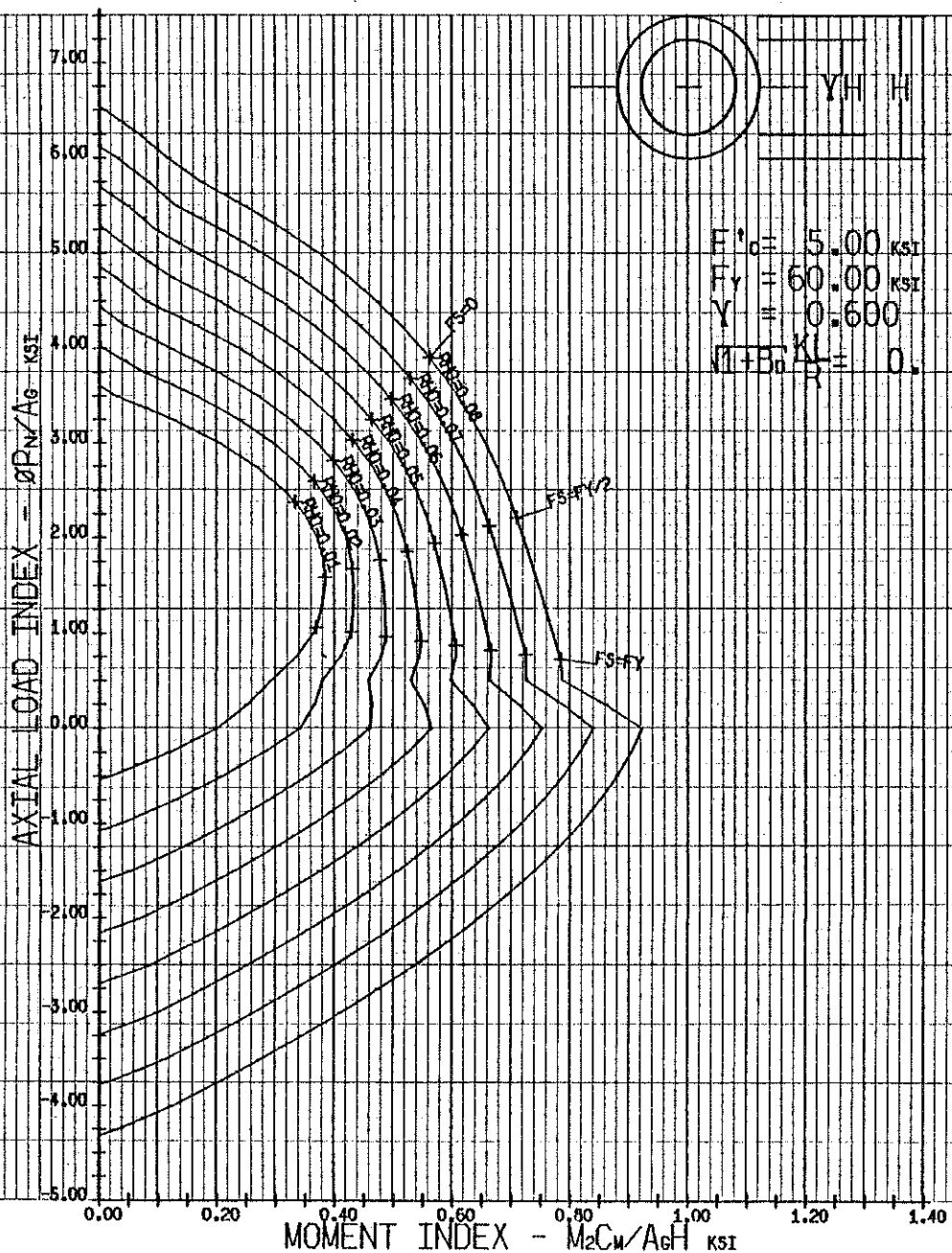


Fig. C5-60.60-55 - Interaction Diagram



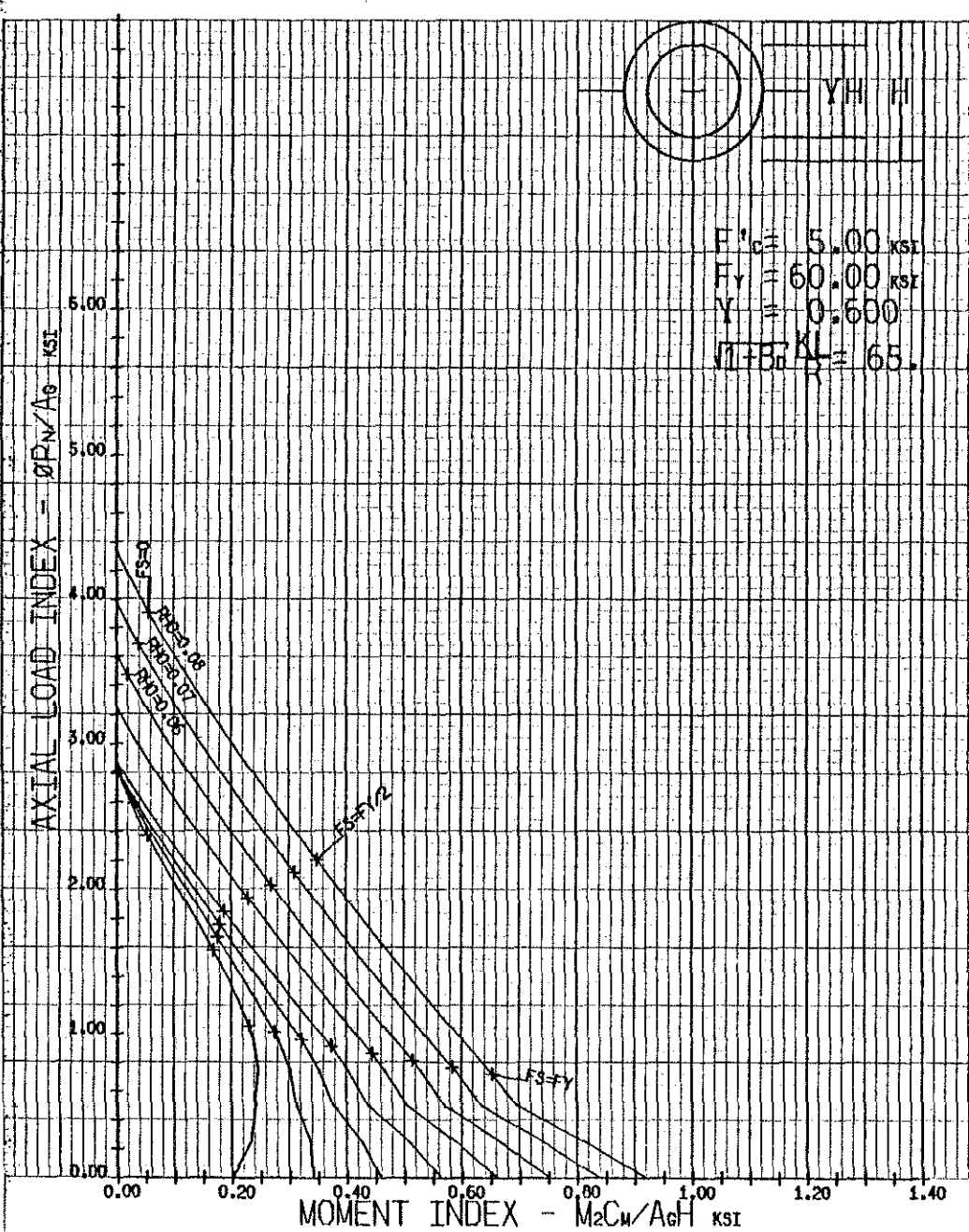


Fig. C5-60.60-65 - Interaction Diagram

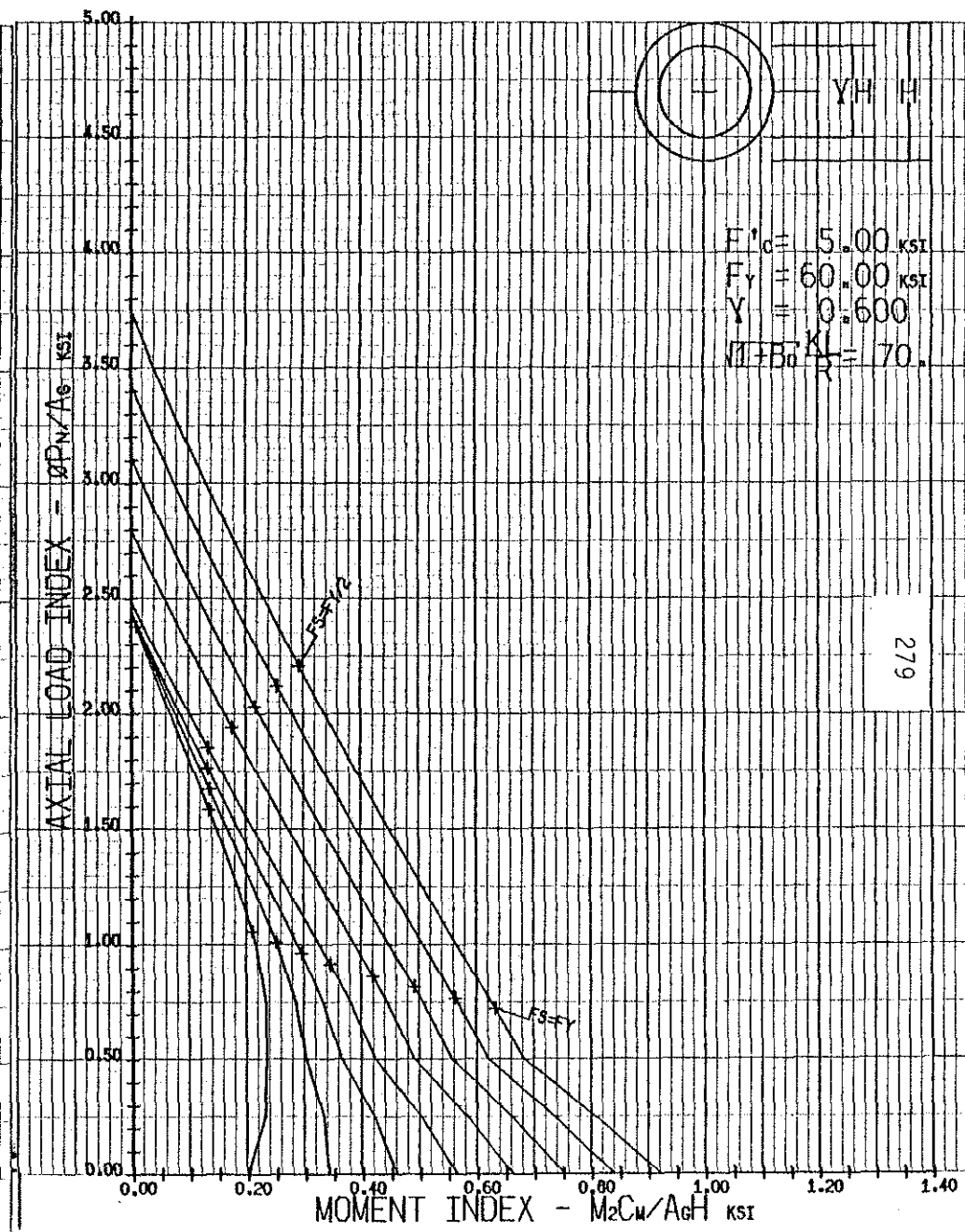


Fig. C5-60.60-70 - Interaction Diagram

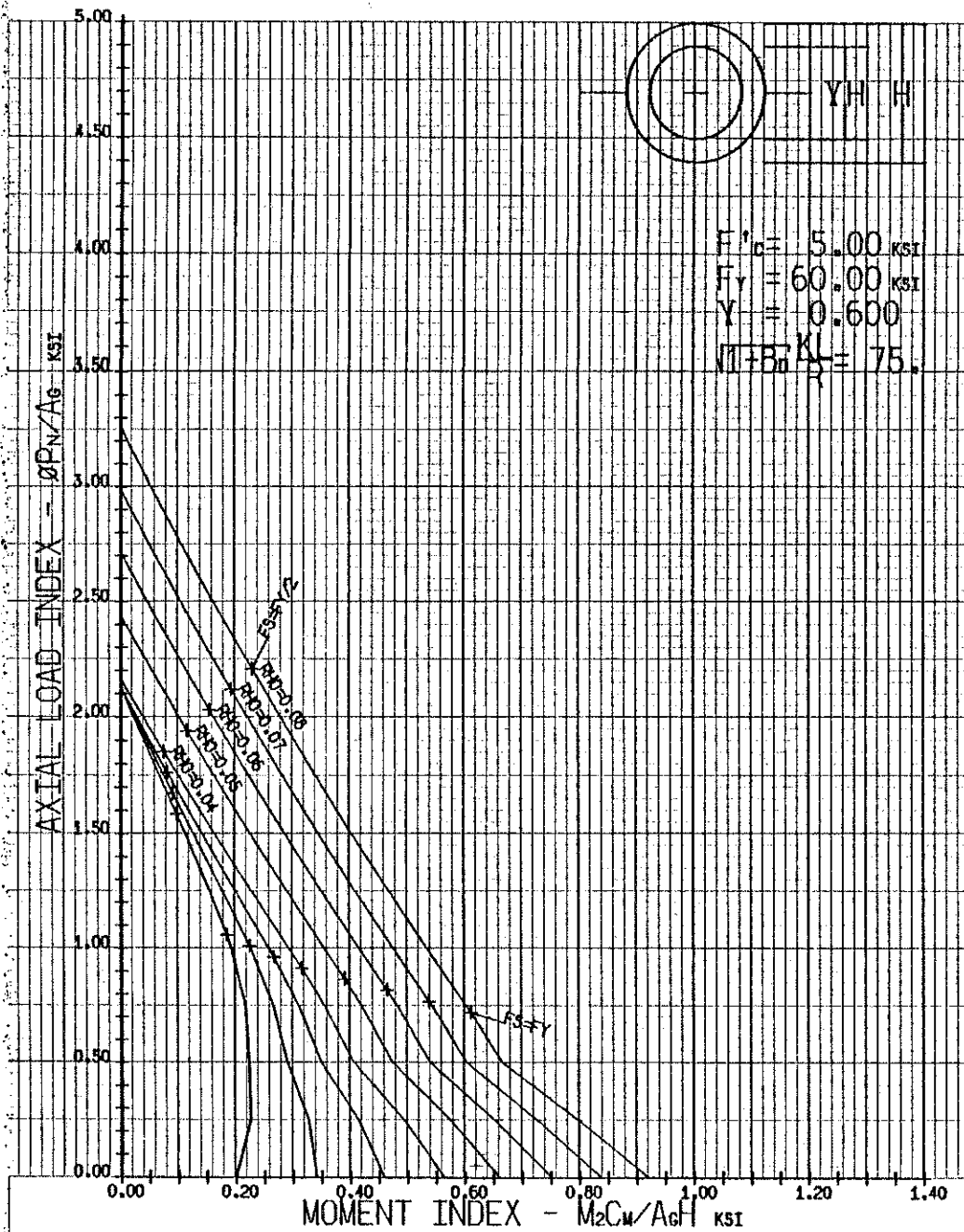


Fig. C5-60.60-75 - Interaction Diagram

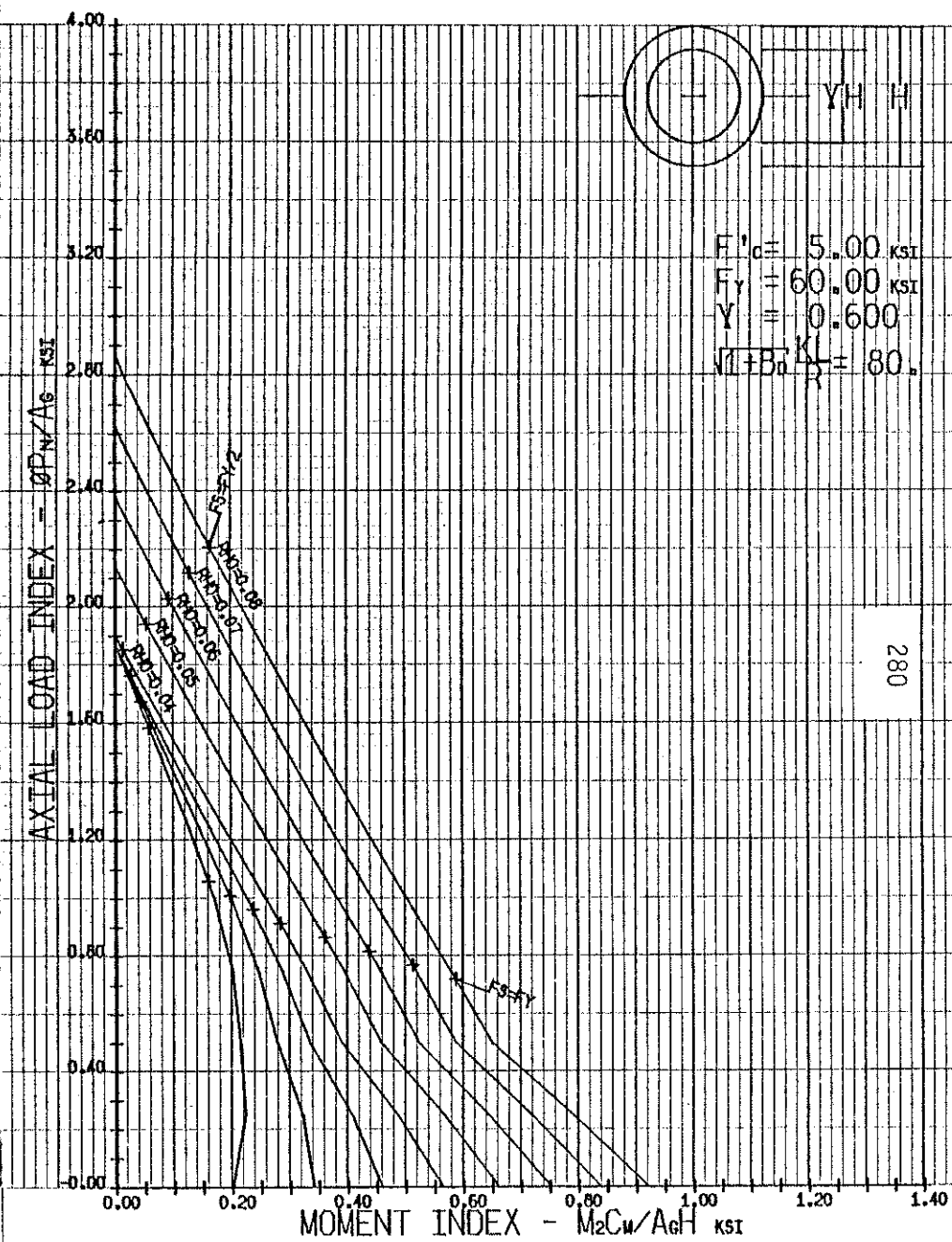


Fig. C5-60.60-80 - Interaction Diagram



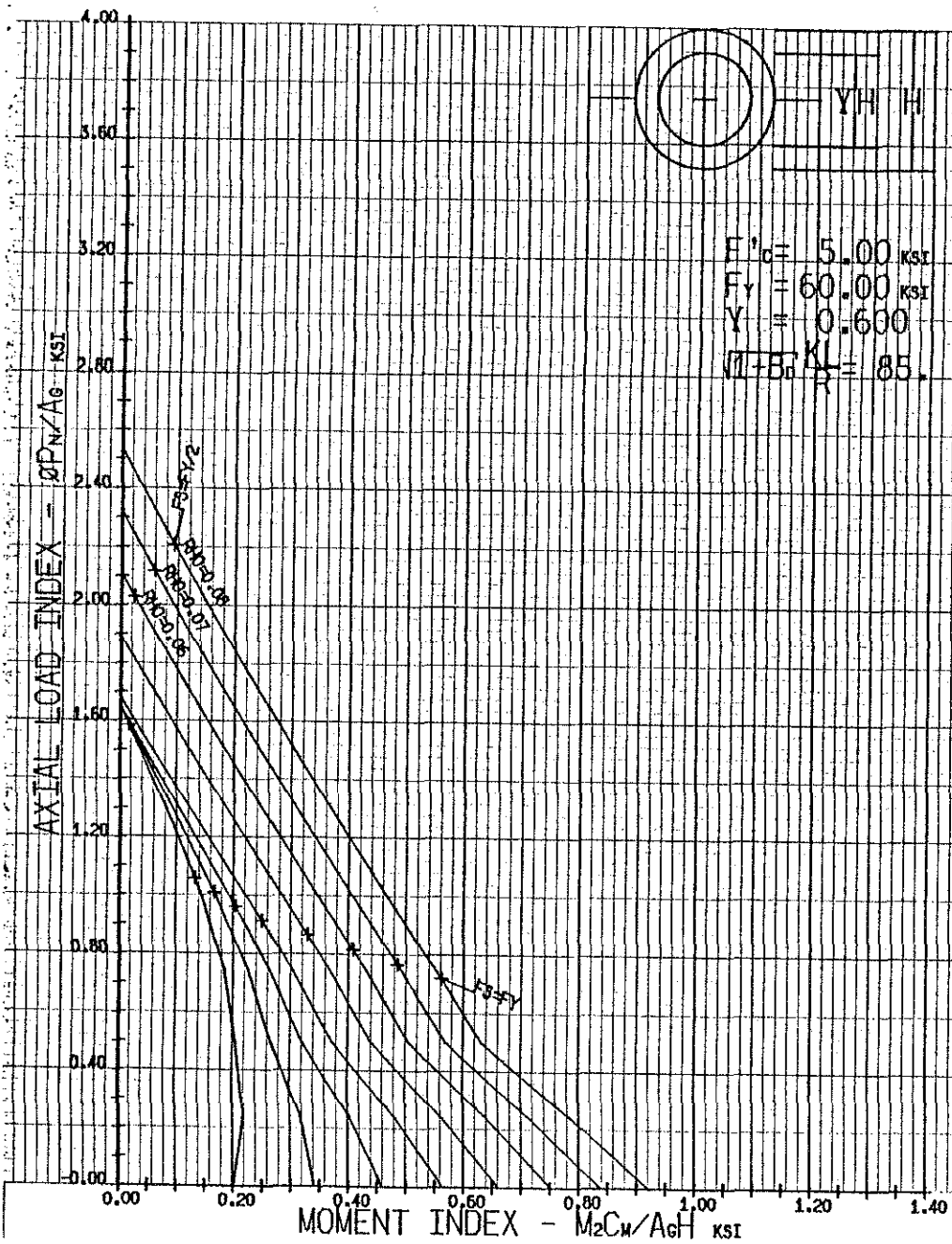


Fig. C5-60.60-85 - Interaction Diagram

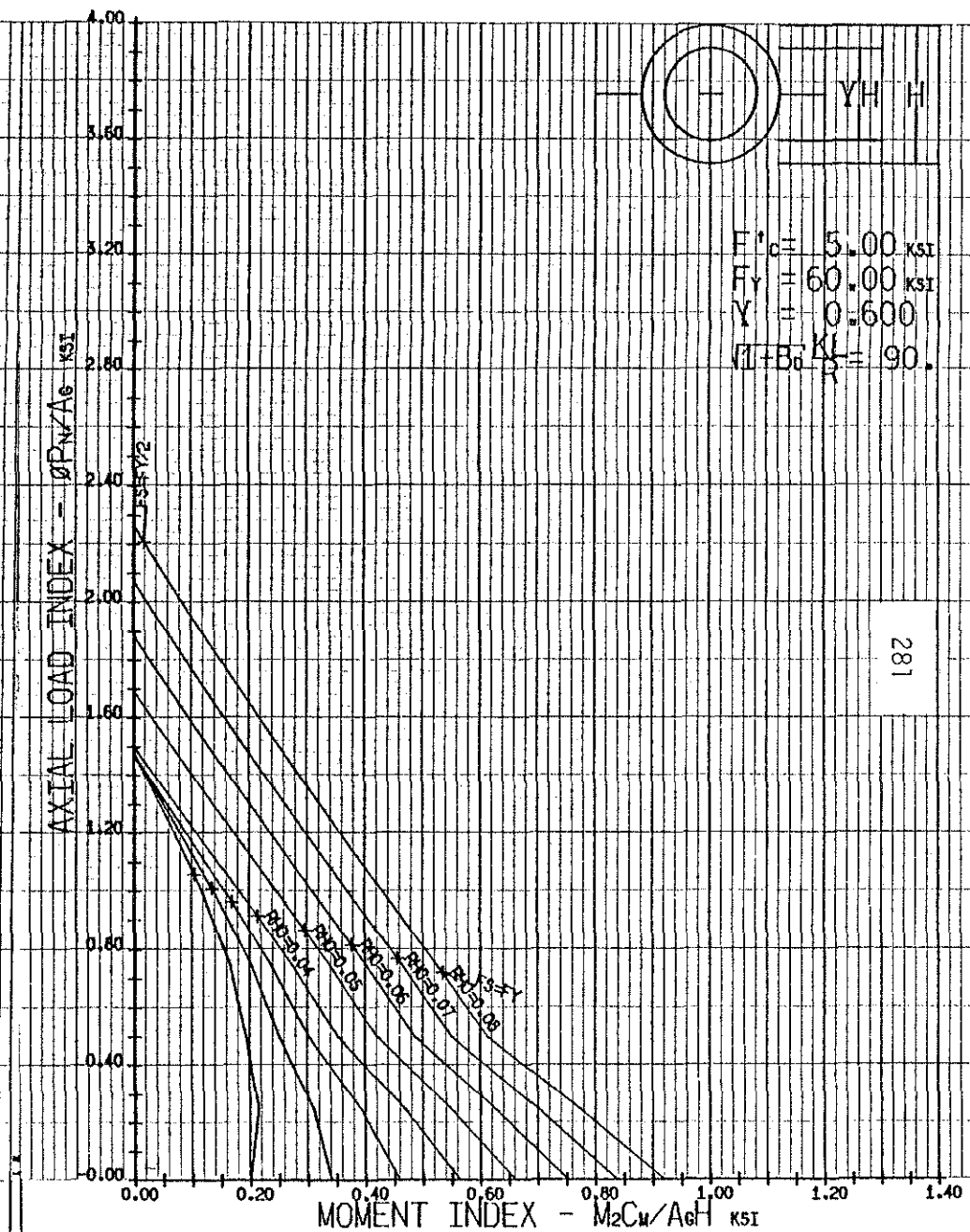


Fig. C5-60.60-90 - Interaction Diagram



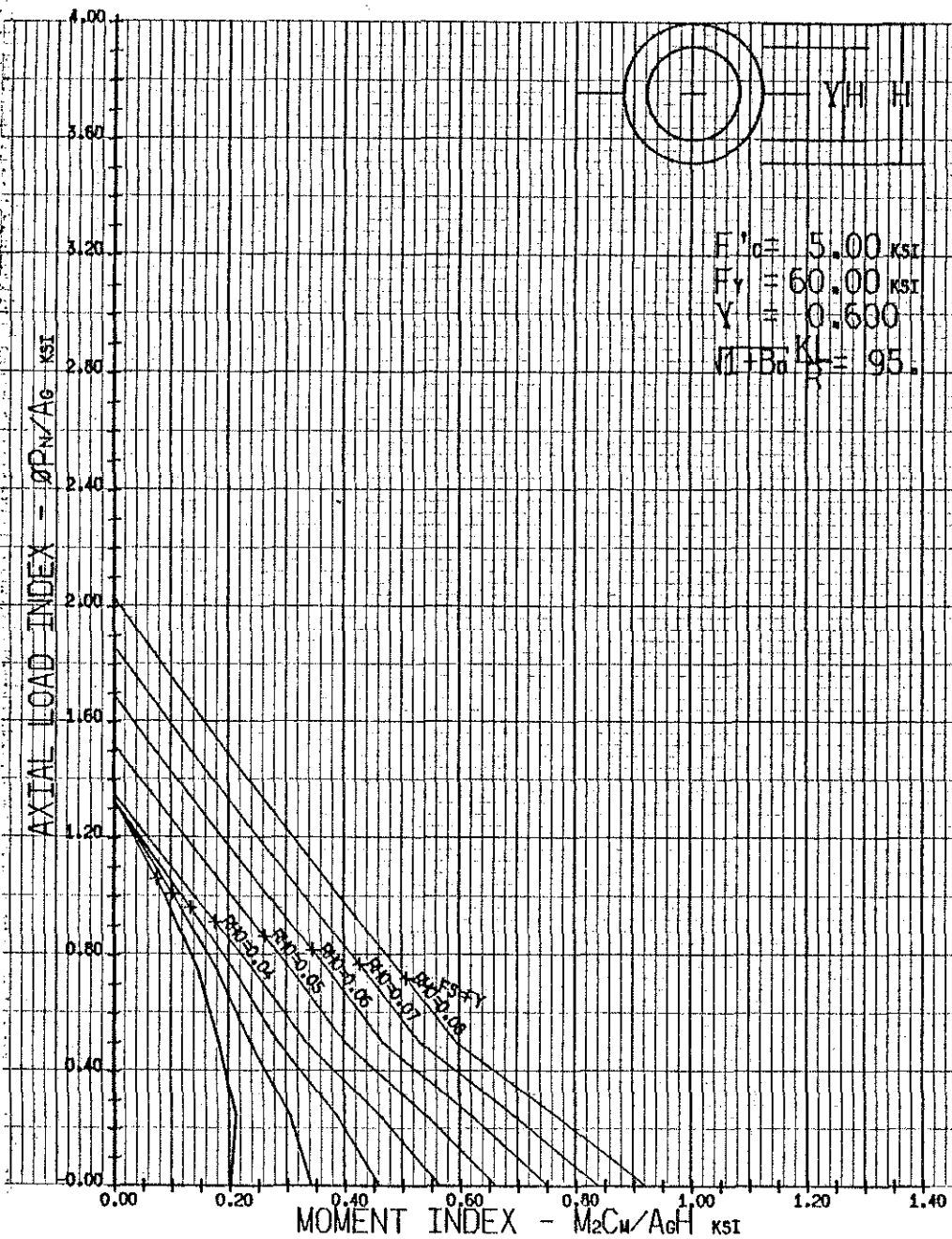


Fig. C5-60.60-95 - Interaction Diagram

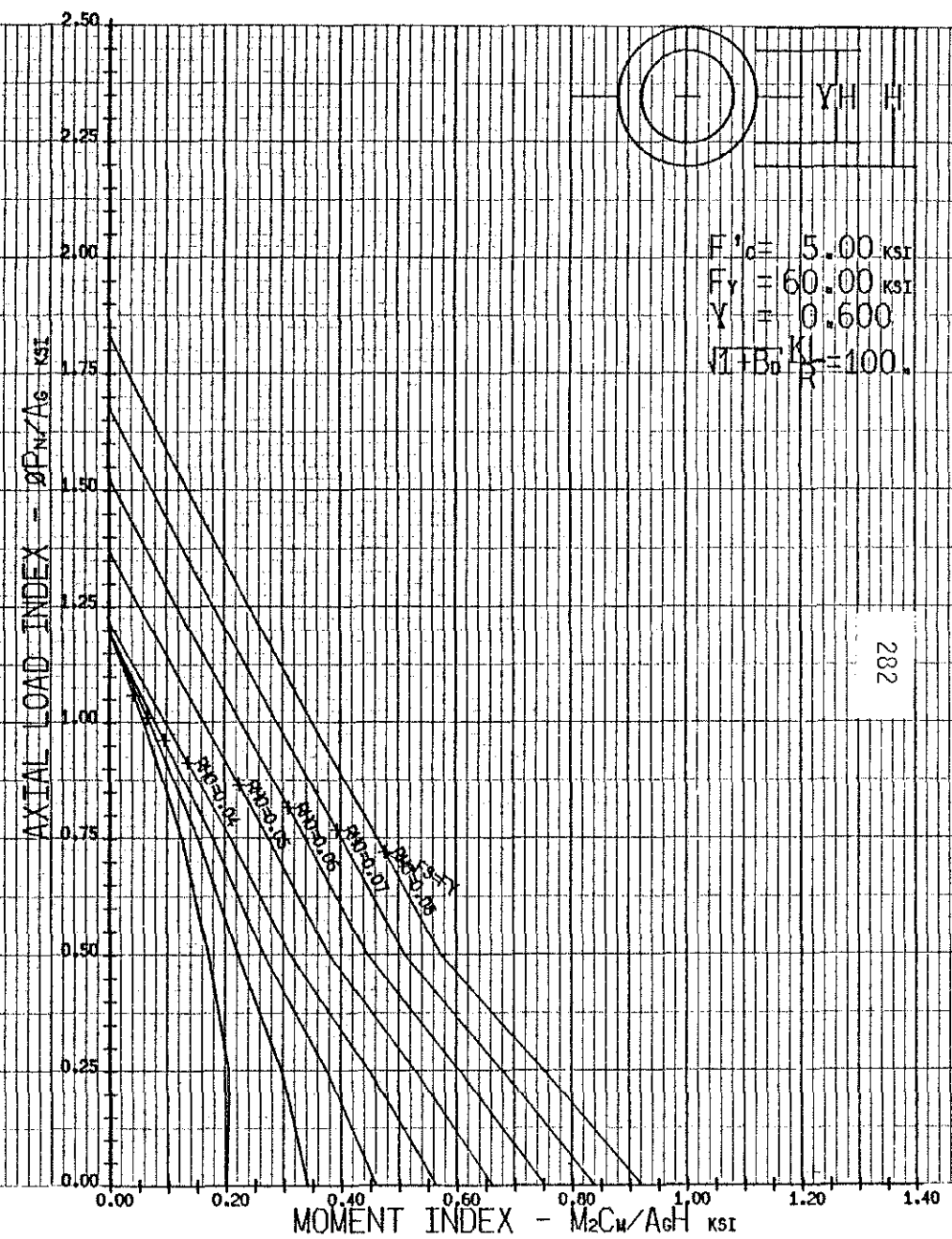


Fig. C5-60.60-100 - Interaction Diagram

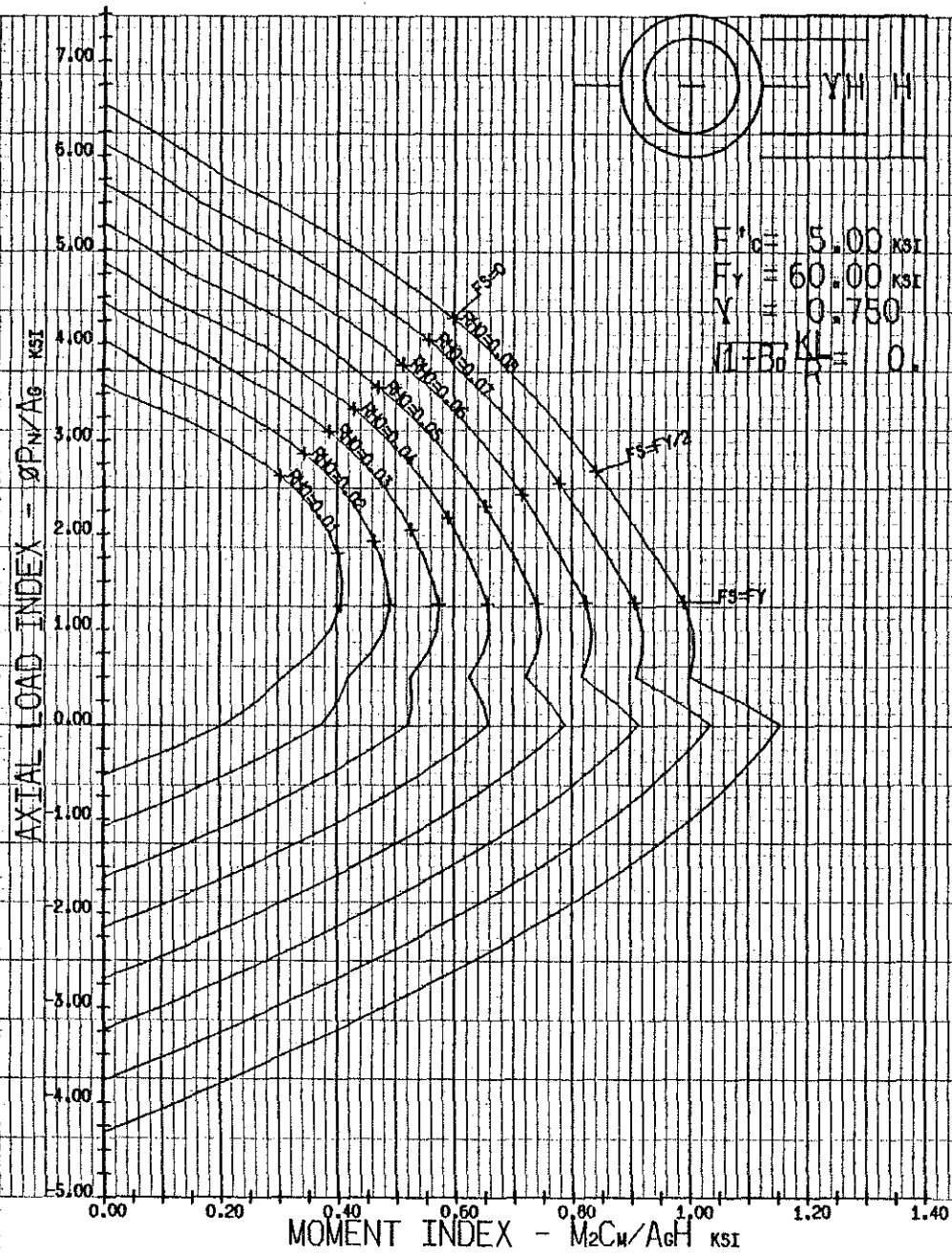


Fig. C5-60.75-0 - Interaction Diagram

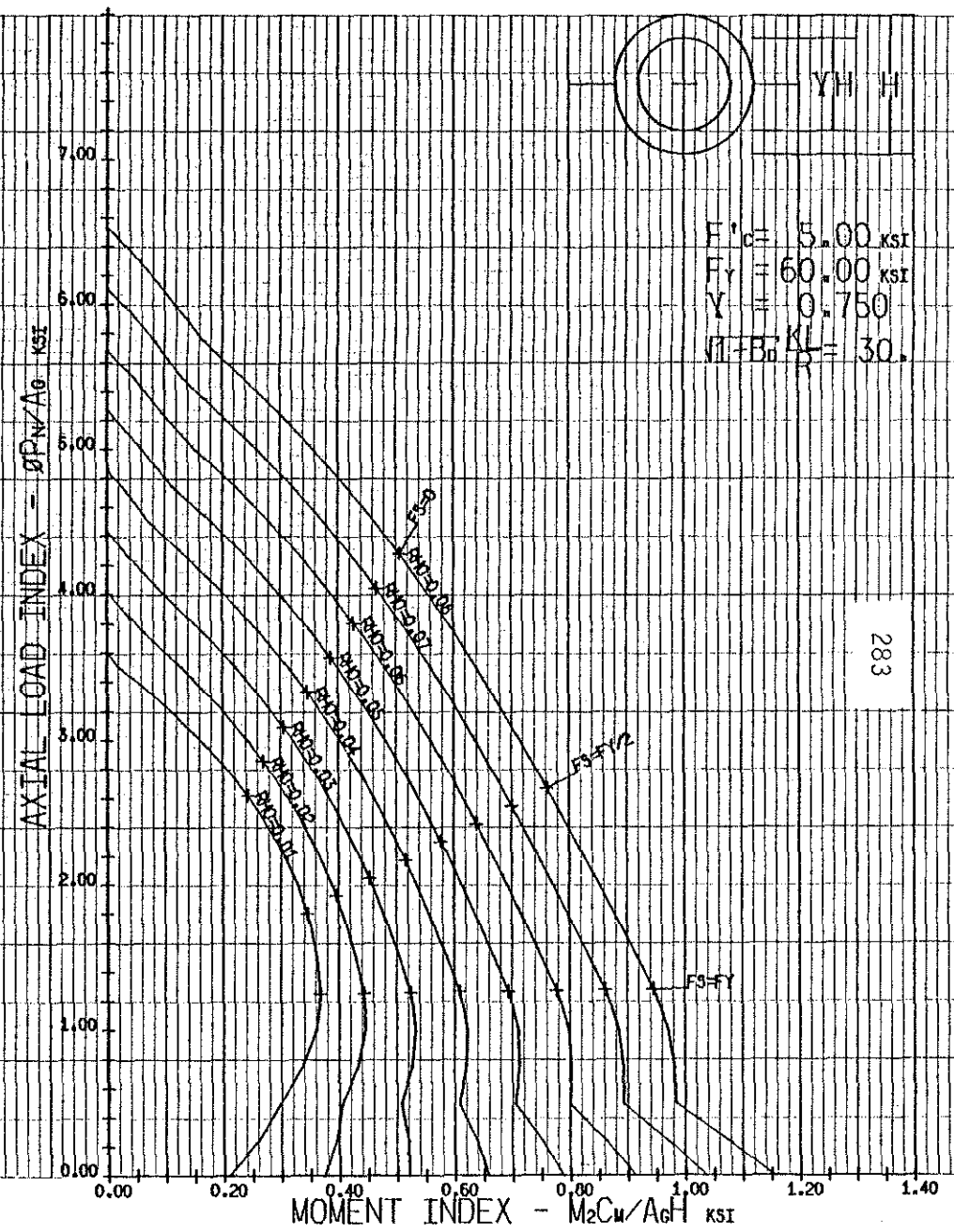


Fig. C5-60.75-30 - Interaction Diagram

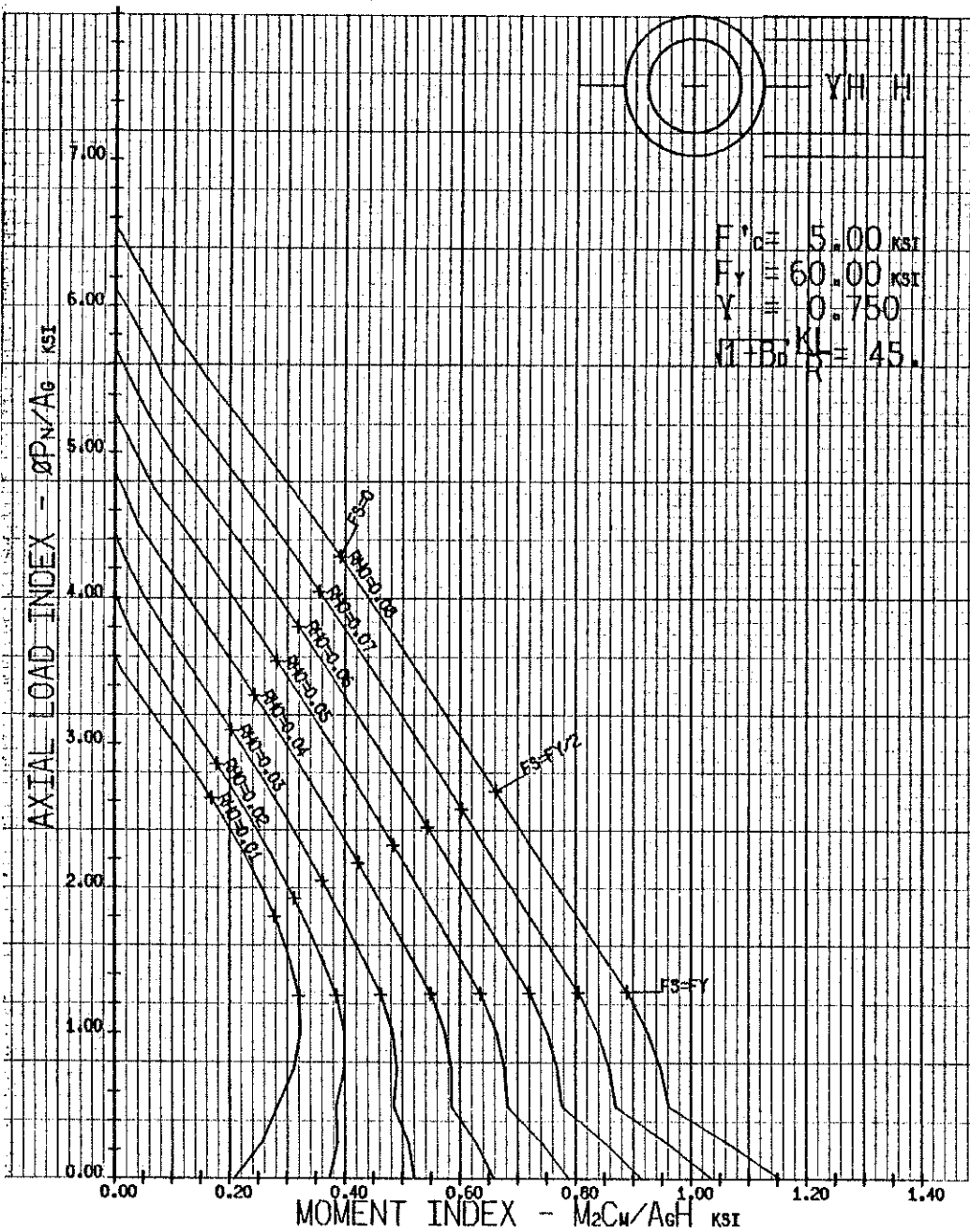


Fig. C5-60.75-45 - Interaction Diagram

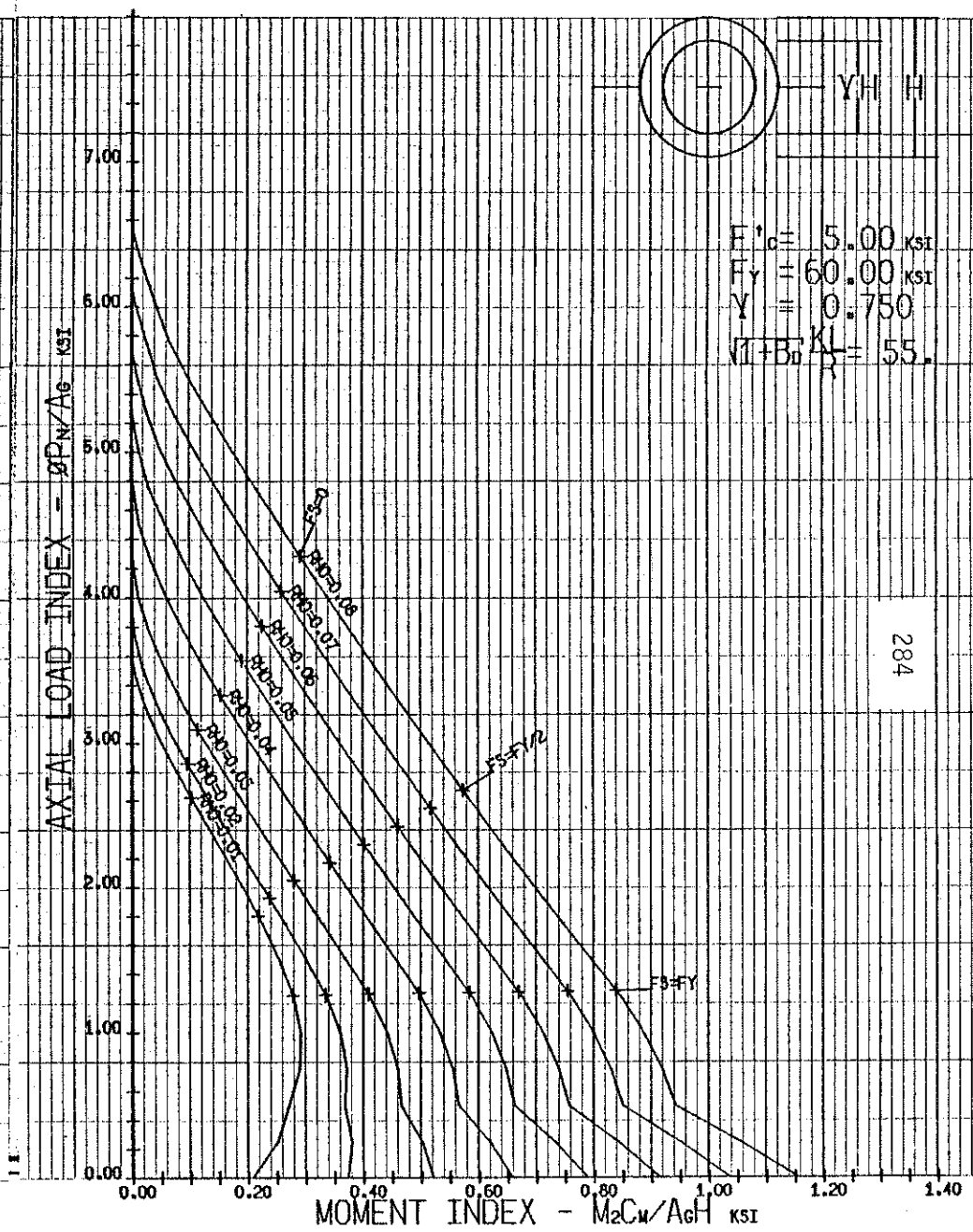


Fig. C5-60.75-55 - Interaction Diagram



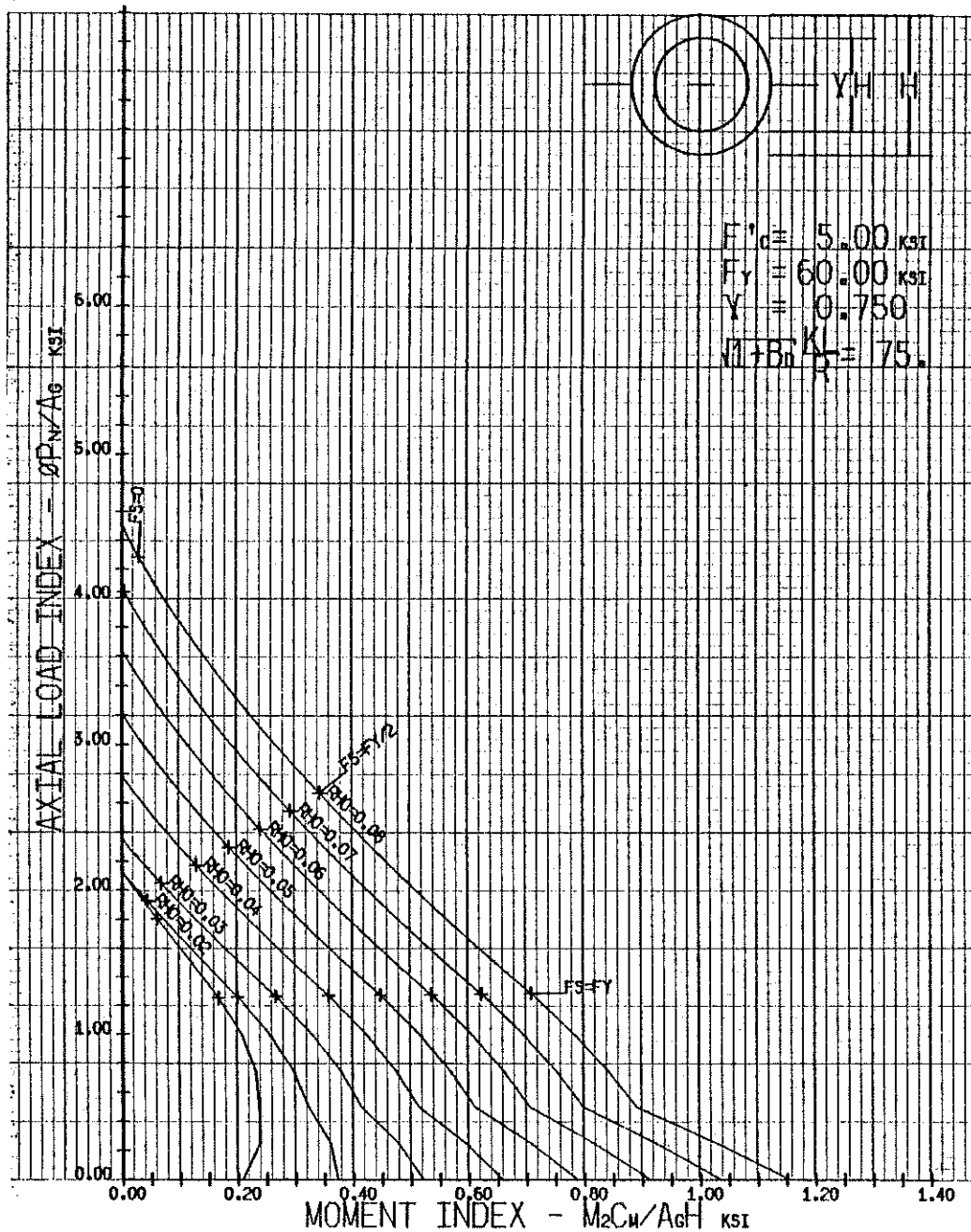


Fig. C5-60.75-75 - Interaction Diagram

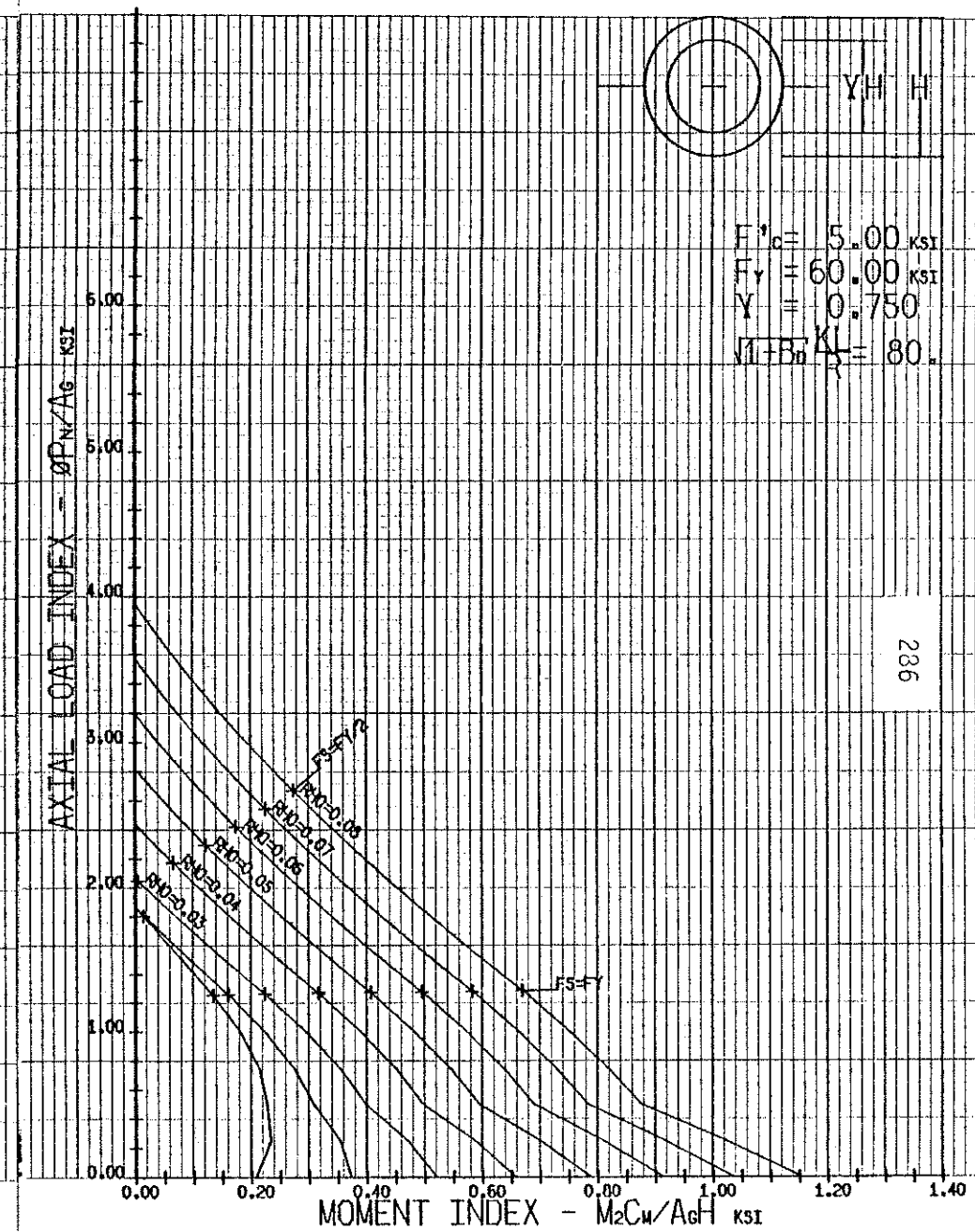


Fig. C5-60.75-80 - Interaction Diagram



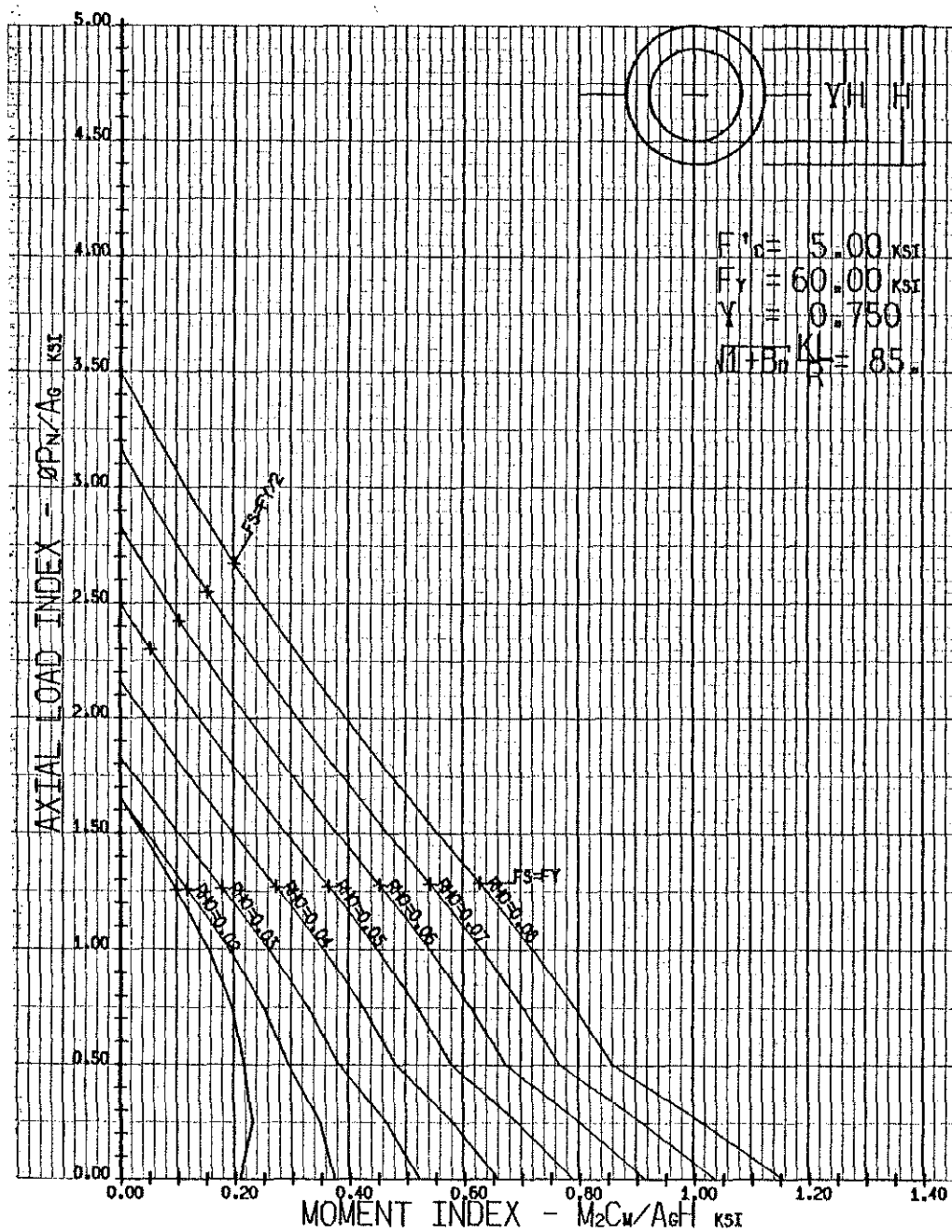


Fig. C5-60.75-85 - Interaction Diagram

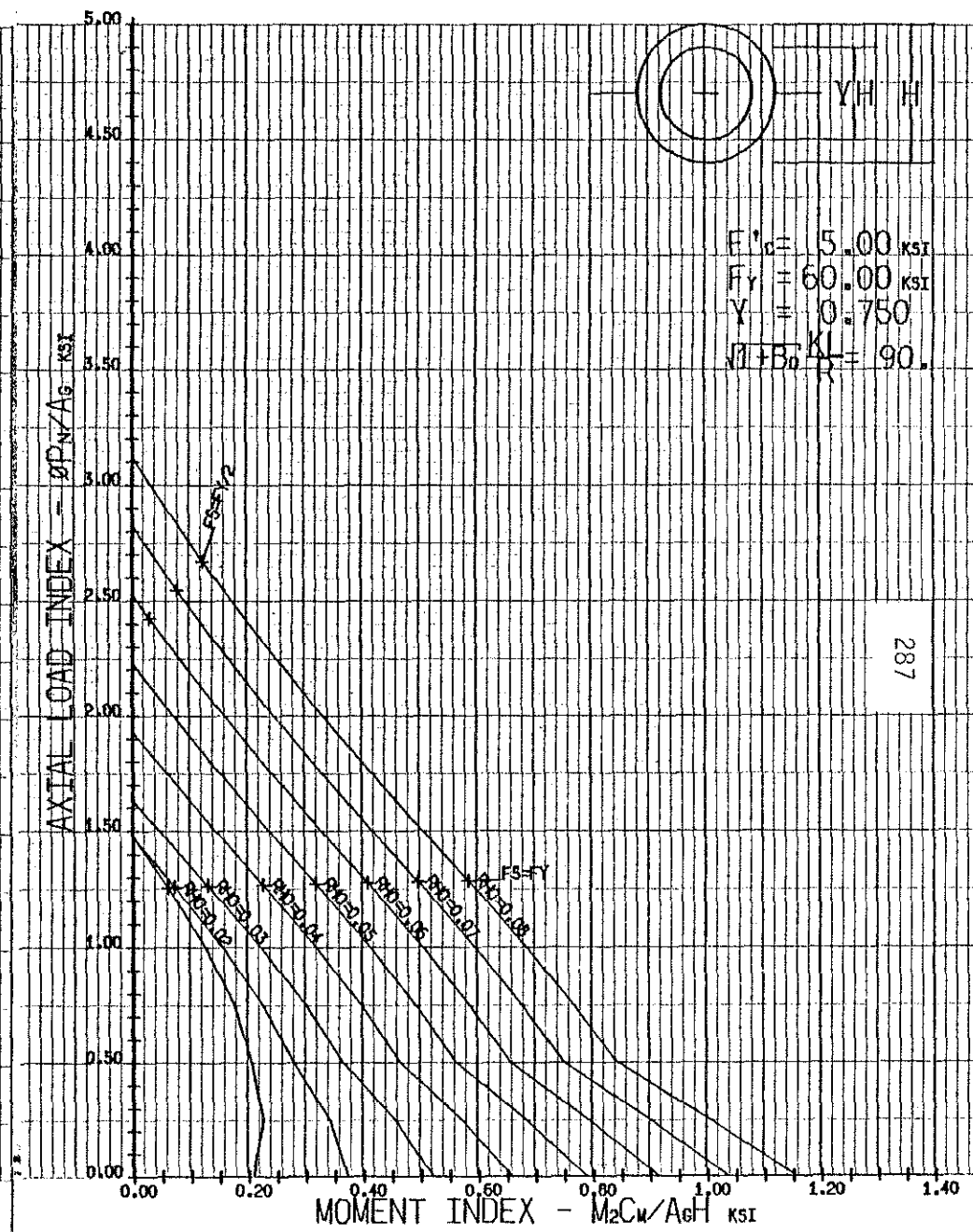


Fig. C5-60.75-90 - Interaction Diagram



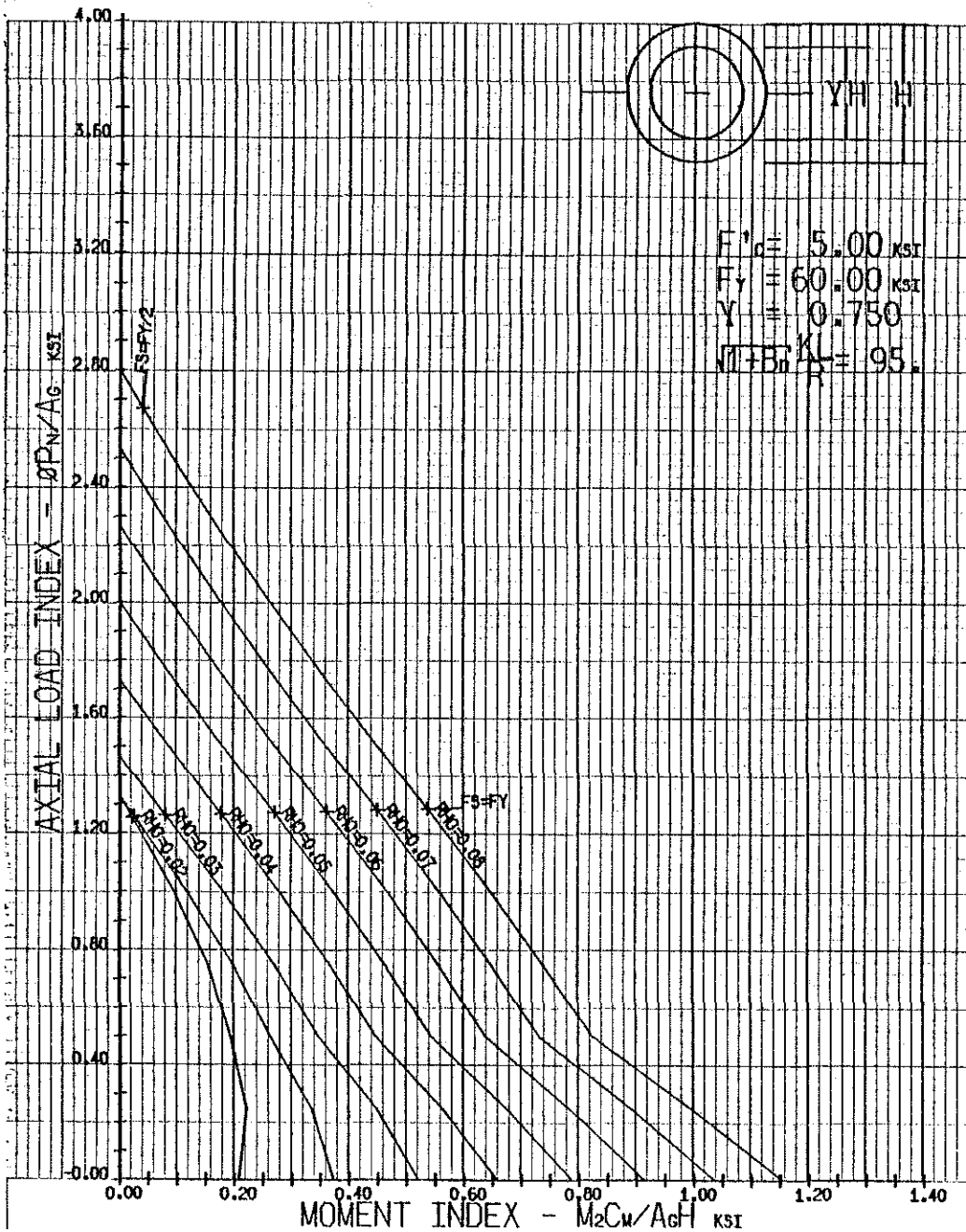


Fig. C5-60.75-95 - Interaction Diagram

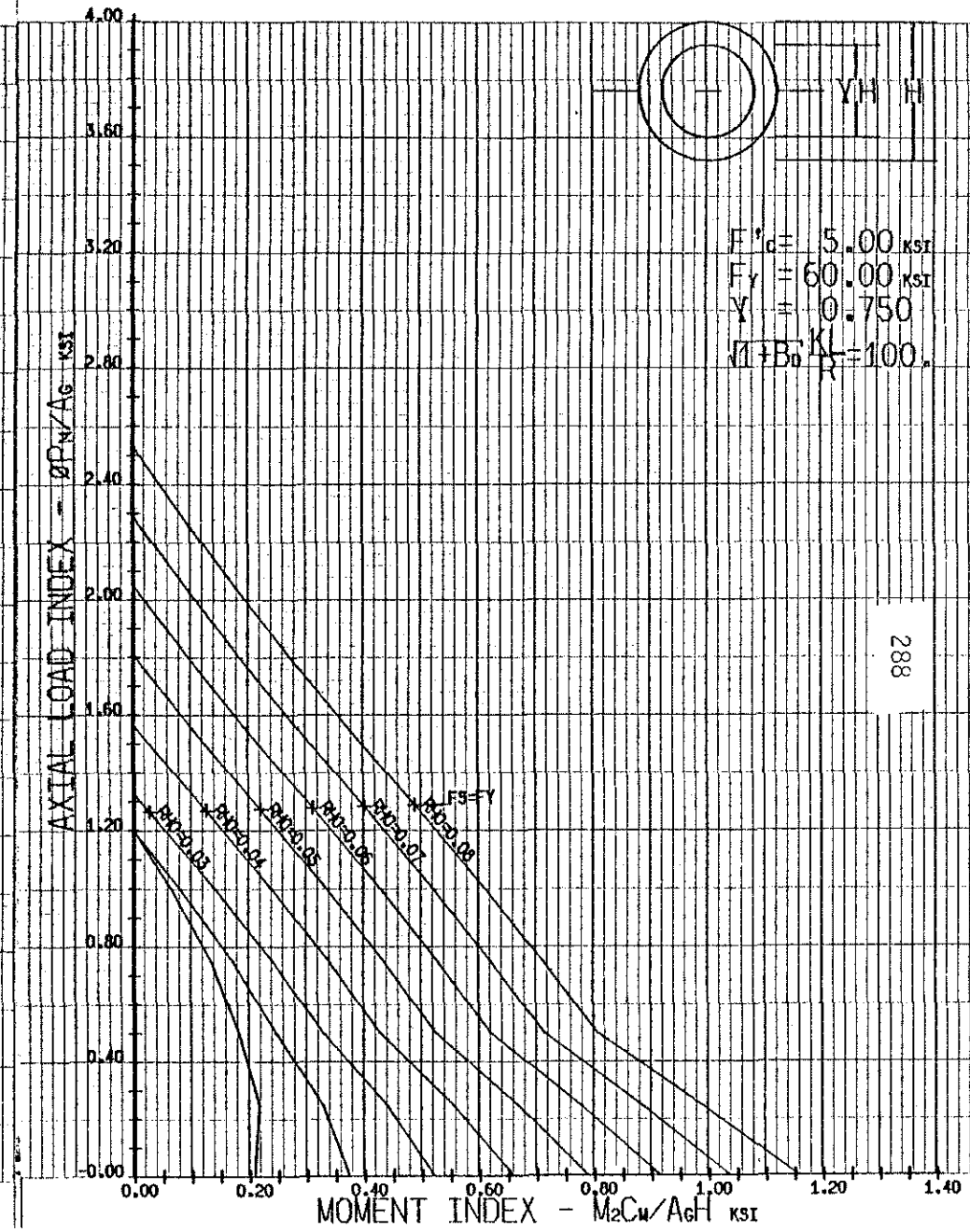


Fig. C5-60.75-100 - Interaction Diagram

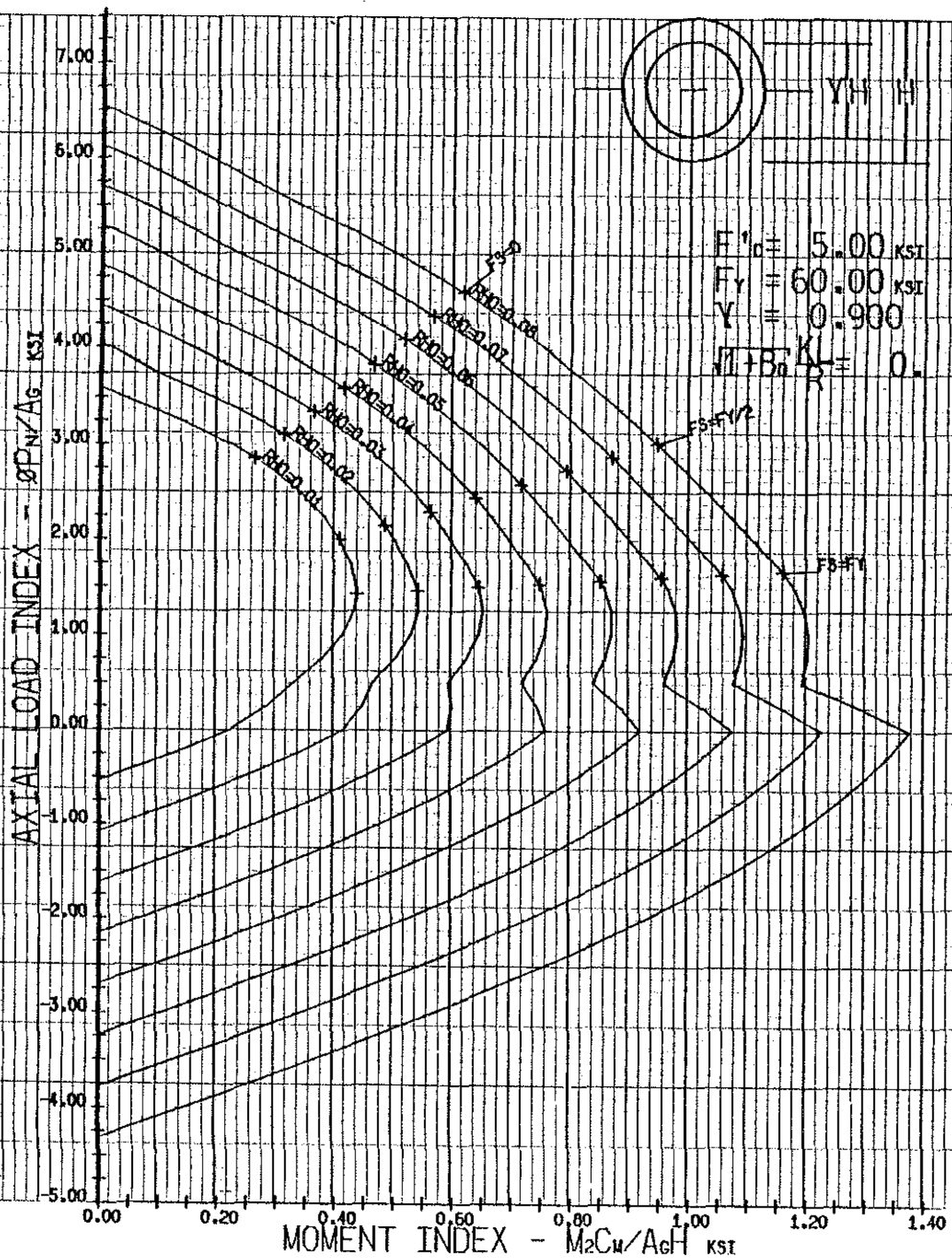


Fig. C5-60.90-0 - Interaction Diagram

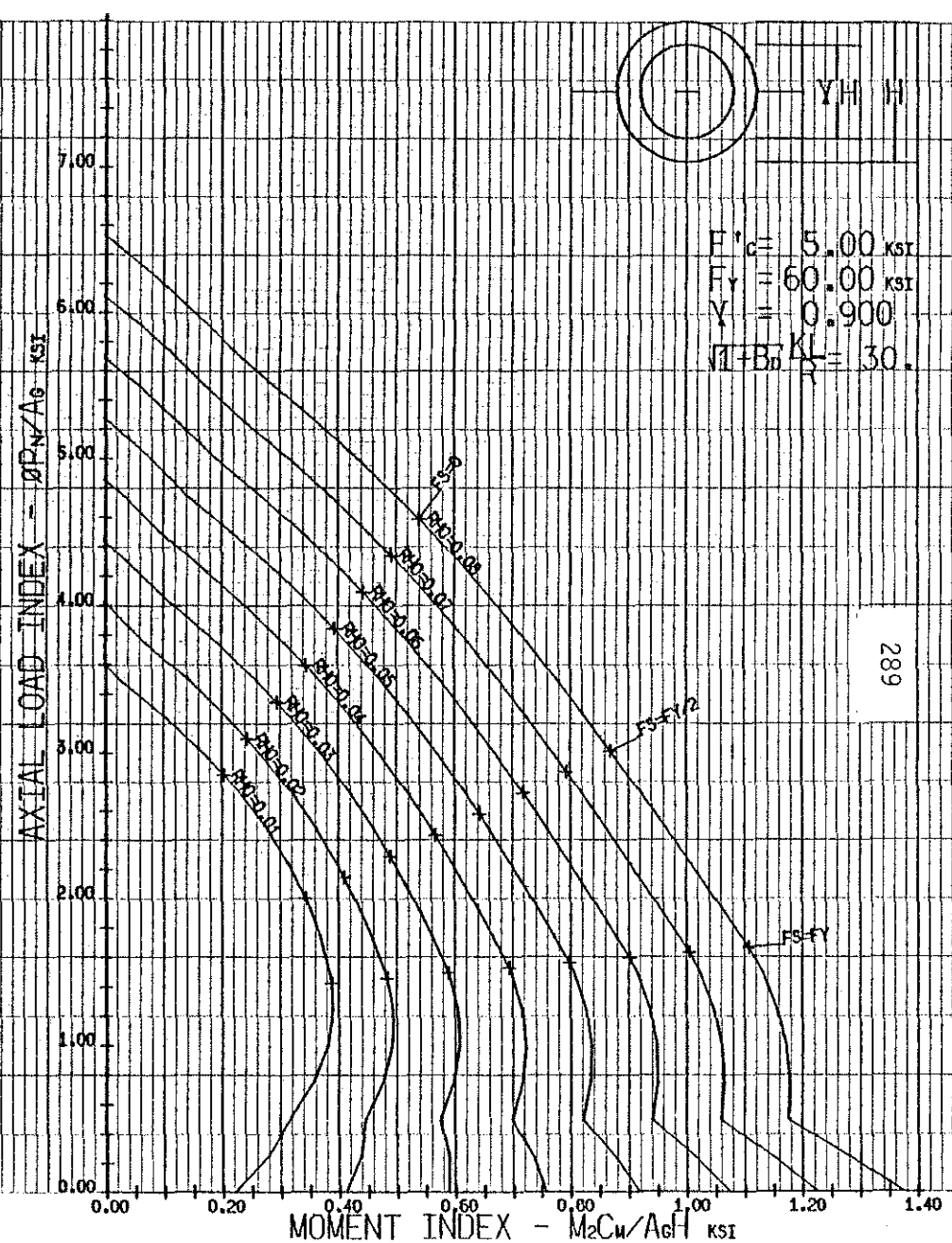


Fig. C5-60.90-30 - Interaction Diagram

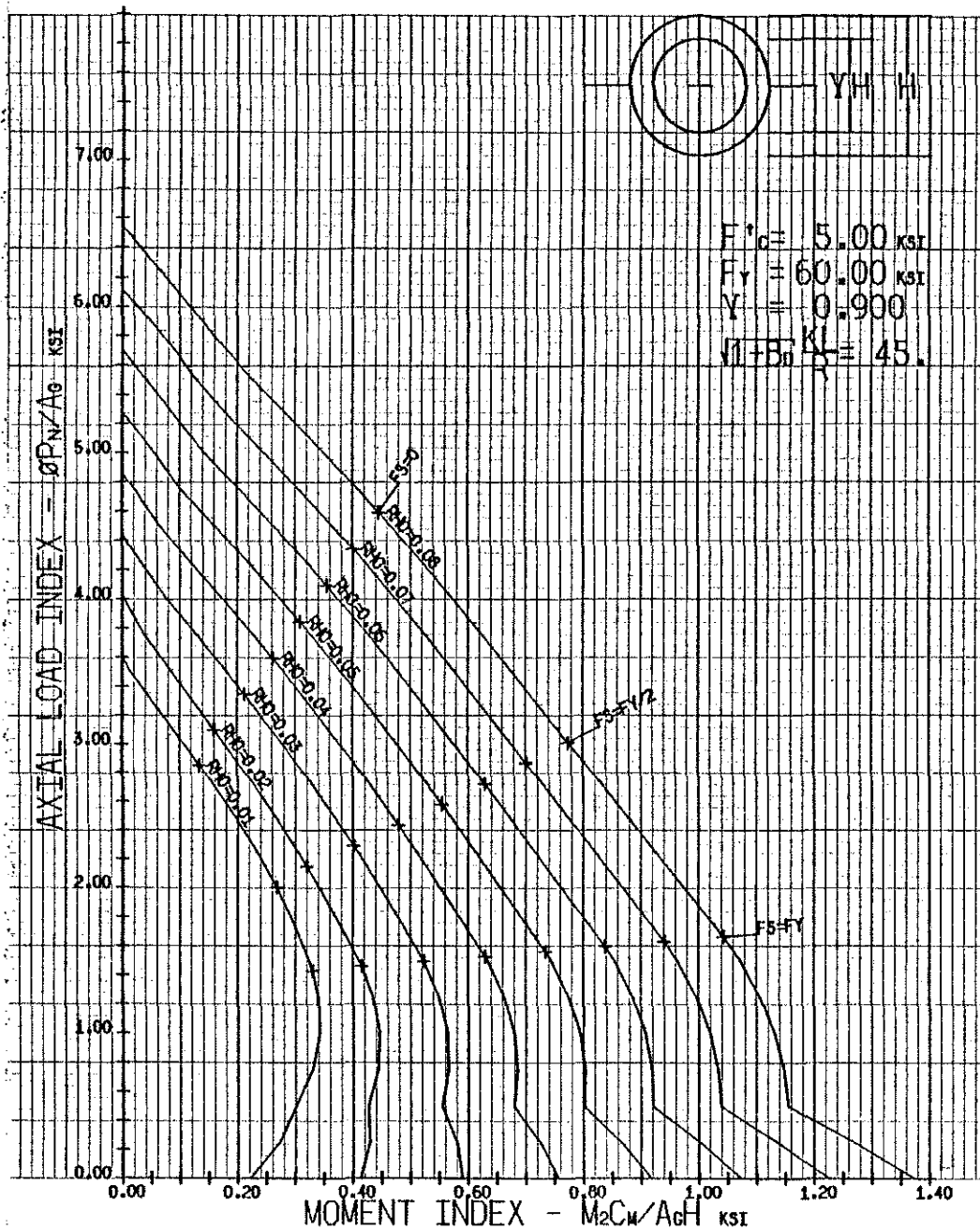


Fig. C5-60.90-45 - Interaction Diagram

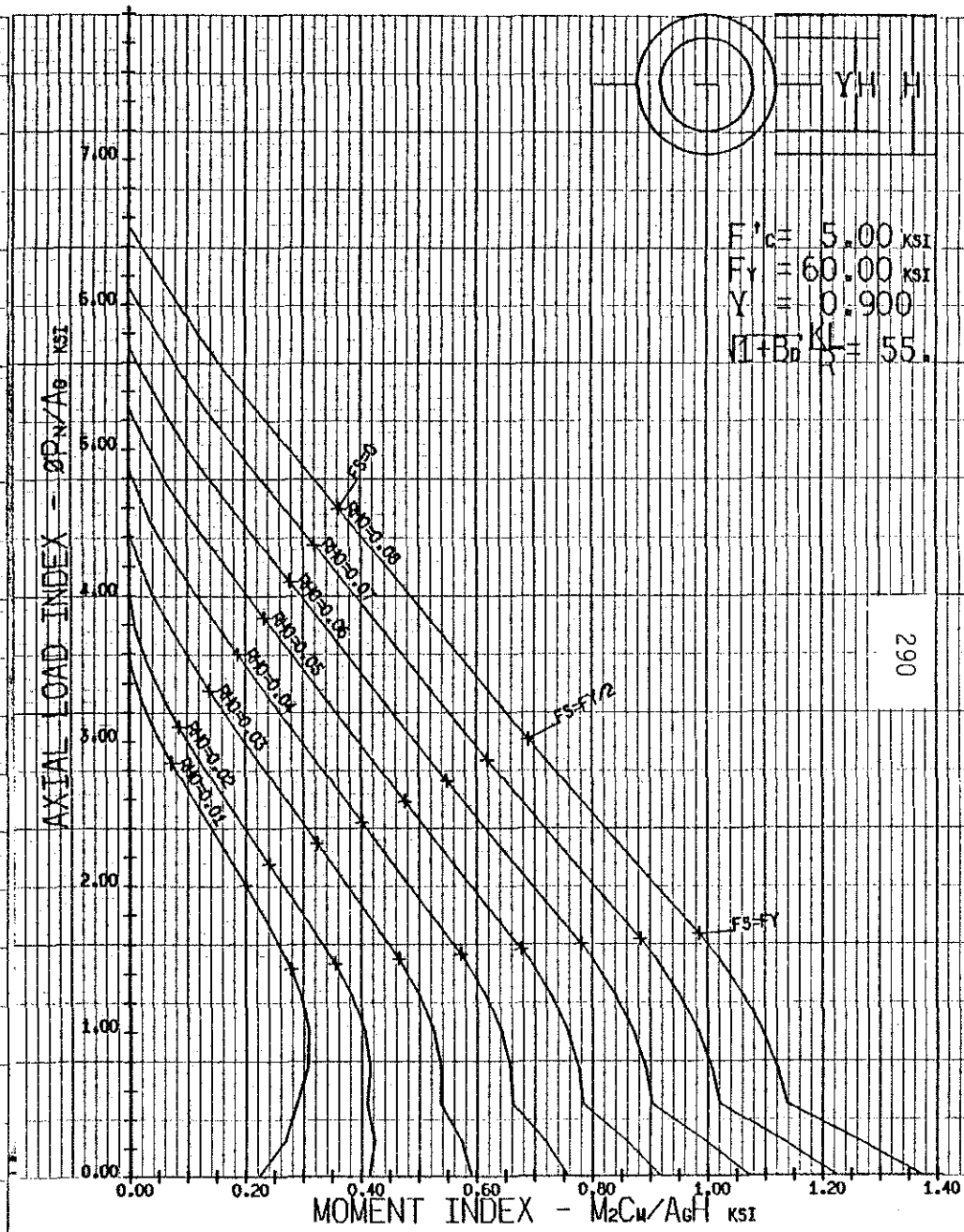


Fig. C5-60.90-55 - Interaction Diagram

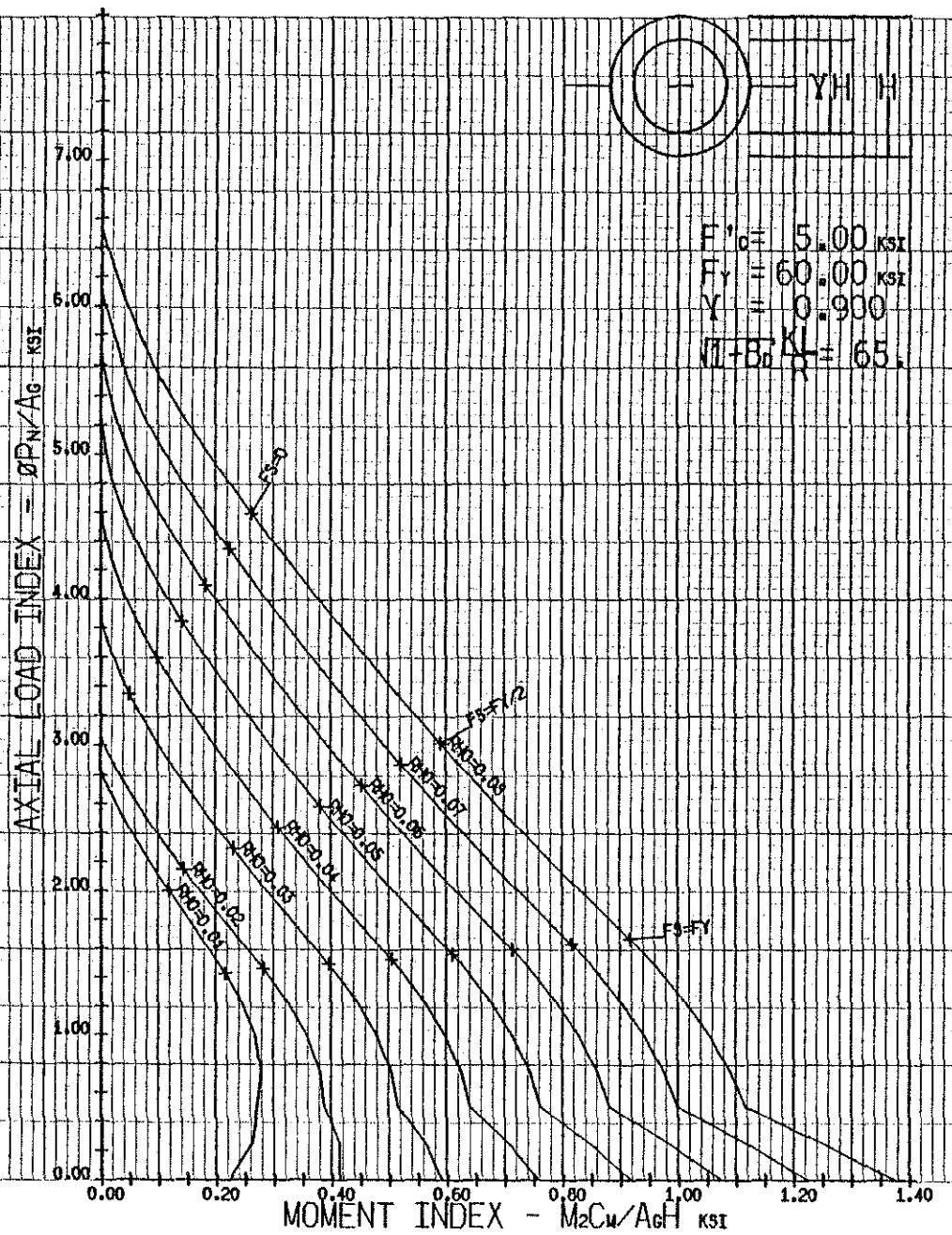


Fig. C5-60.90-65 - Interaction Diagram

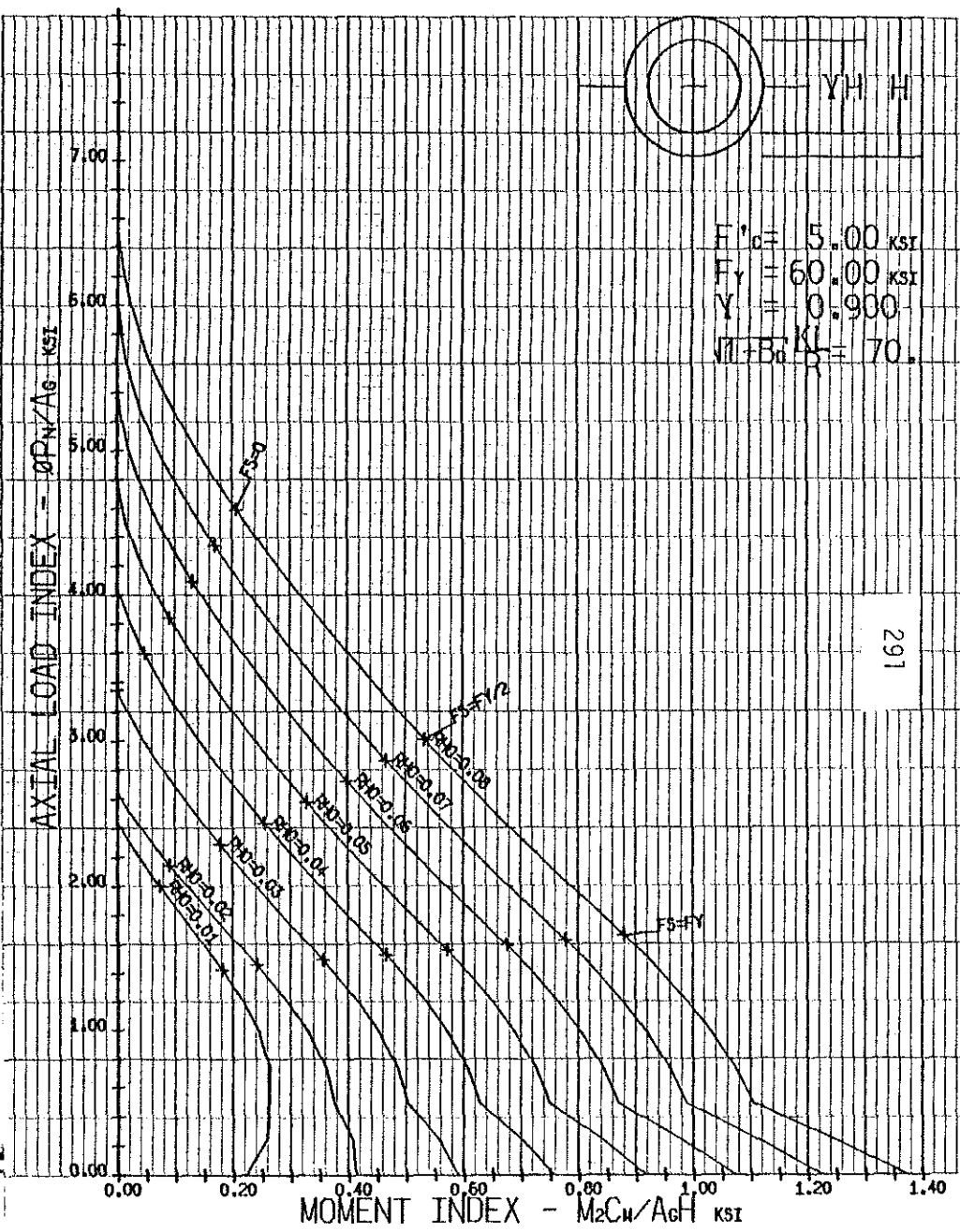


Fig. C5-60.90-70 - Interaction Diagram

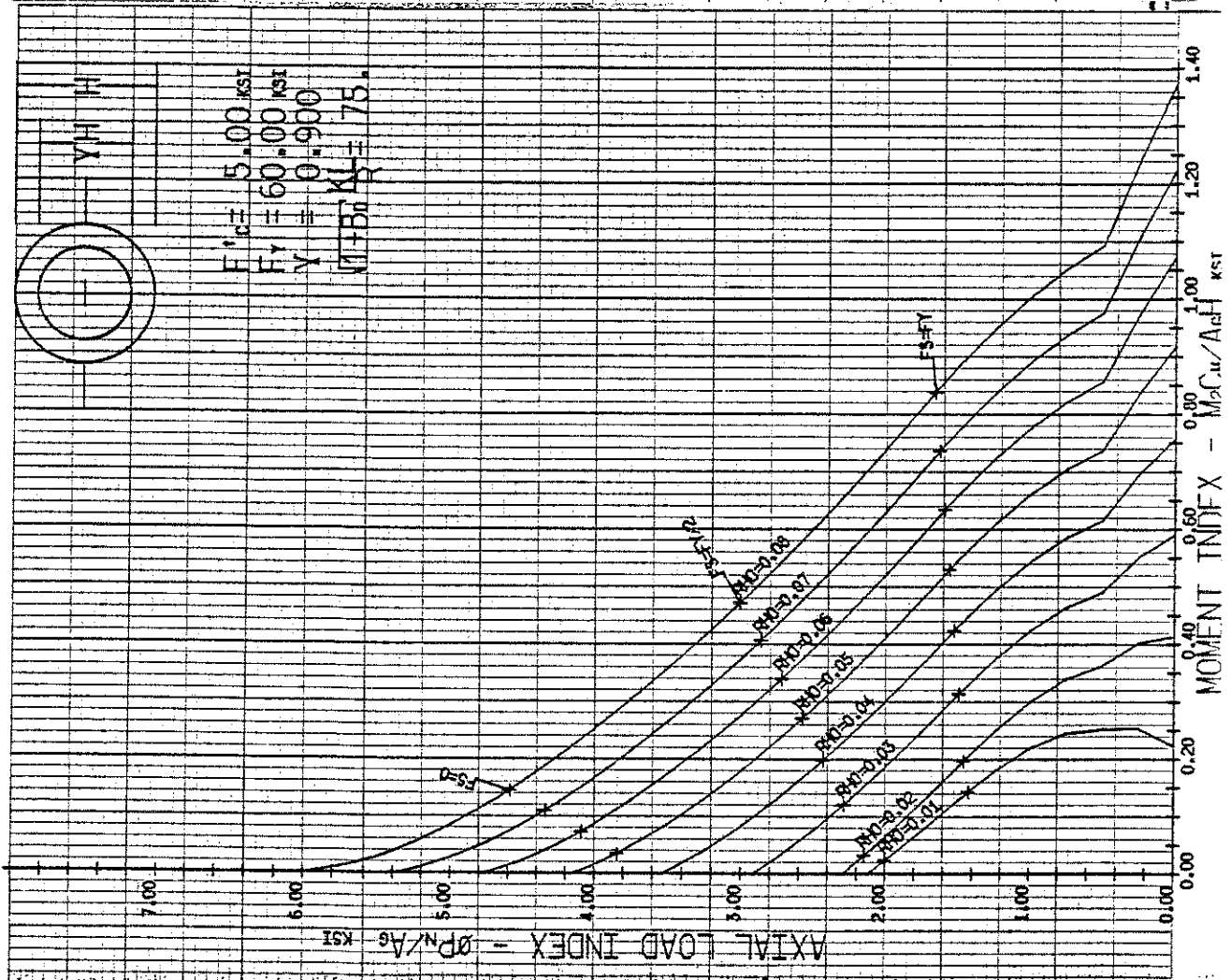


Fig. C5-60.90-75 - Interaction Diagram

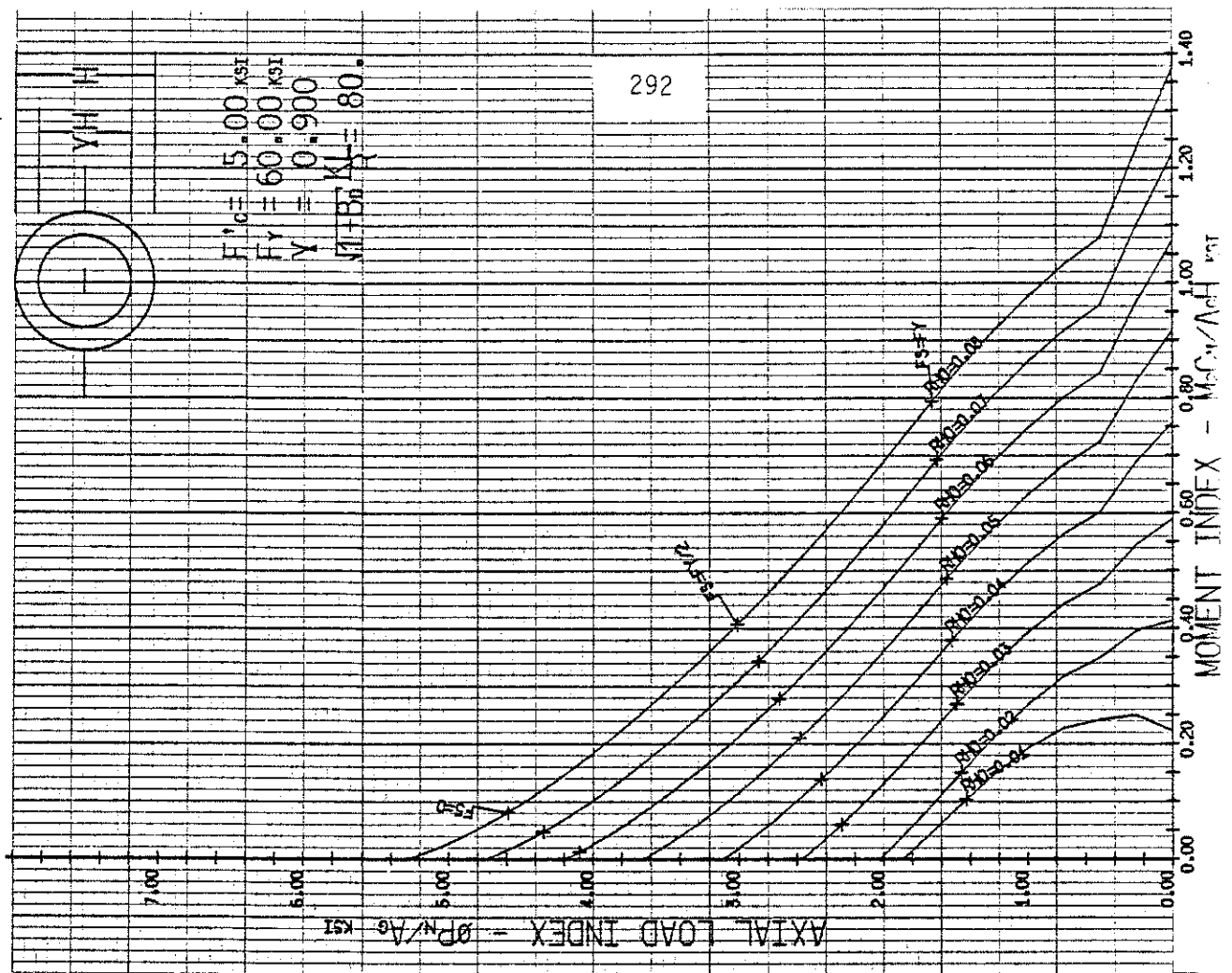


Fig. C5-60.90-80 - Interaction Diagram



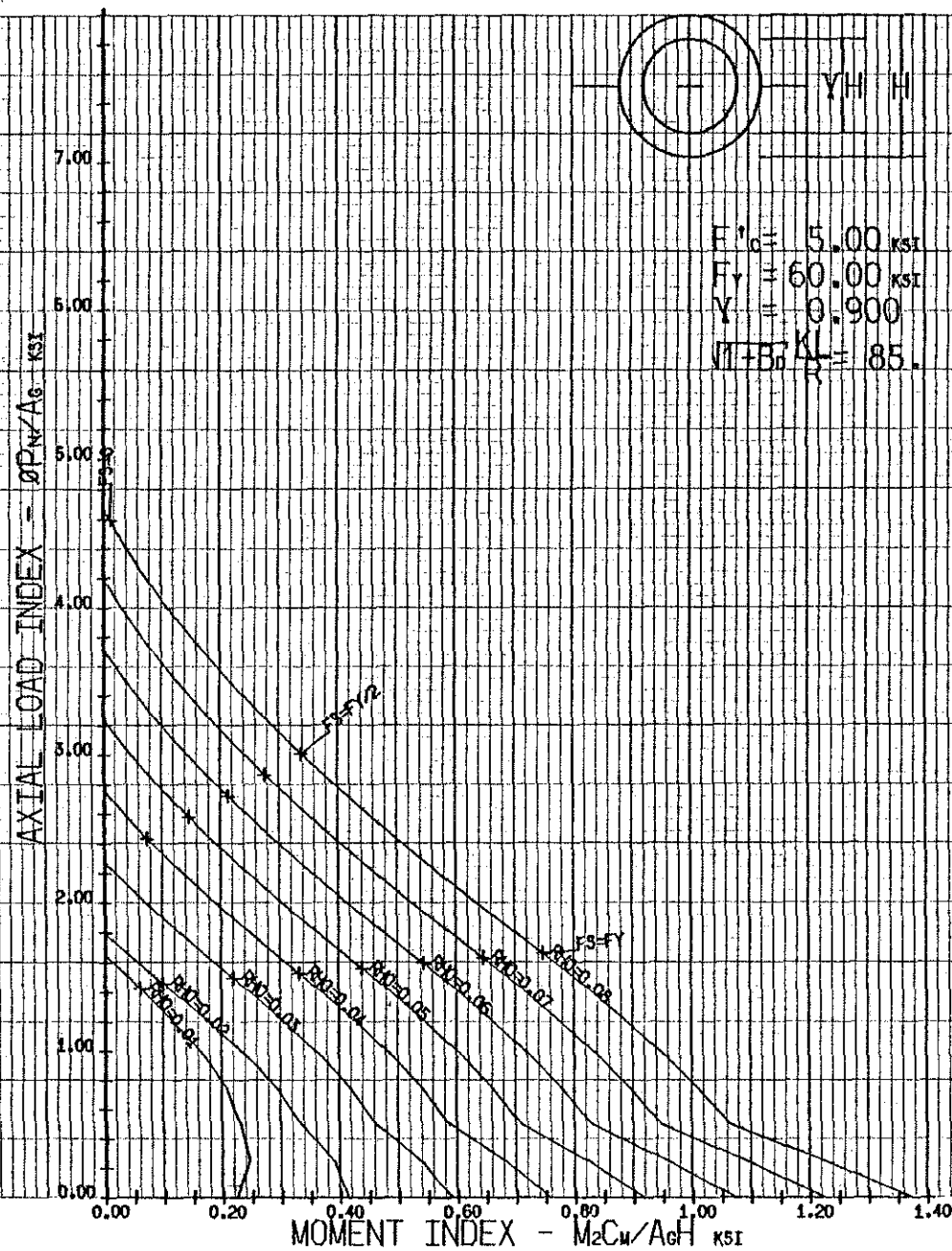


Fig. C5-60.90-85 - Interaction Diagram

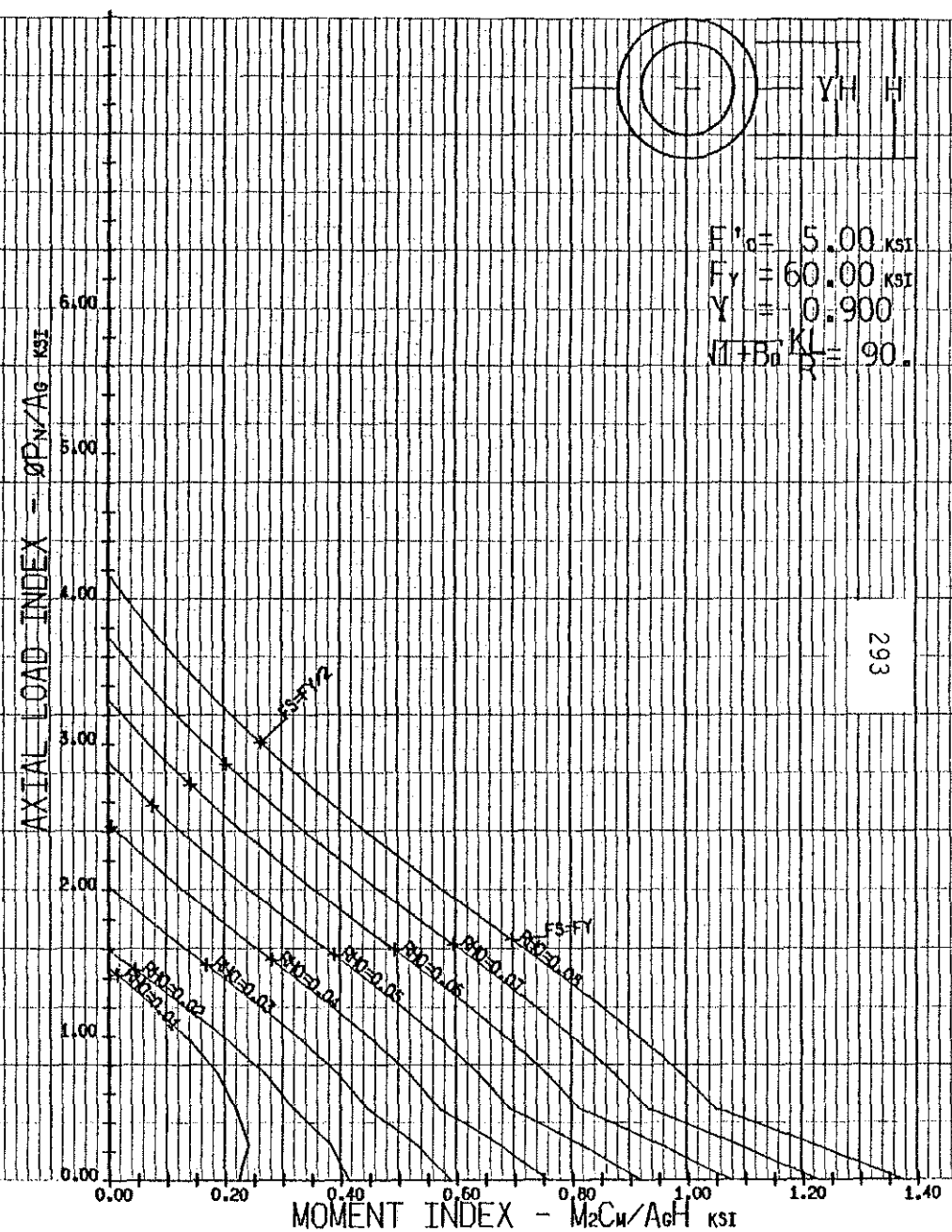


Fig. C5-60.90-90 - Interaction Diagram



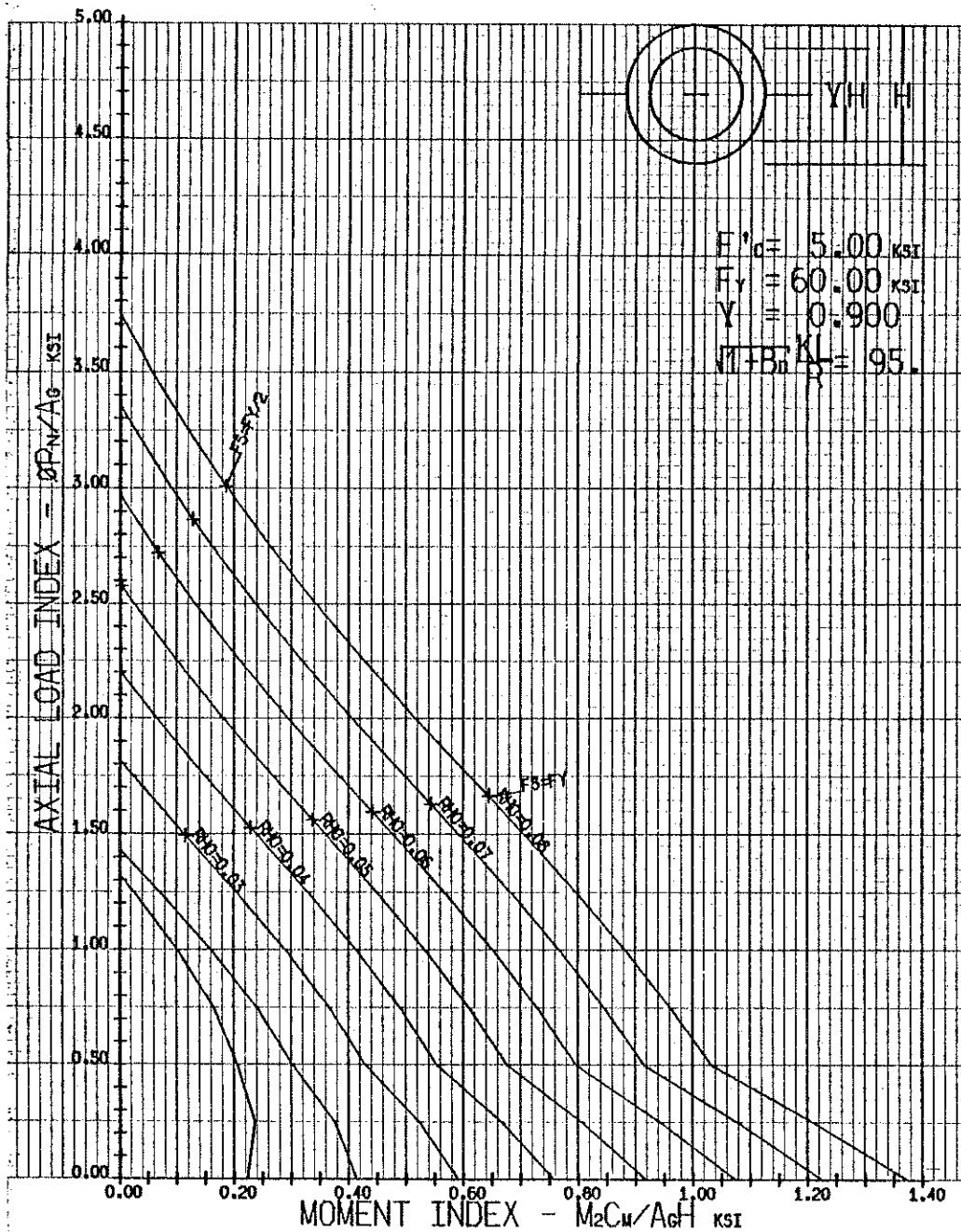


Fig. C5-60.90-95 - Interaction Diagram

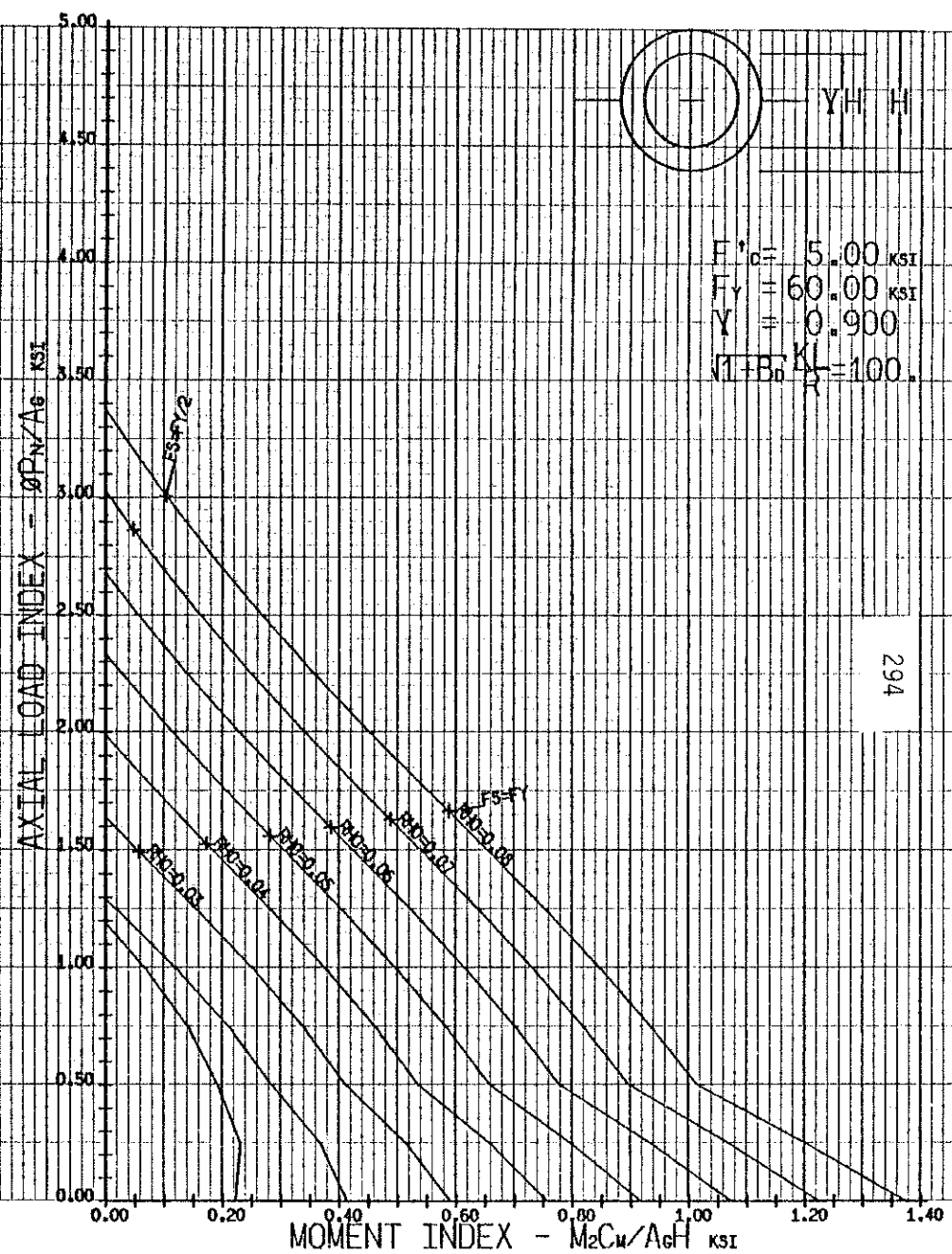


Fig. C5-60.90-100 - Interaction Diagram

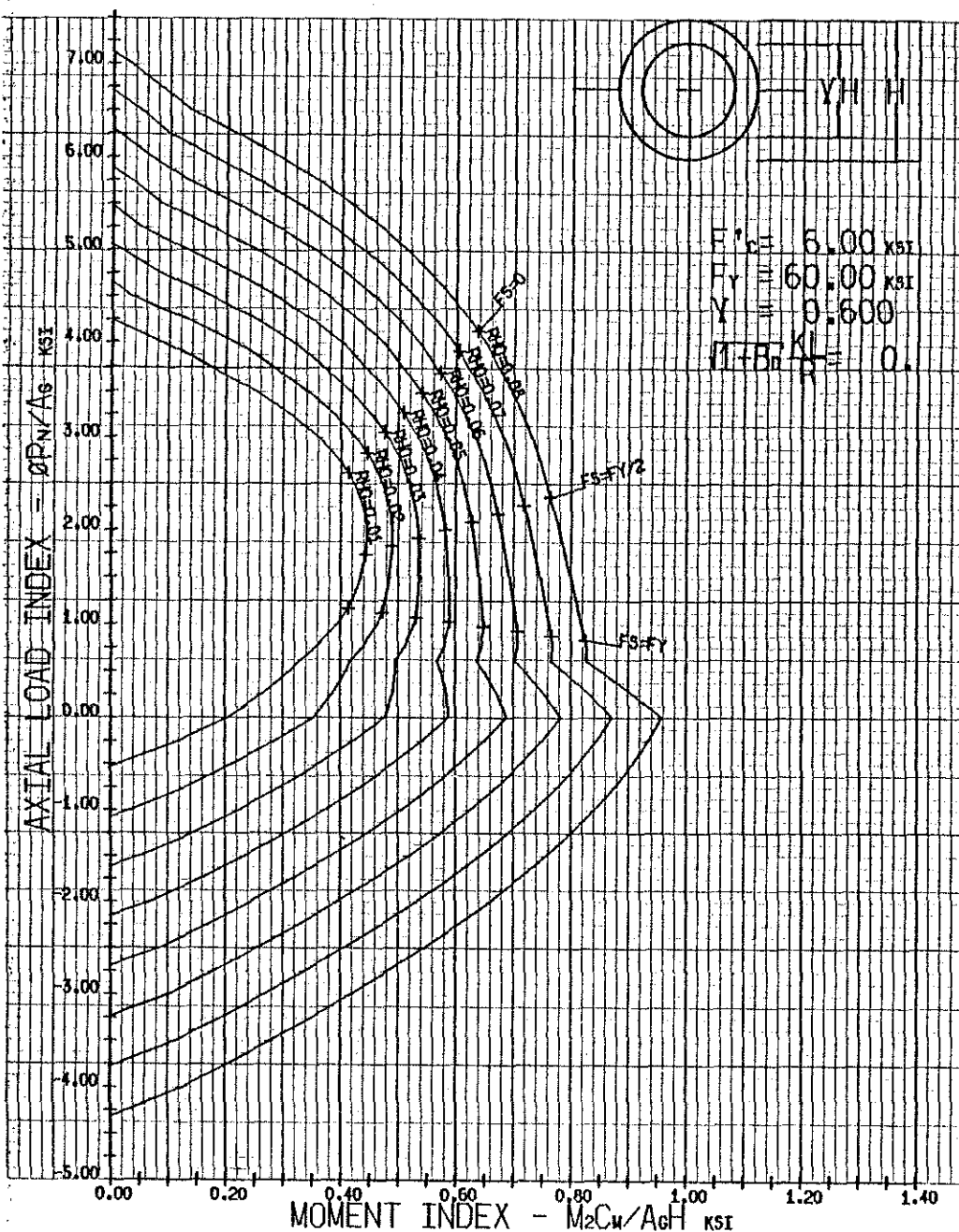


Fig. C6-60.60-0 - Interaction Diagram

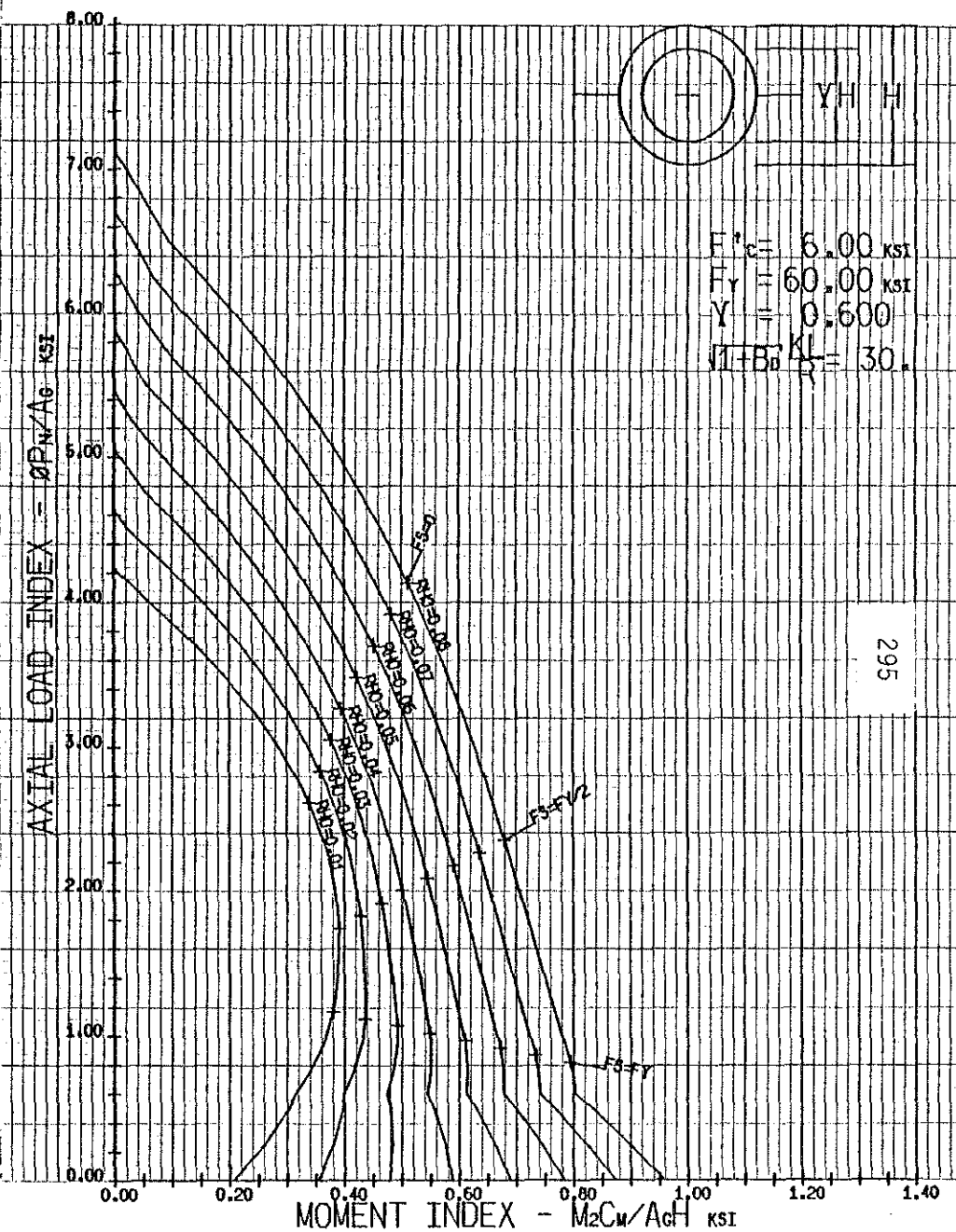


Fig. C6-60.60-30 - Interaction Diagram

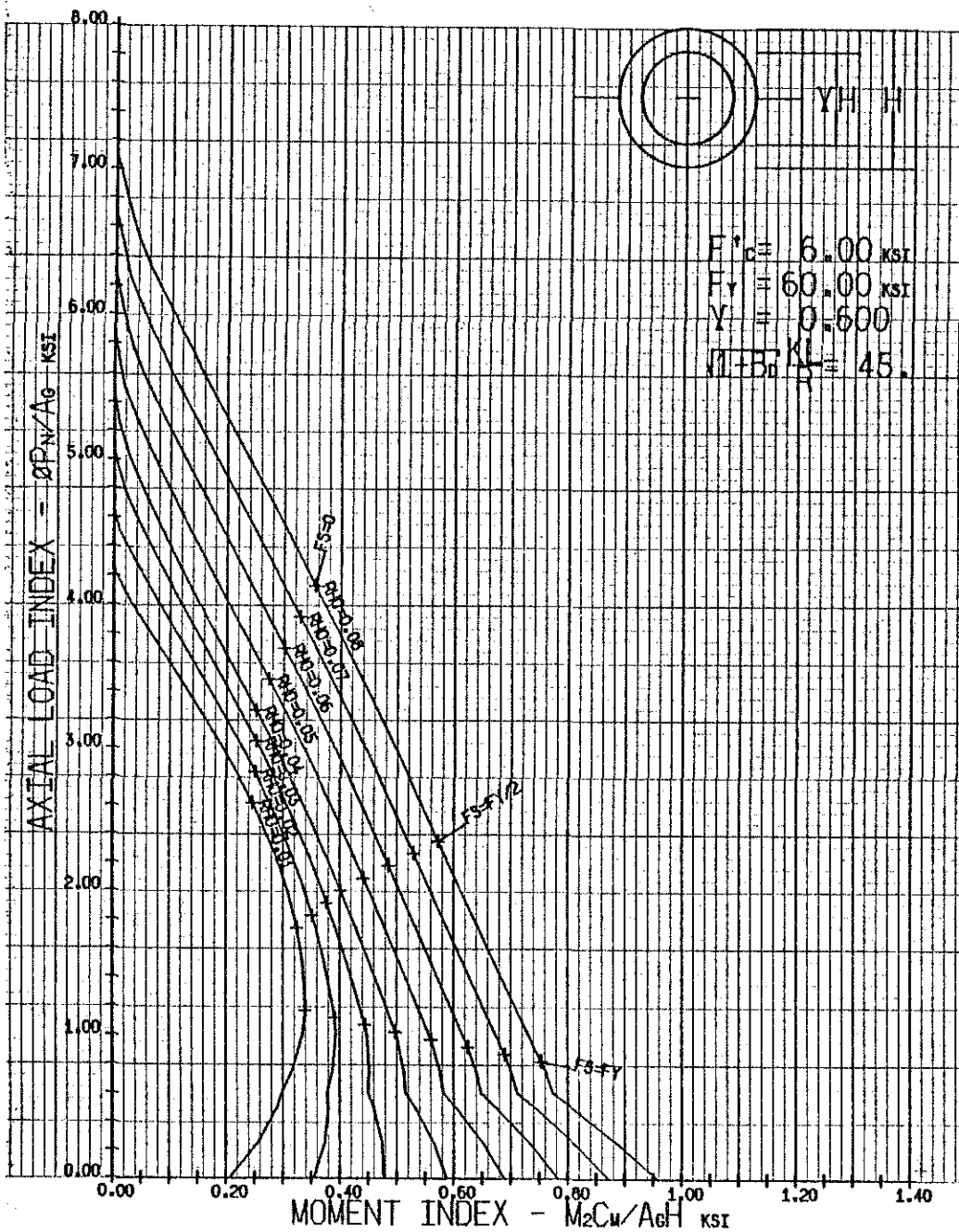


Fig. C6-60.60-45 - Interaction Diagram

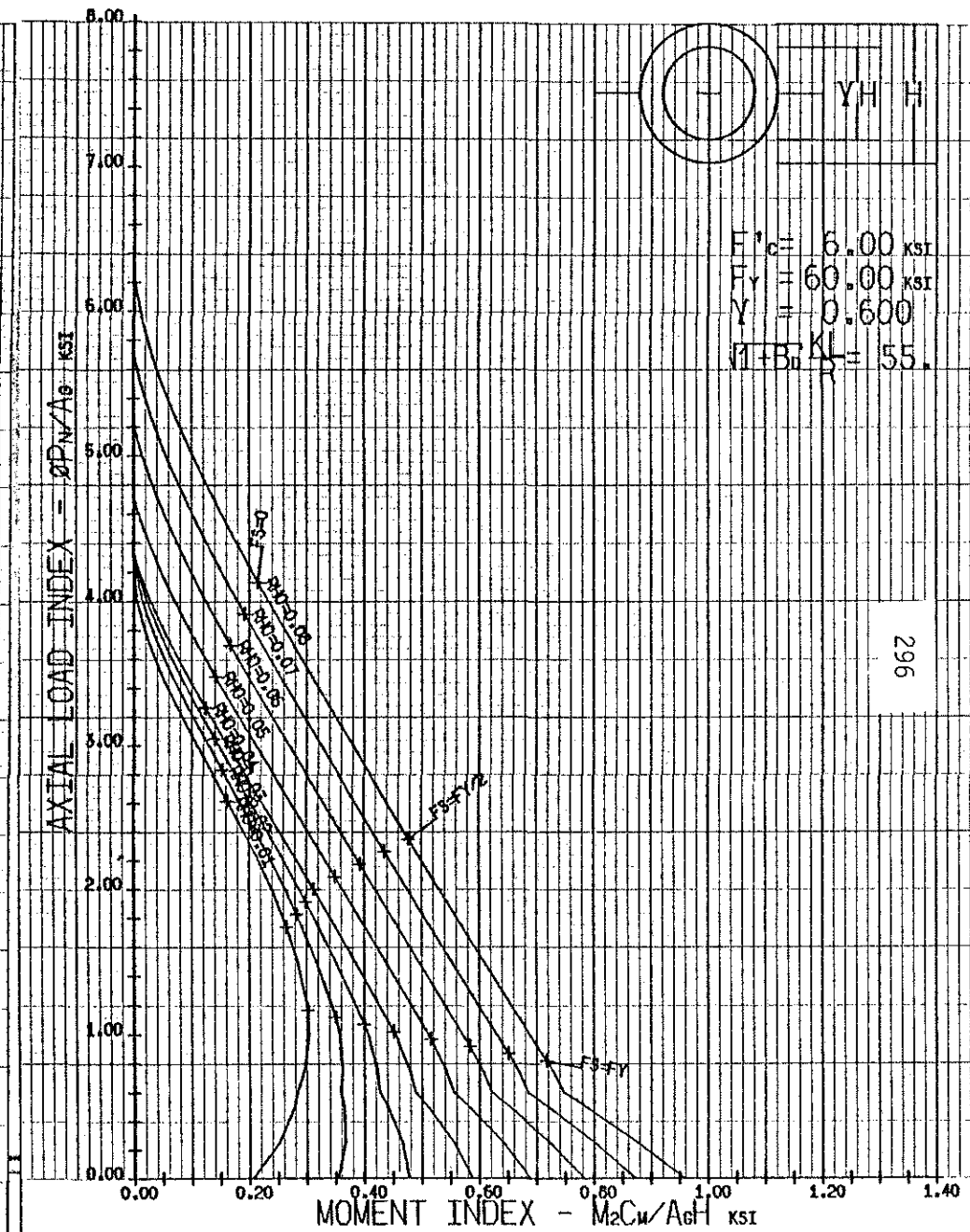


Fig. C6-60.60-55 - Interaction Diagram

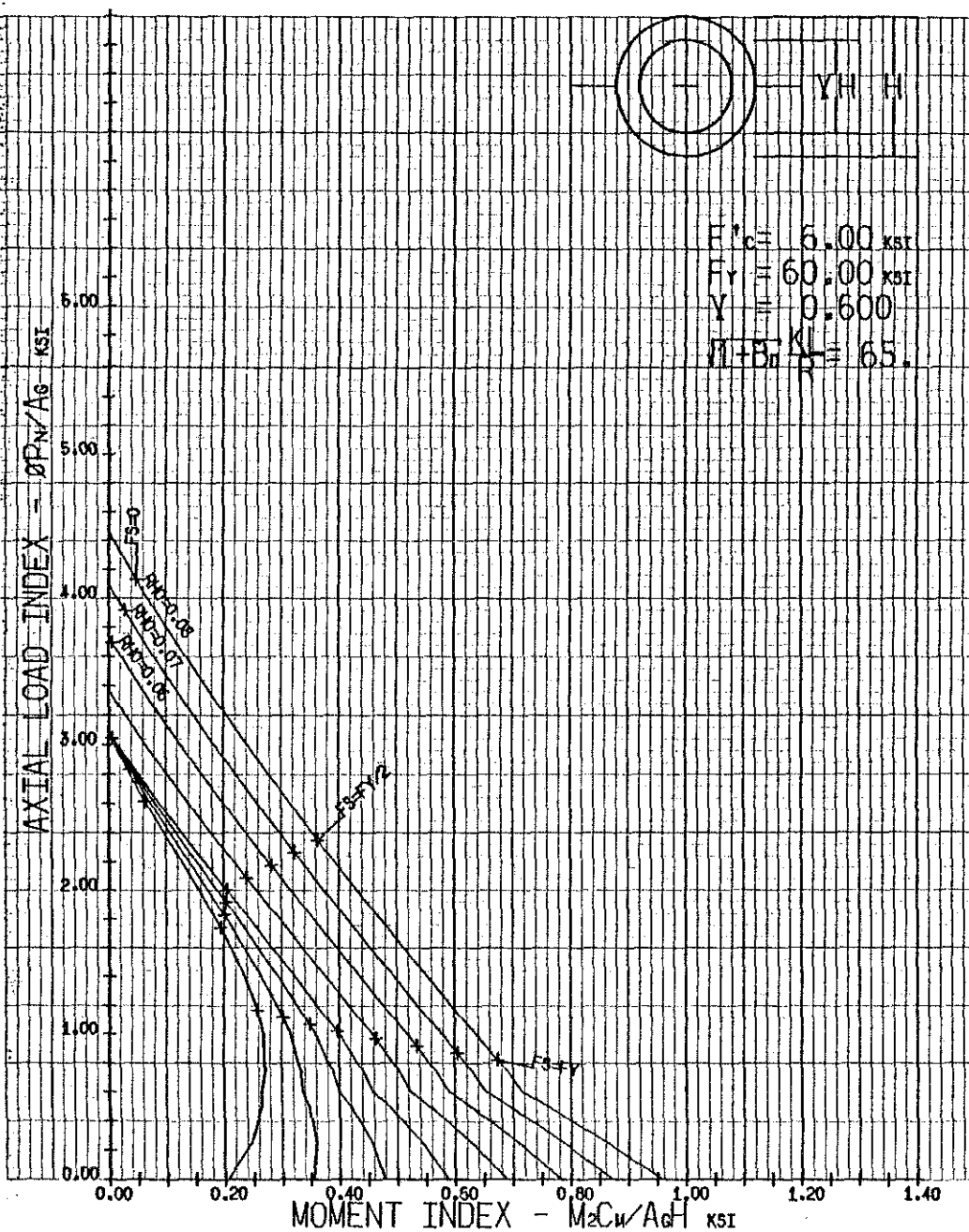


Fig. C6-60.60-65 - Interaction Diagram

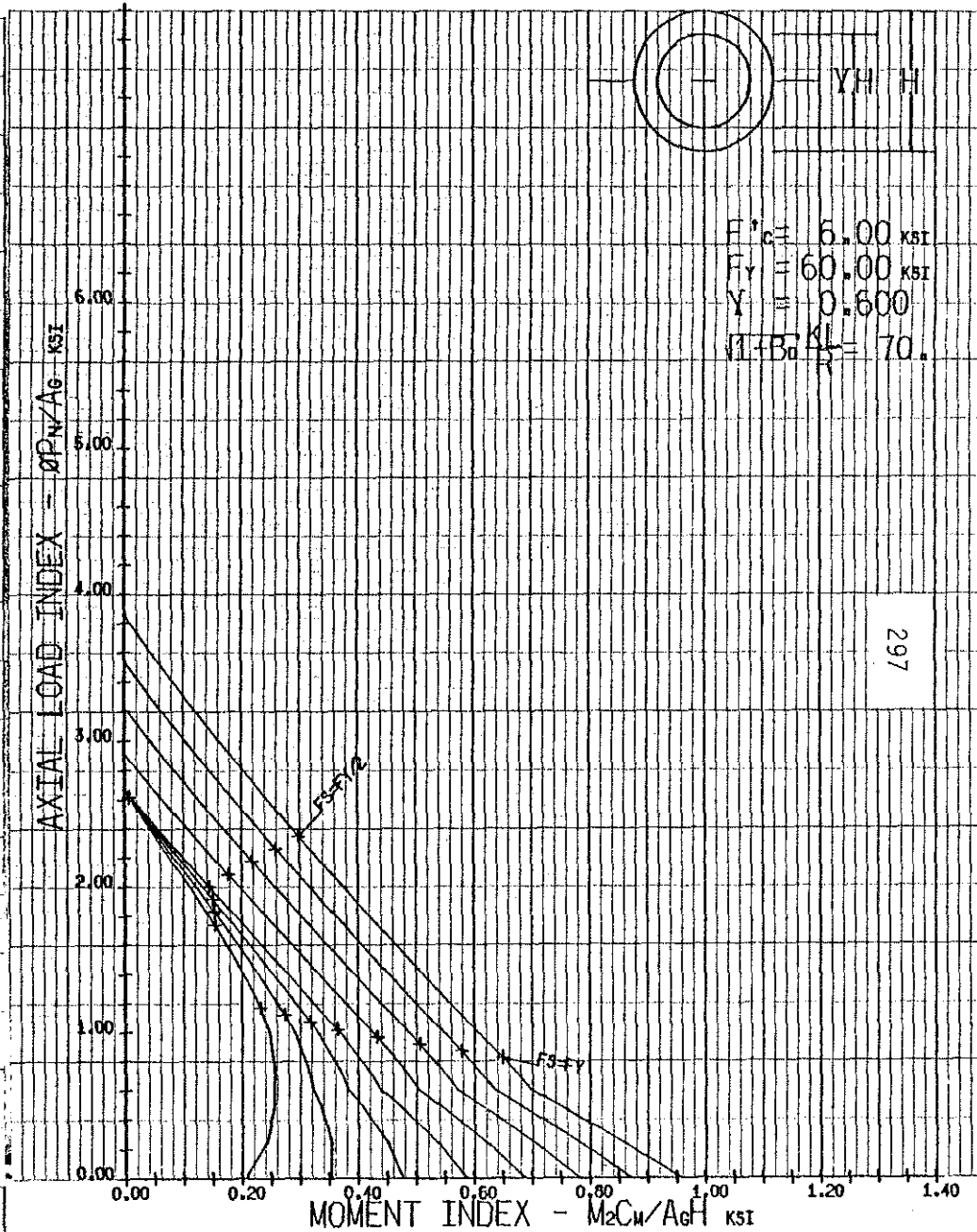


Fig. C6-60.60-70 - Interaction Diagram

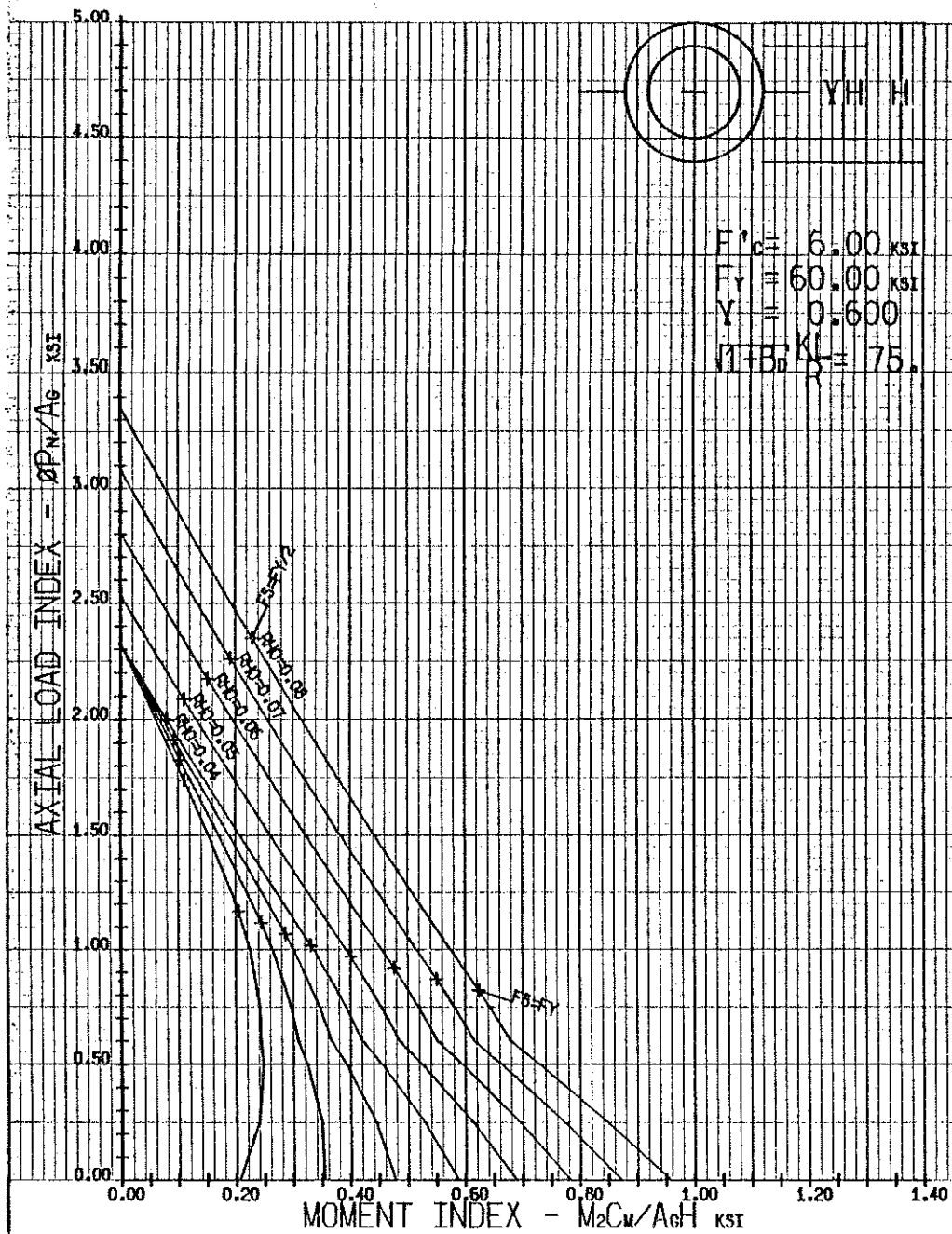


Fig. C6-60.60-75 - Interaction Diagram

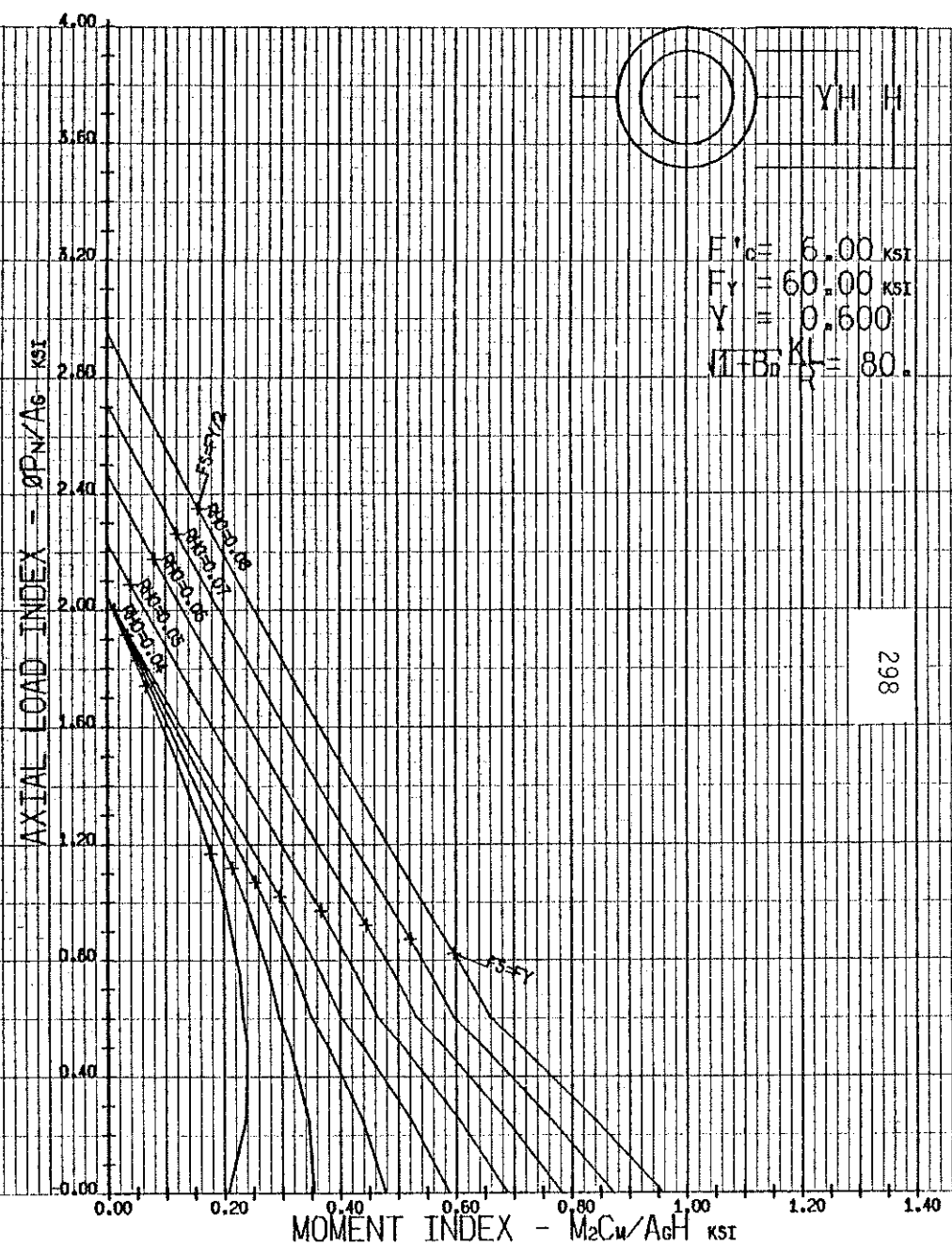
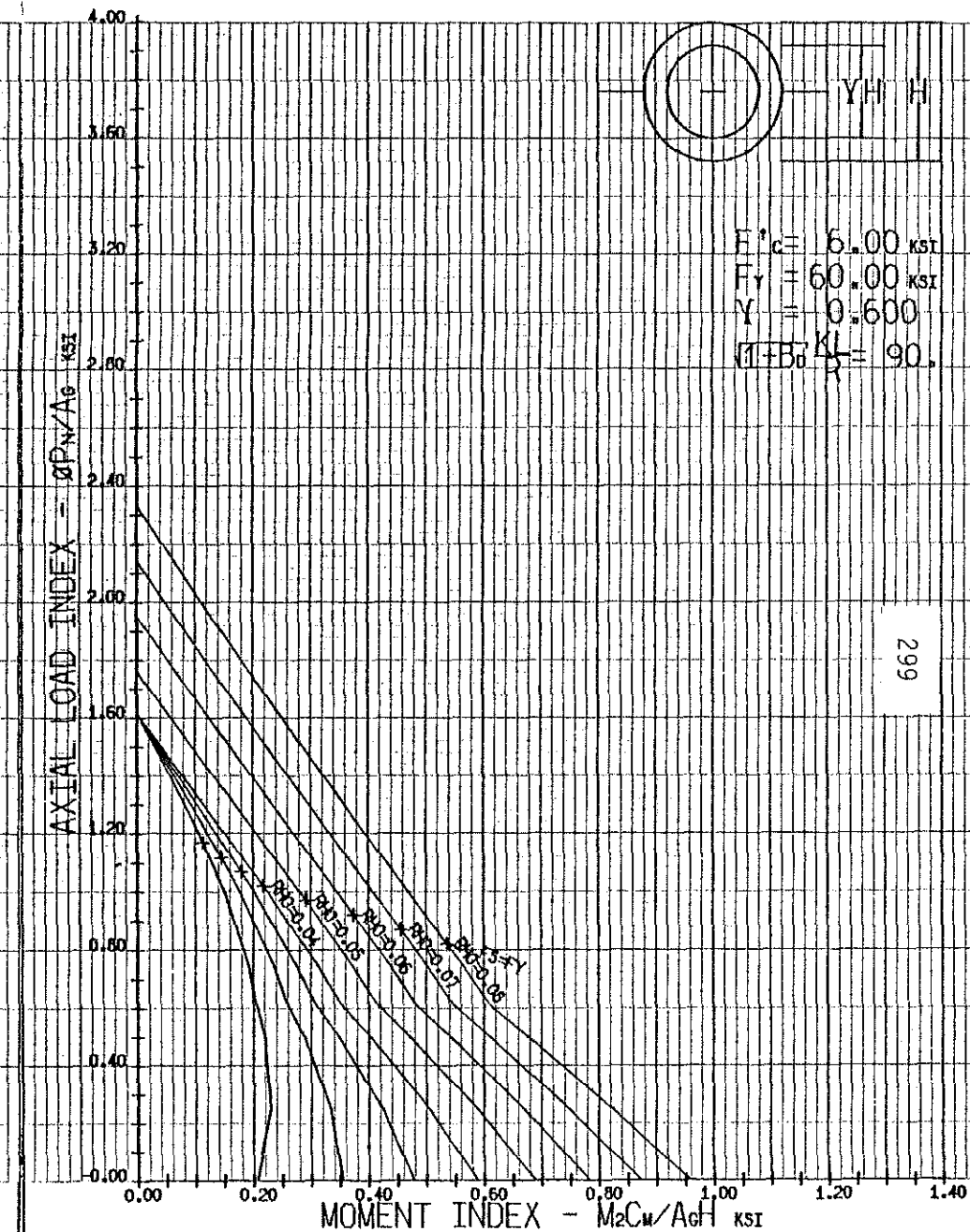
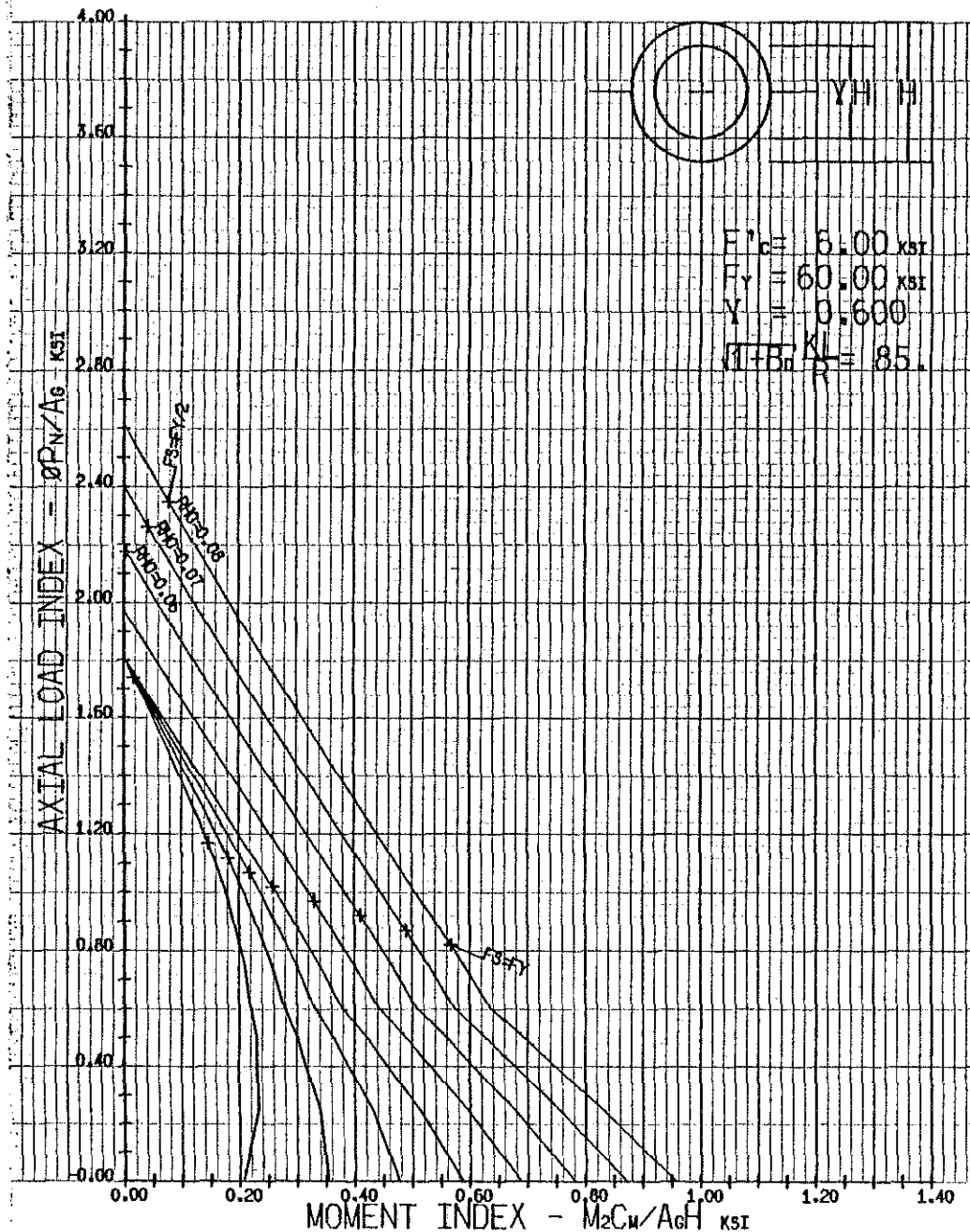


Fig. C6-60.60-80 - Interaction Diagram







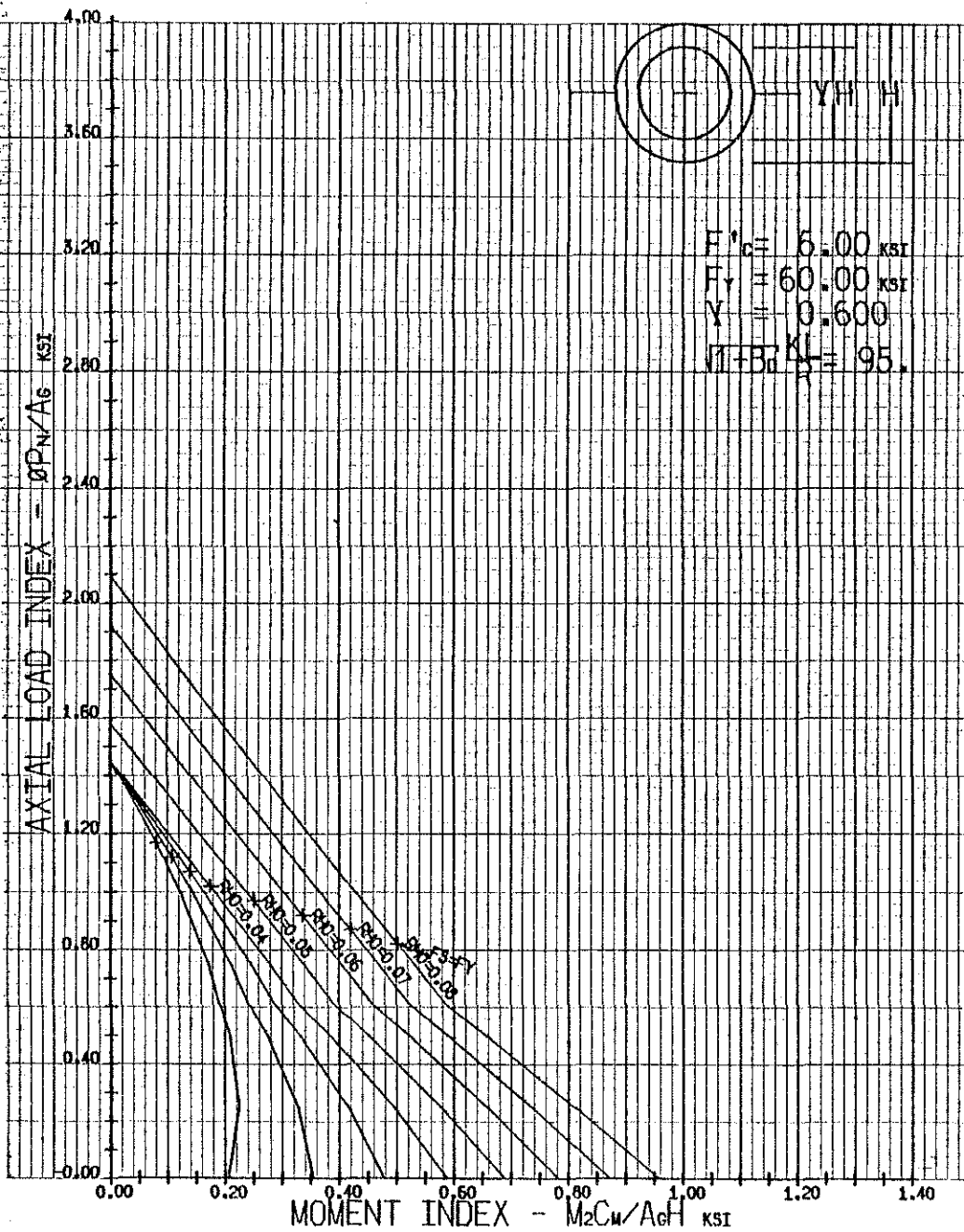


Fig. C6-60.60-95 - Interaction Diagram

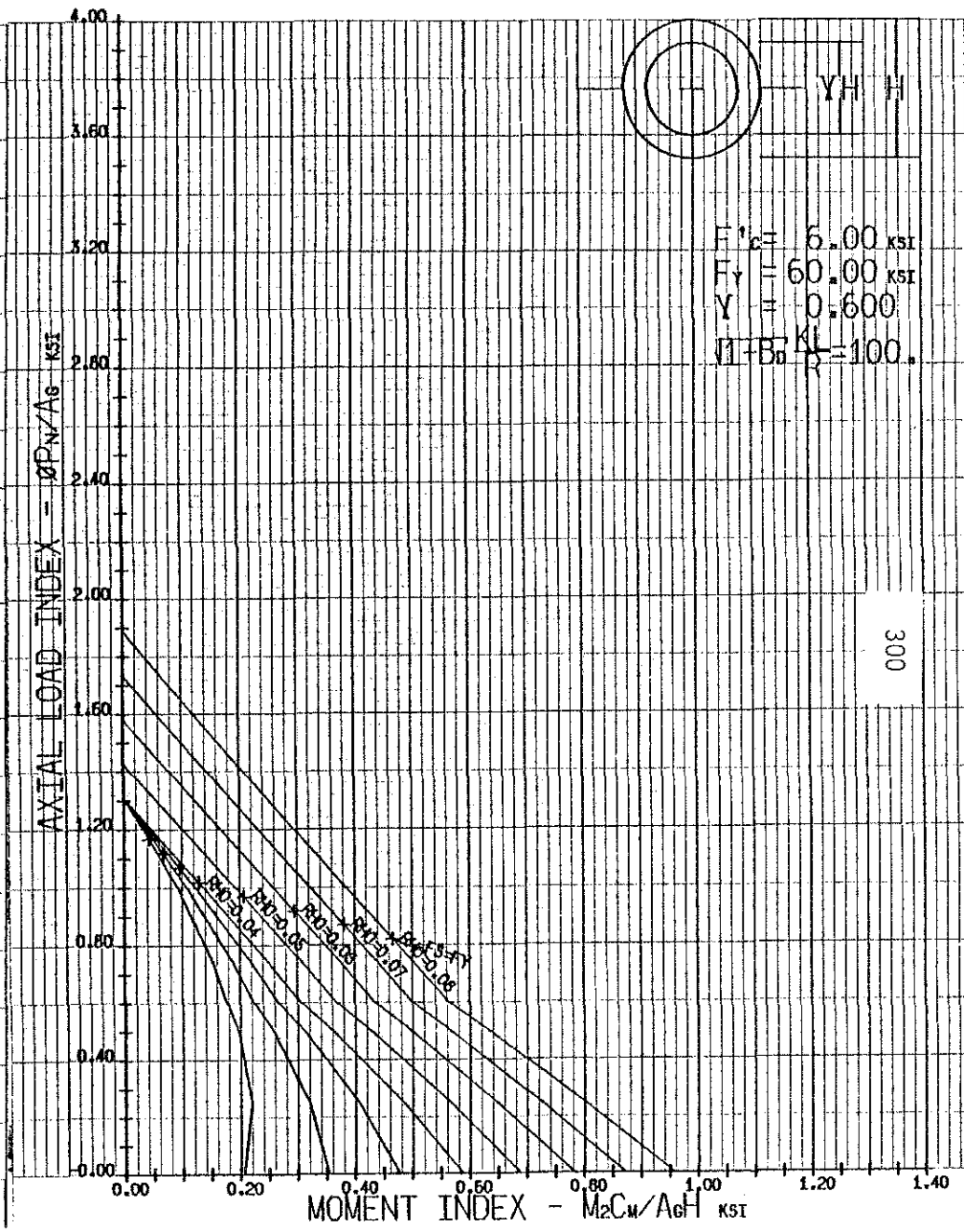


Fig. C6-60.60-100 - Interaction Diagram

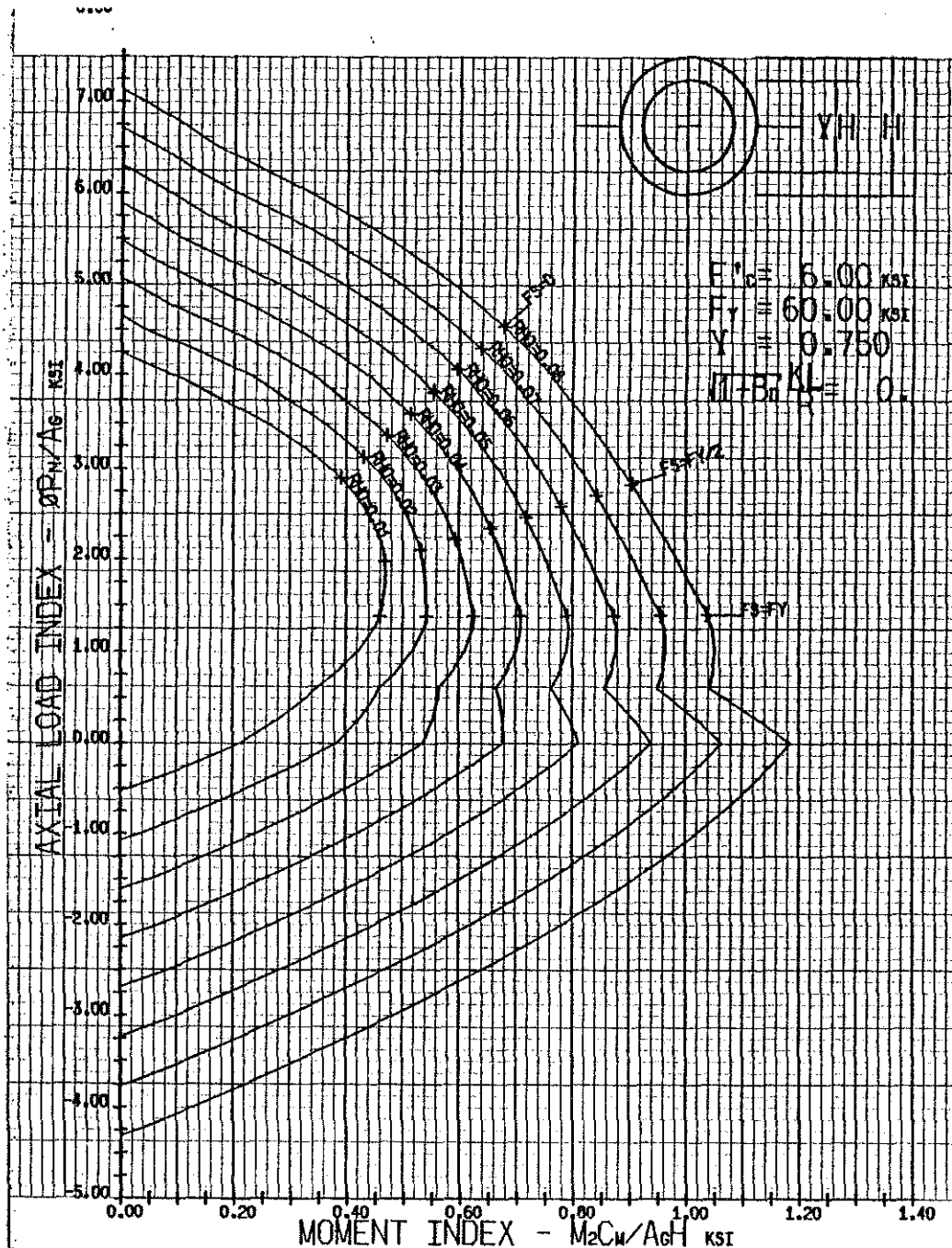


Fig. C6-60.75-0 - Interaction Diagram

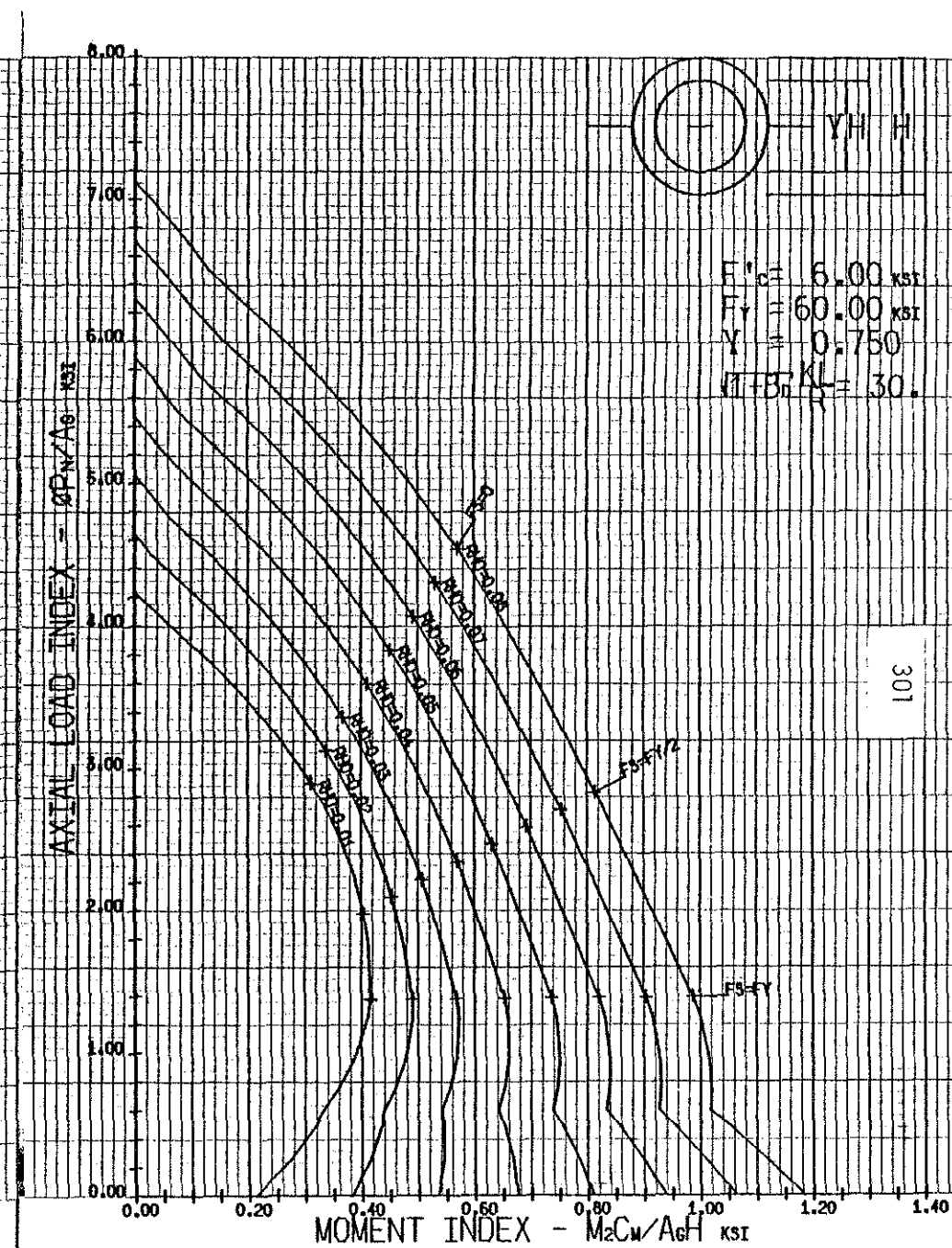


Fig. C6-60.75-30 - Interaction Diagram

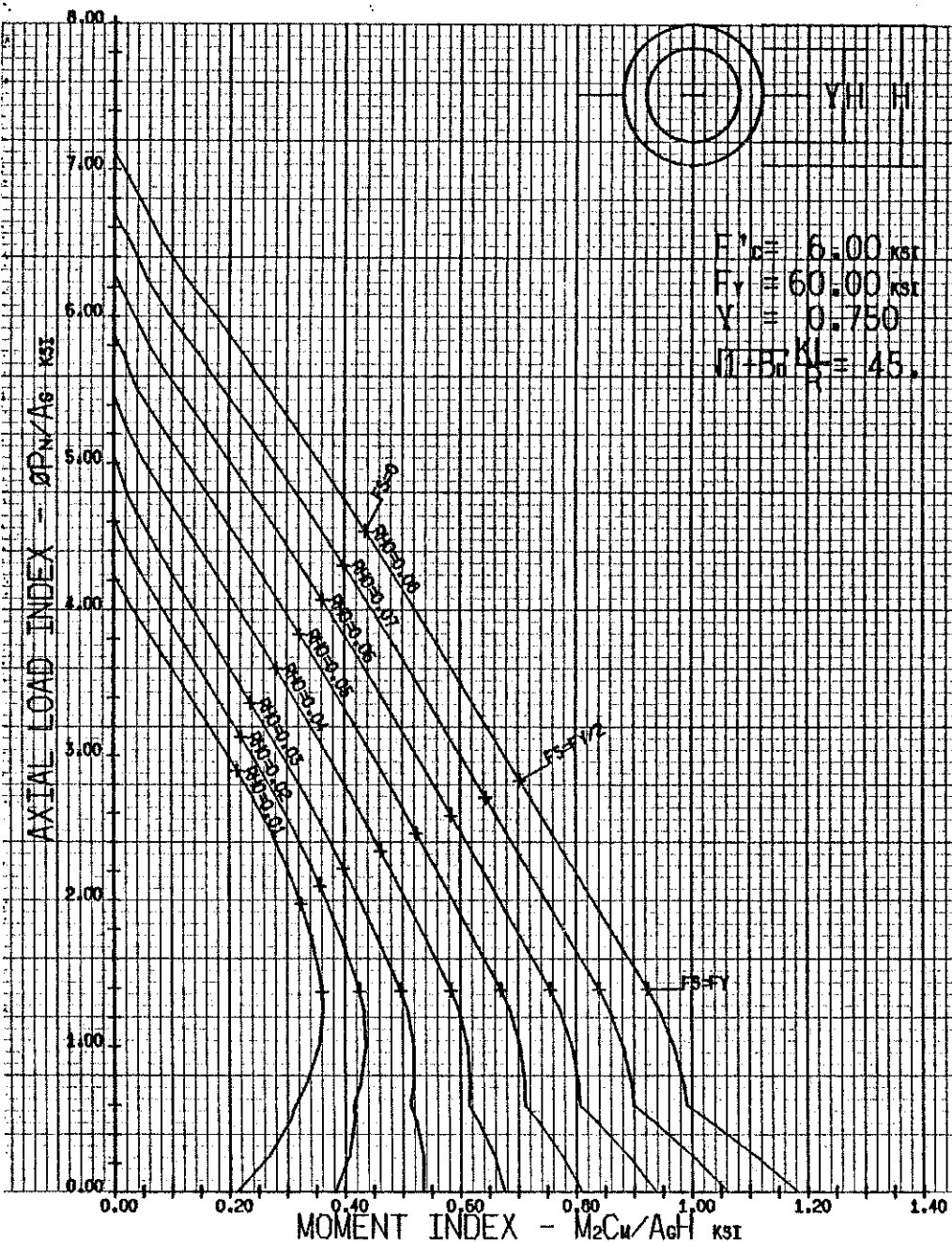


Fig. C6-60.75-45 - Interaction Diagram

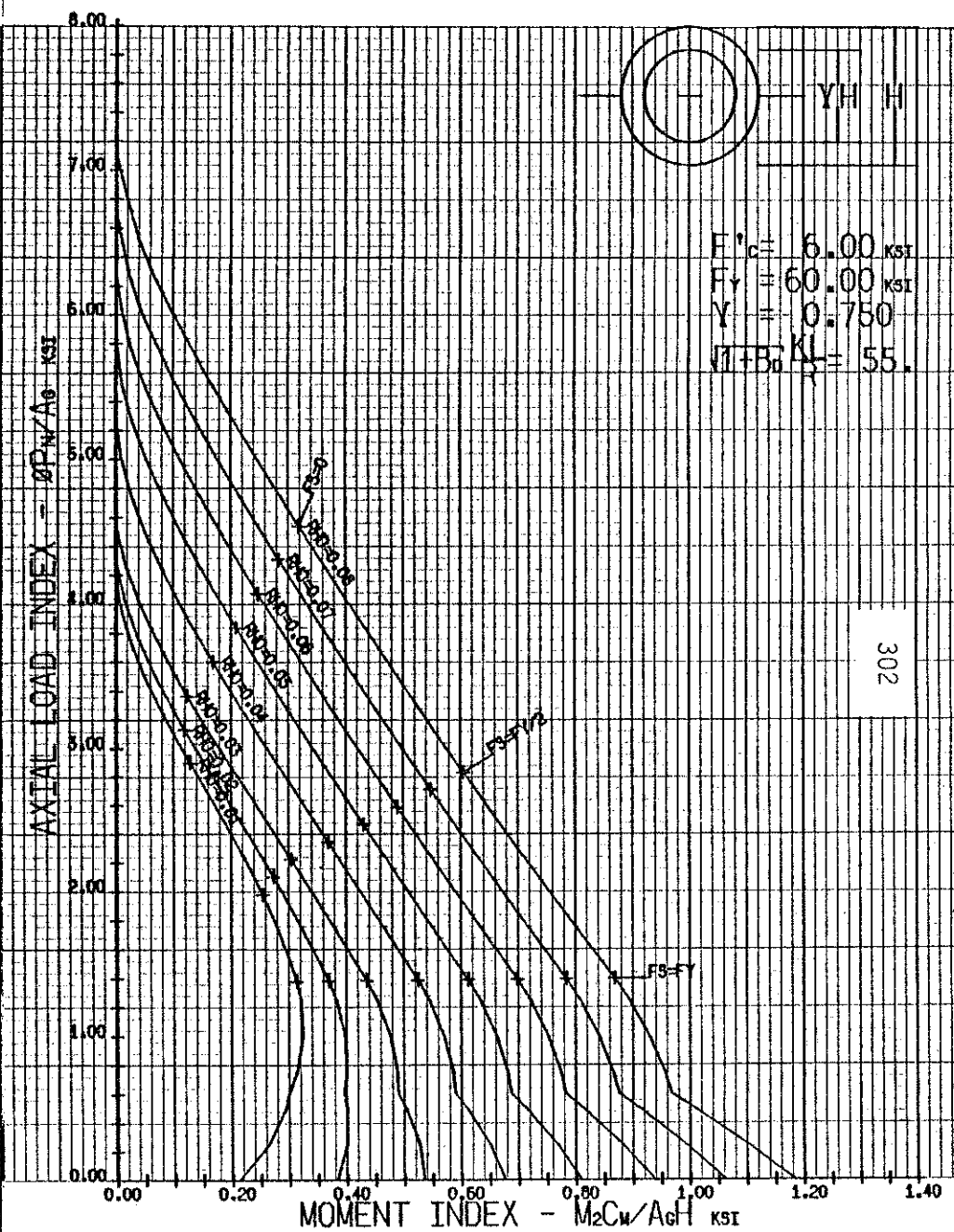


Fig. C6-60.75-55 - Interaction Diagram

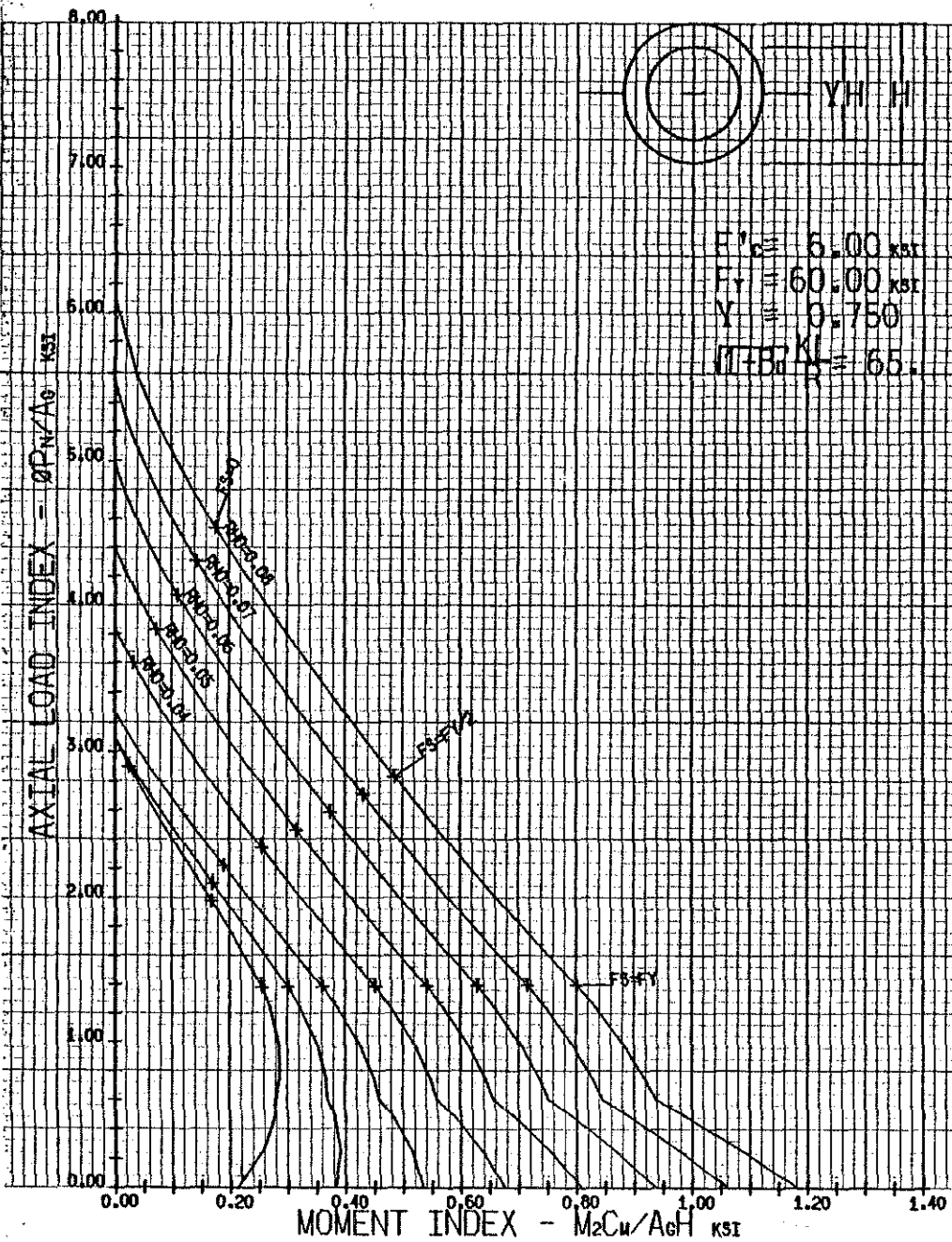


Fig. C6-60.75-65 - Interaction Diagram

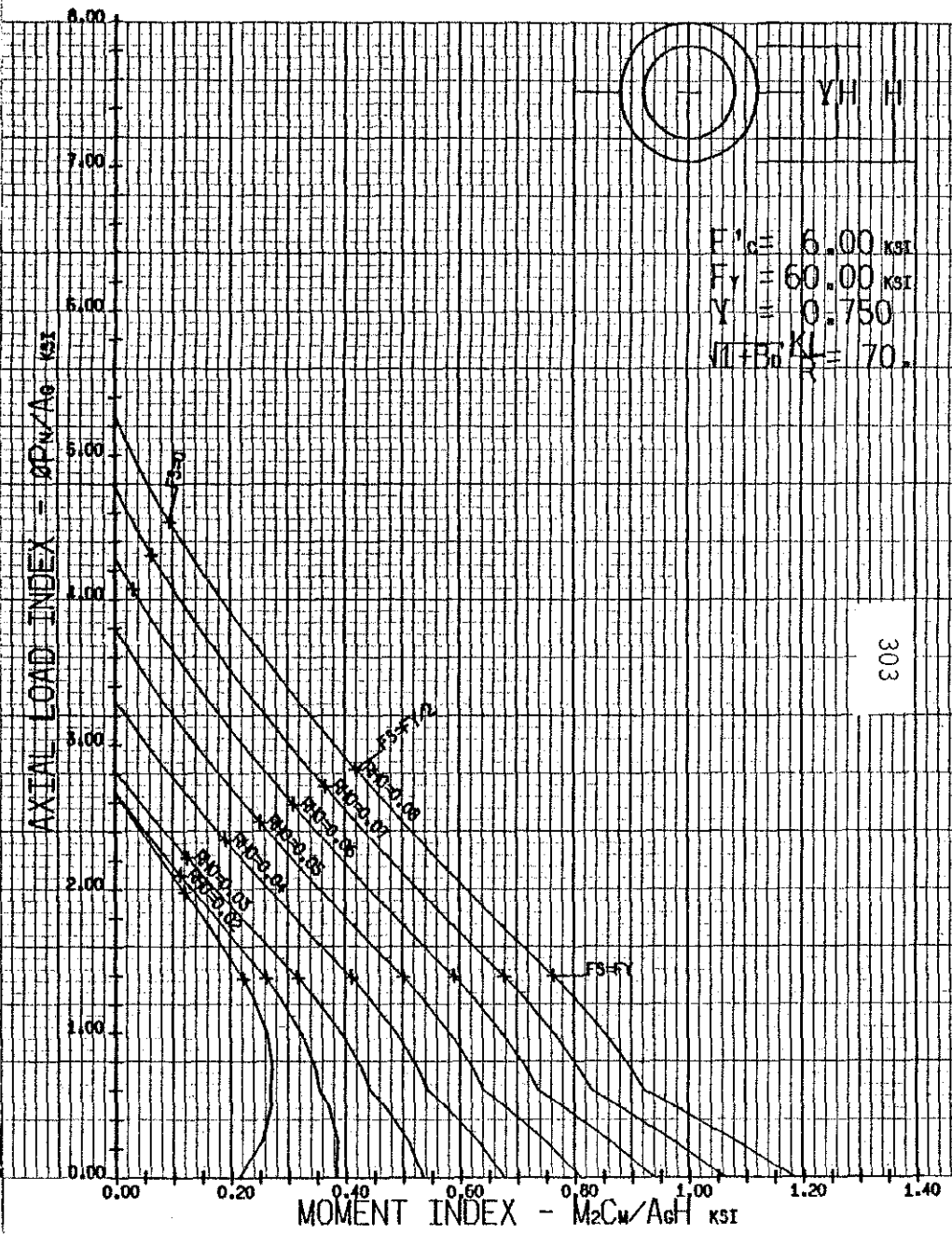


Fig. C6-60.75-70 - Interaction Diagram

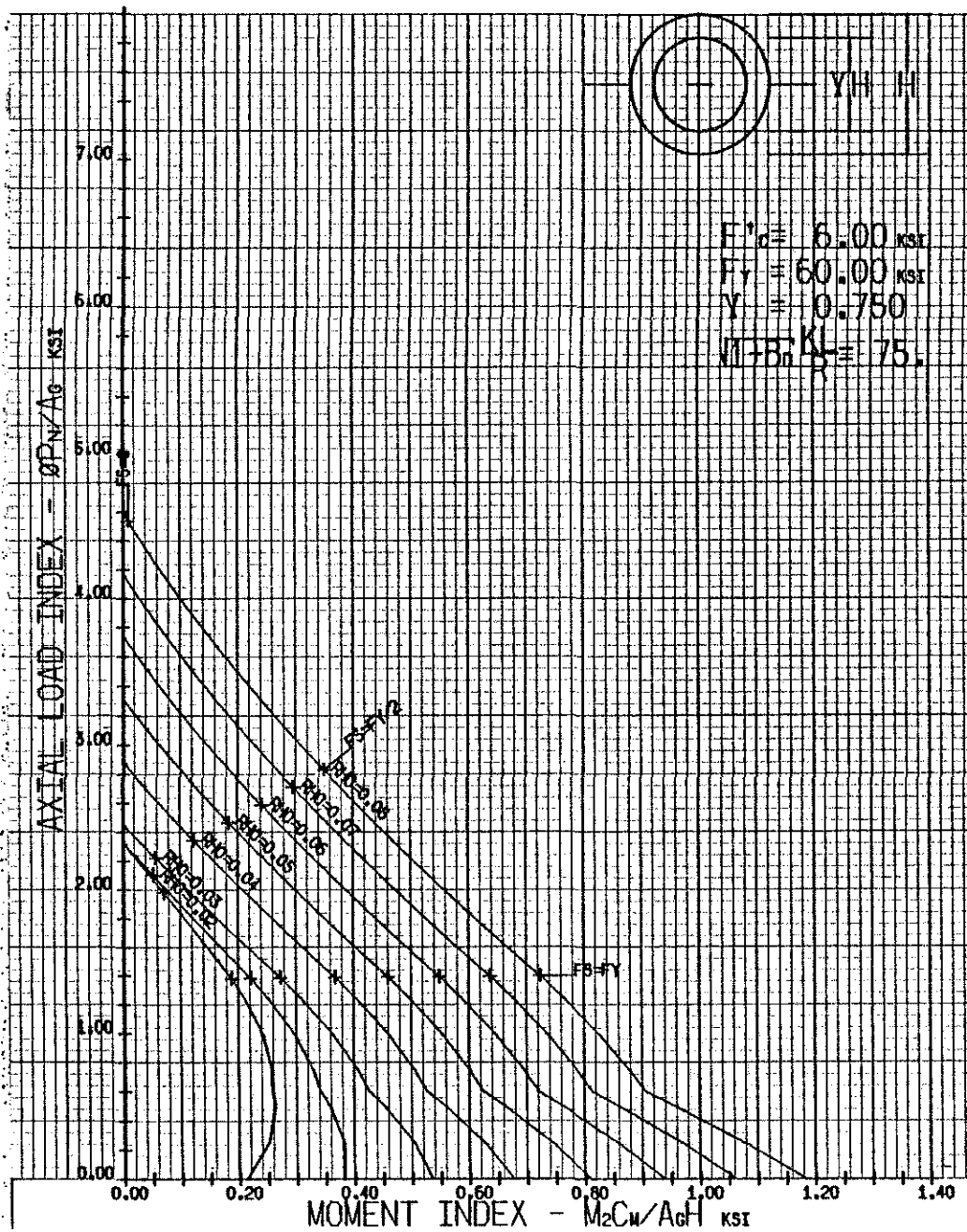


Fig. C6-60.75-75 - Interaction Diagram

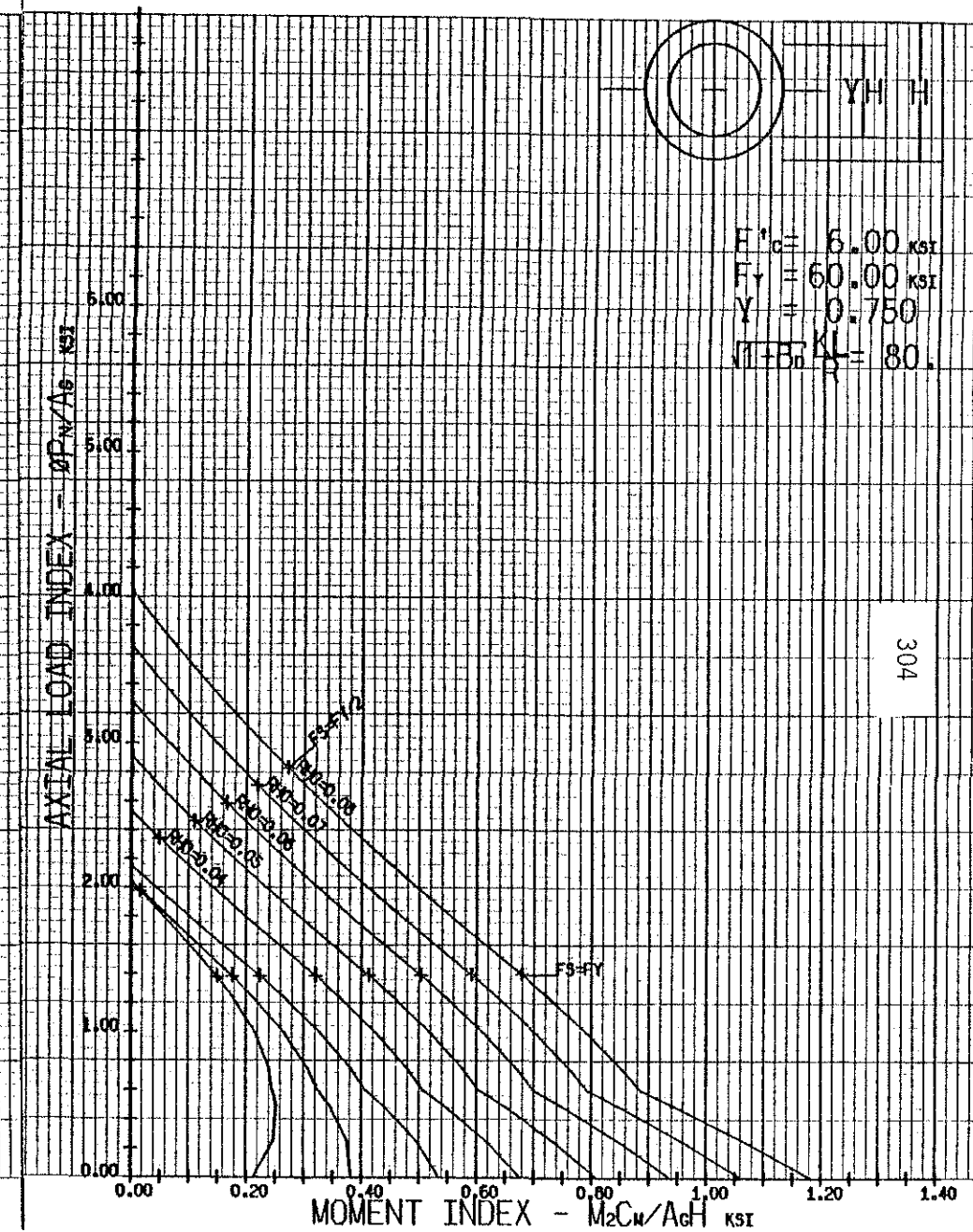


Fig. C6-60.75-80 - Interaction Diagram



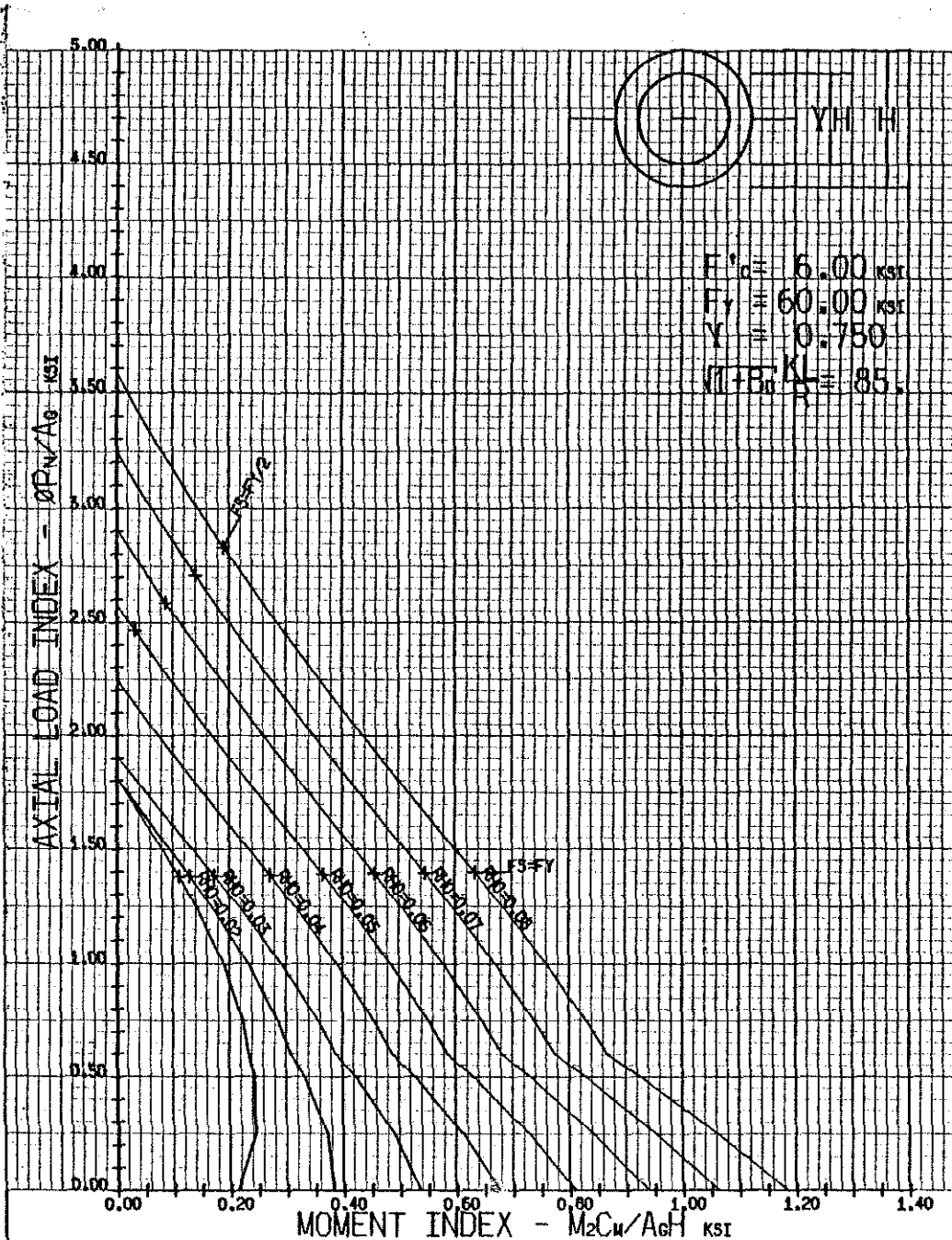


Fig. C6-60.75-85 - Interaction Diagram

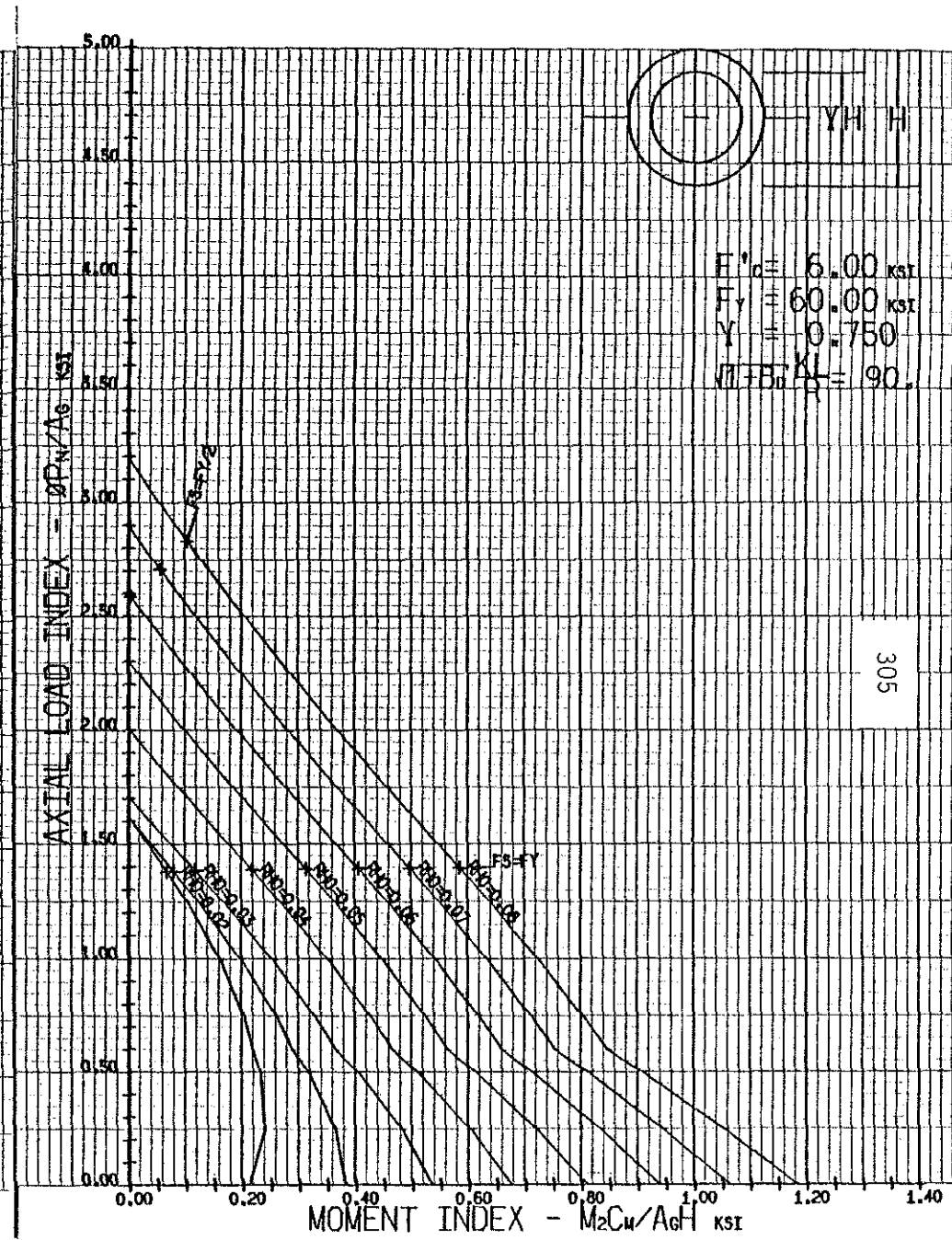


Fig. C6-60.75-90 - Interaction Diagram



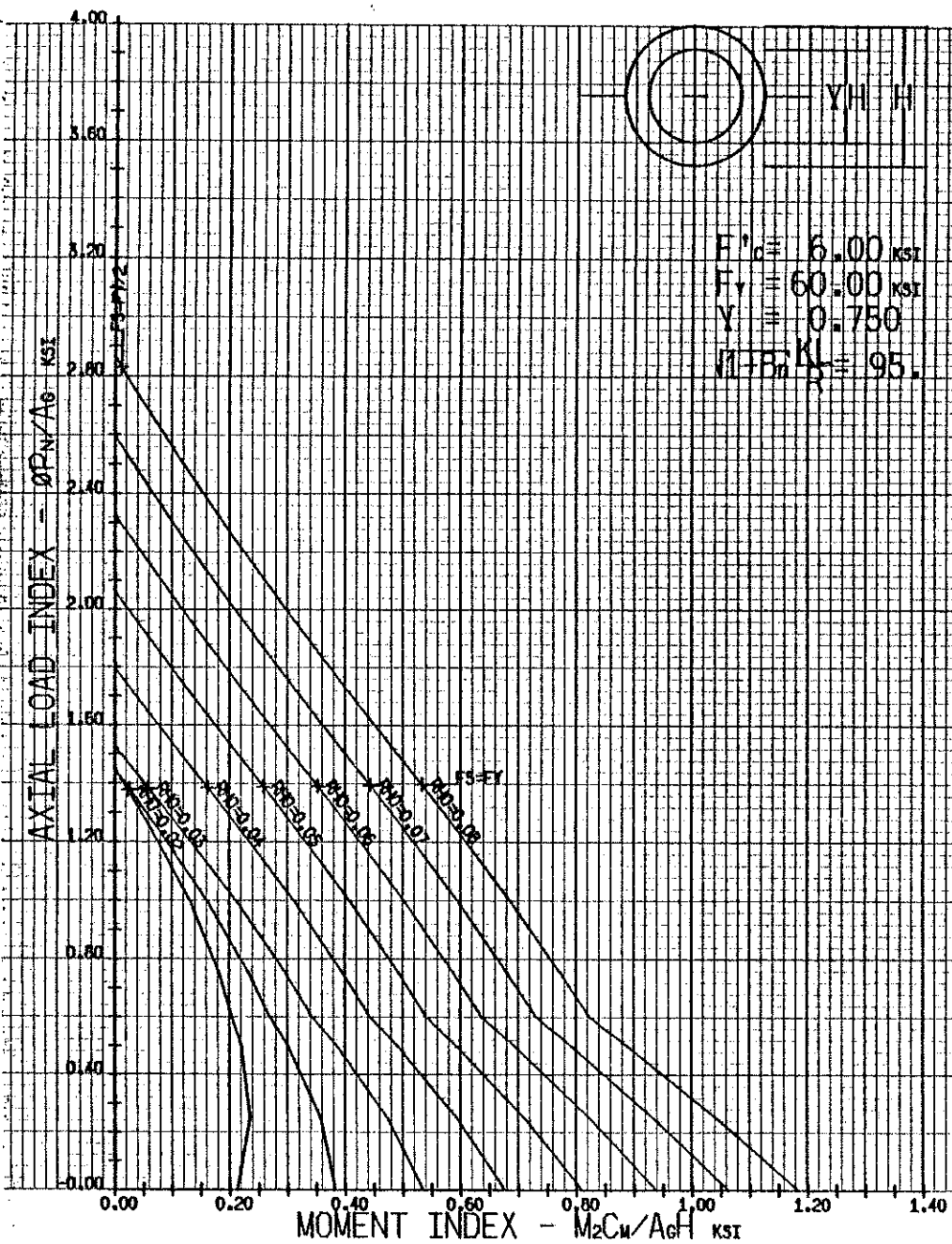


Fig. C6-60.75-95 - Interaction Diagram

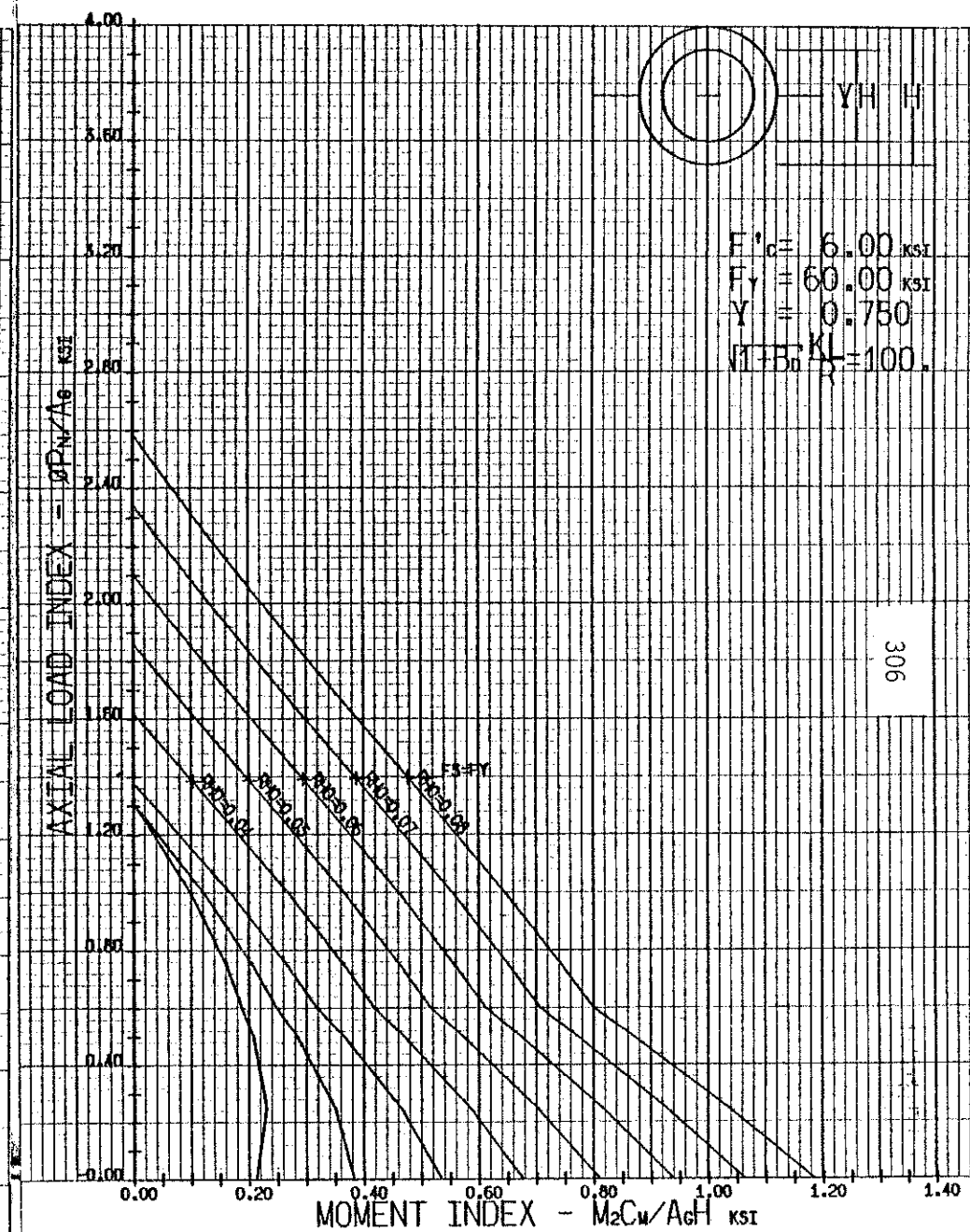


Fig. C6-60.75-100 - Interaction Diagram

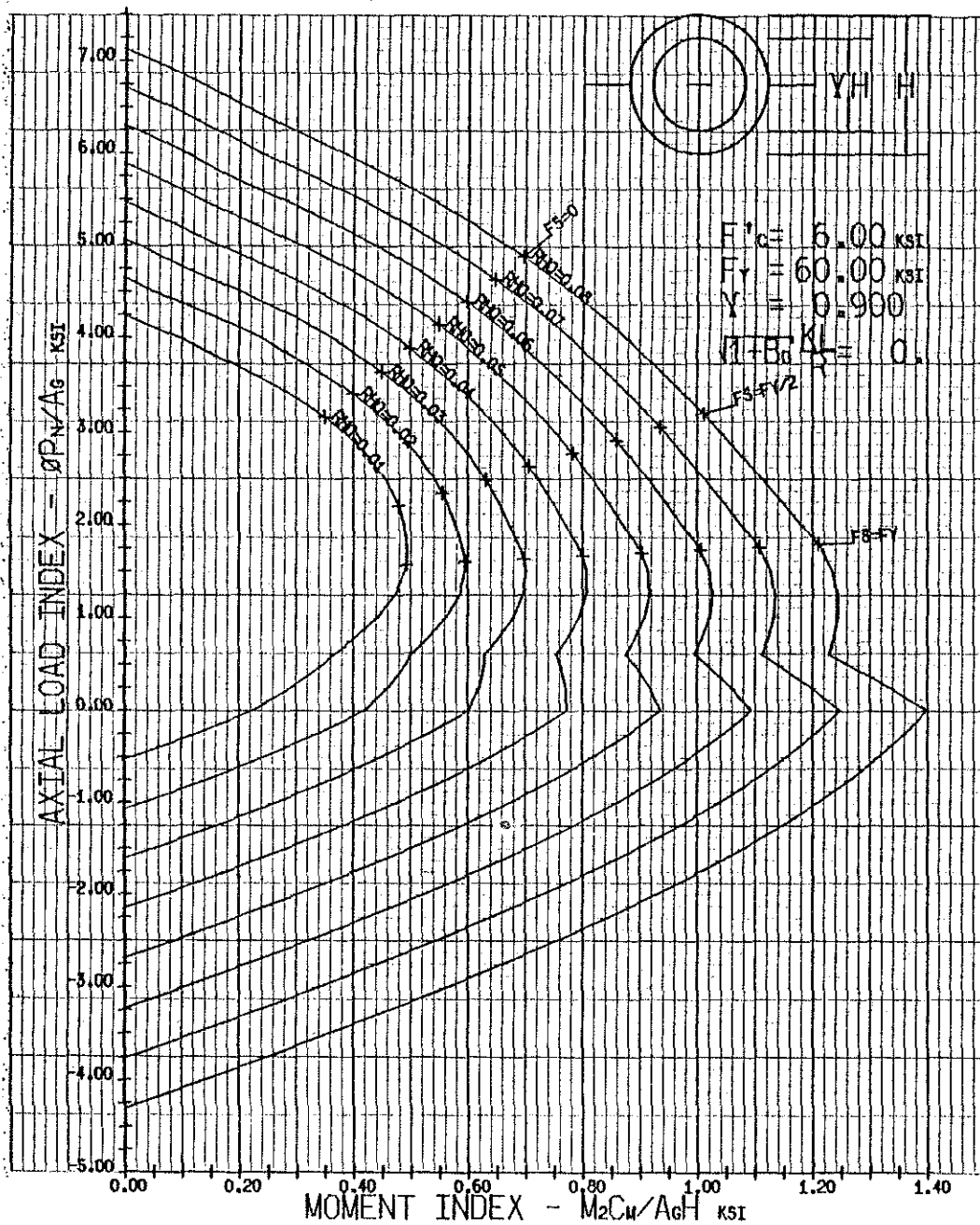


Fig. C6-60.90-0 - Interaction Diagram

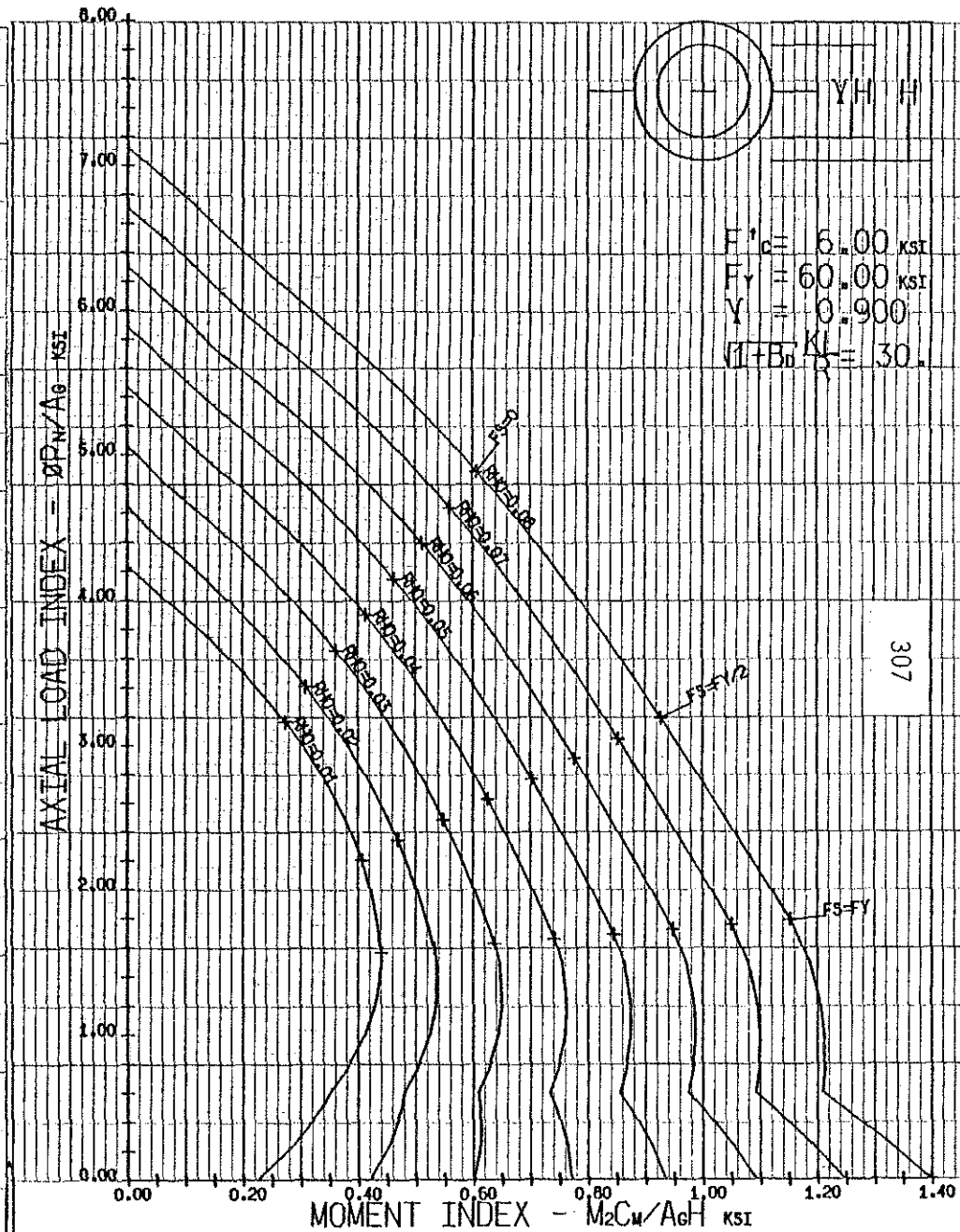


Fig. C6-60.90-30 - Interaction Diagram

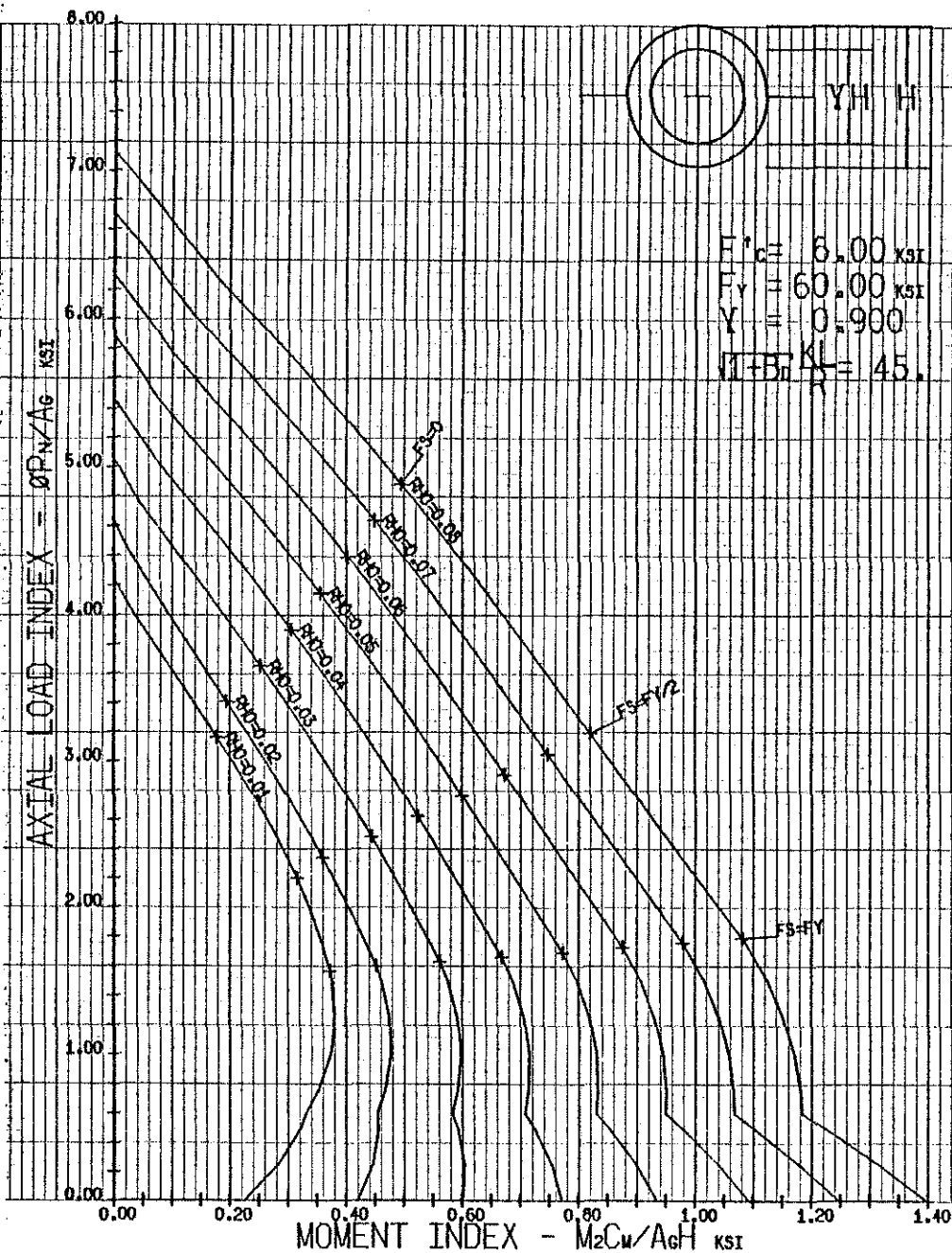


Fig. C6-60.90-45 - Interaction Diagram

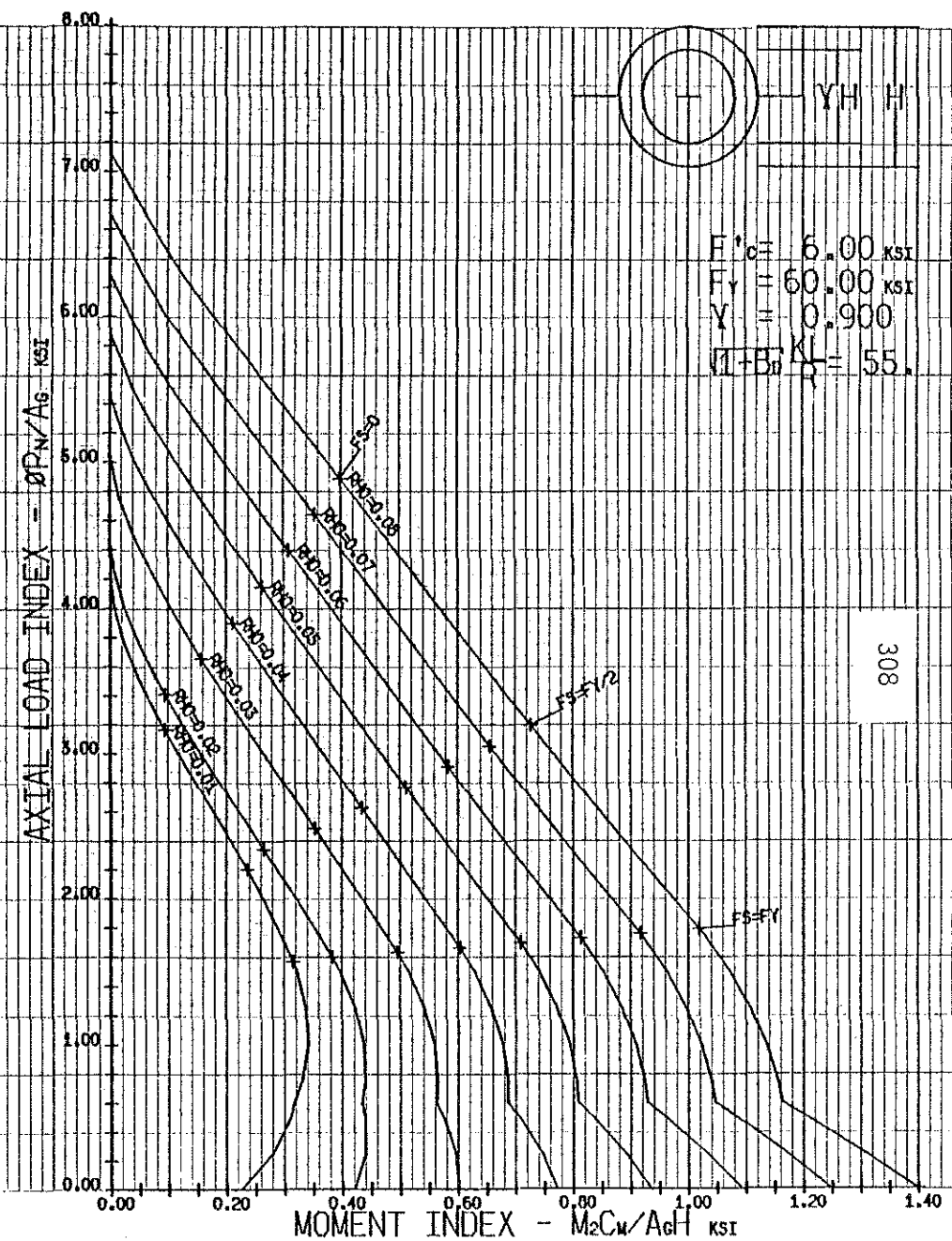


Fig. C6-60.90-55 - Interaction Diagram

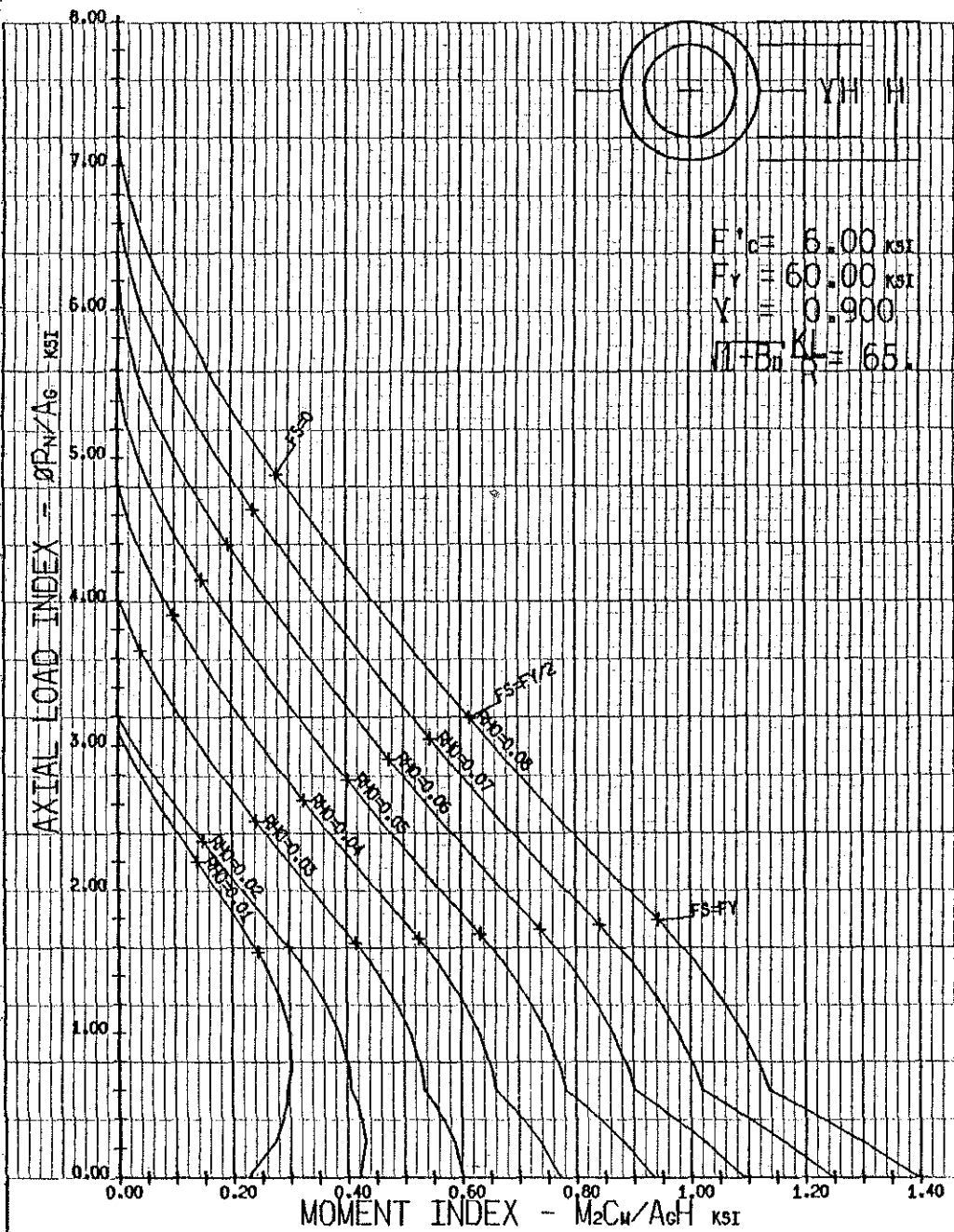


Fig. C6-60.90-65 - Interaction Diagram

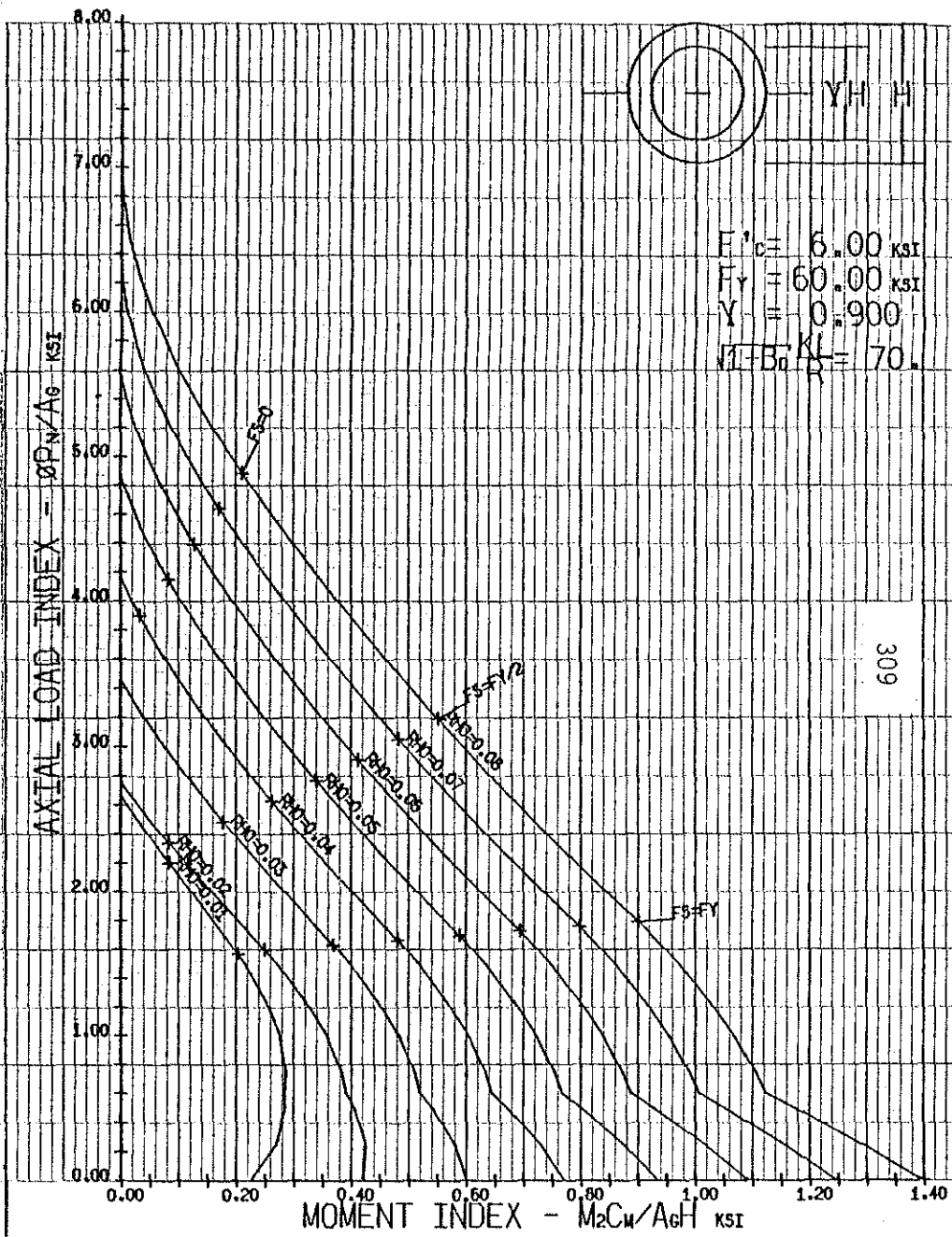


Fig. C6-60.90-70 - Interaction Diagram

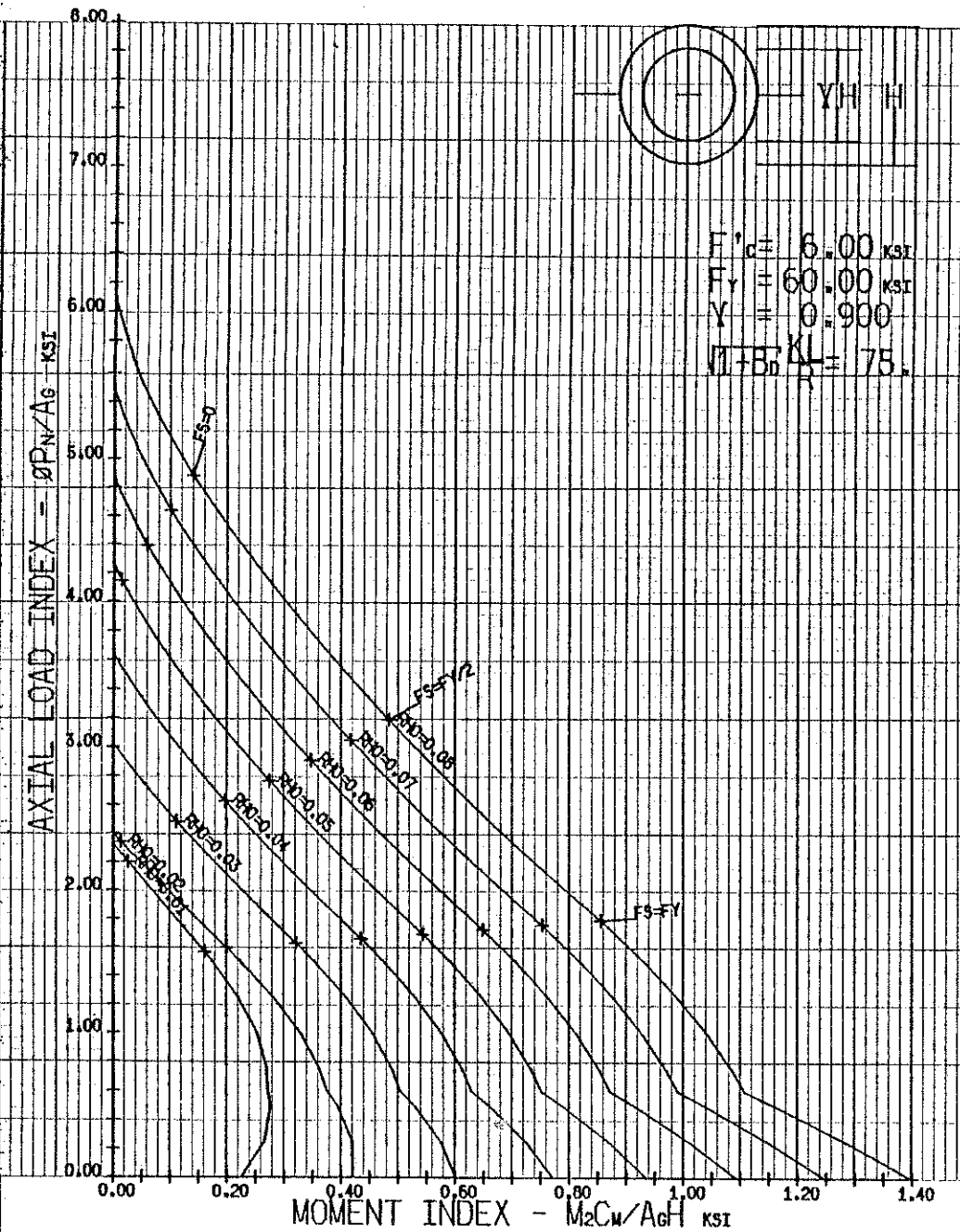


Fig. C6-60.90-75 - Interaction Diagram

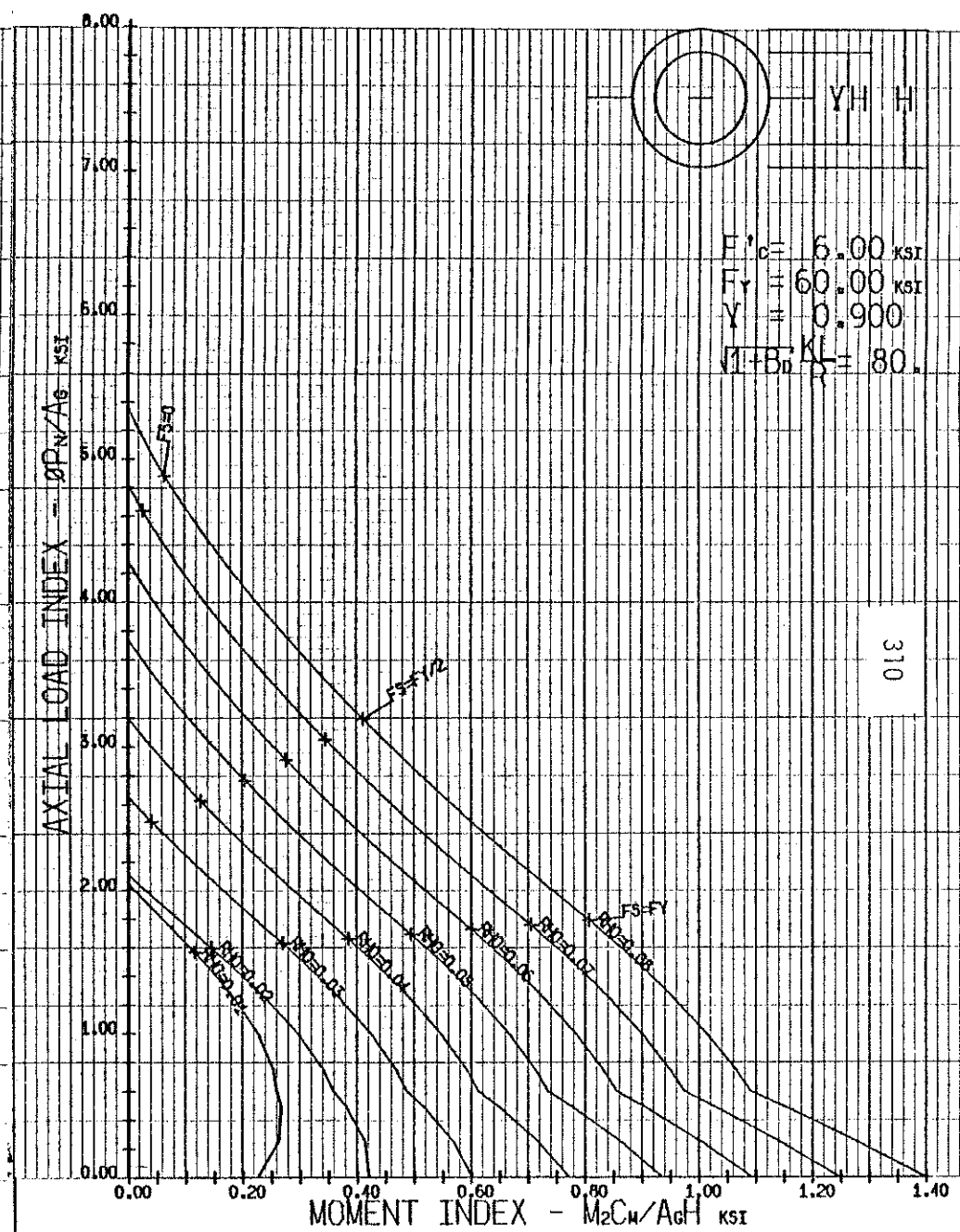


Fig. C6-60.90-80 - Interaction Diagram



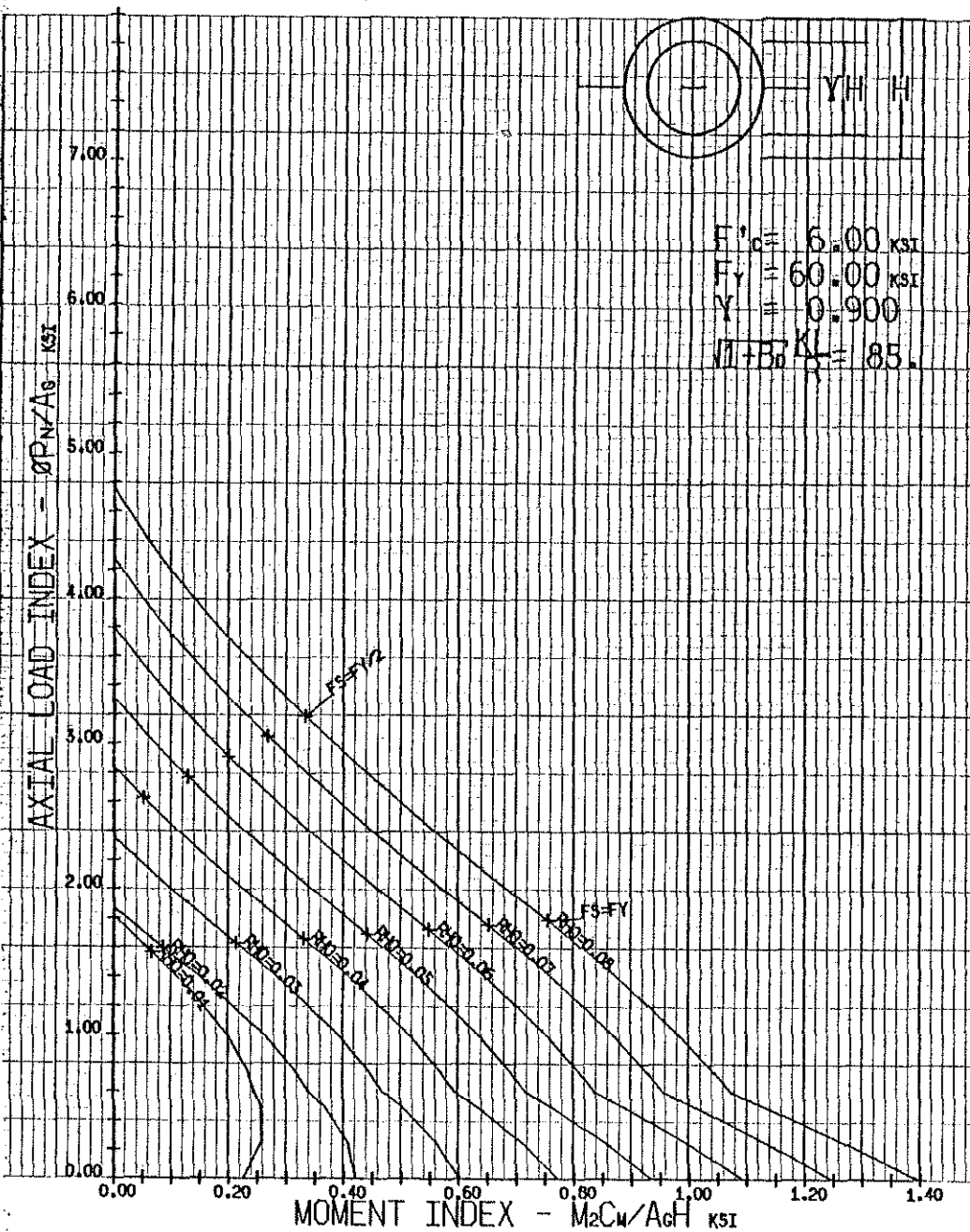


Fig. C6-60.90-85 - Interaction Diagram

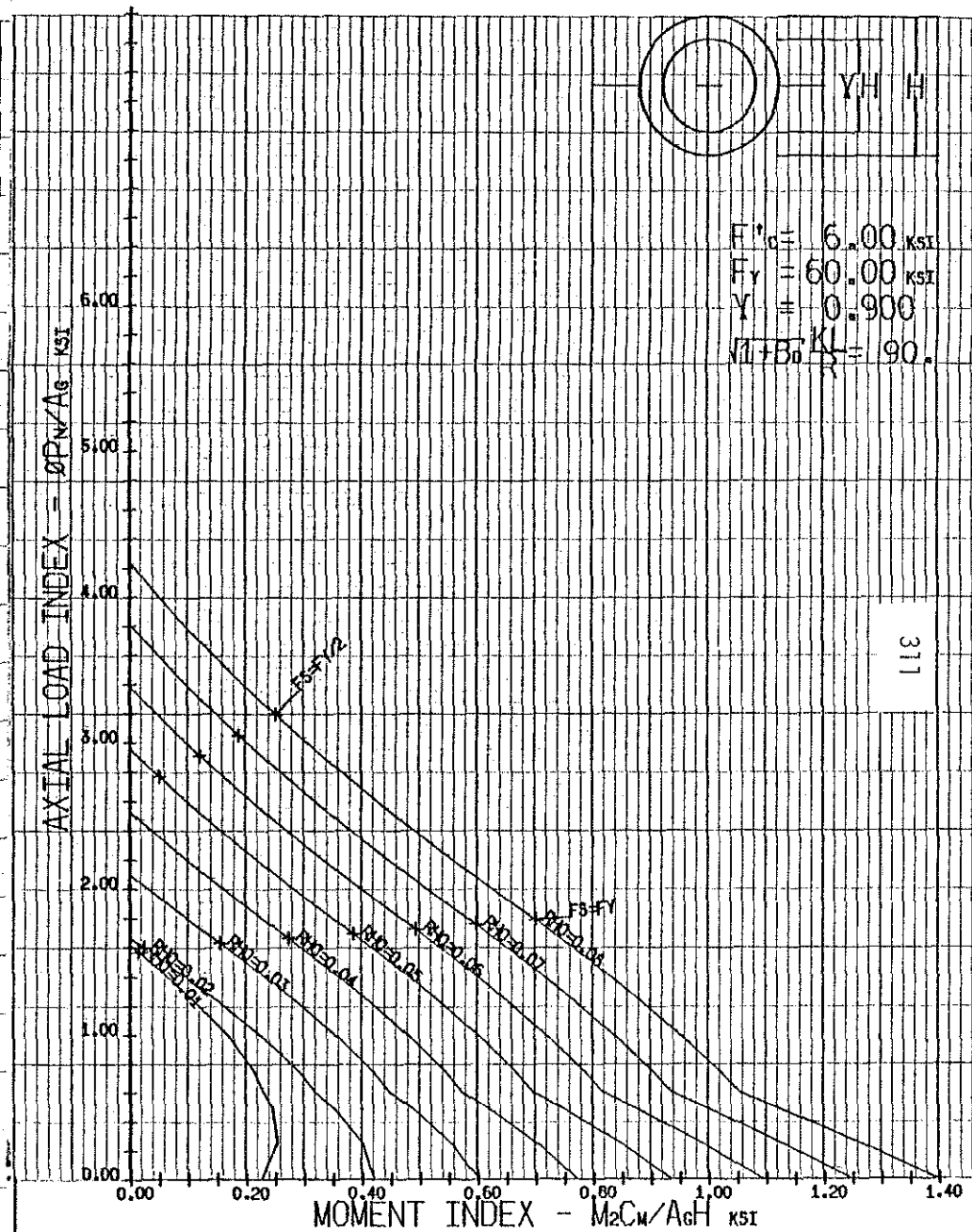


Fig. C6-60.90-90 - Interaction Diagram



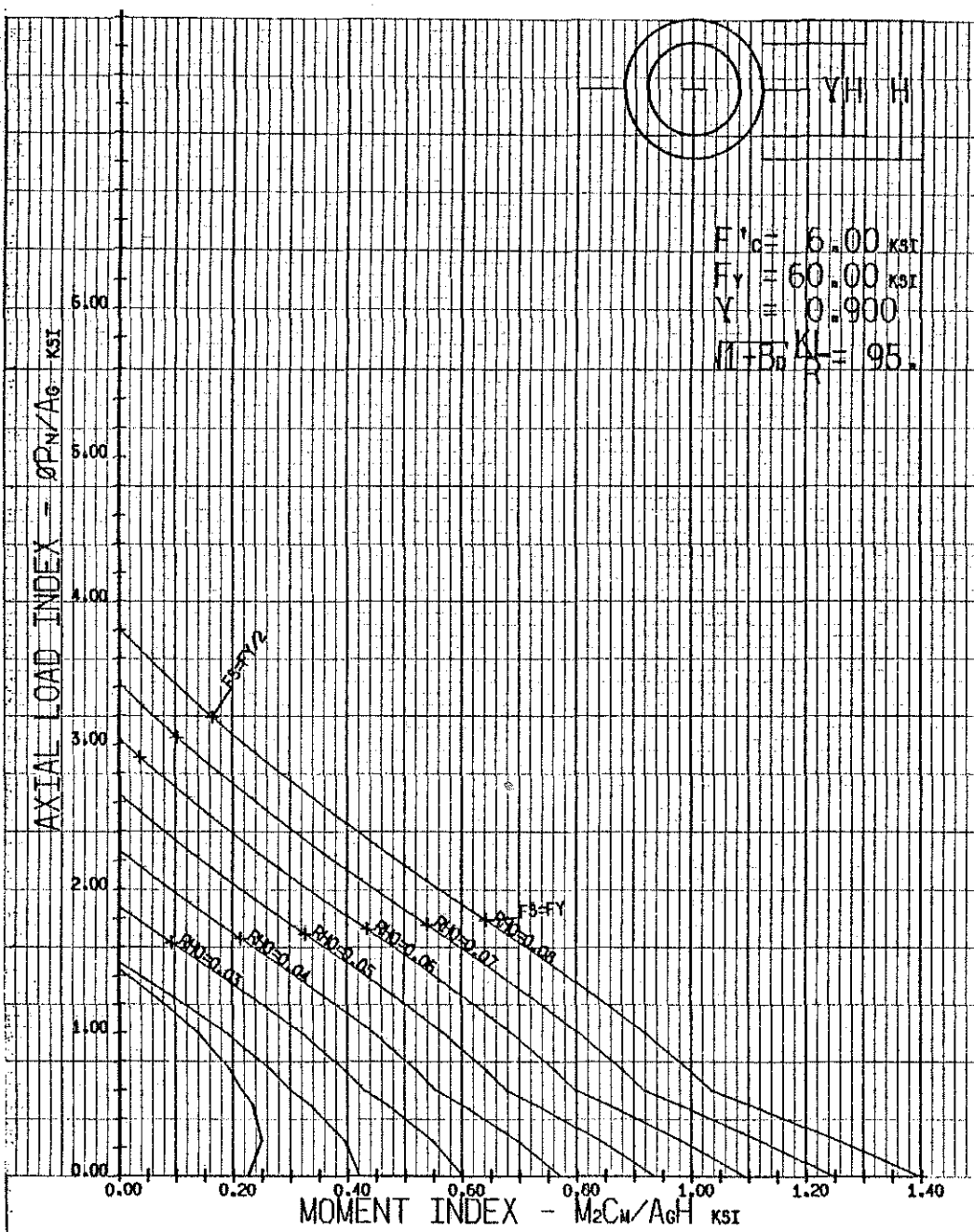


Fig. C6-60.90-95 - Interaction Diagram

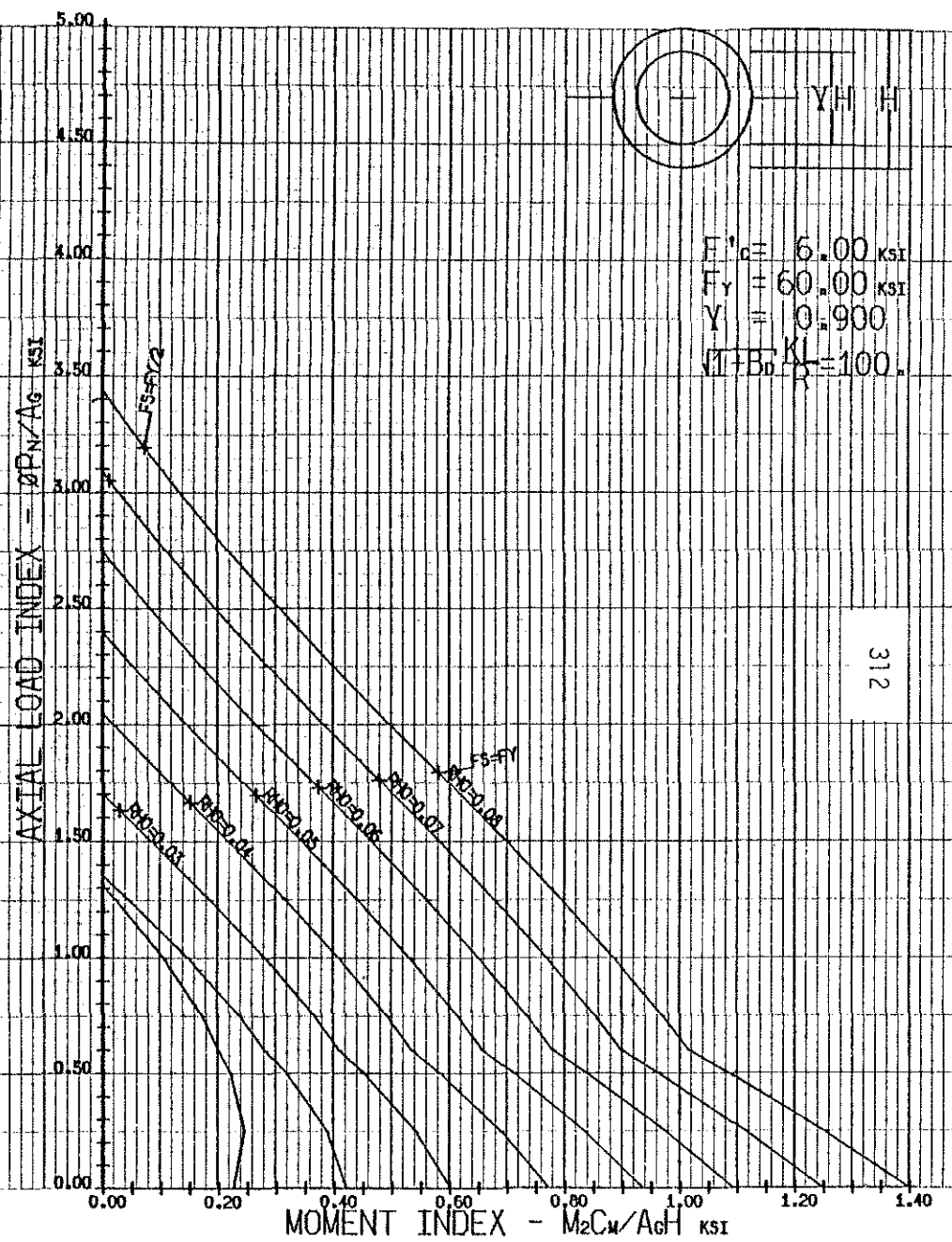


Fig. C6-60.90-100 - Interaction Diagram